National Technical university of Athens

School of applied mathematical and physical sciences

Internship assignment

Study for the Dyson-Schwinger recursive algorithm and production of Feynmann diagrams

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1 Introduction

The first step for the quantum field theory introduction was the derivation of the Klein-Gordon classical field equation as a single-particle wavefunction. In order to perform a proper analysis on such systems it is important to use an analogy that we know, in this case it is the harmonic oscillator.
The basic idea of Quantum field theory is to begin with a classical field theory (classical wavefunction of a particle) and then quantize it. Specifically, it means to replace the dynamical variables with operators, to use the well-known commutation relations (already studied in classical field theory of the harmonic oscillator) and find the eigenvalues and eigenstates of the system using the harmonic oscillator model.
In quantum field theory it is possible treat the mediators of the forces as separate particles and study a multi-particle quantum system based on the idea of the harmonic oscillator as a dynamical system of multiple eigenstates. It combines quantum mechanics and special relativity. The reason for having a multi-particle theory arises not only from Klein-Gordon's negative energy states problem but also from other less obvious inconsistencies (causality problem-solved by introducing antiparticles).
In this assignment, we considered $\hbar = c = 1$.

2 Field theory

A classical field theory is a **continuous** mechanical system with a continuous set of degrees of freedom. There are three main parts that compose the theory[1]:

1. **Hamiltonian and Lagrangian**
   The Hamiltonian density is a functional of fields which can be integrated over all space and return the Hamiltonian.
   $$H = \int d^3x \mathcal{H} \tag{2.0.1}$$
   The Lagrangian density is just a Legendre transformation of the Hamiltonian density.
   $$\mathcal{L}[\phi, \dot{\phi}] = \pi[\phi, \dot{\phi}]\dot{\phi} - \mathcal{H}[\phi, \pi[\phi, \dot{\phi}]] \tag{2.0.2}$$

2. **Least action principle and Euler-Lagrange equations**
   In QFT we almost exclusively use Lagrangian densities because of their Lorentz invariance. The action $\mathcal{S}$ is defined as the time integral of the Lagrangian density, which is a function of one or more fields $\phi(x)$, and their corresponding derivatives $\partial_\mu \phi$:
   $$\mathcal{S} = \int dt \mathcal{L} = \int d^4x \mathcal{L}(\phi, \partial_\mu \phi) \tag{2.0.3}$$
   Considering a variation on the field $\phi \to \phi + \delta\phi$: We can derive from $\delta\mathcal{S}$:
   $$\delta\mathcal{S} = \int d^4x \left(\left[\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right] \delta\phi + \partial_\mu \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi \right] \right) \tag{2.0.4}$$
   The Euler-Lagrange equations of motion:
   $$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0 \tag{2.0.5}$$

3. **Noether's theorem & conserved charges**
   In classical field theory, Noether's Theorem concerns continuous transformations on the fields ϕ. More specifically, if the Lagrangian and thus the equations of motion remain invariant under infinitesimal transformations of the fields ϕ, then we call this transformation a symmetry.
   Noether's theorem implies that if a transformation is a **continuous** symmetry of the Lagrangian we can find a conserved quantity-current known as Noether current:

$$J_\mu = \sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \frac{\delta \phi_n}{\delta \alpha} \rightarrow \partial_\mu J_\mu = 0 \tag{2.0.6}$$

   This means that the total charge derived from Noether current is constant in time:

$$Q = \int d^3x J_0 \rightarrow \partial_t Q = \int d^3x \partial_t J_0 = 0 \tag{2.0.7}$$

   The current is conserved on-shell (when equations of motion are satisfied) and it works for both global (parametrized by numbers α) and local (parametrized by functions $\alpha(x)$) symmetries.
   In general, the currents play a very important role in field theories as they can represent Noether currents, external particle flows, sources of fields or even just a term in the Lagrangian.

To quantize a theory, we rewrite the dynamical variables as operators and use the harmonic oscillator model to find the system's eigenstates and eigenvalues. The theory is composed by the commutation relations of the canonical momentum $\pi(y)$ against the field $\phi(x)$, which hold even in the Heisenberg picture when both operators are considered in the same time.
It is easy to find, for instance, the Klein-Gordon Hamiltonian spectrum using the harmonic oscillator ladder operators α and $\alpha^\dagger$, and express the field and momentum eigenstates:

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_{\vec{p}}}} \left(\alpha_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}} + \alpha^\dagger_{\vec{p}}\, e^{-i\vec{p}\cdot\vec{x}} \right) \tag{2.0.8}$$

$$\pi(x) = \int \frac{d^3p}{(2\pi)^3} (-i) \sqrt{\frac{\omega_{\vec{p}}}{2}} \left(\alpha_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}} - \alpha^\dagger_{\vec{p}}\, e^{-i\vec{p}\cdot\vec{x}} \right) \tag{2.0.9}$$

$$H = \int \frac{d^3p}{(2\pi)^3} \omega_{\vec{p}} \left(\alpha^\dagger_{\vec{p}} \alpha_{\vec{p}} + \frac{1}{2} [\alpha_{\vec{p}}, \alpha^\dagger_{\vec{p}}] \right) \tag{2.0.10}$$

Based on the three main parts of the theory, we can easily derive the Coulomb's law from the Lagrangian, using external currents:

$$\mathcal{L} = -\frac{1}{4} F^2_{\mu\nu} - A_\mu J_\mu \xrightarrow{Euler-Lagrage} \partial_\mu F_{\mu\nu} = J_\nu \tag{2.0.11}$$

The last expression include Maxwell's equations, with a source (current) present. Expanding, the electromagnetic tensor $F_{\mu\nu}$:

$$J_\nu = \Box A_\nu - \partial_\nu(\partial_\mu A_\mu) \xrightarrow[Lorentz-gauge]{\partial_\mu A_\mu = 0} A_\nu = \frac{1}{\Box} J_\nu \tag{2.0.12}$$

This pattern is very common in QFT. A_ν is the field that is determined by the current source J_ν after its propagation with the propagator:

$$\Pi_A = \frac{1}{\Box} \tag{2.0.13}$$

QFT green functions There are two ways of quantization. The canonical quantization, which was historically the first one to be developed and the Feynman path integrals, which is a more general and formal way of quantization. Except from those two analytical ways of performing QFT calculations, one can turn to more old-fashioned perturbation theory or other computational methods such as Dyson-Schwinger recursion for the calculation of scattering amplitudes and phase space generation.

2.1 QED

Quantum Electrodynamics is the quantum field theory that best describes the electromagnetic interactions. The propagator of the force in this case is a photon (massless of spin-1). QED consists of the Maxwell's and Dirac equations determined by relativistic invariance. If we take the Dirac Lagrangian, then add the Maxwell term and consider a covariant derivative of the form:

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu \tag{2.1.1}$$

The final form of the QED Lagrangian through this transformation is:

$$\mathcal{L}_{Dirac} = \bar{\psi}(i\not{\partial} - m)\psi \xrightarrow{+L_{Maxwell}} \mathcal{L}_{QED} = \bar{\psi}(i\not{\partial} - m)\psi - \underbrace{\frac{1}{4}F^{\mu\nu}F_{\mu\nu}}_{L_{Maxwell}} - \underbrace{q\bar{\psi}\gamma^\mu\psi A_\mu}_{\text{interaction term}} \tag{2.1.2}$$

where $F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This Lagrangian is invariant under local U(1) transformations:

$$\psi(x) \to e^{-i\alpha(x)}\psi(x)\ , \quad A_\mu(x) \to A_\mu(x) - \frac{1}{q}\partial_\mu\alpha(x) \tag{2.1.3}$$

The Feynman rules that constitute the QED are shown in 1. We are going to use them in LO-cross sections calculation examples in this assignment:

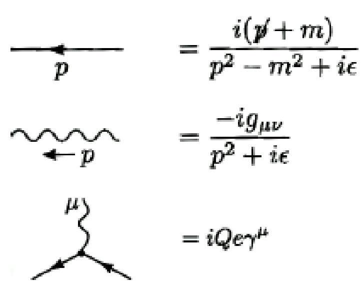


Figure 1: Feynman Rules for QED

2.2 QCD

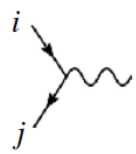
Quantum Chromodynamics is a non-Abelian gauge theory that describes the strong interactions. The mediators in this theory are the W and Z massive bosons. The SU(3) local phase transformation is:

$$\psi(x) \to \psi'(x) = e^{ig\alpha^a(x)t^a}\psi(x) \tag{2.2.1}$$

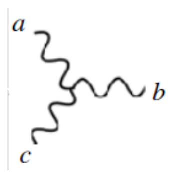
2.3 Feynman rules

2.4 QCD color decomposition- Graphical approach

Let us explain the color decomposition process using a more graphical approach[2]. For each set of operators, we can have graphical representation.
There are $N_c = 3$ colors in QCD which define the identity of quarks and gluons. Quarks and antiquarks use a lower index q_i where $i \in 1, 2, 3$ and gluons which are the mediators of the strong force are denoted g^α where $\alpha \in 1, 2, ..., 8$. For the graphical representation we need to follow the graphical rules, and use an index-only notation for the graphs and the calculations. The general rules for color decomposition are shown below:

- $q\bar{q}g =$  $= T^\alpha_{ij} = \frac{1}{\sqrt{2}}\tau^\alpha_{ij}$

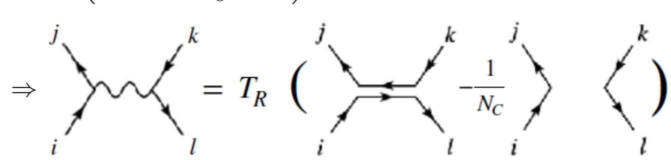
- $q\bar{q} =$  $= \delta_{ij}$

- $ggg =$  $= if^{abc}$

The above operators compose the $SU(3)$ group algebra. Some significant properties are found below, and will be used a lot in the calculations:

- $[T^a, T^b] = if^{abc}T^c$
- $Tr[T^aT^b] = T_R\delta^{ab}$
- Fierz identity:

$$T^\alpha_{ij}T^\alpha_{kl} = T_R\left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{il}\delta_{kj}\right)$$



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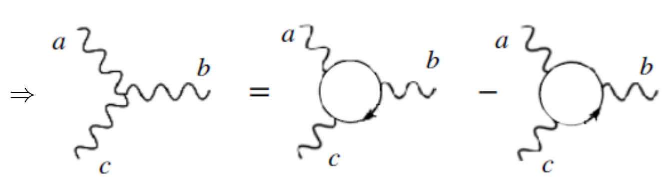
$$if^{abc} = \frac{1}{T_R}Tr[T^aT^bT^c - T^cT^bT^a]$$

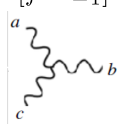


At this point, we will provide a detailed example to illustrate the color decomposition technique. We will prove, using the graphic and algebraic representation, the relation:

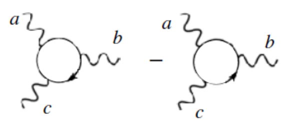
$$[f^{abc}L_1](\tau^{\alpha_1})_{i_1j_1}(\tau^{\alpha_2})_{i_2j_2}(\tau^{\alpha_3})_{i_3j_3} = \left(-\frac{i}{\sqrt{2}}\right)\delta_{i_1j_2}\delta_{i_2j_3}\delta_{i_3j_1} + \left(\frac{i}{\sqrt{2}}\right)\delta_{i_3j_2}\delta_{i_2j_1}\delta_{i_1j_3} \tag{2.4.1}$$

Starting with the first part:

$$1^{st} \text{ part} = \ [f^{abc}L_1]\ (\tau^{\alpha_1})_{i_1j_1}(\tau^{\alpha_2})_{i_2j_2}(\tau^{\alpha_3})_{i_3j_3}$$

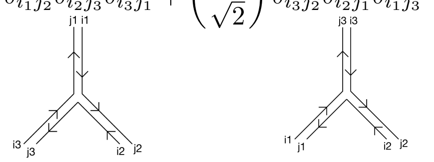


$$= \frac{1}{2i\sqrt{2}}tr[\tau^{\alpha_1}\tau^{\alpha_2}\tau^{\alpha_3} - \tau^{\alpha_3}\tau^{\alpha_2}\tau^{\alpha_1}](\tau^{\alpha_1})_{i_1j_1}(\tau^{\alpha_2})_{i_2j_2}(\tau^{\alpha_3})_{i_3j_3}L_1$$



(2.4.2)

$$\overset{\text{Ward Id. for a, b, c}}{=} \left(-\frac{i}{\sqrt{2}}\right)\delta_{i_1j_2}\delta_{i_2j_3}\delta_{i_3j_1} + \left(\frac{i}{\sqrt{2}}\right)\delta_{i_3j_2}\delta_{i_2j_1}\delta_{i_1j_3} = 2^{nd} \text{ part}$$



The Ward identity was performed for each of the a, b, c gluons giving 2 extra terms every time we contract with each $(\tau^{\alpha})_{ij}$ operator. In the end, though, we are left with only two final results.

2.5 Introduction to SMEFT

In this chapter, we will discuss the Standard Model Effective Field Theory and explore the reasons why it is an important consideration in phenomenology. We will outline the fundamentals of SMEFT[3] to ensure that the results of our study are clear and comprehensible.

Since the discovery of the Higgs boson at the LHC a decade ago, no clear evidence of new physics has emerged at the specific TeV scale at which the LHC operates (with an energy limit of 13.6 TeV). Today, many scientists are convinced that physics beyond the Standard Model possibly exists at higher energy scales than those currently accessible by the LHC. Theoretical evidence continues to motivate researchers to explore these larger scales in the search for new physics.

Let us begin by pointing out the issues in the current field theory.

Assuming that new physics is "heavy" and therefore detectable only at higher energy scales, scientists have explored how the Standard Model would be modified by the presence of this heavier physics making various corrections in the standard model Lagrangian. Using the SMEFT framework, it has been proved that the SMEFT Lagrangian at its low energy limit actually matches the measurements we already have based on the standard model.[4]

Assume $SU(3) \times SU(2) \times U(1)$ gauge theory with no more light particles and that Higgs boson is a part of $SU(2)$ doublet. In this case, only SM particles exist, however new interactions appear. The SMEFT Lagrangian is an expansion of the standard model Lagrangian, which at the same time represents the lower energy term, with some additional terms:

$$\mathcal{L} = \underbrace{\mathcal{L}_{SM}}_{d=4} + \sum_i C_i^{(6)} \frac{\mathcal{O}_i^{d=6}}{\Lambda^2} + \sum_i C_i^{(8)} \frac{\mathcal{O}_i^{d=8}}{\Lambda^4} + ... \tag{2.5.1}$$

The constants $C,_i^{(6)}$,$C_i^{(8)}$... may be many and they are called Wilson coefficients. Correspondingly, the there are the EFT operators $\mathcal{O}_i^{d=6}$,$\mathcal{O}_i^{d=8}$, and the parameter Λ represents the energy scale, whose power depends on the dimension of the operator.
In this assignment we will be dealing only with the dimension-6 operator. The dimension-6 operators $\mathcal{O}_i^{d=6}$ in the Warsaw basis are given in 3. They are categorized into those that interact with bosonic fields and remain invariant under $SU(3)$ transformations, and those that interact with thermionic fields, which can be further classified based on their transformation properties under $SU(3)$ variations.
Although it would be interesting to study each of them individually, our primary focus is on three specific ones, the O_{uG}, O_{uH} and O_{HG}.
In another classification based on the primary types of physical observables or processes that constrain these operators, the first two are EWPO (electroweak precision observables) operators, and the last one is bosonic. This means that the first two operators could affect electroweak processes, potentially causing deviations in measurements, while the last one would influence measurements involving bosonic fields.

X^3		H^6 and H^4D^2		$\psi^2 H^3$	
$\mathcal{O}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_H$	$(H^\dagger H)^3$	$\mathcal{O}_{eH}$	$(H^\dagger H)(\bar{l}_p e_r H)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{uH}$	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
$\mathcal{O}_W$	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{HD}$	$\left(H^\dagger D^\mu H\right)^\star \left(H^\dagger D_\mu H\right)$	$\mathcal{O}_{dH}$	$(H^\dagger H)(\bar{q}_p d_r H)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
X^2H^2		$\psi^2 XH$		$\psi^2 H^2 D$	
$\mathcal{O}_{HG}$	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$	$\mathcal{O}_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$\mathcal{O}_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$	$\mathcal{O}_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$\mathcal{O}_{HW}$	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{H}\, G_{\mu\nu}^A$	$\mathcal{O}_{He}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$\mathcal{O}_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$	$\mathcal{O}_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$\mathcal{O}_{HB}$	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{H}\, B_{\mu\nu}$	$\mathcal{O}_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$\mathcal{O}_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$	$\mathcal{O}_{Hu}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
$\mathcal{O}_{HWB}$	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$	$\mathcal{O}_{Hd}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
$\mathcal{O}_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$	$\mathcal{O}_{Hud}$	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$
$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$		
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$		
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$		
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$				

Figure 2: Dimension-6 operators in the Warsaw basis. The grey cells indicate operators that break flavour $SU(3)$ explicitly.[4]

$$\begin{aligned}
&\text{EWPO:} && \mathcal{O}_{HWB}\,, \mathcal{O}_{HD}\,, \mathcal{O}_{ll}\,, \mathcal{O}_{Hl}^{(3)}\,, \mathcal{O}_{Hl}^{(1)}\,, \mathcal{O}_{He}\,, \mathcal{O}_{Hq}^{(3)}\,, \mathcal{O}_{Hq}^{(1)}\,, \mathcal{O}_{Hd}\,, \mathcal{O}_{Hu}\,,\\
&\text{Bosonic:} && \mathcal{O}_{H\Box}\,, \mathcal{O}_{HG}\,, \mathcal{O}_{HW}\,, \mathcal{O}_{HB}\,, \mathcal{O}_{W}\,, \mathcal{O}_{G}\,,\\
&\text{Yukawa:} && \mathcal{O}_{\tau H}\,, \mathcal{O}_{\mu H}\,, \mathcal{O}_{bH}\,, \mathcal{O}_{tH}\,.
\end{aligned} \tag{2.8}$$

Figure 3: Classification of the operators based on the impacts they will show in different measurements and processes.[4]

In our research we will be dealing with the $t\bar{t}H$ interaction[5]. The top quark and the Higgs boson are the two heaviest elements in the SM until today. As a result, using SMEFT, this interaction is expected to reveal evidence of beyond the SM physics. The SMEFT introduces additional correction terms into the Lagrangian at the Next-to-Leading Order (NLO) approximation. This means that, for certain sets of particles, such as in this case, there will be more interactions, including off-shell ones, beyond those typically observed in the Standard Model.
As mentioned before, the operators that we investigate here are the O_{uG}, O_{uH} and O_{HG}, and some processes that are derived from the SMEFT are shown below:

(a)
(b)
(c)

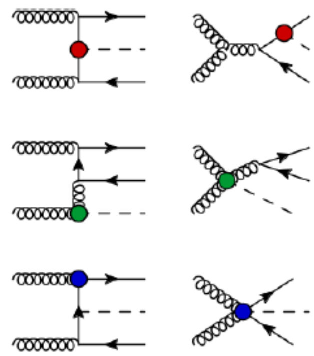


Figure 4: Example diagrams for $t\bar{t}H$ production. (a)$O_{t\phi}$, (a)$O_{\phi G}$ and (a)O_{tG}

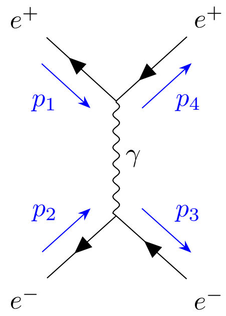
NLO SMEFT predictions in general are important for several reasons. To understand their contribution in physics we will provide a similar example.
In 1930, Fermi predicted the $\mu^-\nu_\mu \to e^-\bar{\nu_e}$ interaction. However, he introduced the coupling. However, at that time, Fermi did not know that this interaction was related to the W-boson exchange, as later described by the modern Standard Model. Similarly, SMEFT operates on the principle of using effective interactions to describe possible new physics. By introducing NLO terms that are fitted to existing SM measurements, SMEFT allows us to explore additional interactions that might occur, even if their mechanisms are not yet fully understood.
In order to start our investigation, in Section 4, we introduce a Python library designed to symbolically calculate the output currents, scalars, and vectors based on the process and operator involved in the interactions. In this way we will be able to implement the symbolically generated currents in HELAC-PHEGAS[6] simulation tool, make the fitting according to standard model and eventually explore the SMEFT interactions.

3 Examples of Analytical calculations

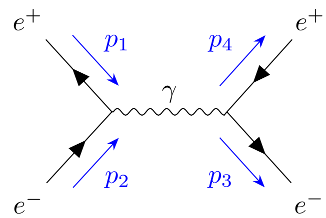
By applying Feynman rules and trace techniques, one can calculate scattering amplitudes for QED and QCD processes. Below, we will present the analytical calculations; however, such calculations are automated using the HELAC interface, which we will discuss further in the next section.

3.1 QED: Bhabha scattering without helicity summation

Using M.D. Schwartz's notation, calculations have been conducted to



(a) scattering: t-channel



(b) annihilation: s-channel

In order to find the S-matrix element for the Bhabha scattering, we need to sum over all the matrix elements of all possible feynmann diagrams of the process. In our case we have a t-channel and an s-channel, so the sum will become:

$$\begin{aligned} \mathcal{M} &= \mathcal{M}_t + \mathcal{M}_s \Rightarrow \\ \langle |\mathcal{M}|^2 \rangle &= \frac{1}{4} \sum_{s,s'} \left(|\mathcal{M}_t|^2 + |\mathcal{M}_s|^2 + \mathcal{M}_s \bar{\mathcal{M}}_t + \mathcal{M}_t \bar{\mathcal{M}}_s \right) \end{aligned} \tag{3.1.1}$$

Following the appropriate convention for the momenta directions, the matrix elements for both t and s-channels are presented below:

$$\begin{aligned} \mathcal{M}_t &= \bar{u}(p_3)(-ie\gamma^\mu)u(p_2) \left(\frac{-ig_{\mu\nu}}{q^2} \right) \bar{v}(p_4)(-ie\gamma^\nu)v(p_1) \\ \mathcal{M}_t &= -\frac{e^2}{t} [\bar{u}(p_3)\gamma^\mu u(p_2)][\bar{v}(p_4)\gamma_\mu v(p_1)] \end{aligned} \tag{3.1.2}$$

$$\begin{aligned} \mathcal{M}_s &= \bar{u}(p_2)(-ie\gamma^\mu)v(p_1) \left(\frac{-ig_{\mu\nu}}{q^2} \right) \bar{v}(p_4)(-ie\gamma^\nu)u(p_3) \\ \mathcal{M}_s &= -\frac{e^2}{s} [\bar{u}(p_2)\gamma^\mu v(p_1)][\bar{v}(p_4)\gamma_\mu u(p_3)] \end{aligned} \tag{3.1.3}$$

We will now calculate the sum over all spins of the matrix element squared:

$$\begin{aligned} \sum_{s,s'} |\mathcal{M}_t|^2 &= \frac{e^4}{q^4} \left([\bar{u}(p_3)\gamma^\mu u(p_2)][\bar{v}(p_4)\gamma_\mu v(p_1)] \right) \left([\bar{u}(p_3)\gamma^\mu u(p_2)][\bar{v}(p_4)\gamma_\mu v(p_1)] \right)^\dagger \\ &= \frac{e^4}{q^4} \left([\bar{u}(p_3)\gamma^\mu u(p_2)][\bar{v}(p_4)\gamma_\mu v(p_1)] \right) \left([\bar{v}(p_1)\gamma_\nu v(p_4)][u(p_2)\gamma^\nu \bar{u}(p_3)] \right) \\ &= \frac{e^4}{q^4} [\bar{u}(p_3)\gamma^\mu u(p_2)][u(p_2)\gamma^\nu \bar{u}(p_3)][\bar{v}(p_4)\gamma_\mu v(p_1)][\bar{v}(p_1)\gamma_\nu v(p_4)] \\ &= \frac{e^4}{q^4} [\bar{u}_a(p_3)\gamma^\mu_{ab} u_b(p_2) u_c(p_2)\gamma^\nu_{cd} \bar{u}_d(p_3)][\bar{v}^i(p_4)\gamma^{ij}_\mu v^j(p_1)\bar{v}^k(p_1)\gamma^{kl}_\nu v^l(p_4)] \end{aligned}$$

At this point, one can notice that for the fermions the trace can be written as:

$$u(p)_i \bar{u}_j(p) = (\not{p} + m) \tag{3.1.4}$$

and correspondingly for the antifermions:

$$v(p)_i\bar{v}_j(p) = (\not{p} - m) \tag{3.1.5}$$

Using those two relations we continue the calculation:

$$\begin{aligned}
\sum_{s,s'}|\mathcal{M}_t|^2 &= \frac{e^4}{t^2}[\bar{u}_a(p_3)\gamma^\mu_{ab}(\not{p}_2+m)\gamma^\nu_{cd}\bar{u}_d(p_3)][\bar{v}^i(p_4)\gamma^{ij}_\mu(\not{p}_1-m)\gamma^{kl}_\nu v^l(p_4)] \\
&= \frac{e^4}{t^2}tr[(\not{p}_3+m)\gamma^\mu(\not{p}_2+m)\gamma^\nu)]tr[(\not{p}_4-m)\gamma_\mu(\not{p}_1-m)\gamma_\nu] \\
&= \frac{e^4}{t^2}\left(tr[\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu]+m^2tr[\gamma^\mu\gamma^\nu)]\right)\left(tr[\not{p}_4\gamma_\mu\not{p}_1\gamma_\nu]-m^2tr[\gamma_\mu\gamma_\nu)]\right) \\
&= \frac{e^4}{t^2}\left(p_{3\rho}p_{2\sigma}tr[\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu]+4m^2g^{\mu\nu}\right)\left(p_{4\rho}p_{1\sigma}tr[\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_\nu]-4m^2g_{\mu\nu}\right) \\
&= \frac{e^4}{t^2}\left(p_{3\rho}p_{2\sigma}(g^{\rho\mu}g^{\sigma\nu}-g^{\rho\sigma}g^{\mu\nu}+g^{\rho\nu}g^{\mu\sigma})+4m^2g^{\mu\nu}\right)\left(p_{4\rho}p_{1\sigma}(g_{\rho\mu}g_{\sigma\nu}-g_{\rho\sigma}g_{\mu\nu}+g_{\rho\nu}g_{\mu\sigma})-4m^2g_{\mu\nu}\right) \\
&= 16\frac{e^4}{t^2}(\rho^\mu_3\cdot\rho^\nu_2-g^{\mu\nu}(\rho_3\cdot\rho_2)+\rho^\nu_3\cdot\rho^\mu_2-m^2g^{\mu\nu})(\rho_{4\mu}\cdot\rho_{1\nu}-g_{\mu\nu}(\rho_4\cdot\rho_1)+\rho_{4\nu}\cdot\rho_{1\mu}-m^2g_{\mu\nu}) \\
&= 16\frac{e^4}{t^2}(\rho^\mu_3\cdot\rho^\nu_2\cdot\rho_{4\mu}\cdot\rho_{1\nu}-g_{\mu\nu}\rho^\mu_3\cdot\rho^\nu_2(\rho_4\cdot\rho_1)+\rho^\mu_3\cdot\rho^\nu_2\cdot\rho_{4\nu}\cdot\rho_{1\mu}-\rho^\mu_3\cdot\rho^\nu_2m^2g_{\mu\nu}+ \\
&-g^{\mu\nu}(\rho_3\cdot\rho_2)\rho_{4\mu}\cdot\rho_{1\nu}+g^{\mu\nu}(\rho_3\cdot\rho_2)g_{\mu\nu}(\rho_4\cdot\rho_1)-g^{\mu\nu}(\rho_3\cdot\rho_2)\rho_{4\nu}\cdot\rho_{1\mu}+g^{\mu\nu}(\rho_3\cdot\rho_2)m^2g_{\mu\nu}+ \\
&+\rho^\nu_3\cdot\rho^\mu_2\cdot\rho_{4\mu}\cdot\rho_{1\nu}-g_{\mu\nu}\rho^\nu_3\cdot\rho^\mu_2(\rho_4\cdot\rho_1)+\rho_{4\nu}\cdot\rho_{1\mu}\rho^\nu_3\cdot\rho^\mu_2-m^2\rho^\nu_3\cdot\rho^\mu_2g_{\mu\nu}- \\
&-m^2g^{\mu\nu}\cdot\rho_{4\mu}\cdot\rho_{1\nu}+m^2g^{\mu\nu}g_{\mu\nu}(\rho_4\cdot\rho_1)-m^2g^{\mu\nu}\rho_{4\nu}\cdot\rho_{1\mu}+m^2g^{\mu\nu}m^2g_{\mu\nu})= \\
&= 16\frac{e^4}{t^2}((\rho_3\cdot\rho_4)(\rho_2\cdot\rho_1)-(\rho_3\cdot\rho_2)(\rho_4\cdot\rho_1)+(\rho_3\cdot\rho_1)(\rho_2\cdot\rho_4)-m^2(\rho_3\cdot\rho_2)- \\
&-(\rho_3\cdot\rho_2)(\rho_4\cdot\rho_1)+4(\rho_3\cdot\rho_2)(\rho_4\cdot\rho_1)-(\rho_3\cdot\rho_2)(\rho_4\cdot\rho_1)+4m^2(\rho_3\cdot\rho_2)+ \\
&+(\rho_3\cdot\rho_1)(\rho_4\cdot\rho_2)-(\rho_3\cdot\rho_2)(\rho_4\cdot\rho_1)+(\rho_3\cdot\rho_4)(\rho_1\cdot\rho_2)-m^2(\rho_3\cdot\rho_2)- \\
&-m^2(\rho_4\cdot\rho_1)+4m^2(\rho_4\cdot\rho_1)-m^2(\rho_4\cdot\rho_1)-4m^4) \\
&= 16\frac{e^4}{t^2}(2(\rho_3\cdot\rho_4)(\rho_1\cdot\rho_2)+(\rho_3\cdot\rho_1)(\rho_4\cdot\rho_2)-2m^2(\rho_3\cdot\rho_2)+2m^2(\rho_1\cdot\rho_4)-4m^4)\xrightarrow{m=0} \\
&= 16\frac{e^4}{t^2}(2(\rho_3\cdot\rho_4)(\rho_1\cdot\rho_2)+2(\rho_3\cdot\rho_1)(\rho_4\cdot\rho_2))
\end{aligned}$$

Having considered p_1, p_2 as the incoming momenta and p_3, p_4, as the outcoming momenta, the mandlestam constants if we consider $m = 0$ are:

$$s = (p_1+p_2)^2 = (p_3+p_4)^2 = 2p_1\cdot p_2 = 2p_3\cdot p_4 \tag{3.1.6}$$

$$t = (p_1-p_4)^2 = (p_3-p_2)^2 = 2p_1\cdot p_4 = 2p_2\cdot p_3 \tag{3.1.7}$$

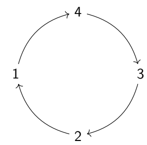
$$u = (p_1-p_3)^2 = (p_4-p_2)^2 = 2p_2\cdot p_4 = 2p_1\cdot p_3 \tag{3.1.8}$$

So, the matrix element is calculated:

$$\frac{1}{4}\sum_{s,s'}|\mathcal{M}_t|^2 = 8\frac{e^4}{t^2}(\frac{s^2+u^2}{4}) = 2e^4\left(\left(\frac{s}{t}\right)^2+\left(\frac{u}{t}\right)^2\right) \tag{3.1.9}$$

To find the s-channel matrix element, one can follow a detailed step-by-step procedure similar to that used for the t-channel matrix element. Alternatively, by comparing the momentum indices of the t-channel matrix element to those of the s-channel matrix element, one can identify the necessary adjustments to transform the t-channel expression into the s-channel expression, and eventually simplify the calculation.

$$\sum_{s,s'} |\mathcal{M}_s|^2 = \frac{e^4}{q^4} \left([\bar{u}(p_2)\gamma^\mu v(p_1)][\bar{v}(p_4)\gamma_\mu u(p_3)]\right) \left([\bar{u}(p_2)\gamma^\mu v(p_1)][\bar{v}(p_4)\gamma_\mu u(p_3)]\right)^\dagger$$



We observe that this is a cyclic permutation: $4 \to 3 \to 2 \to 1 \to 4$:

So, the result is going to be:

$$\sum_{s,s'} |\mathcal{M}_s|^2 = 16\frac{e^4}{s^2}(2(\rho_2 \cdot \rho_3)(\rho_4 \cdot \rho_1) + 2(\rho_2 \cdot \rho_4)(\rho_3 \cdot \rho_1))$$

And by using the same Mandlestam constants s, t and u, as before, we finally get:

$$\frac{1}{4}\sum_{s,s'} |\mathcal{M}_s|^2 = 8\frac{e^4}{s^2}(\frac{t^2+u^2}{4}) = 2e^4 \left(\left(\frac{t}{s}\right)^2 + \left(\frac{u}{s}\right)^2 \right) \tag{3.1.10}$$

$$\begin{aligned}
\sum_{s,s'} \bar{\mathcal{M}}_t \mathcal{M}_s &= -\frac{e^4}{ts}\sum_{s,s'} \left([\bar{u}(p_3)\gamma^\mu u(p_2)][\bar{v}(p_4)\gamma_\mu v(p_1)]\right)^\dagger \left([\bar{u}(p_2)\gamma^\nu v(p_1)][\bar{v}(p_4)\gamma_\mu u(p_3)]\right) \\
&= -\frac{e^4}{ts}\sum_{s,s'} \left([\bar{u}(p_2)\gamma^\mu u(p_3)][\bar{v}(p_1)\gamma_\mu v(p_4)]\right) \left([\bar{u}(p_2)\gamma^\nu v(p_1)][\bar{v}(p_4)\gamma_\mu u(p_3)]\right) \\
&= -\frac{e^4}{ts}\sum_{s,s'} \left([\bar{u}(p_2)\gamma^\mu u(p_3)][\bar{u}(p_2)\gamma^\nu v(p_1)][\bar{v}(p_1)\gamma_\mu v(p_4)][\bar{v}(p_4)\gamma_\mu u(p_3)]\right) \xrightarrow{m=0} \\
&= -\frac{e^4}{ts} tr[\not{p}_2\gamma^\mu \not{p}_3 \gamma^\nu \not{p}_1 \gamma_\mu \not{p}_4 \gamma_\nu]
\end{aligned}$$

Because in helicity formalism $\not{p}$ and γ are 4x4 matrices we can use the trace identities to bring them in a form more convenient:

$$\sum_{s,s'} \bar{\mathcal{M}}_t \mathcal{M}_s = -\frac{e^4}{ts} tr[\not{p}_1\gamma^\nu \not{p}_2 \gamma^\mu \not{p}_3 \gamma_\nu \not{p}_4 \gamma_\mu]$$

We will now use the identities:

- $\gamma^\nu \not{p} = p_\beta \gamma^\alpha \gamma^\beta (2p^\alpha - \not{p}\gamma^\alpha) = (2p^\alpha - \not{p}\gamma^\alpha)$
- $\not{p}\not{p} = p_\alpha p_\beta \gamma^\alpha \gamma^\beta = p^2 = 0$

By replacing: $p_4 = p_1 + p_2 - p_3$, we need to break the trace in 3 parts that we need to calculate separately:

$$\begin{aligned}
\sum_{s,s'} \bar{\mathcal{M}}_t \mathcal{M}_s &= -\frac{e^4}{ts} tr[\not{p}_1\gamma^\nu \not{p}_2 \gamma^\mu \not{p}_3 \gamma_\nu (\not{p}_1 + \not{p}_2 - \not{p}_3) \gamma_\mu] \\
&= -\frac{e^4}{ts}\left(tr[\not{p}_1\gamma^\nu \not{p}_2 \gamma^\mu \not{p}_3 \gamma_\nu \not{p}_1 \gamma_\mu] + tr[\not{p}_1\gamma^\nu \not{p}_2 \gamma^\mu \not{p}_3 \gamma_\nu \not{p}_2 \gamma_\mu] - tr[\not{p}_1\gamma^\nu \not{p}_2 \gamma^\mu \not{p}_3 \gamma_\nu \not{p}_3 \gamma_\mu]\right) \\
&= -\frac{e^4}{ts}\left(T_{p_1} + T_{p_2} - T_{p_3}\right)
\end{aligned}$$

After calculations (the actual calculations are done by hand in the hand-written assignment) we have:

$$T_{p_2} = 0$$

$$T_{p_1} = -32(p_3 \cdot p_1)(p_2 \cdot p_1)$$

$$T_{p_3} = 32(p_2 \cdot p_3)(p_1 \cdot p_3)$$

We used permutation techniques to calculate easily the element T_{p_3} The final result for this matrix element, using the same Mandlestam constant s, t, u as before, is:

$$\begin{aligned}\frac{1}{4}\sum_{s,s'}\bar{\mathcal{M}}_t\mathcal{M}_s &= -\frac{e^4}{4ts}\left(32(p_3 \cdot p_1)(p_2 \cdot p_1) - 32(p_2 \cdot p_3)(p_1 \cdot p_3)\right) \\ &= -\frac{e^4}{4ts}\left(32\frac{-us}{4} - 32\frac{ut}{4}\right) = 2e^4\left(\frac{u}{t} + \frac{u}{s}\right)\end{aligned}$$

Similarly, we calculate the final element:

$$\begin{aligned}\frac{1}{4}\sum_{s,s'}\bar{\mathcal{M}}_s\mathcal{M}_t &= -\frac{e^4}{4ts}\left(32(p_4 \cdot p_2)(p_2 \cdot p_1) - 32(p_1 \cdot p_4)(p_2 \cdot p_4)\right) \\ &= -\frac{e^4}{4ts}\left(32\frac{-us}{4} - 32\frac{ut}{4}\right) = 2e^4\left(\frac{u}{t} + \frac{u}{s}\right)\end{aligned}$$

Eventually, the spin-averaged matrix element is:

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= 2e^4\left(\left(\frac{u}{t}+\frac{u}{s}\right)+\left(\frac{u}{t}+\frac{u}{s}\right)+\left(\frac{t}{s}\right)^2+\left(\frac{u}{s}\right)^2+\left(\frac{s}{t}\right)^2+\left(\frac{u}{t}\right)^2\right) \\ &= 2e^4\left(2\frac{u(s+t)}{ts}+\left(\frac{t}{s}\right)^2+\left(\frac{u}{s}\right)^2+\left(\frac{s}{t}\right)^2+\left(\frac{u}{t}\right)^2\right) \\ &= 2e^4\left(2\frac{u^2}{ts}+\left(\frac{t}{s}\right)^2+\left(\frac{u}{s}\right)^2+\left(\frac{s}{t}\right)^2+\left(\frac{u}{t}\right)^2\right) \\ &= 2e^4\left(u^2\left(\frac{1}{t^2}+2\frac{1}{ts}+\frac{1}{s^2}\right)+\left(\frac{t}{s}\right)^2+\left(\frac{s}{t}\right)^2\right)\end{aligned}$$

$$\langle|\mathcal{M}|^2\rangle = 2e^4\left(u^2\left(\frac{1}{t}+\frac{1}{s}\right)^2+\left(\frac{t}{s}\right)^2+\left(\frac{s}{t}\right)^2\right) \tag{3.1.11}$$

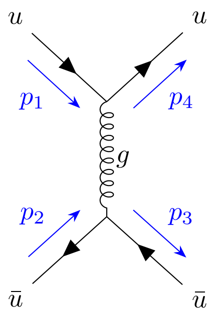
3.2 QCD: elementary processes

3.2.1 Study the elementary process $u\bar{u} \to u\bar{u}$ analogous to Bhabha scattering in QED

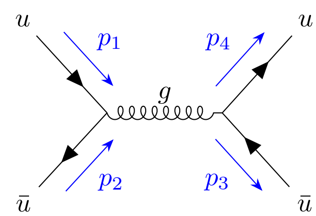
The process of calculating scattering amplitudes and cross-sections has been analyzed in the previous subsection. The steps to calculate cross sections for jet production in hadronic collisions are exactly the same for QCD as it is in QED. However, in QCD it is essential not only to sum over all possible reactions of quarks, antiquarks and gluons, but also to sum over all final and initial colors, since we are in $SU(3)$ group.
Let us remind the cross section for the Bhabha scattering process 3.1.11:

$$\langle |\mathcal{M}|^2 \rangle = 2e^4 \left(u^2 \left(\frac{1}{t} + \frac{1}{s} \right)^2 + \left(\frac{t}{s} \right)^2 + \left(\frac{s}{t} \right)^2 \right)$$

In order to calculate the $u\bar{u} \to u\bar{u}$ cross section, we simply replace the e^2 QED coupling by g^2 times an $SU(3)$ factor for the color summation. This specific process is more easily analyzed using helicity formalism, than using the spin summation technique. The second one is rather difficult as each term receives different color factors so the calculations will be exhausting.



(a) scattering: t-channel



(b) annihilation: s-channel

In order to calculate the total scattering amplitude, we have to distinguish the 3 cases and sum over colors for each case differently.

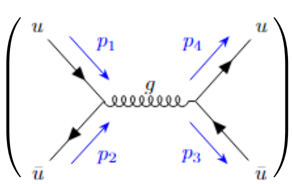
s - channel

We first express the scattering amplitude of the s-channel diagram.

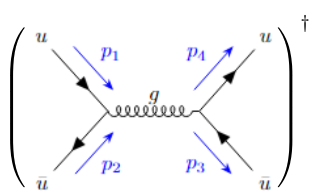
$$\begin{aligned} \mathcal{M}_s &= \bar{v}(p_2)(-ig_s\gamma^\mu T^a_{i_1,i_2})u(p_1)\left(-\delta_{ab}\frac{g_{\mu\nu}}{q^2}\right)u(p_4)(-ig_s\gamma^\nu T^b_{i_4,i_3})\bar{v}(p_3) \\ &= (-ig_s)^2\bar{v}(p_2)\gamma^\mu T^a_{i_1,i_2}u(p_1)\left(-\frac{1}{s^2}\right)u(p_4)\gamma_\mu T^a_{i_4,i_3}\bar{v}(p_3) \\ &= \frac{g_s^2}{s^2}T^a_{i_1,i_2}T^a_{i_4,i_3}\bar{v}(p_2)\gamma^\mu u(p_1)u(p_4)\gamma_\mu\bar{v}(p_3) \end{aligned} \tag{3.2.1}$$

The operation is similar to Bhabha scattering, this means that the scattering amplitudes in the end will be exactly the same, plus an extra term that stems from the color summation which is an independent operation.

$|\mathcal{M}_s|^2 =$



$\times$



We sum over initial and final colors and spins:

$$
\begin{aligned}
\frac{1}{4}\sum_{s,s'}|\mathcal{M}_s|^2 &= \frac{g_s^2}{4s^2}\sum_{s,s'}\sum_{\text{colors}} T^a_{i_1,i_2}T^a_{i_4,i_3}\bar{v}(p_2)\gamma^\mu u(p_1)u(p_4)\gamma_\mu\bar{v}(p_3)\times\left(T^b_{i_1,i_2}T^b_{i_4,i_3}\bar{v}(p_2)\gamma^\mu u(p_1)u(p_4)\gamma_\mu\bar{v}(p_3)\right)^\dagger \\
&= \frac{g_s^2}{4s^2}\sum_{s,s'}\sum_{\text{colors}} T^a_{i_1,i_2}T^a_{i_4,i_3}\bar{v}(p_2)\gamma^\mu u(p_1)u(p_4)\gamma_\mu\bar{v}(p_3)\times T^b_{i_2,i_1}T^b_{i_3,i_4}\bar{u}(p_1)\gamma^\nu v(p_2)v(p_3)\gamma_\nu\bar{u}(p_4) \\
&= \frac{g_s^2}{4s^2}\sum_{s,s'}[\bar{v}(p_2)\gamma^\mu u(p_1)\bar{u}(p_1)\gamma^\nu v(p_2)][u(p_4)\gamma_\mu\bar{v}(p_3)v(p_3)\gamma_\nu\bar{u}(p_4)]\sum_{\text{colors}}[T^a_{i_1,i_2}T^b_{i_2,i_1}][T^a_{i_4,i_3}T^b_{i_3,i_4}] \\
&\overset{m=0}{=} \frac{g_s^2}{4s^2}\left(tr[\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu]\times tr[\not{p}_4\gamma_\mu\not{p}_3\gamma_\nu]\right)\frac{1}{3}\frac{1}{3}\left(tr[T^aT^b]\right)^2
\end{aligned}
$$

We have already calculated for Bhabha scattering:

$$
tr[\not{p}_3\gamma^\mu\not{p}_2\gamma^\nu]tr[\not{p}_4\gamma_\mu\not{p}_1\gamma_\nu] = 32\left((p_3\cdot p_4)(p_2\cdot p_1)+(p_3\cdot p_1)(p_2\cdot p_4)\right) \tag{3.2.2}
$$

By a permutation $1\to 3\to 2\to 1$, we find:

$$
\begin{aligned}
\frac{1}{4}\sum_{s,s'}|\mathcal{M}_s|^2 &= 32\frac{g_s^2}{4s^2}\left((p_2\cdot p_4)(p_1\cdot p_3)+(p_2\cdot p_3)(p_1\cdot p_4)\right)\frac{1}{9}\left(tr[T^aT^b]\right)^2 \\
&\overset{\text{replace } s,t,u}{=} 8\frac{g_s^2}{4s^2}\left(u^2+t^2\right)\frac{1}{9}\left(tr[T^aT^b]\right)^2
\end{aligned}
$$

For the color decomposition we can use the identity:

$$
\frac{1}{9}(tr[T^aT^b])^2 = \frac{1}{9}T_R^2\delta^{ab}\delta^{ab} = \frac{1}{4\cdot 9}8 = \frac{2}{9} \tag{3.2.3}
$$

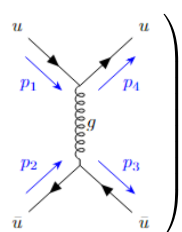
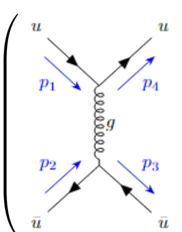
Eventually, the s-channel term is:

$$
\frac{1}{4}\sum_{s,s'}|\mathcal{M}_s|^2 = \frac{4}{9}\frac{g_s^2}{s^2}\left(u^2+t^2\right) \tag{3.2.4}
$$

<u>t - channel</u>

Using the same logic we calculate the t-channel matrix element. The expression for the scattering amplitude of the t-channel diagram is:

$$
\begin{aligned}
\mathcal{M}_t &= \bar{v}(p_3)(-ig_s\gamma^\nu T^b_{i_1,i_3})\bar{v}(p_2)\left(-\delta_{ab}\frac{g_{\mu\nu}}{q^2}\right)u(p_4)(-ig_s\gamma^\mu T^a_{i_4,i_2})u(p_1) \\
&= \frac{g_s^2}{t^2}T^a_{i_1,i_3}T^a_{i_4,i_2}\bar{v}(p_3)\gamma_\mu\bar{v}(p_2)u(p_4)\gamma^\mu u(p_1)
\end{aligned} \tag{3.2.5}
$$

$|\mathcal{M}_t|^2 =$ () $\times$ () †

$$\frac{1}{4}\sum_{s,s'}|\mathcal{M}_t|^2 = \frac{g_s^2}{4t^2}\sum_{s,s'}\sum_{\text{colors}} T^a_{i_1,i_3}T^a_{i_4,i_2}\bar{v}(p_3)\gamma_\mu\bar{v}(p_2)u(p_4)\gamma^\mu u(p_1)) \times \left(T^b_{i_1,i_3}T^b_{i_4,i_2}\bar{v}(p_2)\gamma_\nu\bar{v}(p_3)u(p_4)\gamma^\nu u(p_1)\right)^\dagger$$
$$= \frac{g_s^2}{4t^2}\sum_{s,s'}[u(p_4)\gamma^\mu u(p_1)\bar{u}(p_1)\gamma^\nu\bar{u}(p_4)][\bar{v}(p_3)\gamma_\mu\bar{v}(p_2)v(p_2)\gamma_\nu v(p_3)]\sum_{\text{colors}} T^a_{i_1,i_3}T^b_{i_3,i_1}T^a_{i_4,i_2}T^b_{i_2,i_4}$$
$$= \frac{g_s^2}{4t^2}tr[\not{p}_4\gamma^\mu\not{p}_1\gamma^\nu]tr[\not{p}_3\gamma_\mu\not{p}_2\gamma_\nu]\frac{1}{9}\left(tr[T^aT^b]\right)^2$$

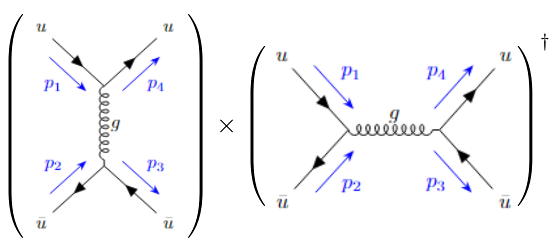
Again, using the relation 3.2.2 for the trace calculation, and 3.2.3 for the color decomposition, the equation becomes:

$$\frac{1}{4}\sum_{s,s'}|\mathcal{M}_t|^2 = 32\frac{g_s^2}{4t^2}\left((p_3\cdot p_4)(p_2\cdot p_1)+(p_3\cdot p_1)(p_2\cdot p_4)\right)\frac{1}{9}\left(tr[T^aT^b]\right)^2$$
$$\overset{\text{replace } s,t,u}{=} 32\frac{g_s^2}{4t^2}\left(\frac{s^2+u^2}{4}\right)\frac{2}{9}$$
$$\frac{1}{4}\sum_{s,s'}|\mathcal{M}_t|^2 = \frac{4}{9}\frac{g_s^2}{t^2}\left(s^2+u^2\right) \tag{3.2.6}$$

Combined terms $\mathcal{M}_t\bar{\mathcal{M}}_s$, $\bar{\mathcal{M}}_t\mathcal{M}_s$

We only need to calculate the first term $\mathcal{M}_t\bar{\mathcal{M}}_s$, since the second term is identical.

$$\mathcal{M}_t\bar{\mathcal{M}}_s =$$



$$\frac{1}{4}\sum_{s,s'}\mathcal{M}_t\bar{\mathcal{M}}_s = \frac{g_s^2}{4ts}\sum_{s,s'}\sum_{colors} T^a_{i_1,i_3}T^a_{i_4,i_2}T^b_{i_2,i_1}T^b_{i_3,i_4}\left[\bar{v}(p_2)\gamma_\mu\bar{v}(p_3)u(p_4)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_1)\gamma^\nu v(p_2)v(p_3)\gamma_\nu\bar{u}(p_4)\right]$$
$$= \frac{g_s^2}{4ts}\sum_{colors}\left(T^a_{i_1,i_3}T^b_{i_3,i_4}\right)\left(T^a_{i_4,i_2}T^b_{i_2,i_1}\right)\sum_{s,s'}\left[\bar{v}(p_2)\gamma_\mu\bar{v}(p_3)v(p_3)\gamma_\nu\bar{u}(p_4)u(p_4)\gamma^\mu u(p_1)\bar{u}(p_1)\gamma^\nu v(p_2)\right]$$
$$\overset{\text{m=0}}{=} \frac{g_s^2}{4ts}\frac{1}{9}tr[T^aT^bT^aT^b]tr\left[\not{p}_2\gamma_\mu\not{p}_3\gamma_\nu\not{p}_4\gamma^\mu\not{p}_1\gamma^\nu\right]$$

Each trace will be calculated separately using previous results or equations from theory:

- Spin summation using the calculations from Bhabha scattering:

$$\begin{aligned} &tr\left[\not{p}_2\gamma_\mu\not{p}_3\gamma_\nu\not{p}_4\gamma^\mu\not{p}_1\gamma^\nu\right] = tr\left[\not{p}_4\gamma^\mu\not{p}_1\gamma^\nu\not{p}_2\gamma_\mu\not{p}_3\gamma_\nu\right] = tr\left[\not{p}_1\gamma^\nu\not{p}_4\gamma^\mu\not{p}_3\gamma_\nu\not{p}_2\gamma_\mu\right] \\ &= 32\left((p_4\cdot p_3)(p_1\cdot p_3)-(p_1\cdot p_3)(p_4\cdot p_1)\right) = 32\left(\frac{su}{4}+\frac{ut}{4}\right) = 8\left(su+ut\right) \end{aligned} \tag{3.2.7}$$

- Color summation, using equations from Peskin:

$$\frac{1}{9}tr[T^aT^bT^aT^b] = \frac{1}{9}\left[\underbrace{C_2(r)}_{\frac{4}{3}} - \frac{1}{2}\underbrace{N_c}_{3}\right]\underbrace{tr[T^a]}_{4} = -\frac{2}{27} \tag{3.2.8}$$

So eventually, the term is found:

$$\frac{1}{4}\sum_{s,s'}\mathcal{M}_t\bar{\mathcal{M}}_s = \frac{g_s^2}{4ts}\left(-\frac{2}{27}\right)8\,(su+ut) = -\frac{4}{27}\frac{g_s^2}{ts}(su+ut) = \frac{1}{4}\sum_{s,s'}\bar{\mathcal{M}}_t\mathcal{M}_s \tag{3.2.9}$$

Now, combining 3.2.4, 3.2.6 and 3.2.9 we can calculate the total scattering amplitude:

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= \frac{4}{9}\frac{g_s^2}{s^2}\left(u^2+t^2\right) + \frac{4}{9}\frac{g_s^2}{t^2}\left(u^2+s^2\right) - 2\cdot\frac{4}{27}\frac{g_s^2}{ts}(su+ut)\\ &= \frac{4g_s^2}{9}\left(\frac{u^2}{t^2}+\frac{s^2}{t^2}+\frac{u^2}{s^2}+\frac{t^2}{s^2}-\frac{2}{3}\frac{u(s+t)}{st}\right)\\ &= \frac{4g_s^2}{9}\left(\frac{u^2}{t^2}+\frac{s^2}{t^2}+\frac{u^2}{s^2}+\frac{t^2}{s^2}-\frac{2}{3}\frac{u(s+t)}{st}\right)\end{aligned}$$

$$\langle|\mathcal{M}|^2\rangle = \frac{4g_s^2}{9}\left(\frac{u^2+s^2}{t^2}+\frac{u^2+t^2}{s^2}-\frac{2}{3}\frac{u^2}{st}\right) \tag{3.2.10}$$

Comparing the result from 3.2.10 to the one in Bhabha scattering 3.1.11, we observe that there is an extra term in the parenthesis which stems from the color summation.

3.2.2 Alternative Color decomposition process - Graphical representation

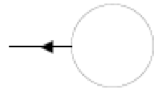
4 Applications on HELAC-PEGAS

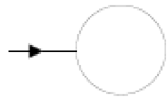
4.1 HELAC program

HELAC is a FORTRAN based package that uses the Dyson-Schwinger recursive algorithm to compute helicity amplitudes for arbitrary scattering processes. The computational cost of this algorithm, that is the steps to solve the recursive equations grows asymptotically as 3^n in contrast to Feynman diagrams approach which grows as $n!$, where n is the number of particles involved in the process. The Dyson-Schwinger equations express recursively the n-point Green functions in terms of 1-,2-,...,(n-1)-point functions.
Before presenting the recursive equations, we need to write down the appropriate definitions. Let P be the sum of all the external momenta of the particles involved in the process $P = \sum_{i\in I} p_i$ where $I \subset 1,...,n$. The sub-amplitude of a vector field is:

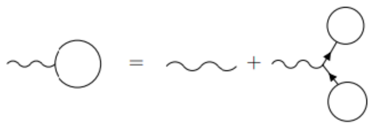
$b_\mu(P) =$ 

Accordingly, for a fermion current and an antifermion with momentum P we note:

$\psi_\mu(P) =$ 

$\bar{\psi}_\mu(P) =$ 

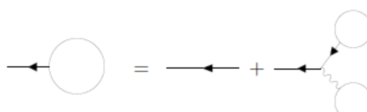
To provide an example, let us consider a QED process where we calculate the output boson in a two particle interaction:



$$b_\mu(P) = \sum_{i=1}^{n} \delta_{P=p_i} b_\mu(p_i) + \sum_{P=P_1+P_2} (ig)\ \Pi^\mu_\nu\ \bar{\psi}(P_2)\gamma^\nu\psi(P_1)\epsilon(P_1, P_2)$$

Where $\Pi^\mu_\nu = \frac{i}{P^2-m^2}\left(-g^\mu_\nu + \frac{P^\mu P_\nu}{P^2-\xi\mu^2}(1-\xi)\right)$ the propagator.

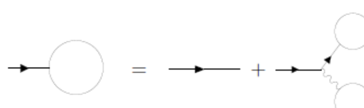
For the fermion with momentum P:



$$\psi_\mu(P) = \sum_{i=1}^{n} \delta_{P=p_i} \psi_\mu(p_i) + \sum_{P=P_1+P_2} (ig)\ P\ \not{b}(P_2)\psi(P_1)\epsilon(P_1, P_2)$$

Where $P = \frac{i}{\not{P}-m}$.

Finally, for the antifermion:



$$\psi_\mu(P) = \sum_{i=1}^{n} \delta_{P=p_i} \psi_\mu(p_i) + \sum_{P=P_1+P_2} (ig)\ \bar{\psi}(P_1)\not{b}(P_2)\ \bar{P}\ \epsilon(P_1, P_2)$$

Where $\bar{P} = \frac{i}{-\not{P}-m}$. For QCD, processes it is essential that color be taken into consideration, therefore the calculations also involve a summation over color degrees.

4.2 1^{st} Project: Symbolically calculated SMEFT operators using python

On GitHub[7], you can find all the necessary Python scripts for performing symbolic calculations with spinors and other mathematical objects such as metrics, vectors, and Lorentz and lightcone representations. Inside the SMEFT folder, one can spot the SMEFT_main.ipynb jupyter notebook file, which illustrates the results of the three SMEFT operators acting on spinors vectors or scalars depending on the situation. This file is using the standard_formulas.py, python library that includes all the standard objects to build our framework, such as the metric, the helicities or the polarizations. One level up from there, the SMEFT_library.py, includes all the necessary functions to calculate the output of the SMEFT operators acting on spinors, vectors or scalars. The calculations and results of the output currents inside the SMEFT_main.ipynb file are quite clear and useful for the next step, which is the implementation in HELAC simulation interface to generate the appropriate feynmann diagrams.

4.3 2^d Project: Implemetnation of Z' boson in HELAC-PHEGAS

In this project, a Z' boson implementation is integrated into the HELAC source code. According to the study by [8], the Z' boson is a color singlet, and its interaction with fermionic currents is described by the Lagrangian:

$$L_{int} = \bar{t}\gamma_\mu(c_L P_L + c_R P_R)tV_1^\mu = c_t\bar{t}\gamma_\mu(cos(\theta)P_L + sin(\theta)P_R)tV_1^\mu \tag{4.3.1}$$

Where $P_{L/R} = (1 \pm \gamma^5)/2$, $c_t = \sqrt{c_L^2 + c_R^2}$ and $tan(\theta) = \frac{C_R}{C_L}$ the projection operators. The decay width of V_1 to a top-antitop pair is given by:

$$\Gamma(V_1 \to t\bar{t}) = \frac{c_t^2 M_{V_1}}{8\pi}\sqrt{1 - \frac{4m_t^2}{M_{V_1}^2}} \times \left[1 - \frac{m_t^2}{M_{V_1}^2}\left(1 - 3sin(2\theta)\right)\right] \tag{4.3.2}$$

Studying the Tree-level top production with V_1-strahlung, we use the simplification $\theta = \frac{\pi}{2}$.

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