15.094J/1.142J Robust Modeling Optimization and Computation

VENTILATOR ADAPTIVE ROBUST ALLOCATION

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1 Introduction

COVID-19 is a novel coronavirus which caused a global pandemic right before 2019 was over. Due to this pandemic thousands of lives where lost, while the fast spread of the disease causes a shortage in some of the most essential to stop spreading health care supplies. Specifically, although little are known of how the virus actually affects one's life, most of the hospitalized patients developed life threatening respiratory symptoms resulting the need of intensive care units and mechanical ventilators for many of those cases. Thus many healthcare facilities across US, and specifically in high density areas like New York, were faced with a ventilator shortage. This, along with the exponential growth of number of cases made the patients care even worse.

However, good news, the virus does not spread to all places with the same rate. In other words, the shortage is not happening at the same time across the states. Not only that but according to [1] the total supply of ventilators is more than the actual demand. This is very promising since it implies that under state cooperation it is possible to transfer ventilators from states with a few intubated cases to those who actually are facing a shortage. In this the healthcare providers will only have to care about the patient's care and not for the supply shortage.

Of course, under a federal management of ventilators in this manner, the new ventilators produced will be more fairly split between states. Such a central policy will require though all the states to agree to share their ventilators. This can create agreement issues, however, a fair policy can be generated in order to create space for the proper agreements.

One of the main issues though is the uncertainty in the ventilators demand. Many models have been created by now to forecast this demand. The lack of accurate data though about this new virus in general might cause those models to be weak. Under worst case scenarios, with overestimated or underestimated forecasts even a fair sharing policy might cause unneeded transfers that result in more bad than good. For example if state s has overestimated demand while state s' underestimated and we request a transfer from s' to s, then there is the case that actually state s' would be better of not sharing. We would not want something like this to happen!

To prevent such scenarios we would need robust ventilator allocation solutions that are balanced under uncertainty. This work will focus on the evaluation of a robust ventilator allocation system. Fortunately, there is a team actively working both on the forecasting and the actual sharing system. So my goal will be extend the existing model in order to provide more robust solutions. In what follows, the allocation problem is introduced in part 2, the model formulation is explained in part 3, the methods used in this work are described in part 4 and finally the results and future improvements are discussed in parts 5 and 6.

2 Problem Statement

The ventilator allocation problem is a multi-period problem of resource allocation. Specifically, creating a sharing network of states s = 1...S (50) we can see this problem as a continuous flow of ventilator units so as to fit ventilator demand from day to day. Main goal is to minimize the shortage of ventilators while create only essential ventilator unit transfers.

In a perfect world, were we know exactly the ventilator demand, we can make the transfers and fit the demand almost immediately. However, since this is not possible, the model is described under the perfect world scenario, while the demand used is either forecasted in the case of a static optimization nominal model or uncertain in the case of a robust model.

Under this perspective, we have available the number of states and their baseline ventilator supply, the distances between states, and a number of ventilators provided by the federal government per day, which can be thought as the total amount of new ventilators produced in a factory. Of course, depending on the distance between states, the number of days to transfer the ventilators varied form 1 to 3.

In what follows, a model close to the one proposed in [1] is presented. Specifically, the nominal model of ventilator sharing is presented along with the constraints to provide robustness that [1] suggest. After that the robust model is explained.

3 Formulation

The nominal formulation is presented:

Sets

$s, s' \in S$	set of US states
$t \in T$	set of days (time horizon)

Data

$D_{st} \in \mathbb{Z}^+$		Ventilator demand in state s on day t. (incoming Demand)
$b_s \in \mathbb{Z}^+$		Baseline #ventilators in state s (initial supply).
$F \in \mathbb{Z}^+$	(=450)	Ventilators produced per day by the factory (Daily federal supply).

Parameters

$dist_{ss'}$		distance between state s and state s'.
$\tau_{ss'}$	(=3)	#days to deliver ventilators from state s to state s'.
t_{min}	(=10)	minimun #days a ventilator is in use.
λ	(=0.1)	lasso regularizer.
M		big M optimization value (* discussed later).

Variables

$V_{st} \in \mathbb{Z}^+$	Ventilators in state s on day t (in use + supply)
$VS_{st} \in \mathbb{Z}^+$	Ventilator shortfall in state s on day t (excess demand)
$x_{ss't} \in \mathbb{Z}^+$	out for delivery #ventilators from state s to state s' on day t
$f_{st} \in \mathbb{Z}^+$	#ventilators state s will receive from factory on day t (federal supply)
$y_{st} \in \{0,1\}$	$\mathbb{1}\{$ state s faces a shortfall on day t$\}$

Objective

$$\min \quad \sum_{s\in S}\sum_{t\in T} VS_{st} + \lambda\Big(\sum_{s\in S}\sum_{s'\in S}\sum_{t\in T} dist_{ss'}x_{ss't} + \sum_{s\in S}\sum_{t\in T} f_{st}\Big) \tag{1}$$

Constraints

$$V_{s0} = b_s,\ \forall s \in S \tag{2}$$

$$\sum_{s \in S} f_{st} \leq F,\ \forall t \in T \tag{3}$$

$$\forall s \in S, \forall t \in T:$$

$$V_{st} \geq (1 - f_{max}) b_s \tag{4}$$

$$VS_{st} \geq d_{st} - V_{st} \tag{5}$$

$$VS_{st} \leq M y_{st} \tag{6}$$

$$\sum_{s' \in S} x_{sst} \leq V_{max}(1 - y_{st}) \tag{7}$$

$$V_{st} = V_{s(t-1)} + f_{st} + \sum_{s' \in S} x_{s's\max(1, t-\tau_{s',s})} - \sum_{s' \in S} x_{ss't} \tag{8}$$

$$V_{st} \geq \sum_{t'=\max(1, t-t_{min})}^{t-1} (f_{st'} + \sum_{s' \in S} x_{s'st'}) \tag{9}$$

$$VS_{st} + \Delta_{st} \geq (1+\alpha) d_{st} - V_{st} \tag{10}$$

$$VS_{st} + \Delta_{st} \leq (1+\alpha) d_{st} y_{st} \tag{11}$$

$$\min \quad \sum_{s \in S} \sum_{t \in T} (VS_{st} + \rho \Delta_{st}) + \lambda (\sum_{s \in S} \sum_{s' \in S} \sum_{t \in T} dist_{ss'} x_{ss't} + \sum_{s \in S} \sum_{t \in T} f_{st}) \tag{12}$$

Very briefly, constraint (2) initializes the model, constraint (5) is the definition constraint of ventilator shortage, constraints (3), (4) and (7) can be seen as capacity/inventory constraints and finally constraints (8), (9), (6) and (7) are the flow constraints. For more details see [1].

Finally in (10) and (11) are the constraints that promise robustness in the buffer model [1], where Δ_{st} is described as the shortage buffer but in another perspective can be though as a "plan ahead demand" constraint, where ρ and α are the parameters that manage this buffer.

Regarding the big M notation constraint (6), in the proposed implementation by [2] the demand of state s on day t is used. However, to reduce the constraints that impose uncertainty under the robust model, this has been set to a high value for the simulations.

4 Methods

4.1 Demand uncertainty

Models like the proposed one need good predictive ability in order to perform well. Basic idea is that the uncertainty is treated in the prediction level. What is good about this perspective is that the model is easily solvable by the available MIP solvers. Also, the solutions might end up being less conservative, especially if used with a slightly over estimated demand.

On the other hand under the robust modeling setting, the demand is treated as uncertain parameter. Thus the uncertainty set acts as the prediction but as part of the model itself. Of course one can try and use a static classic uncertainty set of the ones available. However, given data availability, in the same way predictions can be updated to fit better the real environment, a data driven uncertainty set can be dynamic and adapt as soon as more data become available.

For this reason we introduce the following CLT type of uncertainty to be used for the uncertain demand:

Let $\{D_{st}\}$ the observed demand values form day 1 up to day t. We define:

$$\mu_s = \frac{\sum_{t'=1}^{t} D_{st'}}{t} \qquad \sigma_s = \sqrt{\frac{\sum_{t'=1}^{t} (D_{st'} - \mu_s)^2}{t}}$$

and

$$\mu_t = \frac{\sum_{s\in S} D_{st}}{S} \qquad \sigma_t = \sqrt{\frac{\sum_{t'=1}^{t} (D_{st} - \mu_t)^2}{S}} \quad \mu = \frac{\sum_{t'=1}^{t} \mu'_t}{t} \qquad \sigma = \sqrt{\frac{\sum_{t'=1}^{t} (\mu'_t - \mu)^2}{t}}$$

Now for the uncertain demand $\{d_{st'}\}$, where t' > t, we have:

$$d \in U \equiv \times_s U_s \cup U_g$$

where, for d_s the row vector for state s in demand d

$$U_s = \{d_s : \| \sum_{t'=1}^{T} d_{st} - \mu_s\|_1 \leq \Gamma\sigma_s\sqrt{T}\}$$

and

$$U_g = \{d : \| \sum_{t'=1}^{T} \frac{\sum_{s\in S} d_{st}}{S} - \mu\|_1 \leq \Gamma\sigma\sqrt{T}\}$$

and the uncertain constraint is:

$$d - V - VS \leq 0 \ \forall d \in U$$

The rational behind this uncertainty is that across days the demand of a state may remain close to the mean of the previous days and also the general mean of demand across the states for each day is going to remain close to the mean of previous days' means. In this way we take care both of day time correlations and location correlations without need of correlation matrix thus preserving the linearity of the model. Definitely there are other ways to keep same principles with simpler uncertainty sets, but these type of sets are closer to the CLT (although not the same) proposed uncertainty sets. Γ is a parameter that controls the robustness of the model.

Of course such uncertainty is not anymore constraint-wise. For simpler analysis one can study the impacts of keeping one uncertainty or the other but for this study we consider the combined set.

4.2 Sliding window method:

Since it is hard to solve the full robust model, we propose a sliding window approach. Specifically, since the maximum number of days to transfer a ventilator is 3, to plan ahead responsively we need a window of at least 4 days. Of course depending the computing power, the more days the better.

Sliding window algorithm:

1. Consider a planning horizon and set a window of T days (the horizon need not to be determined)
2. Start on day x
3. Using past data generate the mean and standard deviation parameters required by the uncertainty sets.
4. Solve the robust model for the window of T days.
5. Implement the first day in the solution.
6. Observe the new demands and move to step 3.
7. Repeat until the end of planning horizon.

We hope that along the way we learn the distribution better and adapt to the real demand.

5 Results

Due to the lack of original historical data, for the simulation and comparison of the models the "ihme" ventilator demand predictions available in [2] were used. For this study they are assumed to be the actual demands of those days. Also to speed up the process the parts of code provided in the github repository of [2] were employed.

To produce the results and fairly compare the methods, the first two days were used to form the first uncertainty set and all the simulations start from day 3 and on. In this way all initial shortages are the same, thus neglected here. Specifically the baseline model were the predictions are used as such, the buffer proposed in [1] model, were uncertainty is treated with the used of a buffer, and finally the proposed sliding window model were simulated for the total of 33 days from 4/18 until 20/5. Finally, all the basic parameters used by all models remained in the values used in the implementation provided in the github repository of [2] and the linear relaxation was used to speed up the simulation (It was observed that the solutions were very close anyways).

Critical for the comparison of the models were assumed the following: Total ventilator shortage, number of days US faces shortage in at least one place, the federal new ventilator supplies provided and finally the between states ventilator total ventilator transfers. These are four crucial factors for the decision of the policy used. For example if the federal funding for new supplies is not sufficient for a specific level of production, then solutions that require high federal supply might not be a good fit. In the other hand, if ventilator shortage is a priority then the model that reduces the number of ventilators the most can be assumed as the best policy making device no matter what. The table contains the simulation results.

As one can see in the table, we can definitely benefit from a robust optimization model. Important to notice though is that we substantially increased the number of transfers and new ventilators maybe more than we decreased the shortage. So one needs to balance these two things out. Assuming that the best model is ROSW with parameter $\Gamma = 2$, since it is

	Baseline	***Buffer***	***ROSW***					***without intervention***
	-	-	Γ=0	Γ=1	Γ=2	Γ=2.5	Γ=3	-
Total shortage	5352	5351	21544	3710	3670	3937	3940	41790
#Days in shortage	6	4	23	5	5	4	5	33
Federal transfers	1202	1144	639	3808	7160	8490	11063	-
State transfers	788	1141	1161	2632	3564	4888	6840	-
Total transfers	1990	2285	1800	6430	10624	13378	11903	-

the one that reduces the shortage the most, we can see that in best case we can achieve a decrease of 31% in the price of over 300% increase of the number of transfers compared to the buffer model. Staying in a more comparable environment for the value of $\Gamma = 1$ we have a 30% for a 180% increase of ventilator transfers, which again is not negligible.

Interesting to see is that the Γ can in cases cause more bad than good since higher values lead to more transfers that in reality cause more shortage probably due to unnecessary relocation of ventilator units. Finally the $\Gamma = 0$ test case will force the demand to equal the respective means to the extend possible.

What is impressive though, is the difference of the baseline model to the buffer model. They both result in similar level of total shortage with the buffer model actually causing more transfers than needed. However the total shortage lasts only 4 days in this case. Notice though, that it takes maximum 3 days to transfer ventilators, while it is also the minimum number of days in the simulations perspective. So the model tends to increase the federal supplies in order to take care of this delay, while the baseline model just takes 2 more days to fit the demand with less federal support needed.

There are certainly tradeoffs that the policy makers need to consider in order to apply any of the above. It is good though that using such a policy we are actually able to make the shortage vanish in less than a week!

6 Future improvements

The study presented here is the first step towards a more robust solution. As we can see it has a potential to improve provided solutions. However there are more paths yet to be explored.

First of all, an experimentation with simpler or decoupled uncertainty sets would help create a better perspective of data driven approaches. A possible thought is to decouple the aforementioned uncertainties or create simpler budget uncertainties with the use of the mean and standard deviation but in the place of nominal demand and coefficient matrix of a normal distributed noise z. This will also help in the analysis of the model.

What is more, since the model is by default adapting, we might benefit of an adaptive optimization model. In this case all transfers are adaptive variables and we can wait and observe the demand in order to trigger transfers, thus avoid unnecessary transfers. Also one might benefit by slightly reformulating the model so that we plan for next day transfers only using the today's demand as input, which specifically adapts the future to the present with robustness. (So that we implement the next day's transfer and not the same day's transfers based on observed demand)

Due to the integer nature of this problem, this is a hard goal. So a sliding window approach with proper here and now variables might reduce the complexity.

7 References

[1] https://www.covidanalytics.io/ventilator_documentation_pdf

[2] https://www.covidanalytics.io/ventilator_allocation