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CS 550 – Homework Assignment 1

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Problem 1

Part a:

To calculate the performance of each processor expressed in instructions per seconds, we need to use the classic CPU Performance Equation:

$$CPU\ Time = \frac{(Instruction\ Count) * (CPI)}{(Clock\ Rate)}$$

From that formula, we can get the Performance, which is:

$$Performance = \frac{1}{CPU\ Time}$$

For Processor P1:

$$Clock\ Rate = 3.2\ GHz = 3.2 * 10^9\ Hz$$
$$CPI = 1.5$$

Plugging these values into the formulas above to get the Performance:

$$Performance = \frac{Clock\ Rate}{(Instructions\ Count) * (CPI)}$$

$$Performance = \frac{(3.2 * 10^9\ Hz)}{(1.5)} = 2133333333\ \frac{instructions}{second}$$

For Processor P2:

$$Clock\ Rate = 2.0\ GHz = 2.0 * 10^9\ Hz$$
$$CPI = 1.0$$

Plugging these values into the formulas above to get the Performance:

$$Performance = \frac{(2.0 * 10^9\ Hz)}{(1.0)} = 2.0 * 10^9\ \frac{instructions}{second}$$

For Processor P3:

$$Clock\ Rate = 4.0\ GHz = 4.0 * 10^9\ Hz$$
$$CPI = 2.3$$

Plugging these values into the formulas above to get the Performance:

$$Performance = \frac{(4.0 * 10^9\ Hz)}{(2.3)} = 1739130435\ \frac{instructions}{second}$$

Part b:

To get the CPU Clock Cycles for each Processor, we use the formula below:

$$CPU\ Clock\ Cycles = (CPU\ Time) * (Clock\ Rate)$$

And given for all processors:

$$CPU\ Time = 10\ seconds$$

For Processor P1:

$$CPU\ Clock\ Cycles = (10s) * (3.2 * 10^9 Hz) = 3.2 * 10^{10}\ cycles$$

For Processor P2:

$$CPU\ Clock\ Cycles = (10s) * (2.0 * 10^9\ Hz) = 2.0 * 10^{10}\ cycles$$

For Processor P3:

$$CPU\ Clock\ Cycles = (10s) * (4.0 * 10^9\ Hz) = 4.0 * 10^{10}\ cycles$$

Computing the instructions for each processor, we will use the following formula:

$$Instruction\ Count = \frac{(CPU\ Clock\ Cycles)}{(CPI)}$$

For Processor P1:

$$Instruction\ Count = \frac{(3.2 * 10^{10}\ cycles)}{(1.5)} = 2.133333333 * 10^{10}\ instructions$$

For Processor P2:

$$Instruction\ Count = \frac{(2.0 * 10^{10}\ cycles)}{(1.0)} = 2.0 * 10^{10}\ instructions$$

For Processor P3:

$$Instruction\ Count = \frac{(4.0 * 10^{10}\ cycles)}{(2.3)} = 1.739130435 * 10^{10}\ instructions$$

Part C:

For Processor P2 (since this part revolves around Processor P2):

Reducing the execution time of Processor P2 by 30 percent means the following:

$$New\ CPU\ Time = (0.70) * (Old\ CPU\ Time) = (0.70) * (10) = 7\ seconds$$

Increase of 20 percent in the CPI means the following:

$$New\ CPI = (1.20) * (Old\ CPI) = (1.20) * (1.0) = 1.20$$

Calculating the New Clock Rate:

$$New\ Clock\ Rate = \frac{(Instruction\ Count) * (New\ CPI)}{(New\ CPU\ Time)}$$

$$New\ Clock\ Rate = \frac{(2.0 * 10^{10}) * (1.20)}{(7)} = 3.428571429 * 10^{9}\ Hz$$

So,

$$New\ Clock\ Rate = 3.428571429\ GHz$$

Problem 2

	CPI for each instruction class			
	A	B	C	D
CPI for P1	1	2	3	3
CPI for P2	2	2	2	2

	Percentage of the total instruction counts for each instruction class			
	A	B	C	D
Instruction Percentage	30 %	20 %	30 %	20 %

Question: which is faster P1 or P2?

To determine which processor is faster, we need to find the performance for each processor:

$$Performance = \frac{(Clock\ Rate)}{(Instruction\ Count) * (CPI)}$$

For Processor P1:

$$Performance = \frac{(2.5 * 10^9\ Hz)}{[(0.30 * 10^6) * (1) + (0.20 * 10^6) * (2) + (0.30 * 10^6) * (3) + (0.20 * 10^6) * (3)]} = 1136.363636$$

For Processor P2:

$$Performance = \frac{(3.0 * 10^9\ Hz)}{[(0.30 * 10^6) * (2) + (0.20 * 10^6) * (2) + (0.30 * 10^6) * (2) + (0.20 * 10^6) * (2)]} = 1500$$

Performance of Processor P2 is greater than performance of Processor P1, so Processor P2 is faster than Processor P1

Part a:

To compute the Total CPI for each processor, we use the following formula:

$$CPI = \frac{(Clock\ Rate)}{(Instruction\ Count) * (Performance)}$$

For Processor P1:

$$CPI = \frac{(2.5 * 10^9)}{(10^6) * (1136.363636)} = 2.2$$

For Processor P2:

$$CPI = \frac{(3.0 * 10^9)}{(10^6) * (1500)} = 2.0$$

Part b

To calculate the CPU Clock Cycles for each processor, use the following formula:

$$CPU\ Clock\ Cycles = (CPI) * (Instructions\ Count)$$

For Processor P1:

$$CPU\ Clock\ Cycles = (0.30 * 10^6) * (1) + (0.20 * 10^6) * (2) + (0.30 * 10^6) * (3) + (0.20 * 10^6) * (3)$$

$$CPU\ Clock\ Cycles = 2200000\ cycles$$

For Processor P2:

$$CPU\ Clock\ Cycles = (0.30 * 10^6) * (2) + (0.20 * 10^6) * (2) + (0.30 * 10^6) * (2) + (0.20 * 10^6) * (2)$$

$$CPU\ Clock\ Cycles = 2000000\ cycles$$

Problem 3

Part a:

Let’s compute for the Clock Rate since we are given the Clock Cycle Time:

$$Clock\ Rate = \frac{1}{(Clock\ Cycle\ Time)}$$

$$Clock\ Rate = \frac{1}{(1.0 * 10^{-9}\ seconds)} = 10^9\ Hz$$

To compute the CPI, we need to use the following formula:

$$CPI = \frac{(CPU\ Time) * (Clock\ Rate)}{(Instructions\ Count)}$$

For Compiler A:

$$CPI = \frac{(1.2\ seconds) * (10^9\ Hz)}{(10^9\ instructions)} = 1.2$$

For Compiler B:

$$CPI = \frac{(1.5\ seconds) * (10^9\ Hz)}{(1.2 * 10^9\ instructions)} = 1.25$$

Part b:

First, let’s start by writing the following equation:

$$CPU\ Time = (Instructions\ Count) * (CPI) * (Clock\ Cycle\ Time)$$

Assuming that the execution times on the two processors are the same as indicated per the question, we can write the following (As a side note, PA means Processor running Compiler As code and PB means Processor running Compiler Bs code):

$$CPU\ Time_{PA} = CPU\ Time_{PB}$$

$$(Instructions\ Count)_{PA} * (CPI)_{PA} * (Clock\ Cycle\ Time)_{PA} = (Instructions\ Count)_{PB} * (CPI)_{PB} * (Clock\ Cycle\ Time)_{PB}$$

$$\frac{(Clock\ Cycle\ Time)_{PA}}{(Clock\ Cycle\ Time)_{PB}} = \frac{(Instructions\ Count)_{PB} * (CPI)_{PB}}{(Instructions\ Count)_{PA} * (CPI)_{PA}}$$

$$\frac{(Clock\ Cycle\ Time)_{PA}}{(Clock\ Cycle\ Time)_{PB}} = \frac{(1.2 * 10^9) * (1.25)}{(10^9) * (1.2)} = 1.25$$

$$(Clock\ Cycle\ Time)_{PA} = 1.25 * (Clock\ Cycle\ Time)_{PB}$$

So, if the execution times on the two processors are the same, then the processor running Compiler As code is 25 percent faster than the processor running Compiler Bs code.

Part c:

First, let's start by writing the following formula below:

$$CPU\ Time = \frac{(Instructions\ Count) * (CPI)}{(Clock\ Rate)}$$

As a side note, A means Compiler As code, B means Compiler Bs code, and C means Compiler Cs code (Compiler C is the new Compiler).

The speedup of using this new Compiler (Compiler C) versus using Compiler A on the original processor is as follows:

$$\frac{(CPU\ Time)_A}{(CPU\ Time)_C} = \frac{(Instructions\ Count)_A * (CPI)_A}{(Instructions\ Count)_C * (CPI)_C}$$

$$\frac{(CPU\ Time)_A}{(CPU\ Time)_C} = \frac{(10^9) * (1.2)}{(6 * 10^8) * (1.1)} = 1.82$$

The speedup of using this new Compiler (Compiler C) versus using Compiler B on the original processor is as follows:

$$\frac{(CPU\ Time)_B}{(CPU\ Time)_C} = \frac{(Instructions\ Count)_B * (CPI)_B}{(Instructions\ Count)_C * (CPI)_C}$$

$$\frac{(CPU\ Time)_B}{(CPU\ Time)_C} = \frac{(1.2 * 10^9) * (1.25)}{(6 * 10^8) * (1.1)} = 2.27$$

From the equations above, we can see that Compiler C execution time is 45 % faster than Compiler A and that Compiler C execution time is 56 % faster than Compiler B. These percentages were obtained as follows:

$$\left(1 - \left(\frac{1}{1.82}\right)\right) * (100\ \%) = 45\ \%$$

$$\left(1 - \left(\frac{1}{2.27}\right)\right) * (100\ \%) = 56\ \%$$

Problem 4

Part a:

The formula to calculate the yield is as follows:

$$Yield = \frac{1}{\left(1 + \left(Defects * \left(\frac{(Die\ Area)}{2}\right)\right)\right)^2}$$

However, we need to compute the Die Area. To compute the Die Area, this formula below will be used:

$$Die\ Area = \frac{(Wafer\ Area)}{(Dies\ Per\ Wafer)}$$

For the 15 cm diameter wafer:

$$Die\ Area = \frac{(pi * (r^2))}{(84)} = \frac{pi * (7.5^2)}{84} = 2.103745081\ cm^2$$

$$Yield = \frac{1}{\left(1+\left((0.022)*\left(\frac{2.103745081}{2}\right)\right)\right)^2} = 0.9552759784$$

For the 20 cm diameter wafer:

$$Die\ Area = \frac{((pi) * (10^2))}{(100)} = pi\ cm^2$$

$$Yield = \frac{1}{\left(1 + \left((0.031) * \left(\frac{pi}{2}\right)\right)\right)^2} = 0.9092888491$$

Part b:

To compute the Cost Per Die, the formula below should be used:

$$Cost\ Per\ Die = \frac{(Cost\ Per\ Wafer)}{(Dies\ Per\ Wafer) * (Yield)}$$

For the 15 cm diameter wafer:

$$Cost\ Per\ Die = \frac{(\$\ 12)}{(84) * (0.9552759784)} = \$\ 0.1495454152$$

For the 20 cm diameter wafer:

$$Cost\ Per\ Die = \frac{(\$\ 15)}{(100) * (0.9092888491)} = \$\ 0.1649640817$$

Part c:

For the 15 cm diameter wafer:

Number of dies per wafer is increased by 10 percent means the following:

$$New\ Dies\ per\ Wafer = (1.10) * (84) = 92.4\ dies$$

Defects increases by 15 percent means the following:

$$defects = (1.15) * (0.022) = 0.0253\ defects/cm^2$$

Computing the Die Area (using same formula we used in Part b):

$$Die\ Area = \frac{(pi) * (7.5^2)}{92.4} = 1.912495528\ cm^2$$

Computing for the Yield (using same formula we used in Part b):

$$Yield = \frac{1}{\left(1 + \left(defects * \left(\frac{Die\ Area}{2}\right)\right)\right)^2}$$

$$Yield = \frac{1}{\left(1 + \left((0.0253) * \left(\frac{1.912495528}{2}\right)\right)\right)^2} = 0.9533148001$$

For the 20 cm diameter wafer:

Same analogy as the one above (15 cm diameter wafer) - number of dies per wafer is increased by 10 percent and defects increases by 15 percent. So,

$$New\ Dies\ Per\ Wafer = (1.10) * (100) = 110\ dies$$

$$Defects = (0.031) * (1.15) = 0.03565\ \frac{defects}{cm^2}$$

Computing the Die Area (using same formula we used in Part b):

$$Die\ Area = \frac{((pi) * (10\ cm)^2)}{(110\ dies)} = 2.855993321\ cm^2$$

Computing for the Yield (using same formula we used in Part b):

$$Yield = \frac{1}{\left(1 + \left(defects * \left(\frac{Die\ Area}{2}\right)\right)\right)^2}$$

$$Yield = \frac{1}{\left(1 + \left((0.03565) * \left(\frac{2.855993321}{2}\right)\right)\right)^2} = 0.9054626424$$

Problem 5:

Part a:

To get the capacitive load of the processor, we need to use the power equation as follows:

$$Dynamic\ Power = \left(\left(\frac{1}{2}\right) * (Cpacitive\ Load) * (Voltage)^2 * (Clock\ Rate)\right)$$

We are given the voltage, clock rate and dynamic power, so we can go ahead and compute for the capacitive load.

Computing for the capacitive load as follows:

$$Capacitive\ Load = \frac{(2) * (Dynamic\ Power)}{(Voltage)^2 * (Clock\ Rate)}$$

$$Capacitive\ Load = \frac{(2) * (48\ W)}{(1.2\ V)^2 * (3.2 * 10^9\ Hz)} = 2.083333333 * 10^{-8}\ F$$

Part b:

Computing the Old Total Power:

$$Old\ Total\ Power = Static\ Power + Dynamic\ Power$$

$$Old\ Total\ Power = (12\ W) + (48\ W) = 60\ W$$

The total power (including both dynamic power and static power) has been reduced by 10 percent:

$$New\ Total\ Power = (0.90) * (Old\ Total\ Power)$$

$$New\ Total\ Power = (0.90) * (60\ W) = 54\ W$$

Since we have the Old Voltage value, we can compute for the leakage current using the following formula:

$$Old\ Total\ Power = (Leakage\ Current) * (Old\ Voltage)$$

Computing for Leakage Current:

$$Leakage\ Current = \frac{(Old\ Total\ Power)}{(Old\ Voltage)}$$

$$Leakage\ Current = \frac{(60\ W)}{(1.2\ V)} = 50\ A$$

Computing for the new voltage value (since leakage current is kept the same):

$$New\ Total\ Power = (Leakage\ Current) * (New\ Voltage)$$

$$New\ Voltage = \frac{(New\ Total\ Power)}{(Leakage\ Current)}$$

$$New\ Voltage = \frac{(54\ W)}{(50\ A)} = 1.08\ V$$

Computing for the Voltage Reduction Percentage:

$$Voltage\ Reduction\ Percentage = \left(1 - \left(\frac{(New\ Voltage)}{(Old\ Voltage)}\right)\right) * (100\ \%)$$

$$Voltage\ Reduction\ Percentage = \left(\left(1 - \left(\frac{1.08}{1.2}\right)\right)\right) * (100\ \%) = 10\ \%$$

So, if the total dissipated power (including dynamic power and static power) has been reduced by 10 percent, the voltage must be reduced by 10 percent to maintain the same leakage current

Problem 6

	FP	INT	L/S	B
CPI	1	1	4	2
Instruction Count	50*10^6	100*10^6	80*10^6	16*10^6

Part a:

First, let's get the old CPU Execution Time as follows:

$$Old\ CPU\ Time = \frac{(Instruction\ Count) * (CPI)}{(Clock\ Rate)}$$

$$Old\ CPU\ Time = \frac{(10^6) * \big((50 * 1) + (100 * 1) + (80 * 4) + (16 * 2)\big)}{(2.0 * 10^9\ Hz)}$$

$$Old\ CPU\ Time = 0.251\ seconds$$

CPIs of FP and INT has been reduced by 40 percent and CPIs for L/S and B has been reduced by 30 percent. So, let's compute the new CPIs values and then compute for the New CPU Time:

$$New\ INT\ CPI = (0.60) * (1) = 0.60$$

$$New\ FP\ CPI = (0.60) * (1) = 0.60$$

$$New\ LS\ CPI = (0.70) * (4) = 2.8$$

$$New\ B\ CPI = (0.70) * (2) = 1.4$$

Computing the New CPU Time:

$$New\ CPU\ Time = \frac{(10^6) * \big((50 * 0.60) + (100 * 0.60) + (80 * 2.8) + (16 * 1.4)\big)}{(2.0 * 10^9\ Hz)}$$

$$New\ CPU\ Time = 0.1682\ seconds$$

To get how much faster we can execute the program, do as follows:

$$\frac{New\ CPU\ Time}{Old\ CPU\ Time} = \frac{(0.1682)}{(0.251)} = 0.6701195219$$

$$New\ CPU\ Time = 0.6701195219 * (Old\ CPU\ Time)$$

$$Percentage\ Time\ Faster = (1 - 0.67) * (100) = 33\ \%$$

This means that we can execute the program 33 percent faster if the CPIs of INT and FP has been reduced by 40 percent and the CPIs of L/S and B has been reduced by 30 percent.

Part b:

Let’s start by computing for the New CPU Time:

Since the program will be executed 20 percent faster then:

$$New\ CPU\ Time = (0.80) * (Old\ CPU\ Time)$$

$$New\ CPU\ Time = (0.80) * (0.251\ seconds)$$

$$New\ CPU\ Time = 0.2008\ seconds$$

All CPIs and Instructions Count are the same except the CPI for FP, which what the question wants us to change. So, using the formula below, we can compute for CPI of FP:

$$New\ CPU\ Time = \frac{(Instructions\ Count) * (CPI)}{(Clock\ Rate)}$$

$$(Instructions\ Count) * (CPI) = (New\ CPU\ Time) * (Clock\ Rate)$$

$$(10^6) * \left((50 * CPI_{FP}) + (100 * 1) + (80 * 4) + (16 * 2)\right) = (0.2008) * (2.0 * 10^9\ Hz)$$

$$(50 * CPI_{FP}) + (452) = \frac{(0.2008) * (2.0 * 10^9\ Hz)}{(10^6)}$$

$$50 * CPI_{FP} = (401.6) - (452)$$

$$CPI_{FP} = -1.008$$

This is incorrect as CPI cannot be negative. So, this means that we cannot make the program run 20 percent faster by improving only CPI of FP instructions.

Part c:

Let's start by computing for the New CPU Time:

Since the program will be executed 30 percent faster, then:

$$New\ CPU\ Time = (0.70) * (Old\ CPU\ Time)$$

$$New\ CPU\ Time = (0.70) * (0.251\ seconds)$$

$$New\ CPU\ Time = 0.1757\ seconds$$

All CPIs and Instructions Count are the same except the CPI for L/S, which what the question wants us to change. So, using the formula below, we can compute for CPI of L/S:

$$New\ CPU\ Time = \frac{(Instructions\ Count) * (CPI)}{(Clock\ Rate)}$$

$$(Instructions\ Count) * (CPI) = (New\ CPU\ Time) * (Clock\ Rate)$$

$$(10^6) * \big((50 * 1) + (100 * 1) + (80 * CPI_{LS}) + (16 * 2)\big) = (0.1757) * (2.0 * 10^9\ Hz)$$

$$(80 * CPI_{LS}) + (182) = \frac{(0.1757) * (2.0 * 10^9\ Hz)}{(10^6)}$$

$$80 * CPI_{LS} = 351.4 - 182$$

$$CPI_{LS} = 2.1175$$

So, Percentage CPI LS Improvement is as follows:

$$Percentage\ CPI_{LS}\ Improvement = \left(1 - \left(\frac{(CPI_{LS})}{(Old\ CPI_{LS})}\right)\right) * (100\ \%)$$

$$Percentage\ CPI_{LS}\ Improvement = \left(1 - \left(\frac{2.1175}{4}\right)\right) * (100\ \%) = 47\ \%$$

So, we must improve the CPI of L/S instructions by 47 percent if we want the program to run 30 percent faster.