



KINROSS WOLAROI  
— SCHOOL —

**2023**

## **HSC Mathematics Advanced**

Trial HSC Examination

Teacher Setting Paper: Mr Doyle

Head of Department: Mr Doyle

### **General Instructions**

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculator may be used
- Write your answers for Section I on the multiple-choice answer sheet provided
- A reference sheet is provided at the back of this paper
- In Questions 11- 34, show relevant mathematical reasoning and/or calculations.

**Total marks – 100**

### **Section I – Multiple-Choice**

10 marks

Attempt Questions 1-10

Allow 15 minutes for this section

### **Section II – Free Response**

90 marks

Attempt questions 11 - 34

Allow 2 hours and 45 minutes for this section

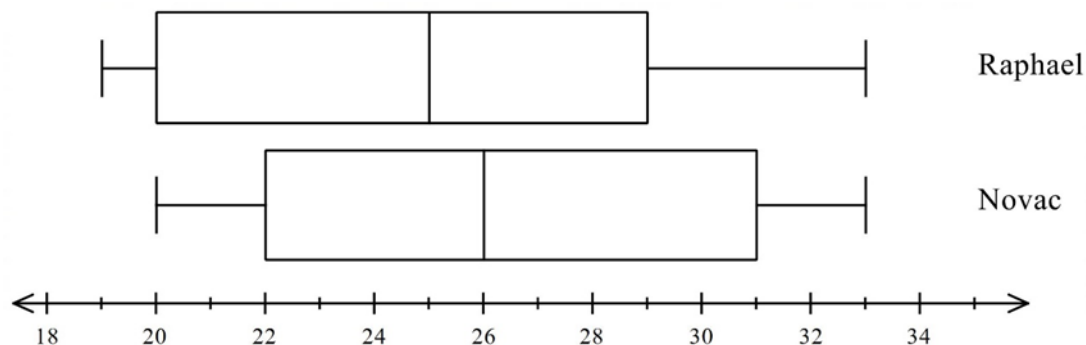
*This examination paper does not necessarily reflect the content or format of the Higher School Certificate Examination in this subject.*

**Section I****10 marks****Attempt Questions 1 – 10****Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1 - 10.

**QUESTION 1**

The parallel box plots are used to compare the time taken to complete a set of tennis (to the nearest minute), by two players over their last 40 games.

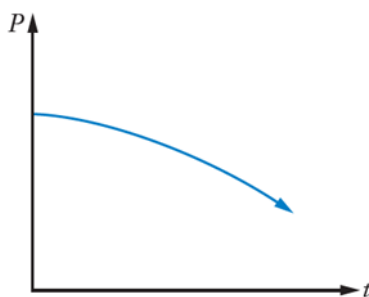


Which is a true statement about the two sets of data?

- (A) They have the same interquartile range.
- (B) They have the same median.
- (C) They have the same range.
- (D) They have the same upper quartile value.

**QUESTION 2**

The population ( $P$ ) of Algeville is shown over time ( $t$ ) by the graph below:



Which statement best describes the population of Algeville?

- (A) The population is increasing at an increasing rate.
- (B) The population is increasing at a decreasing rate.
- (C) The population is decreasing at an increasing rate.
- (D) The population is decreasing at a decreasing rate.

**QUESTION 3**

What is  $\int 3^x dx$  ?

- (A)  $\frac{3^x}{(\ln 3)} + C$
- (B)  $\frac{1}{(\ln 3)3^x} + C$
- (C)  $3^x(\ln 3) + C$
- (D)  $3^x + C$

**QUESTION 4**

The curve  $y = \ln x$  is horizontally dilated by a scale factor of 3 and is then translated to the left by  $\pi$  units.

What is the equation which describes this new curve?

- (A)  $y = \ln\left(\frac{x}{3} + \pi\right)$
- (B)  $y = \ln\left(\frac{x + \pi}{3}\right)$
- (C)  $y = 3 \ln(x + \pi)$
- (D)  $y = \ln\left(\frac{x}{3} - \pi\right)$

**QUESTION 5**

If  $f(x) = 3x^2 - 5$  and  $g(x) = 8 - x^3$ , what is  $g(f(1))$ ?

- (A)  $-8$
- (B)  $0$
- (C)  $8$
- (D)  $16$

**QUESTION 6**

A spinner is designed with the numbers 1 to 5. The outcomes are presented in the discrete probability distribution below:

$x$	1	2	3	4	5
$P(X = x)$	0.35	0.3	$m$	0.1	0.05

What is the value of  $m$  ?

- (A) 0.2
- (B) 0.4
- (C) 0.5
- (D) 0.1

**QUESTION 7**

An infinite geometric series has a first term of 10 and a limiting sum of 30.

What is the common ratio?

- (A)  $\frac{1}{3}$
- (B)  $\frac{1}{2}$
- (C)  $\frac{2}{3}$
- (D)  $\frac{3}{4}$

**QUESTION 8**

What is the radius of the circle  $x^2 + y^2 - 2y = 2$  ?

- (A)  $\sqrt{2}$
- (B)  $\sqrt{3}$
- (C) 2
- (D) 3

**QUESTION 9**

On a test, Elsa's mark was better than around 11% of her class. The test scores had a mean of 67 and a standard deviation of 7. Which of the following is most likely to have been her mark in the test?

- (A) 53
- (B) 58.5
- (C) 74.5
- (D) 81

**QUESTION 10**

What is the  $\int (\sin^2 x + \cos^2 x + \tan^2 x) dx$ ?

- (A)  $\tan x + c$
- (B)  $\cot x + c$
- (C)  $x + \sec x + c$
- (D)  $x - \log_e \cos^2 x + c$

**END OF SECTION I**

**Section II****90 marks****Attempt Questions 11-34****Allow about 2 hours 45 minutes for this section**

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response. Extra writing space is provided at the end of Section II. If you use this space, clearly indicate which question you are answering.

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**Marks****QUESTION 11 (2 marks)**Find the vertex of the parabola  $y = x^2 - 4x - 12$ .**2**

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**QUESTION 12 (2 marks)**Solve in simplest exact form the equation  $2\sin x = -1$  for  $0 \leq x \leq 2\pi$ .**2**

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**QUESTION 13 (2 marks)**Evaluate  $\int_0^{\frac{\pi}{4}} \cos 2x \, dx$ .**2**

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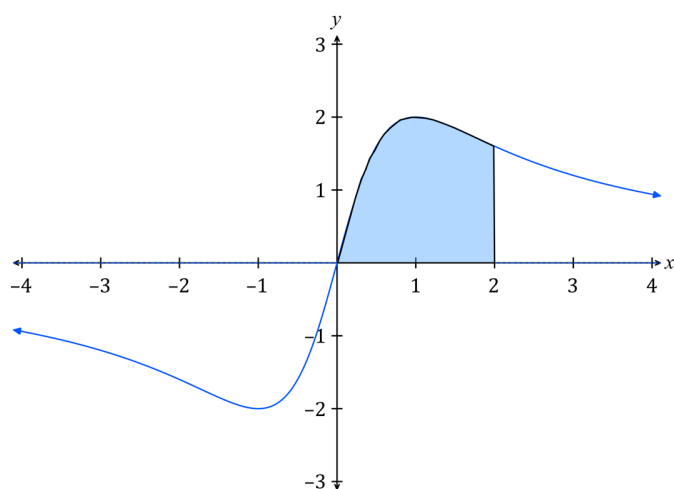
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**QUESTION 14** (2 marks)

The diagram below shows the graph of  $y = \frac{4x}{x^2 + 1}$ .



The region enclosed by the graph, the  $x$ -axis and the line  $x = 2$  is shaded. Calculate the exact value of area of the shaded region.

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**QUESTION 15** (3 marks)

Solve the equation  $\log_2(1-x) + \log_2(3-x) = 3$ .

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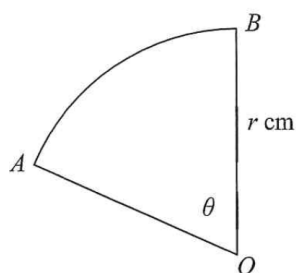
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**QUESTION 16** (3 marks)



NOT TO SCALE

In the diagram,  $AOB$  is a sector of a circle of radius  $r$  cm that contains an angle  $\theta$  radians at the centre  $O$  of the circle. The area of the sector is  $9 \text{ cm}^2$  and its perimeter is 5 times its arc length. Find the values of  $r$  and  $\theta$ .

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**QUESTION 17** (4 marks)

Find the equation of the tangent to the curve  $y = x \sin x$  at the point where  $x = \pi$  .  
Give your answer in general form.

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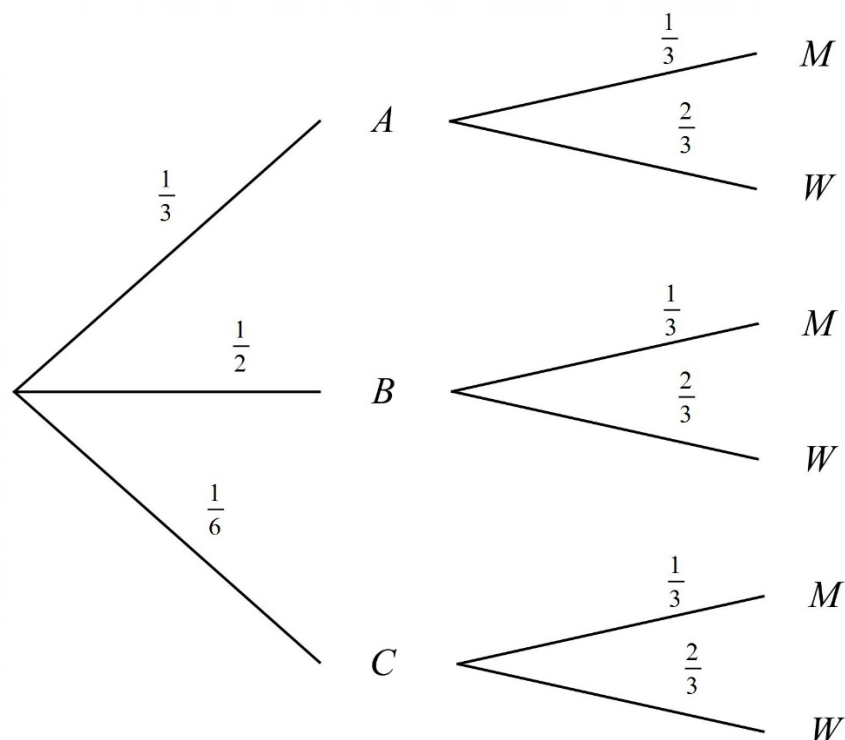
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**QUESTION 18** (3 marks)

There are three candidates for the Gordon house prefect, Andie, Bill and Cathy.

There are two candidates for the Gordon house captain, Milo and Wanda.

Based on a survey of the house, the probability tree shows the likelihood of the candidates being chosen for each position.



- (a) What is the probability that Andie will be the Prefect with Milo as the captain? 1

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- (b) What is the probability that Andie will be the Prefect or Milo will be the captain, but not both? 2

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**QUESTION 19** (4 marks)

Differentiate the following with respect to  $x$ .

(a)  $3x^2 - 4x + 1$  **1**

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(b)  $\frac{\log_e x}{x^2}$  **2**

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(c)  $\tan^2 x$  **1**

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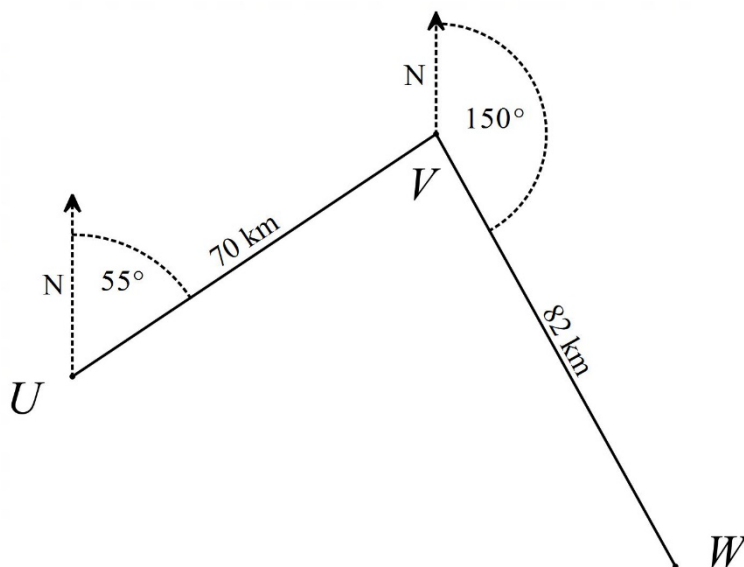
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**QUESTION 20** (5 marks)

A helicopter leaves Underwood and flies 70km on a bearing of  $055^\circ$  to Vanna Beach. It then flies 82km on a bearing of  $150^\circ$  to Weston.

The diagram below illustrates the journey.



- (a) The pilot then plans to fly directly back to Underwood.  
Calculate, to the nearest km, the total distance the helicopter will fly.

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- (b) Determine the bearing (from Weston) on which the pilot should fly to return to Underwood.  
Give your answer to the nearest degree.

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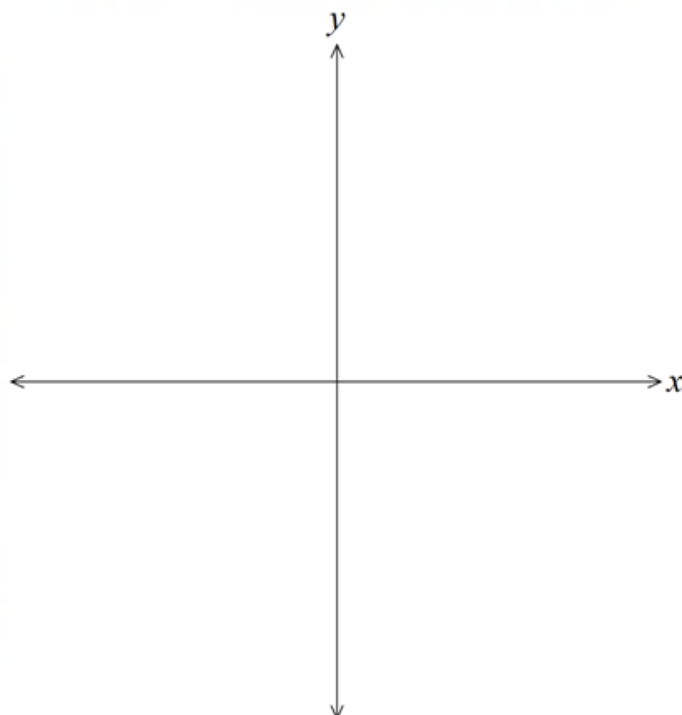
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**QUESTION 21** (2 marks)

Sketch the graph of  $y = |x + 3| - 1$ , showing any intercepts.

**2**



**QUESTION 22** (4 marks)

The ages of the 26 million residents of Noland are normally distributed with a mean of 45 years and standard deviation 11 years.

(a) Find how many more people are younger than 34 than are older than 67.

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(b) A person is chosen at random from the residents who are younger than 56. Find the probability that this person is also younger than 34. Answer to the nearest whole number.

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**QUESTION 23** (3 marks)

$$A = \int_1^5 \frac{10}{x+1} dx$$

Use the trapezoidal rule with four sub-intervals to find an estimate of the area evaluated by the above integral. Answer correct to two decimal places.

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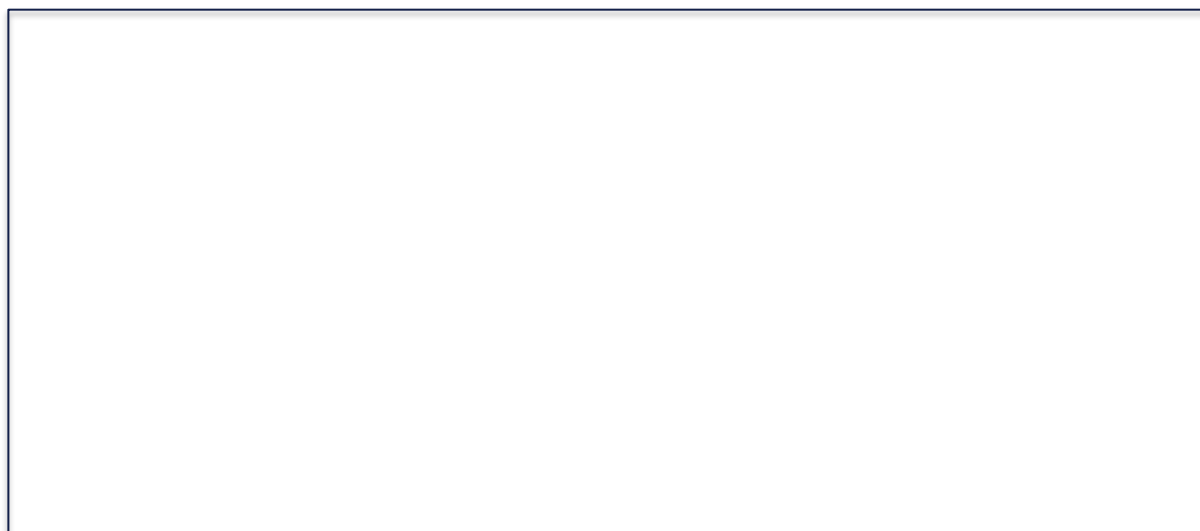
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**QUESTION 24** (3 marks)

- (a) On the same set of axes, sketch the graphs of  $y = 1 + \cos x$  and  $y = \sin x$  over the domain  $0 \leq x \leq \pi$ .

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- (b) Hence, or otherwise, find the values of  $x$  for which  $1 + \cos x = \sin x$  in the domain  $0 < x < \pi$ .

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**QUESTION 25** (3 marks)

For an arithmetic series the third term ( $T_3$ ) is 31 and the sixth term ( $T_6$ ) is 22.

(a) Find the common difference.

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(b) Find the sum of the terms from  $T_{10}$  to  $T_{20}$  inclusive.

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**QUESTION 26** (4 marks)

At the end of each year, Adam puts 3.5% of his salary into a retirement fund. He started work at the beginning of 2001 with an initial salary of \$80 000 and receives a salary increase of \$2000 each year.

(a) Find the year in which his yearly retirement fund contribution first exceeds \$5000.

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(b) Find the total amount that he will have put into his retirement fund when he retires at the end of 2045.

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**QUESTION 27** (5 marks)

At 7pm on a Wednesday evening after a storm, Jack's water tank was full. The capacity of the tank was 3000 litres. The tap on the tank, however, was leaking such that the change in volume at any time ( $t$ ) hours was proportional to the volume ( $V$ ) of the tank and can be modelled by the equation below.

$$V = V_0 e^{-kt}$$

- (a) Show that  $V_0$  is 3000L.

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- (b) Given that the volume of the tank after 3 hours is 1900 litres, find the value of  $k$  correct to 4 decimal places.

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- (c) By the time Jack discovered that the tap was leaking, there was only 250 litres of water remaining. At what time, and on which day, did Jack discover the leak? (Answer correct to the nearest minute.)

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**QUESTION 28** (4 marks)

The table below gives the future value of an annuity of \$1 per period for various periods and interest rates.

Table of Future Value Interest Factors								
	Interest rate per period							
Number of Periods	0.25%	0.30%	0.35%	0.40%	0.45%	0.50%	0.55%	0.60%
53	56.5961	57.3530	58.1230	58.9063	59.7033	60.5141	61.3391	62.1785
54	57.7376	58.5250	59.3264	60.1419	60.9719	61.8167	62.6765	63.5516
55	58.8819	59.7006	60.5340	61.3825	62.2463	63.1258	64.0212	64.9329
56	60.0291	60.8797	61.7459	62.6280	63.5264	64.4414	65.3733	66.3225
57	61.1792	62.0624	62.9620	63.8786	64.8123	65.7636	66.7329	67.7204
58	62.3322	63.2485	64.1824	65.1341	66.1040	67.0924	68.0999	69.1267
59	63.4880	64.4383	65.4070	66.3946	67.4014	68.4279	69.4744	70.5415
60	64.6467	65.6316	66.6359	67.6602	68.7047	69.7700	70.8565	71.9647
61	65.8083	66.8285	67.8692	68.9308	70.0139	71.1189	72.2463	73.3965
62	66.9729	68.0290	69.1067	70.2065	71.3290	72.4745	73.6436	74.8369
63	68.1403	69.2331	70.3486	71.4874	72.6499	73.8368	75.0487	76.2859
64	69.3106	70.4408	71.5948	72.7733	73.9769	75.2060	76.4614	77.7436
65	70.4839	71.6521	72.8454	74.0644	75.3098	76.5821	77.8820	79.2101
66	71.6601	72.8670	74.1004	75.3607	76.6487	77.9650	79.3103	80.6854

- (a) Jacinta invests \$250 per month in an annuity which pays 5.4% p.a. compounding monthly.  
What will be the value of the annuity after 5 years?

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- (b) Robyn finds that she can only get an interest rate of 3.6% p.a. compounding monthly.

If she wants to achieve the same total amount as Jacinta after the same time period, what amount should she invest each month?

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**QUESTION 29** (7 marks)

Let  $f(x) = x^3 + x^2 - x$ .

(a) Find the stationary points on the graph  $y = f(x)$  and determine their nature.

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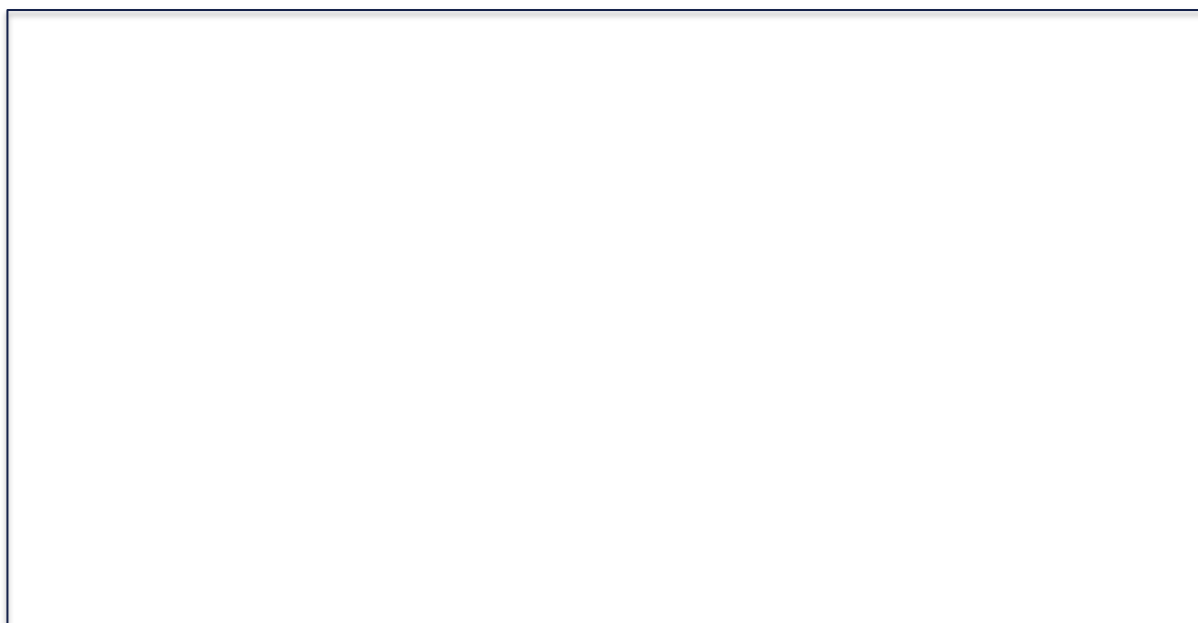
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(b) Hence, sketch the graph of  $f(2x)$ . (You do not need to show  $x$ -intercepts)

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(c) Find all possible values of  $a$  such that the graph of  $y = f(2x) + a$  has only one  $x$ -intercept.

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**QUESTION 30** (4 marks)

If  $y = 2e^{4x}$

(a) Express  $x$  in terms of  $y$  . **2**

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(b) Show that  $\frac{dy}{dx} \times \frac{dx}{dy} = 1$  . **2**

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**QUESTION 31** (3 marks)

If  $k > 0$  is a constant, find the integer  $n$  such that  $\int_k^{3k} \frac{6}{3x-k} dx = \log_e n$  . **3**

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**QUESTION 32** (7 marks)

A continuous random variable  $X$  has a probability density function  $f(x)$  given by

$$f(x) = e^{-\frac{1}{2}x} \text{ for } 0 \leq x \leq \log_e 4 \text{ and } f(x) = 0 \text{ otherwise.}$$

(a) Verify that  $f(x)$  is a probability density function.

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(b) Find the cumulative distribution function  $F(x)$  of  $X$ .

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(c) Find, in simplest exact form, the median of  $X$ .

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**QUESTION 33** (6 marks)

The velocity, in cm/s, of a particle moving in a straight line is given by the equation:

$$v = 4t^3 - 24t^2 + 20t \text{ where } t \text{ is the time in seconds.}$$

At  $t = 0$  the particle is 3cm to the right of the origin.

- (a) Find an expression for  $x$ , the position of the particle, as a function of time. **2**

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- (b) Determine the type of point that occurs on the graph of  $x(t)$  when  $t = 1$ . **2**

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- (c) How far will the particle travel in the first 2 seconds? **2**

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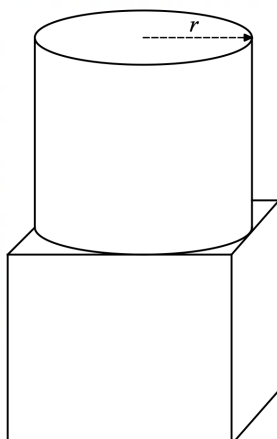
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**QUESTION 34** (5 marks)

An engineering assignment requires students to model a cylindrical pillar of radius  $r$  metres, standing atop a cube whose top face circumscribes the base of the pillar as shown.



The final model must be formed using exactly 1.5 cubic metres of concrete. All of the exposed faces of the solid are then coated in a waterproofing sealant.

(a) Show that the surface area to be coated is given by  $A = 24r^2 + \frac{3-16r^3}{r}$ .

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Question 34 continues over page

**QUESTION 34 (Continued)**

(b) Hence, find the minimum surface area that must be coated in such a model, correct to two decimal places. **2**

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KINROSS WOLAROI  
— SCHOOL —

**2023**

## **HSC Mathematics Advanced**

Trial HSC Examination

### **MULTIPLE-CHOICE ANSWER SHEET**

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

1.    ☐ A   ☐ B   ☐ C   ☐ D
2.    ☐ A   ☐ B   ☐ C   ☐ D
3.    ☐ A   ☐ B   ☐ C   ☐ D
4.    ☐ A   ☐ B   ☐ C   ☐ D
5.    ☐ A   ☐ B   ☐ C   ☐ D
6.    ☐ A   ☐ B   ☐ C   ☐ D
7.    ☐ A   ☐ B   ☐ C   ☐ D
8.    ☐ A   ☐ B   ☐ C   ☐ D
9.    ☐ A   ☐ B   ☐ C   ☐ D
10.   ☐ A   ☐ B   ☐ C   ☐ D