

Distribution of The Largest Eigenvalues of Randomly Generated Matrices

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1 Introduction

The purpose of this assignment was to investigate the distribution of the largest eigenvalues of a randomly populated square matrices, and using high-performance computing techniques to achieve it. The code was developed during the lecture in partnership with all the classmates, and a big part of it was completed by Prof Lennaert at the end.

The aim is to generate a matrix of a given size $N \times N$ where it is populated by values generated from a pseudo-random number generator with a Gaussian distribution, then calculate the eigenvalues of the matrix and save the largest eigenvalue found. This process is to be repeated until a large dataset is created containing the largest eigenvalue of randomly generated matrices, and finally study the distribution of these values.

Given the computational cost and repetitiveness of the task, the code was parallelized using MPI. The program was run on the Digital Alliance of Canada (SHARCNET) cluster to produce two datasets shown in figures 1 and 3.

The distribution of such eigenvalues is theorized to have a Tracy-Widom Distribution (TWD), which deviates from a normal distribution on one tail. For comparison, a TWD was generated in Python and plotted (figure 2) with matching parameters to figure 1. The skewed right tail is more pronounced in figure 3 confirming that the distribution is not Gaussian.

2 Acknowledgments

This project was completed in collaboration with classmates (Amin, Abe, Darryen) and Prof Lennaert Vanveen.

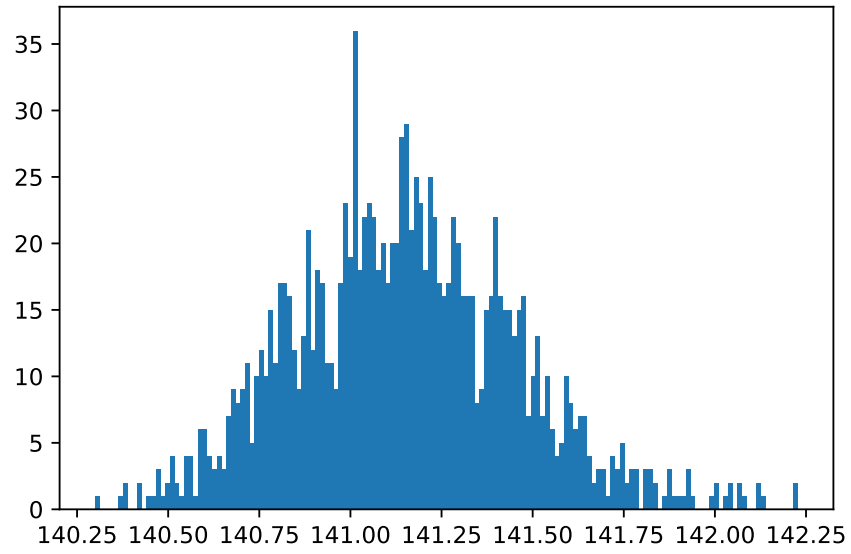


Figure 1: 1300 eigenvalues from randomly generated matrices of size 5000x5000.

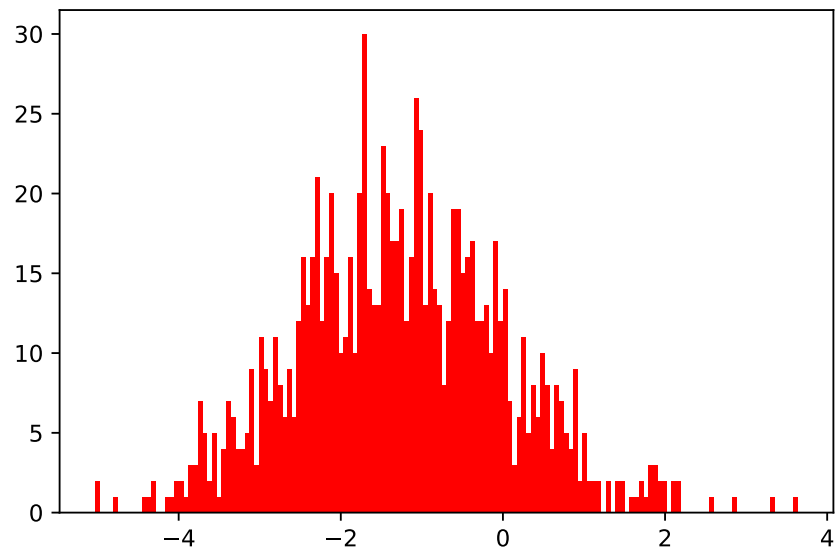


Figure 2: Tracy Widom Distribution using 1300 data points.

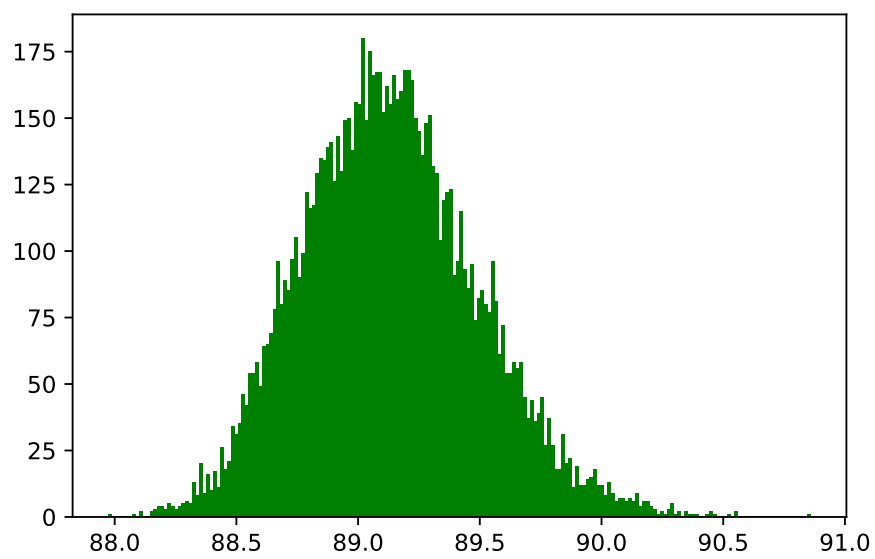


Figure 3: 10000 eigenvalues from randomly generated matrices of size 2000x2000.