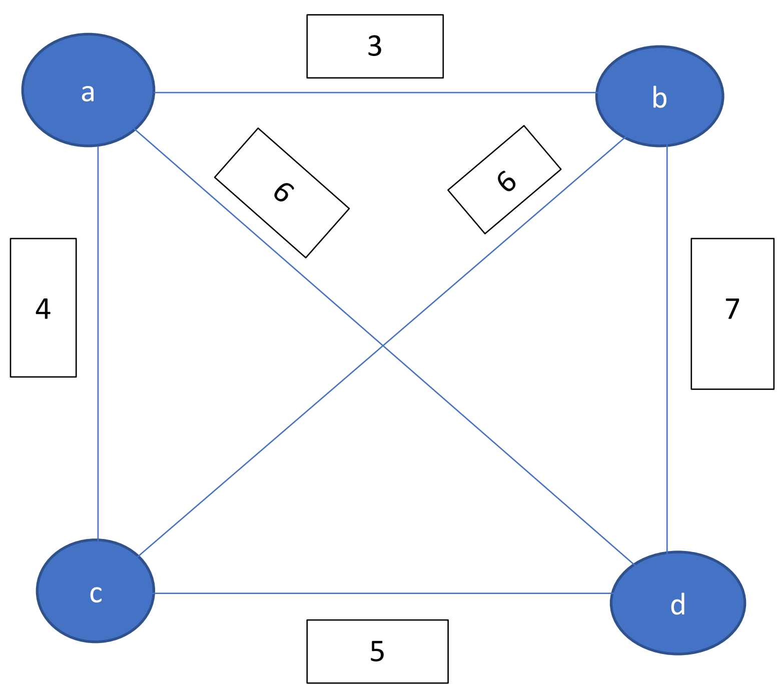
Name: Georges Hatem

CS 590 Homework Assignment 13: Minimum Spanning Trees Reinforcement Exercises

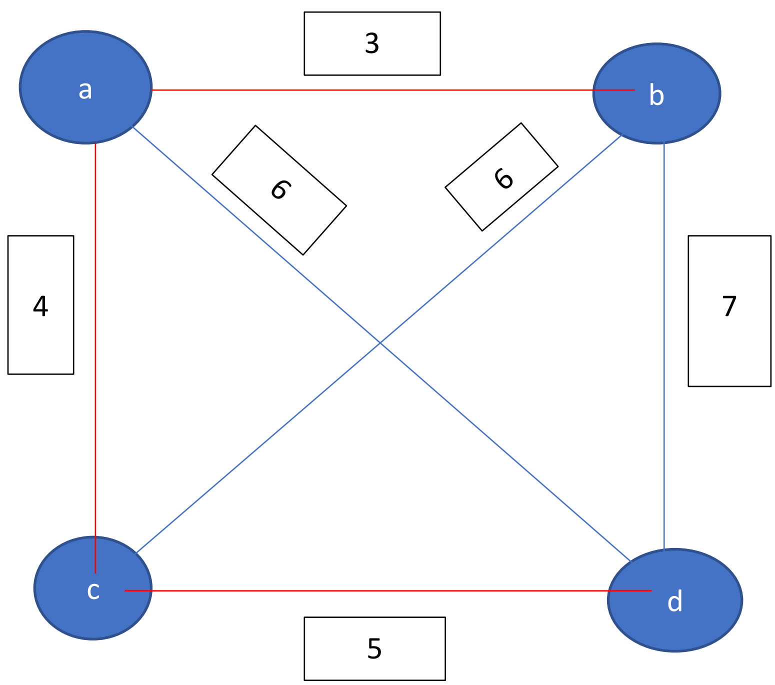
Due Date: April 24, 2022

Problem 15.6.9:

An example of a weighted, connected, undirected graph, G, such that the minimum spanning tree for G is different from every shortest-path tree rooted at a vertex of G is as follows below:



Let's get the Minimum Spanning Tree for the Graph above (highlighted below is the Minimum Spanning Tree (T) that we get from the Graph (G) above):



The highlighted in red above shows the MST Tree (T).

This is an explanation of the MST Tree above:

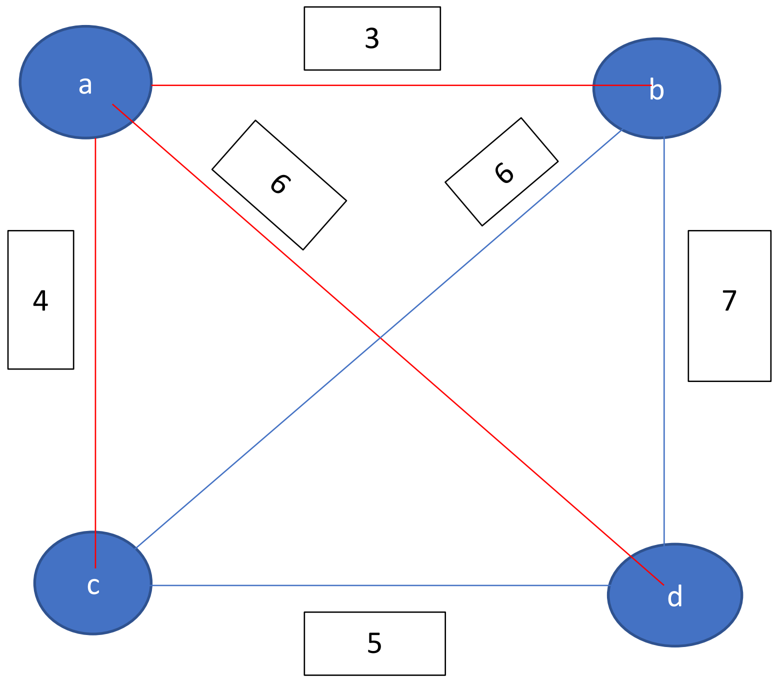
First, let's suppose we are applying Kruskal Algorithm's. So, we start with 4 separate clusters {a}, {b}, {c}, {d}.

The minimum edge in the Graph (G) above is (a,b). So, we add a and b to the same cluster and we add them to the MST Tree (T). After that, the second minimum edge in the Graph (G) is (a,c). So, we add a and c to the same cluster. So, now we have {a,b,c} in the same cluster. After that, the third minimum edge in the Graph (G) is (c,d). So, we add c and d to the same cluster. So, now, we have {a,b,c,d} in the same cluster. So, the algorithm goes for (a,d), (c,b), and (d,b). However, {a,b,c,d} are all in the same clusters. So, nothing happens when the algorithm goes for (a,d), (c,b), and (d,b).

Now, let's do the shortest path (let's pick a as the start vertex), we get the following:

1) If b is the destination, then the shortest path would be (a,b) – 3
2) If If c is the destination, then the shortest path would be (a,c) – 4
3) If d is the destination then the shortest path would be (a,d) – 6

These 3 steps illustrate all the shortest paths that takes us from vertex a to vertices b, c, or d as described above.



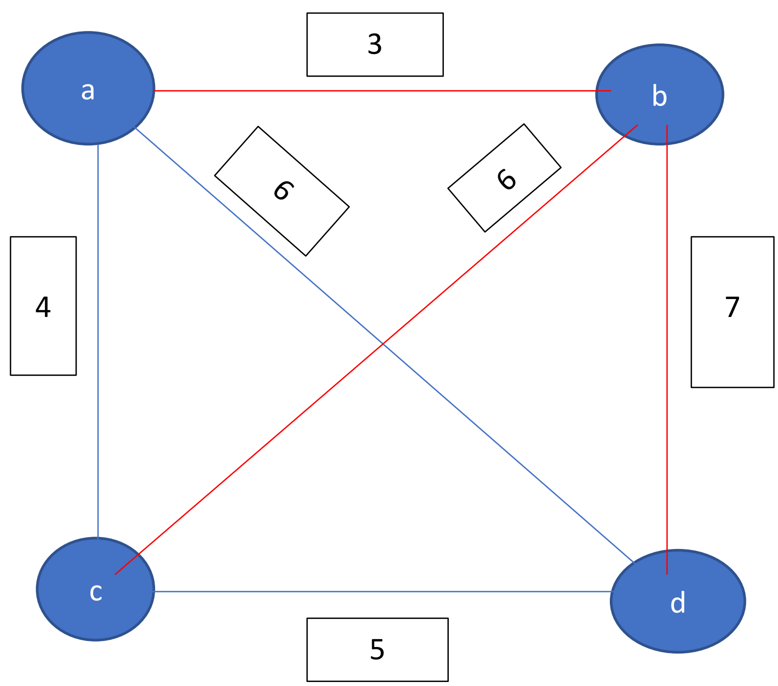
As you can see from above, the shortest paths taken from a to get to b, c, and d is different than the minimum spanning tree (T) path taken.

Right now, we will do the same analysis as above but with b as the starting vertex:

1) If a is the destination, then the shortest path to get from b to a is (b,a)

2) If d is the destination, then the shortest path to get from b to d is (b,d)
3) If c is the destination, then the shortest path to get from b to c is (b,c).

These 3 steps that takes us from the starting vertex b to a destination vertex such as a, d, or c is as follows:

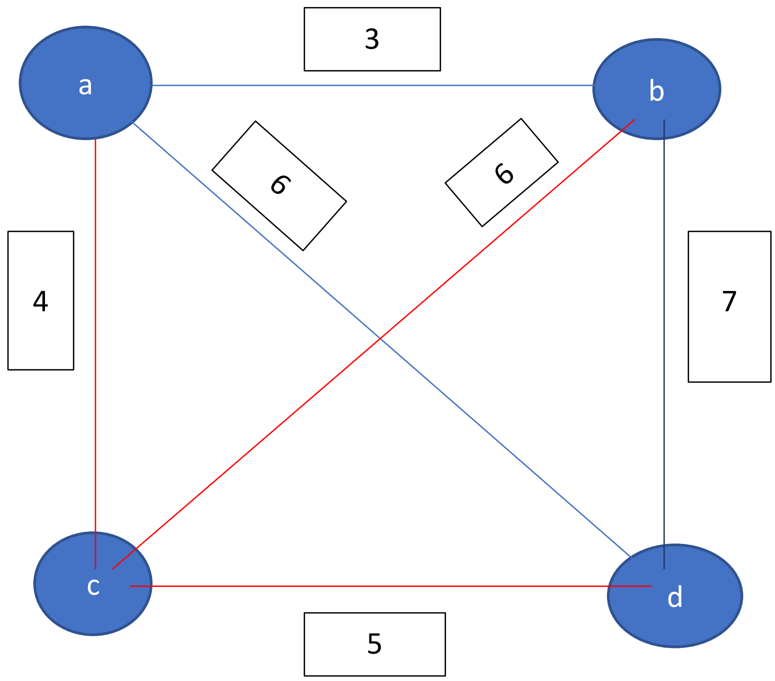


As you can see from above, the shortest paths taken from b to get to a, c, and d is different than the minimum spanning tree (T) path taken.

Right now, we will do the same analysis as above but with c as a starting vertex:

1) If a is the destination, then the shortest path to go from c to a is (c,a)
2) If d is the destination, then the shortest path to get from c to d is (c,d)
3) If b is the destination, then the shortest path to get from c to b is (c,b)

These 3 steps above take us from the starting vertex c to the destination vertexes b, d, or a as follows:

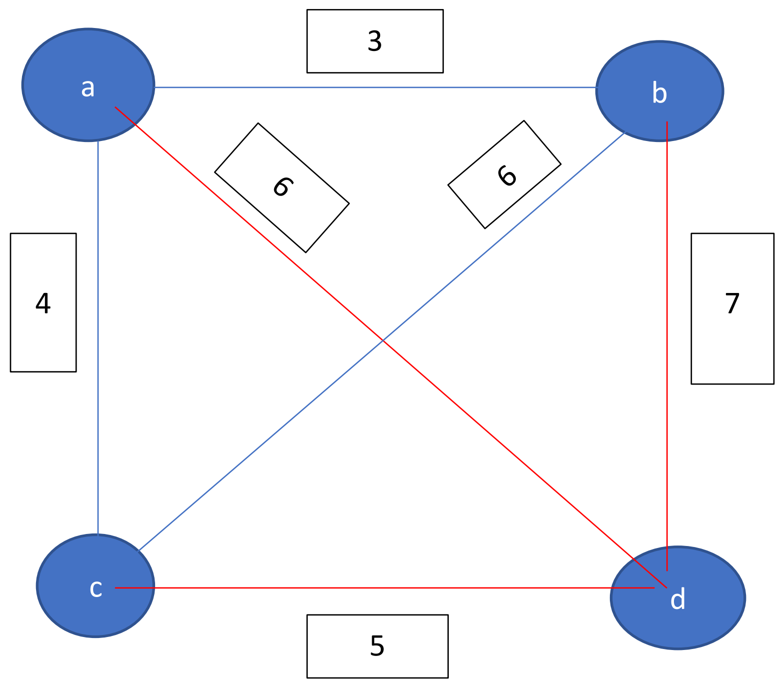


As you can see from above, the shortest paths taken from c to get to a, b, and d is different than the minimum spanning tree (T) path taken.

Right now, we will do the same analysis as above but with vertex d as the starting vertex:

1) If c is the destination vertex, then the shortest path to go from d to c is (d,c)
2) If a is the destination vertex, then the shortest path to go from d to a is (d,a)
3) If b is the destination vertex, then the shortest path to go from vertex d to vertex b is (d,b)

These 3 steps above take us from the starting vertex d to the destination vertices a, b, and d as follows:



As you can see from above, the shortest paths taken from d to get to a, b, and c is different than the minimum spanning tree (T) path taken.

From the above analysis, you can see that the weighted, connected, undirected graph, G, described above has the minimum spanning

tree (T) different from every shortest-path tree rooted at a vertex of G.