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CS 590 Homework 1: Creativity Exercises

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Problem 1.6.39:

We have the following recurrence equation for a function T(n) as follows:

$$T(n) = \begin{cases} 1 & when\, n = 0 \\ 2T(n-1) & otherwise \end{cases}$$

We need to prove by induction that:

$$T(n) = 2^n$$

Base Case:

From the definition of the function, for n = 0 we have:

$$T(0) = 1$$

Let's see if we get 1 when we plug n = 0 in T(n) = 2^n:

$$T(0) = 2^0 = 1$$

This means that $T(n) = 2^n$ works for n = 0

Inductive Hypothesis:

For the hypothesis, I assume that $T(n) = 2^n$ is true

Inductive Step:

So, now let's work with the other value of T(n) when T(n) is not equal to zero:

$$T(n) = 2T(n-1) \; when \; n \neq 0$$

If $T(n) = 2^n$, then $T(n+1) = 2^{n+1}$

So, let's plug compute for T(n+1) from the recursion equation when $n \neq 0$:

$$T(n+1) = 2T(n+1-1) = 2T(n)$$

To check if $T(n) = 2^n$ is true, we will substitute it in T(n+1) above to check what we get as a result:

$$T(n+1) = 2 * T(n) = 2 * (2^n) = 2^{n+1}$$

Therefore, $T(n) = 2^n$ is true

<u>Conclusion:</u>

$T(n) = 2^n$ satisfies the recursion equation

$$T(n) = \begin{cases} 1 & when\ n = 0 \\ 2T(n-1) & otherwise \end{cases}$$

as you can see from above. **Therefore, by induction** $\boldsymbol{T(n) = 2^n}$

<u>Problem 1.6.42:</u>

To show that $\sum_{i=1}^{n} (i)^2\ is\ O(n^3)$ the following below will be done:

If n = 1, then i = n. Otherwise i < n. Therefore,

$$i \leq n$$

So, I can write the following:

$$\sum_{i=1}^{n}(i)^2 \leq \sum_{i=1}^{n}(n^2)$$

Let's solve for $\sum_{i=1}^{n}(n^2)$:

First thing we take the (n^2) out of the sum:

$$\sum_{i=1}^{n}(n^2) = (n^2)\sum_{i=1}^{n}(1)$$

Then, $\sum_{i=1}^{n}(1)$ is as follows:

$$\sum_{i=1}^{n}(1) = (1 + 1 + 1 + \cdots + 1)$$

This means that $\sum_{i=1}^{n}(1)$ is basically summing 1 n times. So, we can write the following:

$$\sum_{i=1}^{n}(1) = (1) * (n) = n$$

So, now getting back to solving $\sum_{i=1}^{n}(n^2)$:

$$\sum_{i=1}^{n}(n^2) = (n^2) * \sum_{i=1}^{n}(1) = (n^2) * (n) = n^3$$

Therefore, we can deduce the following:

$$\sum_{i=1}^{n}(i)^2 \leq \sum_{i=1}^{n}(n)^2 = (n)^2 \sum_{i=1}^{n}(1) = n^3$$

And this means that:

$$\sum_{i=1}^{n}(i^2)\ is\ O(n^3)$$