Detection of Fake News in Tree Propagation Networks

A Thesis

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Dedication

To my family.

Acknowledgements

I would like thank and express my gratitude to my supervisor, Prof. Spyridon Kontogiannis for his guidance and support throughout my research. I would also like to thank those who supported me all those years, especially my family.

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Abstract

Georgiadis Ioannis, M.Sc. in Data and Computer Systems Engineering, Department of Computer Science and Engineering, School of Engineering, University of Ioannina, Greece, August 2021.
Detection of Fake News in Tree Propagation Networks.
Advisor: Spyridon Kontogiannis, Associate Professor.

The proliferation of fake news in online social media platforms has opened up novel, multidisciplinary directions of research trying to achieve automated mechanisms for the timely identification and containment of fake news, and mitigation of its widespread impact on public opinion. While much of the earlier research was focused on identification of fake news based on its contents (e.g., writing style of the story, stance of involved reactions to it, linguistic analysis, etc.), or on the related context (e.g., exploitation of users' engagement and their reputation within the social media platform, etc.), which are mostly based on AI-enabled techniques, there has been a rising interest in the provision of proactive intervention strategies which are mostly based on the analysis of the spatio-temporal characteristics of the evolving story within the underlying propagation network infrastructure. Most of these works mainly focus on the analysis of the time-series of the reactions to the stories. Some recent works focus on the structural characteristics of the propagation network. For example, it has been experimentally observed that a typical fake-news story evolves faster, deeper and farther than a typical true-story, within the social network platform.

In this thesis we continue the line of research focusing on the structural characteristics of the underlying propagation network. Our first differentiation from the literature is that we adopt a probabilistic model for the creation of stories, in which each story is created either by an expert (and is perceived as a *true story*, or by some propagandist (and is then perceived as a fake story). Experts have a high probability of providing the correct answer to the question posed by the story, e.g., because they

are based on concrete arguments and scientific evidence. Propagandists, on the other hand, simply try to promote a particular stance (in favor of, or against the ground-truth answer) with the story, irrespective of the ground-truth. It should be noted that both an expert and a propagandist might provide either a correct or a false answer, but the expert is highly likely to be correct.

The above mentioned probabilistic model was proposed by Papanatasiou (2019), and was then studied and analyzed for a very simplified case in which the underlying propagation network is a simple directed path. In this thesis we provide a similar analysis for the case in which the underlying propagation network is a rooted directed tree. This is a much more challenging case, since the sequential nature of the users' reactions to an emergent story (and the direct consequences of their own actions to the entire story) no longer holds.

We first provide a careful analysis of the users' behavior during the evolution of the story, assuming that they behave rationally, i.e., they are expected-utility maximizers based on their own prior and posterior beliefs for the ground-truth value and for the type (true/fake) of the story. We then proceed with the involvement also of the platform, as an independent observer of the entire propagation network. Our goal is to determine an efficient mechanisms for the platform in order to decide in real-time whether and when exactly to intervene to the evolution of an emerging story, while only observing in the underlying propagation tree.

Εκτεταμενη Περιληψη

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Detection of Fake News in Tree Propagation Networks.
Επιβλέπων: Σπυρίδων Κοντογιάννης, Αναπληρωτής Καθηγητής.

Τα μέσα κοινωνικής δικτύωσης έχουν γίνει σημαντικό μέρος της καθημερινής μας αλληλεπίδρασης λόγω της εύκολης προσβασιμότητάς τους από τους χρήστες. Ως εκ τούτου, έχουν γίνει ένας μαζικός κόμβος για την ανταλλαγή πληροφοριών και πολλοί χρήστες επιλέγουν να καταναλώνουν ειδήσεις από πλατφόρμες κοινωνικής δικτύωσης όπως το Twitter, το YouTube και το Facebook. Αυτή η ευκολία πρόσβασης στις πλατφόρμες κοινωνικής δικτύωσης σε συνδυασμό με τη δυνατότητα δημοσίευσης πληροφοριών με τη μορφή ειδησεογραφικού άρθρου, η οποία δίνεται και στους απλούς χρήστες, δημιουργεί το φαινόμενο της διασποράς παραπληροφόρησης. Πρόσφατες σημαντικές περιπτώσεις ψευδών ειδήσεων, που έφεραν τα φώτα της δημοσιότητας σε αυτό το πρόβλημα, ήταν οι προεδρικές εκλογές στις ΗΠΑ το 2016, ένα από τα πιό σημαντικά γεγονότα το οποίο ανέδειξε το πρόβλημα σε όλη την έκταση του.

Οι έρευνες γύρω από τη διάδοση ψευδών ειδήσεων προσελκύουν την ακαδημαϊκή κοινότητα. Οι προσεγγίσεις γύρω από την ανίχνευση και τον περιορισμό τους ποικίλλουν, αλλά υπάρχουν δύο βασικοί στόχοι που είναι κοινοί. Πρώτον, θα πρέπει να υπάρχει ένας τρόπος να περιγραφούν και να διατυπωθούν οι ανθρώπινες αλληλεπιδράσεις και δεύτερον, πρέπει να διατυπωθεί ο τρόπος με τον οποίο μοιράζονται πληροφορίες στο οικοσύστημά, όπως μια πλατφόρμα κοινωνικής δικτύωσης. Με βάση τη διατύπωση που περιγράφει αυτές τις αλληλεπιδράσεις, επινοούμε αποτελεσματικές μεθόδους που ανιχνεύουν, μετριάζουν ή και αποτρέπουν τη διάδοση φημών και ψευδών ειδήσεων.

Μια πολύ συχνή προσέγγιση αυτού του προβλήματος είναι η προσαρμογή του σε ένα μοντέλο επιδημιολογίας. Τα μοντέλα αυτά τείνουν να περιγραφούν με φυσικό τρόπο το φαινόμενο μετάδοσης ψευδούς πληροφορίας. Ο ρυθμός με τον οποίο *μολύνει* η πληροφορία τους χρήστες του δικτύου, είναι μια ένδειξη με την οποία μπορούμε να την κατατάξουμε ως ψευδή ή αληθή. Παρόλο που η απλότητα αυτού του μοντέλου είναι και η δύναμη του, αδυνατεί να συγκλίνει νωρίς από άποψη χρόνου διότι, συνήθως απαιτείται χρόνος για να παρατηρήσουμε το πώς μεταβάλεται. Πέρα από τα μοντέλα που βασίζονται στην δομή του δικτύου, υπάρχουν και προσεγγίσεις, οι οποίες επεξεργάζονται το περιεχόμενο της είδησης. Τέτοιου είδους μοντέλα αποτελούν μεγάλο κομμάτι της αντιμετώπισης του φαινόμενου και είναι υπό λειτουργιά σε ορισμένες πλατφόρμες. Η πρόκληση σε τέτοιου είδους μοντέλα είναι τα δεδομένα εκπαίδευσης τα οποία σπανίζουν στον τομέα και μερικές φορές είναι αναξιόπιστα με αποτέλεσμα το μοντέλο να μην είναι τόσο ακριβές.

Στην παρούσα θέση αναπτύσσουμε ένα μοντέλο, το οποίο παρατηρεί την εξέλιξη της εξάπλωσης μιας ιστορίας και εκμεταλλεύεται την δομή της για να αποφανθούμε εάν πρόκειται για μια ψευδή ιστορία. Το οικοσύστημα που εξετάζουμε περιέχει τρεις βασικές οντότητες, τους χρήστες ενός κοινωνικού δικτύου, οι οποίοι μοιράζονται μεταξύ τους μια ιστορία και την πλατφόρμα κοινωνικής δικτύωσης, η οποία είναι υπεύθυνη να αποτρέπει την μετάδοση ύποπτων ιστοριών. Η τρίτη και τελευταία οντότητα είναι ένας εξωτερικός μηχανισμός από επαγγελματίες δημοσιογράφους οι όποιοι ταυτοποιούν αν η ιστορία είναι αληθής η ψευδής. Υποθέτοντας πως οι χρήστες είναι κοινωνικά υπεύθυνοι παράγοντες και ότι η πλατφόρμα επιδιώκει να αποτρέπει μετάδοση ψευδών ειδήσεων, οι οντότητες του μοντέλου μας προσπαθούν να μεγιστοποιήσουν την ωφέλεια τους με την κατάλληλη επιλογή στρατηγικής. Σε κάθε χρονική στιγμή η πλατφόρμα, βασιζόμενη στο πώς αντιδρούν οι χρήστες, επιλεγεί ανάμεσα στο να αφήσει την διαδικασία να συνεχιστεί ή να την διακόψει και να προβεί σε δημοσιογραφικό έλεγχο.

Το μοντέλο που εισάγουμε σε αυτή την εργασία βασίζεται σε ένα προϋπάρχον μοντέλο το οποίο ακολουθεί την παραπάνω λογική σε γραφήματα τα οποία έχουν την μορφή μονοπατιού. Εν τούτοις, επεκτείνουμε το μοντέλο αυτό και εισάγουμε τις κατάλληλες υποθέσεις εργασίας, οι οποίες γενικεύουν το μοντέλο, με σκοπό να λειτουργεί υπό πραγματικές συνθήκες. Η μετάβαση από γραφήματα μονοπάτια σε δενδρικά δίκτυα διάδοσης φέρνει κάποιες προκλήσεις τις οποίες αντιμετωπίζουμε σε αυτή την διατριβή, με την εισαγωγή και απόδειξη κατάλληλων υποθέσεων εργασίας. Τέλος προσφέρουμε μια προσεγγιστική λύση η οποία βασίζεται και αυτή στην μεγιστοποίηση ωφελείας με κατάλληλες οριακές τιμές.

Chapter 1

Introduction

1.1 Motivation

Social media has become an important part of our daily interactions due to its easy accessibility for users. According to [1] the user base of social media such as Facebook, YouTube, Twitter and Reddit is doubled since 2015. Hence, social media has become a massive hub for information sharing and many users choose to consume news from social media platform such the above mentioned. This ease of access to social media platforms accompanied with the ability to publish information in form of news article, which is given to regular users as well, creates the phenomenon of misinformation spreading. Most recent important cases of fake news, that brought spotlight to this problem, are the U.S. presidential elections of 2016 [2] and similar incidents in Germany's election of 2017[3].

Researches on fake news and rumor propagation attract the academic community. There is a collection of surveys that provide an overview of the problem, techniques and challenges, such as [2, 3, 4]. Approaches on fake news detection and mitigation

[1] https://datareportal.com/reports/digital-2021-april-global-statshot
[2] https://news.stanford.edu/2017/01/18/stanford-study-examines-fake-news-2016-presidential-election/
[3] https://www.theguardian.com/world/2017/jan/09/

might vary, but there are two core concepts that are common. First, there should be a way to describe and formulate human interactions and how they share information in their ecosystem, such as a social media platform. The second core concept is based on the above formulation, that describes those interactions. Based on these formulas, an approach should devise efficient methods that detect, mitigate or even prevent the spread of rumors and fake news.

There are many interesting challenges related to the topic of fake news and rumor propagation. First and foremost, the human nature that is hard to describe or formulate for a system to process it. Understanding human behavior on the topic of fake news is important in order to improve algorithms or other ways of automation in order to prevent this phenomenon. There is a plethora of researches, similar to [5, 4, 6] that provide us with hints and information in order to understand human behavior on fake news detection. A second challenge is the motive behind the spread of misinformation. The reasons might be financial, political or even based on satire. This challenge is similar to that which concerns human behavior but we specify this explicitly because the development of such model rely heavily on those motives. Such an example is YouTube where users acquire advertisement revenue based on number of views. An attractive video that contains rumors, or misinformation in general, increases the income that it generates. The fact that more users have access to social media platforms creates another issue, that is the classification of rumors. The amount of posts shared within those networks is hard to monitor and classify in order to be used for training in machine learning models. Many platforms rely on fact checking from professional journalist, that specialize on this domain.

On the topic of fake news detection, there are several techniques from different perspectives that deal with the phenomenon of fake news detection and mitigation. One of the most common models used to deal with fake news is the epidemic model. Epidemics tend to describe precisely the propagation of fake news inside social networks because of the similarities they have concerning structure of such networks as well the propagation dynamics. Some notable researches that refine and adjust the basic epidemic models presented in [1], are [7, 8, 9, 10, 11, 12]. The drawback with epidemics is that they are time inefficient since they depend on observing the rates at which the population transition occurs between different states. Aside from propagation analysis, there are linguistics-based techniques that use the content of the information in order to detect fake news. Those approaches can be effective in some cases but they suffer from the fact that most of the time we do not have the exact values of ground truth in order to train those models [4].

1.2 Objectives

Propagation analysis seems prominent approach in order to solve the problem of detecting fake news in online social media platforms. An interesting model is provided in [13] which is a sequential model that consists of a network of agents and a platform that monitors behaviors in that network [5]. Although social networks in real applications are by far more complex, the philosophy of this sequential model can be extended to more complex case studies. Our main objective in this thesis is to improve and adjust this model in order to work for tree propagation networks which is more representative version of a real world scenario and mitigate the problem faster than the approaches in related literature.

This modification comes with challenges concerning complexity of calculations that arise from the fact that social networks are complex structures. For this challenge, we assume in this thesis that the network we are working on is an m-ary tree, which is a more realistic representation of a social structure than the sequential model based on paths provide. This transition form a simple path to an m-ary tree comes with challenges such the formulation of propagation dynamics and the complexity of calculations from platform's side. We deal with this issue by providing the appropriate assumptions. First and foremost, we assume that platform, which can be seen as an *super* agent, possess some distributional information about the other agents' prior beliefs of the story's type (true or fake) but they otherwise have access only to the events revealed to them by the structure of the propagation network. For example, neither an agent nor the platform may actually know what another agent truly believes about the evolving story. They can only observe that this agent, transmitted the story to its followers, but not the reasoning of an action (e.g., she could blindly transmit the story, or she might have conducted a private fact-check and then realized that the story is true). To simplify the analyses in our model, we also assume

[4] Models that do linguistic analysis are leveraging machine learning models.

[5] We provide every detail that we use from [13] but we strongly suggest the reader to study for deeper insight.

that this is a given Gaussian distribution. Another assumption that helps us deal with complexity, is the knowledge that each entity posses throughout the process. Those two assumptions make a natural transition from a sequential model that works for paths, to a more general model that represents tree propagation networks.

Another objective that we have in this thesis is that our model assumed to work under uncertainty. As we already mentioned in the previous paragraph, we make assumptions for the knowledge that agents and platform possesses. This is very important for two reasons. First, it makes our model more general. Reducing the amount of knowledge each entity posses makes the model more general and can work in many scenarios. The second reason is that it respects privacy of personal information and the opinions of agents fall under that category. There are many regulations such as the European Union general data protection regulation that protect personal information and many social media platforms are taking precautions in order to adjust to those regulations. By limiting the amount of knowledge to a platform, we can have such models that can be used in real life scenarios.

1.3 Structure

This thesis consists of five chapters and it is structured as follows. In chapter 2 we provide background for two basic topics, branching processes on trees and Bellman equations for dynamic decision problem, that will be mentioned and used extensively in the analysis of our news-propagation model. Chapter 3 provides the details of the proposed news-propagation model followed by an analysis of agent's dynamics of the news sharing process with the appropriate propositions and lemmas. In chapter 4 we formulate the platform's dynamics and we describe our solution for the optimal inspection time. Finally, in chapter put conclusion chapter we have a discussion on how to generalize the model in more realistic structures followed by our concluding remarks.

Chapter 2

Preliminaries

In this chapter we provide to the reader an introduction to the basic concepts of the theory we are using in order to develop our news-propagation model. Most of the topics in this chapter are provided more analytically in [1, 14, 15, 16] but we include the necessary background to make this thesis complete and provide the reader with a basic knowledge of the tools we used. We once again suggest the reader to further study the chapters from the above bibliography for more details.

2.1 Branching Processes

Epidemics is the most common structure based mitigation technique that is widely used in order to combat fake news propagation. Those models not only describe spread of viruses, but we can use them to formulate computer malware in networks and also information propagation such as the virality of social media posts. In this thesis, our building block is a refined version of branching process. Branching process is a simplistic version of epidemic and it works as follows:

- **First wave** A person that is carrying a disease enters a population with n

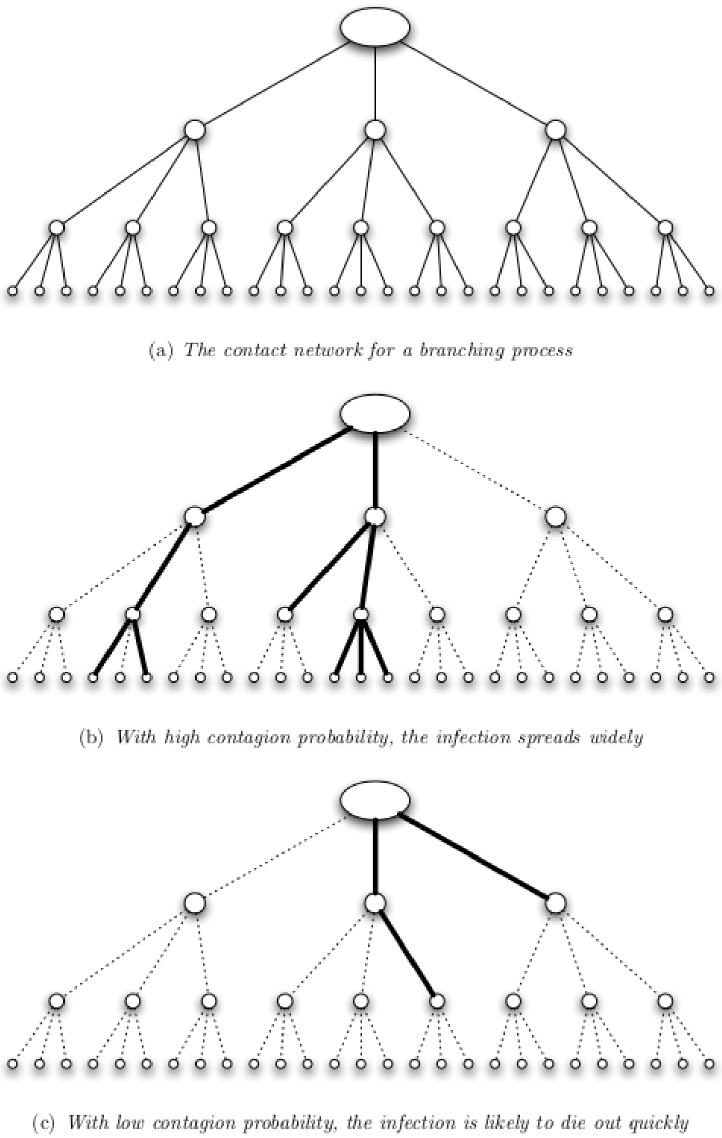


Figure 2.1: Branching process example with $k = 3$. Reference in [1].

individuals. With probability p he transmits the disease to k independently, i.e he meets three people and he infects only the first one.

- **Subsequent waves** Now, each infected person transmits the disease to their contacts, so the amount of susceptible people we have in the second wave is k^2 and in the n-th wave it is k^n by induction.

The above rules define a simple epidemic model where the probability of infections represents the rate and k is the an average amount of a person's contacts. The question on those models is if the disease will survive (i.e., turn into a pandemic) or eventually stop spreading and die. The basic reproductive number determines whether a virus will continue spreading or if it fails. We have the next proposition from [1]:

Proposition 2.1. *Let $R_0 = pk$ be the basic reproductive number where k is the average people an individuals meets and p is the probability that the virus spreads. If $R_0 \geq 1$, then with probability greater than zero the virus persists. If the basic reproductive number is less than 1 then with probability 1 the virus with stop spreading after some waves.*

The proof is provided more analytically in the related reference and it is based on geometric sequences, which will concern the more complex branching process later on the main body of this thesis. The reproductive number in our study can be translated as a prediction where we can tell if the process will trigger a cascade, given the contagion probability and the average people that a person *meets*.

In our thesis we use a more complex version of that model. First of all, we do not have a fixed probability for infection. Every time nodes are added in the propagation tree, this probability is affected. Another modification we make on that model is the time that the process takes place. Instead of waves, we assume that each node contacts his neighbors at some time t, more later at the appropriate chapter. Although branching process seems significantly simpler than epidemics, it captures more with the correct modifications and that is the fact that it micro manage the contagion inside a network. In a nutshell, epidemics translate only the ratio at where entities move from a state to another and it does not account a change of probabilities.

2.2 Bayesian Inference

Bayesian inference is a statistical inference that update beliefs about uncertain parameters as more information becomes available. The Bayesian inference is one of the most successful methods used in decision theory, builds over Bayes' theorem:

$$\mathbb{P}(H|E) = \frac{\mathbb{P}(E|H)\mathbb{P}(H)}{\mathbb{P}(E)}$$

which expresses the conditional probability of the hypothesis H conditional to the event E with the probability that the event, or evidence, E occurs given the hypothesis H. In the previous expression, the posterior probability $\mathbb{P}(H|E)$ is inferred as an outcome of the prior probability $\mathbb{P}(H)$ on the hypothesis, the model evidence $\mathbb{P}(E)$ and the likelihood $\mathbb{P}(E|H)$ of the evidence E occurring, given the validity of the hypothesis H. Bayes' theorem has been widely used as an inductive learning model to transform prior and sample information into posterior information and is widely used in decision theory. In order to visualize the concept of Bayes theorem, we provide a simple example. Suppose people are tested for some disease. If the test is 99% accurate, then this means that $\mathbb{P}(Positive-test|Positive)=0.99$. However, the most relevant information is $\mathbb{P}(Positive|Positive-test)$, namely the probability of having the disease if the test is positive, using Bayes theorem. If the proportion $\mathbb{P}(Positive)$ of infected people in the total population is 0.001, then if we have the value of normalizing factor, i.e. $\mathbb{P}(Positive-test)=0.01$, and we conclude that $\mathbb{P}(Positive|Positive-test)=0.099$, which provides different information and more relevant, using evidence rather than a simply using the accuracy of the test. It is obvious that both prior observations and the observable data, contain information, and so neither should be neglected. Bayes theorem has a general form as well, that works with multiple variables. The formula for discrete multiple variables, which we use in our thesis, is:

$$\mathbb{P}(H_k|E)=\frac{\mathbb{P}(E|H_k)\mathbb{P}(H_k)}{\sum_{i\in I}\mathbb{P}(E|H_i)\mathbb{P}(H_i)}$$

where $H_{i\in I}$ is partition of the sample space and H_k is an observation inside that partition.

The process of drawing conclusions from available information is called inference. However, in many cases the available information is often insufficient to reach certainty through reasoning. In these cases, one may use different approaches for doing inductive inference. The strength of Bayesian inference, which is the method of using Bayes theorem to deduce a claim, is that it requires minimum but relevant information in order to work. This ability makes Bayesian inference an appropriate method to express how opinions are formed inside a social structure. Other notable applications are in medicine, machine learning, data analysis and many more.

Chapter 3

Agents Propagation Model

In this chapter we describe the information propagation process among a group of rational agents taking place in discrete time. We first provide a probabilistic representation of the information shared by this group of individuals under our assumptions followed by the propagation rules of our model. Finally, we conclude this chapter with an analysis of agents news-sharing process dynamics and the structural properties that arise from those. Although this thesis is provides all the basic background, we suggest the reader to study [13] which is the basis of our model.

3.1 A Probabilistic Representation of News

We assume that each event in the world is characterized by a binary state $\Theta \in \{Y, N\}$ [1] which is unobservant by a group of individuals (we call them agents) and they are of interest to find the exact value of Θ. Agents may create informative content in forms of news articles (we call them stories) that contain information over the ground

[1]This model supports events with more than two states but for the sake of simplicity we stay in a binary model.

Table 3.1: The probability distribution of truthful stories ($V = T$ left) and fake stories($V = F$ right) with respect to the ground truth.

$P(m \mid (\Theta, V))$	$\Theta = Y$	$\Theta = N$
$m = y$	a	$1 - a$
$m = n$	$1 - a$	a

$P(m \mid (\Theta, V))$	$\Theta = Y$	$\Theta = N$
$m = y$	β	β
$m = n$	$1 - \beta$	$1 - \beta$

truth. These stories are described by $m \in \{y, n\}$ and state the creators realization of the ground truth Θ. Each story can be characterized by another unobservant variable $V \in \{F, T\}$ which declares the validity of its content. If the stance of a story aligns with the actual event, we have $V = T$ with respect to Θ (truthful story). In the other hand we have $V = F$ with respect to Θ (fake story) whenever we are dealing with stories that are uninformative or misleading in respect to Θ. [2]

In table 3.1 we have the probability distributions for both possible types of messages when we are characterizing them with their validity V. The signal-generating process for truthful stories is described by the system of equations:

$$\begin{aligned} \mathbb{P}(y \mid (Y, T)) = \mathbb{P}(n \mid (N, T)) = a \\ \mathbb{P}(y \mid (N, T)) = \mathbb{P}(n \mid (Y, T)) = a \end{aligned} \tag{3.1}$$

where a can be translated as the persuasiveness of source without any other prior knowledge (i.e. the sharing history) or the probability of a story that shares the same stance as the ground truth thus having truthful validity. We assume that the persuasiveness is $a \in (0.5, 1)$ and it is known parameter in our ecosystem. On contrary, when we are dealing with fake stories the signal-generating process is:

$$\begin{aligned} \mathbb{P}(y \mid (Y, F)) = \mathbb{P}(y \mid (N, F)) = \beta \\ \mathbb{P}(n \mid (Y, F)) = \mathbb{P}(n \mid (N, F)) = 1 - \beta \end{aligned} \tag{3.2}$$

From the above equations we notice that the probabilities of stories m in both cases remain the same over different values of Θ. This reflects the fact that the content of a story is totally uninformative and is randomly aligned with one of the possible states for ground truth, i.e if $\beta > 0.5$ the author of a the story pushes his own agenda with a false narrative described by a story of type $m = y$. Another observable variable, that

[2]We should mention that our model does not capture the intention of creating fake stories, i.e. deliberately creating fake information or bad journalism.

we assume it is common knowledge, is the percentage of fake stories that circulate in our social infrastructure. Platform can monitor the validity of previous stories and calculate the frequency, or an approximation, of fake stories denoted as v. We can also express this quantity as the probability of a newly created story being fake without any prior evidence, $v = \mathbb{P}(V = F)$. Those parameters conclude all possible outcomes of a message type, in terms of validity and stance, that describes a binary state of an event. In the next section we provide an example in order to illustrate how those parameters work in a real scenario.

3.1.1 Example

We now provide an illustrative example in order to understand the role of those parameters named in the previous section. There are several real world incidents we can reference from fact checking sites, but we use a fabricated example for the sake of simplicity. [3]

Suppose we have the next claim about COVID-19:

"University of Ioannina study finds that mortality rate of COVID-19 is lower in countries with warmer climate."

In our example, we have the ground truth, described by $\Theta \in \{Y, N\}$ where Y aligns with the content of the sentence, in other words Y is interpreted as *"Yes, mortality rate is lower in warmer climates"*, while N presents no link between climate and the mortality rate of COVID-19. The content of the article support $\Theta = Y$ so we have that $m = y$. Also we expect that a claim made by university of Ioannina would be highly persuasive since it would be researched with valid methods so a is expected to be over 85%, taken at face value. This means that we are likely to adopt the validity of that claim no matter the content of the research, only by attributing the persuasiveness of the source as an authoritative entity in the research domain. Another case is when we expect from the source of a story to be in interest to push a story that aligns with a narrative. For example, if we knew that university of Ioannina had benefit from supporting Y, then it might be the case that it produces a fake story, whose stance $m = y$ holds with probability β, independently of the actual value of the ground-truth variable Θ. Another scenario is when we expect that the source is not

[3] Some examples fact checking sites are snopes `https://www.snopes.com/` and politifact `https://www.politifact.com/`.

biased but we distrust their journalistic effort or the claim is more likely to be false. Remember that β expresses the bias of a source to produce a fake story that supports either Y or N, while $\beta = 0.5$ means that the fake story is produced randomly to align with one side.

Now let's assume that all stories circulating our social structure is probably truthful. This means that v is approaching 0% and in order to simplify things, let $v = 0$. An agent with prior belief $\theta = \mathbb{P}(\Theta = Y)$ will update his posterior belief as:

$$\theta_{posterior} = \mathbb{P}(\Theta = Y \mid m = y) = \frac{a\theta}{a\theta + (1-a)(1-\theta)}$$

which is greater that his prior belief θ since $a \in (0.5, 1)$. In other words, the fact that all stories are truthful strengthens agent posterior belief that $\Theta = Y$ or equivalent that the story is truthful, $V = T$ and $m = y$. On the contrary, if all the stories shared within the social network are almost certainly fake, $v = 1$, we have that:

$$\theta_{posterior} = \frac{\beta\theta}{\beta\theta + (1-\theta)\beta} = \theta_{prior}$$

We see that posterior belief remains the same as our prior opinion since any new information over previous actions would not affect agents posterior belief.

3.2 Sharing Process and Agents Behavior

We assume that a group of infinite rational agents act in a discrete time $t \in \{1, 2, ...\}$ (we refer time as round t) based on various parameters. Each agent is characterized by its own prior belief over ground truth which we assume that is an *iid* draw from a Gaussian distribution. We denote agent i's prior belief as $\theta_{i0} := \mathbb{P}(\Theta = Y)$. [4] The process begins with an emerging story from the first node [5]. Each agent can choose between the next strategies:

- React to the story by sharing it to its out-neighbors (follower list), without inspecting its content. We refer to this action as **send**.

[4] Without loss of generality, we assume that prior opinions describe each agent's belief over one opinion, namely $\Theta = Y$. Proofs for theorems and corollaries in this work are similar for $\Theta = N$ and we mention if otherwise.

[5] We can assume that the first node is $i = 0$ since we can rearrange those *id*'s, which is planted from an external source. We notice that t and i are similar in paths but this statement does not hold for other structures such as trees.

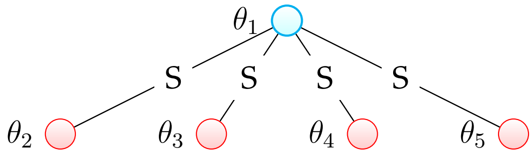


Figure 3.1: Agent θ_1 shares by sending without checking $(A_{1t} = S)$ the story via broadcast to a group of agents (red).

- Refuse to share the story, namely **block**, without checking the content.
- To **check** the story and choose one of the next actions:
  - Share the story is she\he finds that the article is truthful.
  - Not share if she\he finds that the story is fake.

The strategy of each agent at round t is denoted as $A_{it} = \{S, B, C\}$ for each of the above actions respectively. From the above strategies we can see that each one of them splits into 2 steps, namely the inspection choice and the sharing choice. We assume that if an agent picks $A_{it} = C$, the inspection yields a perfect result and the next action dependents on it, i.e. we have socially responsible agents that do not share fake stories. Thus we do not include the option to inspect and share a story if it is evaluated as fake. In advance, we mention that if an agent chooses to share the story either by **send** or **check**, we assume that he shares the message by broadcasting to a group of agents as we see in figure 3.1. Next, we define the probabilities of the above actions in strategy set A_{it}, as:

Definition 3.1. The probabilities of each action A_{it} that agent i selects at round t are:

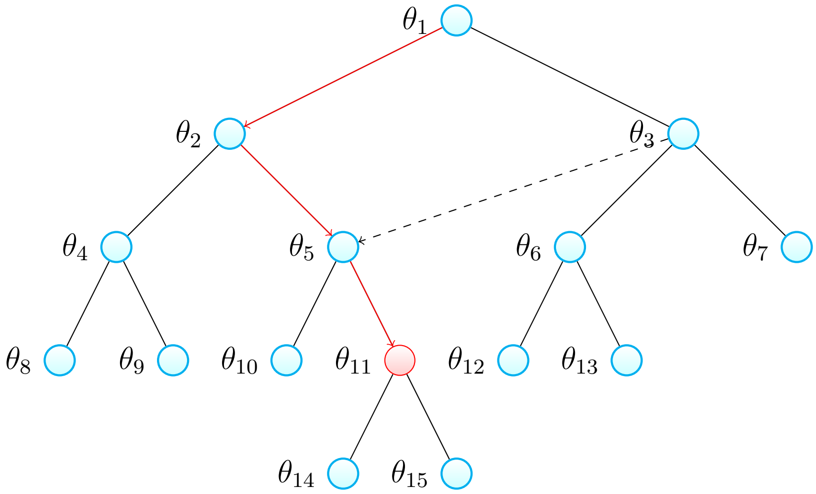
- $S_{it} = \mathbb{P}_i\{A_{it} = S \mid m = y, H_i\}$
- $B_{it} = \mathbb{P}_i\{A_{it} = B \mid m = y, H_i\}$
- $C_{it} = \mathbb{P}_i\{C_{it} = C \mid m = y, H_i\}$ and $C_{it} = 1 - S_{it} - B_{it}$

for actions $\{S, B, C\}$ respectively.

In the analysis of this chapter about agents' sharing process dynamics, we provide closed form equations of the above probabilities.

At each round, we assume that an agent acts and picks one of the actions mentioned in the previous paragraph. This sequential process forms a sub-network $G' \subseteq G$

(a) Sharing path



(b) Sharing Tree

Figure 3.2: Sharing process of a story in different structures (a) Path and (b) Binary tree.

of the original network $G = (V, E)$. We refer this sub-graph as *sharing sub-network* or *sharing tree*, when we are dealing with tree structures, and this sub-graph is known to the platform. For agent i we assume that the *sharing history* he perceives, namely H_i, is the unique path from the originator of the story (source) up to agent i. As we see in figure 3.2, we have the sharing network in which the red edges indicate the sharing history for agent k, with prior belief θ_k, in a path and a tree. The sharing network and history for agent i are the same for a path in contrast with a tree, where we assume that agent i knows that the story is shared only by his predecessors. In our analysis we assume that if agent i receives a story from a agent j, he cannot receive the same story from another node l. As we see in figure 3.2, θ_5 receives the story from θ_2 thus θ_3 will not broadcast the story to θ_5 again.[6]

Throughout the process, agents form and update their beliefs about the validity of the story that is being shared in the network, and their validity. There are two basic probabilities that express the agents opinions. First, we have the agents' own posterior belief over the value of ground-truth as well the agents' posterior belief that

[6]We make a discussion for DAG's in general in a final chapter of this thesis.

an article is fake, respectively:

$$\begin{aligned} b_{it} &= P_i\{\Theta = Y|m = y, H_i\} \\ q_{it} &= P_i\{V = F|m = y, H_i\} \end{aligned} \tag{3.3}$$

where he receives a message $m = y$ with a history path H_i, $\forall t \geq 0$. At each given round t agent updates his own posterior beliefs based on a group of parameters that is common knowledge. The parameters that each agents knows at the round t, when he receives the story are:

- The sharing history H_i up to him from the source.
- His own prior belief θ_i for the ground-truth.
- The ratio of fake news and truthful news that circulate in the platform annotated as v.
- The credibility a of a true story delivering a correct message about the ground-truth value.
- The probability β that a fake story promotes the stance $m = y$ for the ground-truth variable, irrespectively of its actual value.

With the list of above parameters we calculate the closed form equations for 3.3 that are the posterior beliefs of an agent i.

Proposition 3.1. *For each i agent at each round $t \geq 0$ it holds that:*

$$b_{it} = \mathbb{P}_i\{\Theta = Y|m = y, H_i\} = \frac{\theta_i[\beta v w_{it} + a(1-v)]}{\beta v w_{it} + [a\theta_i + (1-a)(1-\theta_i)](1-v)}$$

$$q_{it} = \mathbb{P}_i\{V = F|m = y, H_i\} = \frac{\beta v w_{it}}{\beta v w_{it} + [a\theta_i + (1-a)(1-\theta_i)](1-v)}$$

where $w_{it} = \prod_{k=0}^{t-1} \frac{S_{ik}}{S_{ik} + C_{ik}}$ *and* $w_{i0} = 1$

Proof. We construct the equation for q_{it} since it is the main quantity that we will consider mostly in our work and the proof for b_{it} is equivalent. The beliefs of this model are defined over the set $G = \{(\Theta, V)\}_{\Theta \in \{Y,N\}, V \in \{T,F\}}$ and according to our model for fake news and the independence of V and Θ, we have the prior opinions of an agent as:

- $\mathbb{P}_{i0}\{(Y,T)\} = \theta_i(1-v)$
- $\mathbb{P}_{i0}\{(Y,F)\} = \theta_i v$
- $\mathbb{P}_{i0}\{(N,T)\} = (1-\theta_i)(1-v)$
- $\mathbb{P}_{i0}\{(N,F)\} = (1-\theta_i)v$

Suppose an agent i receives a story in round t, he uses all the information available to him in order to form his posterior belief that a story is fake. The only quantity that changes as time passes is H_i. For an opinion $g \in G$ we have the agents' posterior belief:

$$\begin{aligned}\mathbb{P}_{it}\{g \mid m=y, H_i\} &= \frac{\mathbb{P}\{m=y, H_i \mid g\}\mathbb{P}(g)}{\sum_{g' \in G} \mathbb{P}\{m=y, H_i \mid g'\}\mathbb{P}(g')} = \\ &= \frac{\mathbb{P}\{H_i \mid m=y, g\}\mathbb{P}\{m=y \mid g\}\mathbb{P}(g)}{\sum_{g' \in G} \mathbb{P}\{H_i \mid m=y, g'\}\mathbb{P}\{m=y \mid g'\}\mathbb{P}(g')}\end{aligned} \tag{3.4}$$

which is the generalized Bayesian inference of an opinion g among a set of opinions G. Now we calculate each probability in that expression in order to form q_{it}. From section 3.1 we have the probabilities below:

$$\mathbb{P}\{m=y \mid (Y,T)\} = a, \mathbb{P}\{m=y \mid (N,T)\} = 1-a,$$

$$\mathbb{P}\{m=y \mid (N,F)\} = \mathbb{P}\{m=y \mid (Y,F)\} = \beta$$

Now we calculate the quantity $\mathbb{P}\{H_i \mid m=y, g\}$ which is the probability of a history given a $m=y$ story and an opinion. We are given that the story is y, thus agent i considers his strategy only on V. This means that a fake story is shared only by action S and a truthful story either by S or C:

$$\mathbb{P}\{H_i \mid m=y, (Y,F)\} = \mathbb{P}\{H_i \mid m=y, (N,F)\} =$$

$$\prod_{k=0}^{t-1} \mathbb{P}\{A_{it} = S \mid m=y, H_i\} = S_{it}$$

and

$$\mathbb{P}\{H_i \mid m=y, (Y,T)\} = \mathbb{P}\{H_i \mid m=y, (N,T)\} =$$

$$= \prod_{k=0}^{t-1} [\mathbb{P}\{A_{it} = S \mid m=y, H_i\} + \mathbb{P}\{A_{it} = C \mid m=y, H_i\}] = S_{it} + C_{it}$$

Combining all the above equations, we can compute q_{it}:

$$q_{it} = \mathbb{P}_{it}\{g \mid m = y, H_i\} = \mathbb{P}_{it}\{(Y, F) \vee (N, F) \mid m = y, H_i\} = \\ = \frac{\beta v w_{it}}{\beta v w_{it} + [a\theta_{i0} + (1-a)(1-\theta_{i0})](1-v)}$$

Using the above calculations we can calculate b_{it} in the same way as:

$$b_{it} = \mathbb{P}_{it}\{g \mid m = y, H_i\} = \mathbb{P}_{it}\{(Y, T) \vee (Y, F) \mid m = y, H_i\} = \\ = \frac{\theta_i[\beta v w_{it} + a(1-v)]}{\beta v w_{it} + [a\theta_i + (1-a)(1-\theta_i)](1-v)}$$

where $w_{it} = \prod_{k=0}^{t-1} \frac{S_{ik}}{S_{ik} + C_{ik}}$, $w_{i0} = 1$ is the proportion probability that a story creates a sharing tree G' that is made with $A_{it} = S$ up to round t where S_{ik} and C_{ik} are given by definition 3.1.

□

Throughout the sharing process, each agent acts only once and he receives the appropriate reward for his action. We define the utility of agent i, over his action A, the function $U_i(A_{it})$. Agent i receives reward either if he blocks a fake story or chooses to share a truthful story. On the other hand, he receives no reward if he blocks a truthful story or shares a fake story. Finally, an agent that chooses to inspect a story, does so by paying a price. Notice that the above function is evaluated only for the variables V and A_{it}. Collectively, the induced evaluation function is the following:

$$U_i(A_{it}) = \begin{cases} 1, & (A_{it} = S \wedge V = T) \vee (A_{ih} = B \wedge V = F) \\ 0, & (A_{it} = B \wedge V = T) \vee (A_{ih} = S \wedge V = F) \\ 1-K, & A_{it} = C \end{cases} \tag{3.5}$$

where K is the cost of inspection. We assume that inspection occurs with a cost $K < 0.5$ to avoid trivial cases where inspection is never optimal strategy. Agents are choosing actions that maximize their utility at t-round and in order to do so, they rely on their beliefs for values about the ground truth that we defined in proposition 3.1. Each agent chooses the strategy that maximizes his expected utility, given his belief that the story is fake, q_{it}. The expected utility can be calculated with the next proposition:

Proposition 3.2. *The expected utility of agents action is given by the function:*

$$\mathbb{E}[U_i(A_{ih})] = \begin{cases} 1 - q_{ih}, & A_{ih} = S \\ q_{ih}, & A_{ih} = B \\ 1 - K, & A_{ih} = C \end{cases}$$

Proof. And agent receives reward if he shares truthful stories or blocks fake according to his belief. Thus, agents j expectation for action send, according to his belief q_{jt} in round t, is $\mathbb{E}[U_j(S)] = (1 - q_{jt})U_j(S \wedge T) + q_{jt}U_j(S \wedge F) = 1 - q_{jt}$. In similar manner we have that expected utility gained from blocking a story, based on agents j belief that is fake, is $\mathbb{E}[U_j(S)] = (1 - q_{jt})U_j(B \wedge T) + q_{jt}U_j(B \wedge F) = q_{jt}$. Finally, if a rational agent chooses to inspect a story based on his updated belief, the expected utility can be calculated as $\mathbb{E}[U_j(C)] = (1-q_{jt})U_j(S\wedge T)+q_{jt}U_j(B\wedge F)-K = 1-q_{jt}+q_{jt}-K = 1-K$ where the subtraction of K presents the cost that agent j pays in order to inspect a story and decide whether to send a truthful story or block it otherwise. □

In figure 3.3 we have an illustration of the utility curves over different values of θ_i. Figure 3.3 is also a visual justification for the assumption that we made for $K < 0.5$. We can clearly see that if we assigned a value greater than 0.5, then $1 - K < 0.5$ which implies that $U_i(B) > U_i(C)$, $\forall \theta_i \in (0, \overline{\theta})$ where $U_i(B)$ and $U_i(S)$ intersect, and after that point it holds that $U_i(S) > U_i(C)$, $\forall \theta_i \in (\mu, 1)$.

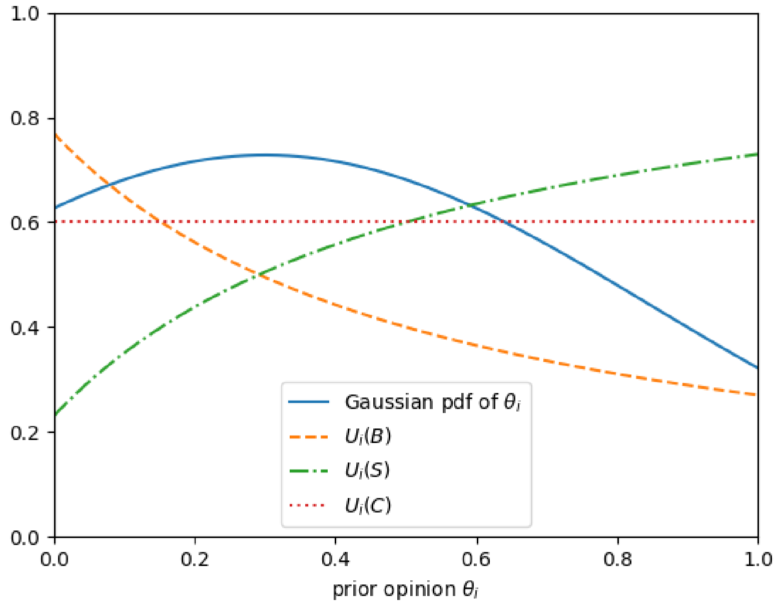


Figure 3.3: The expected utility curves over different prior beliefs that follow a Gaussian distribution at first round. Parameters: $v = K = 0.4$, $a = 90\%$ and $\theta_i \sim \mathcal{N}(\mu = 0.3, \sigma^2 = 0.09)$.

3.3 Propagation Dynamics

In this section we will focus on the dynamics of agents propagation and the properties of our setup, in order to help us tackle the main problem which is the optimal time that we can intervene to stop the propagation of a fake story. In order to introduce the properties, we describe each individual step of the process in a sharing tree. The proofs for theorems, propositions and lemmas that are mentioned in related work are provided in A.

At each round t an agent i is up to decide if he will inspect a story $m = y$ and then if he will share it via a broadcast on a group of agents, i.e. his followers.[7] According to his belief, he evaluates each strategy and picks the one with that maximizes his expected utility. If he decides to share the story, he broadcasts the story to the set of all its out-neighbors (i.e., his\her followers) in the underlying social-network graph. In round $t + 1$, we pick another agent to act in a sense that round t is a counter that determines how many agents have reacted. On the other hand, if an agent j

[7]The choice of agents that are going to react and how it affects the sharing tree and the inspection time is further studied later on this thesis.

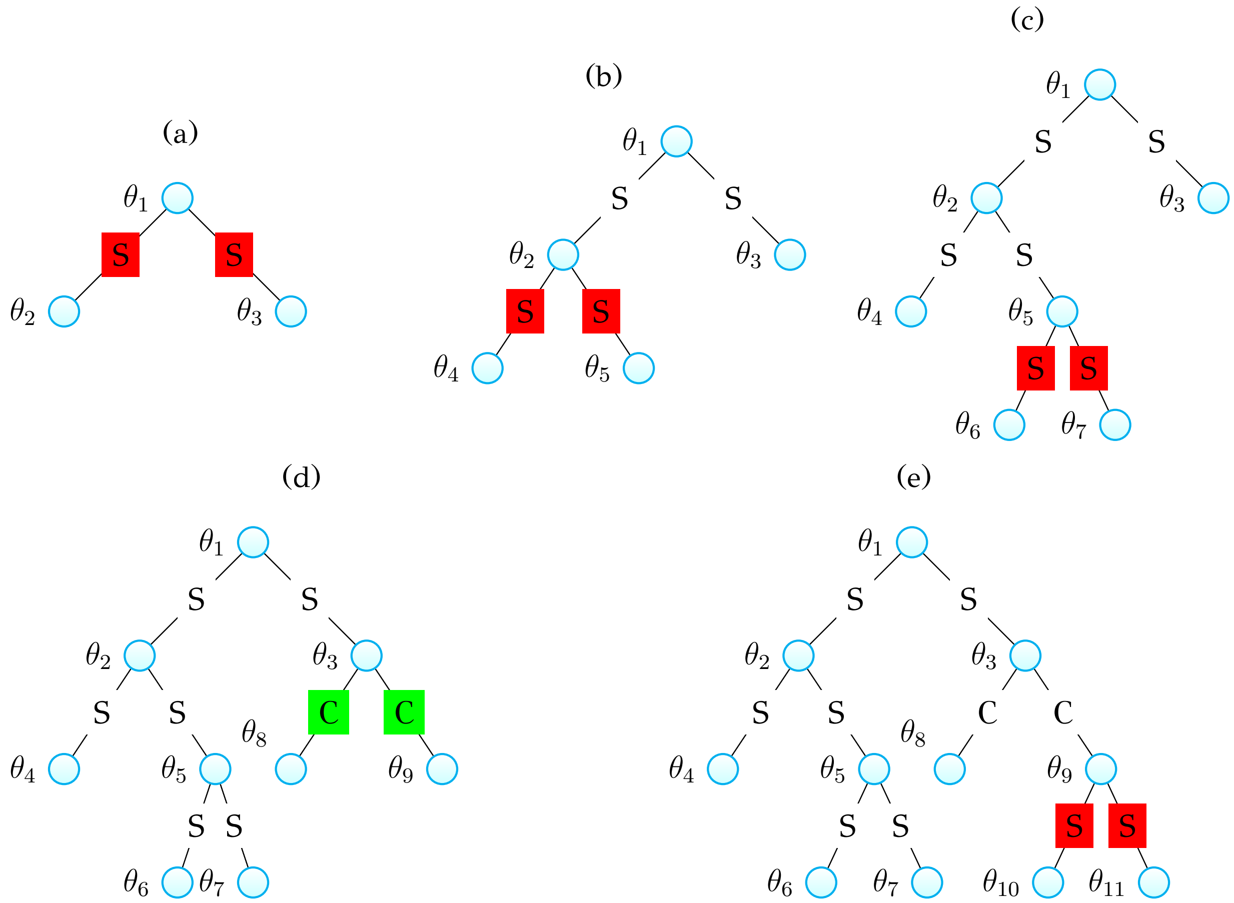


Figure 3.4: The resulting sharing tree after five reactions, from (a) to (e).

decides to not share the story, no matter if he inspects or not, the sharing history H_j is discontinued at his path. Notice that in case of paths, time and agents are equivalent in the sense that we can refer to each agent as t agent that is up to act. Additionally, if an agent decides not to share the story, the whole process is discontinued while in an m-ary tree it stops only in the particular sub-tree whose root node decides not to share.

Before we further continue our study, it is time to illustrate each individual step of a sharing process with an extended example. Using figure 3.4, we begin with agent 1 with θ_1 that chooses to share the story $m = y$ with his neighbors. In order to do that, evaluates his expected utility for each action and picks the appropriate that has the maximum value. In order to do so, he calculates his posterior belief. In the beginning of the process, there is no sharing history since he is the first that reacts to the story on his unique path from the source up to him (i.e. he is the root). From proposition 3.1 we have that $q_{11} = \dfrac{\beta v w_{11}}{\beta v w_{11} + [a\theta_1 + (1-a)(1-\theta_1)](1-v)}$

where $w_{11} = S_0/(S_0 + C_0)$ since there is no previous sharing history. Continuing with the next agent θ_2 in (b) he also evaluates his own posterior belief with the expected utility and he finds that it maximizes when he chooses to share with out checking as well. The difference this time is that the sharing history of his ancestors is non trivial, thus he has to make an estimation about w_{it}. In order to do so, he must calculate $w_{22} = \prod_{k=0}^{1} \frac{S_{ik}}{S_{ik} + C_{ik}}$ which translates to the probability *that a story reaches to agent i without any inspection* and it is a proportion of q_{it}. We notice that this quantity demands the knowledge of probabilities that concern the sharing actions of previous agents in his sharing history. It is obvious that if agent i knew the prior beliefs of their ancestors, then agent i would have a best response strategy and the choice of action would be deterministic. Since we want to study a model that works under uncertainty, we introduce the next two assumptions:

Assumption 3.1. *Aside for the parameters that we assumed in section 3.2 is shared throughout all agents and in order for agent to update his belief by calculating $w_{it} = \prod_{k=0}^{t-1} \frac{S_{ik}}{S_{ik} + C_{ik}}$, we have two options:*

- *Agents presume that all prior opinions follow a normal distribution, $\theta_i \sim \mathcal{N}(\theta^*, \sigma^2)$, and θ^* is common knowledge in our social network $\forall i$.*
- *Agents presume that all prior opinions follow a normal distribution and each agent creates his own normal distribution centralized around him such that $\theta_j \sim \mathcal{N}_i(\theta^*, \sigma^2)$ with $\theta^* = \theta_i$, $\forall j$.*

The first option translates to a setup that agents determine past reactions based on a average prior opinion, i.e. an average value based on a similar subject that platform historically recorded in the social network. This approach creates a normalized sharing behavior because agents are assumed to be completely homogeneous, in the sense that they all sample exactly the same distribution when considering the prior beliefs of other agents. On the other hand, when agent i uses his prior opinion to determine the reaction history up to him, e centers the normal distribution for sampling the other agents' prior beliefs to its own (actual) prior belief. I.e., the agents are now assumed to be heterogeneous.

Continuing in figure 3.4, at (d) and (e) derived sub-trees in our sharing process, we see that agent θ_3 is choosing to inspect the information and share it with his neighbors, θ_8 and θ_9. For our model, and since we assumed that the inspection yields perfect outcome, it is sufficient to find an agent that inspected the story and chose to share it. This observation is useful in the next chapter that is focused on platform's inspection problem and the solution of interrupting probable viral stories that are shared with suspicious reactions.

In order to separate the actual probabilities that an agent i chooses a strategy between $\{B, C, S\}$ from his estimated value about those probabilities for his ancestors, we introduce the next annotations:

Definition 3.2. We define as $B_{it}^j, C_{it}^j, S_{it}^j$ the probabilities that agent i observes for agent j and his actions $\{B, C, S\}$ respectively for round t. More specifically, we have the equations below:

$$B_{it}^j = \mathbb{P}_i\{A_{jt}(\theta_i^*) = B \mid m = y, H_j\}$$

$$C_{it}^j = \mathbb{P}_i\{A_{jt}(\theta_i^*) = C \mid m = y, H_j\}$$

$$S_{it}^j = \mathbb{P}_i\{A_{jt}(\theta_i^*) = S \mid m = y, H_j\}$$

where $A_{jt}(\theta_i^*)$ is the action that agent i believes that is optimal for agent j according to the assumption 3.1 where agent i presumes that his predecessors acting with some prior θ^*.

Now we provide closed form equations for probabilities in definitions 3.1 and 3.2, derived from the above proposition.

Proposition 3.3. *Let be agent i that is up to react in round t. Then for the probabilities B_{it}, C_{it}, S_{it} it holds that:*

$$B_{it} = \mathcal{F}\left(\frac{1}{2\alpha - 1}\left[\frac{\beta v w_{it} K}{(1-v)(1-K)} - (1-\alpha)\right]\right)$$

$$S_{it} = 1 - \mathcal{F}\left(\frac{1}{2\alpha - 1}\left[\frac{\beta v w_{it}(1-K)}{(1-v)K} - (1-\alpha)\right]\right)$$

$$C_{it} = 1 - B_{it} - S_{it}$$

where $\mathcal{F}$ is the cumulative distribution function (cdf) from where θ_i's are drawn.

It is important to notice that in proposition 3.3, the formulation is independent of the cdf $\mathcal{F}$ that distribution has. In our model we assumed that tall the agents, and the platform, presume that the other agents' prior beliefs are drawn independently from a normal distribution $N(\mu, \sigma^2)$

$$\mathcal{F}(x) = \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

where $erf(x)$ is the error function specified in the background.

Now that we have the appropriate tools and specified the all the assumptions about how an agent processes information and how he acts, we are ready to analyze each component of our setup and find useful properties that will help us tackle the platform's inspection problem. One important property that will help in the analysis of this model is the monotonicity of q_{it}. The following lemmas from [13] are useful tools throughout the analysis.

Lemma 3.1. *Suppose that an agent receives a story in round t. The agent's posterior belief that the story is fake, q_{it}, is strictly decreasing in her prior opinion over ground truth, θ_i.*

Lemma 3.2. *Suppose that an agent receives a story in round t with a sharing history H_i and a prior belief over Θ, θ_i. The agent's posterior belief that this story is fake is decreasing as t increases.*

The probability q_{it} can be seen as a sequence of θ_i, $\{q_{it}\}_{\theta_i \sim \mathcal{N}(\mu,\sigma)}$ for fixed t's, or as a sequence of t, $\{q_{it}\}_{t\in\mathbb{N}}$ by fixing agents to their prior opinions. Lemma 3.1 describes the behavior of q_{it} as we adjust the prior opinion of an agent, while 3.2 expresses the monotonicity of q_{it} as time approaches to infinity. From 3.1 we notice that for a given round t_0, if agents' prior belief that $\Theta = Y$ is closer to 1 then it is less likely that he will perceive that the story is fake. In addition, in lemma 3.2 we see that the later an agent receives a story he is less likely to believe that it is fake. In other words, as the length of H_i increases[8] up to an agent i, he is more willing to believe that some agent j, between the originator of the story up to him, has inspected the story and found its content truthful. Lemmas 3.1 and 3.2 are very important to extract properties for the behavior of agents as well for the optimal solution for the platform that we will analyze in the next chapter.

As soon as an agent receives a story and based on 3.2, his best response is the one that maximizes his expected utility thus, every action A_{it} occurs with probability that depends on his belief that a story is fake q_{it}. Using definition 3.1 for those probabilities combined the above proposition we derive bounds of our sharing process.

[8]We remind that the sharing history H_i is a path from the root to a node in a sharing tree $G' \subseteq G$.

Proposition 3.4. *In any round t, there exists a lower and an upper bound $\underline{Z}_{it}, \overline{Z}_{it} \in [0,1]$ respectively, for agent i, such that:*

1. *If $\theta_i < \underline{Z}_{it}$ then $A_{it} = B$.*

2. *If $\theta_i > \overline{Z}_{it}$ then $A_{it} = S$.*

3. *If $\underline{Z}_{it} \leq \theta_i \leq \overline{Z}_{it}$ then $A_{it} = C$.*

where the thresholds $\underline{Z}_{it}$ and $\overline{Z}_{it}$ expressed as:

$$\underline{Z}_{it} = \frac{1}{2\alpha - 1}\left[\frac{\beta v w_{it} K}{(1-v)(1-K)} - (1-\alpha)\right]$$

$$\overline{Z}_{it} = \frac{1}{2\alpha - 1}\left[\frac{\beta v w_{it}(1-K)}{(1-v)K} - (1-\alpha)\right]$$

Proposition 3.4 shows that at any given time, the optimal choice of strategy that maximizes the expected utility of an agent is bounded within the above thresholds $\underline{Z}_{it}$and $\overline{Z}_{it}$. Notice that we can slightly modify proposition 3.4 in a way such that they become sequences over θ_i for a given round t. This brings us to the next corollary:

Corollary 3.1. *For all agents i, there exists thresholds with θ_i as argument such that:*

$$\overline{W}_i = \frac{1-K}{K}\frac{1-v}{v}\frac{2\alpha-1}{\beta}\left(\theta_i + \frac{1-a}{2\alpha-1}\right) < w_{it}$$

$$\underline{W}_i = \frac{K}{1-K}\frac{1-v}{v}\frac{2\alpha-1}{\beta}\left(\theta_i + \frac{1-a}{2\alpha-1}\right) > w_{it}$$

equivalent to $\underline{Z}_{it}\overline{Z}_{it} \in [0,1]$ respectively.

The last corollary is an alternative expression of thresholds $\underline{Z}_{it}$ and $\overline{Z}_{it}$ with the advantage that they are not dependent on time t. With the help of corollary 3.1, instead monitoring the sliding window where it is formed from $\underline{Z}_{it\to\infty}, \overline{Z}_{it\to\infty}$, we can calculate the round t where w_{it_0} escapes out of $\underline{W}_i$, $\overline{W}_i \in [0,1]$ thresholds. We remind that w_{it} is a proportion of the actual belief, for agent i, that the story as fake. The sequence $w_{it} \propto q_{it}$ calculates the amount of shares that made without inspection in the process, namely $A_{it} = S$. Corollary 3.1 is very useful for platforms mechanism in order to answer which stories are probably fake based on the propagation and we further exploit their properties in chapter 4.

In figure 3.5, we have a Gaussian pdf and the bounds $\underline{Z}, \overline{Z}$ in first round $t = 0$. We have three cases where:

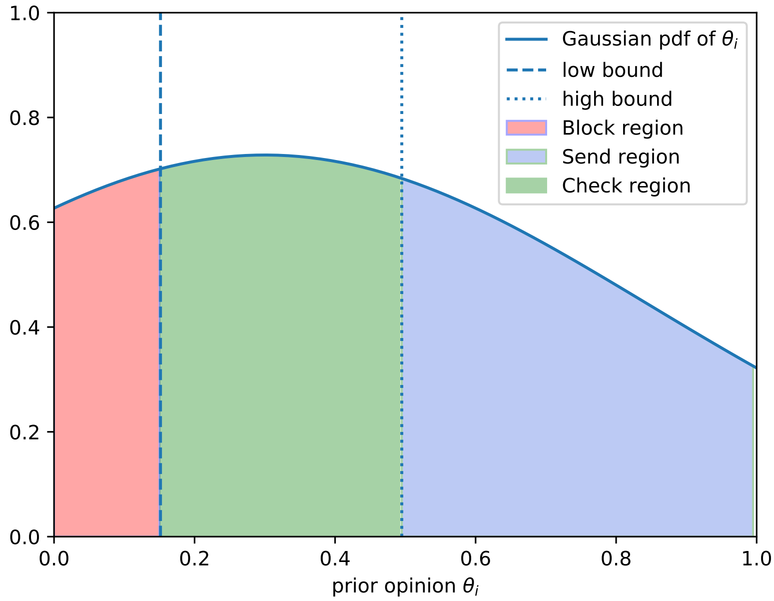


Figure 3.5: Regions (areas according to high and low thresholds) that characterize agents actions for different prior opinions in first round. Parameters: $v = K = 0.4$, $a = 90\%$ and $\theta_i \sim \mathcal{N}(\mu = 0.3, \sigma^2 = 0.09)$.

- Priors θ_i that are picked below $\underline{Z}$, inside the red area, consists of users that prefer to block the sharing process without even inspecting it.
- The green area, that is between the thresholds $\underline{Z}, \overline{Z}$, consists of agents with prior opinions such that they are more likely to check the story in first round.
- The blue area, above $\overline{Z}$ threshold, belongs to agents with such prior opinions that they are going to share the message with no inspection.

Now we proceed, with the next proposition, by proving that those thresholds are non increasing in t and also the difference $|\overline{Z}_{it} - \underline{Z}_{it}|$ is non increasing and non zero while $t \to \infty$.

Proposition 3.5. *Given an agent j and thresholds $\underline{Z}_{jt}$, $\overline{Z}_{jt} \in (0, 1)$, it holds that $\underline{Z}_{jt}$ and $\overline{Z}_{jt}$ are decreasing in t as well as the difference $|\overline{Z}_{jt} - \underline{Z}_{jt}|_{t\to\infty}$*

Proof. The claim is complete if we prove that w_{it} is non increasing. That is obvious since $w_{it} = \prod_{k=0}^{t-1} \frac{S_{ik}}{S_{ik} + C_{ik}}$ is an product of quantities such that $S_{it}, C_{it} \in (0, 1)$, thus

$w_{jt+1} < w_{jt}$, $\forall t$. For the difference $|\overline{Z}_{it} - \underline{Z}_{it}|$ let's assume that we have a agent j with θ_j, we compare the difference in round t with the next round $t+1$. We have that:

$$|\overline{Z}_{j(t+1)} - \underline{Z}_{j(t+1)}| - |\overline{Z}_{jt} - \underline{Z}_{jt}| =$$

$$= \frac{1}{2\alpha - 1}\left[\frac{\beta v w_{j(t+1)}(1-K)}{(1-v)K} - (1-\alpha)\right] - \frac{1}{2\alpha - 1}\left[\frac{\beta v w_{j(t+1)}K}{(1-v)(1-K)} - (1-\alpha)\right] -$$

$$\left\{\frac{1}{2\alpha - 1}\left[\frac{\beta v w_{jt}(1-K)}{(1-v)K} - (1-\alpha)\right] - \frac{1}{2\alpha - 1}\left[\frac{\beta v w_{jt}K}{(1-v)(1-K)} - (1-\alpha)\right]\right\} =$$

$$\frac{1}{2\alpha - 1}\left[\frac{\beta v(1-K)}{(1-v)K}(w_{j(t+1)} - w_{jt}) - \frac{\beta v K}{(1-v)(1-K)}(w_{j(t+1)} - w_{jt})\right] =$$

$$= \delta(w_{j(t+1)} - w_{jt})$$

where δ is a constant such that $\delta > 0$ for $K < 0.5$ and $a > 0.5$. Since $w_{j(t+1)} < w_{jt}$ we have that the difference is decreasing and it is non zero.

The proof is equivalent using the assumption 3.1 where S_{it} and C_{it} are replaced by S_{it}^j and C_{it}^j respectively, $\forall j \in path(root, i)$. $\square$

With proposition 3.5, we have the next important corollary for the sharing process, that expands the cascading behavior in [13] for sharing trees.

Corollary 3.2. *Let G' be a sharing tree as specified in section 3.2. There exists a certain depth h_c at which the best response is to share without inspect. More specifically, there is a depth h_c such that $\overline{Z}_{ih_c} \leq 0$ for some agent i or equivalent, $h_c = min\left\{h \mid S_{ih} = 1 \text{ or } \overline{Z}_{ih_c} \leq 0\right\}$.*

Since each path expands as an independent experiment (agents know only how many of their predecessors shared), for each of these paths there is a certain depth that is critical. In the case of a path, where agents react in sequential manner, the round represents the number of agents that react. In other words, we have a critical agent positioned in a specific spot of the path such that, after a certain amount of reactions, at round T_c the best response of $T_c + 1$ agent is to send without inspecting the story. In figure 3.6 we have an example of a sharing process alongside with the propagation network. The sequence of agents $\{\theta_1, \theta_2, \theta_5, \theta_{11}, \theta_15\}$ reached in such

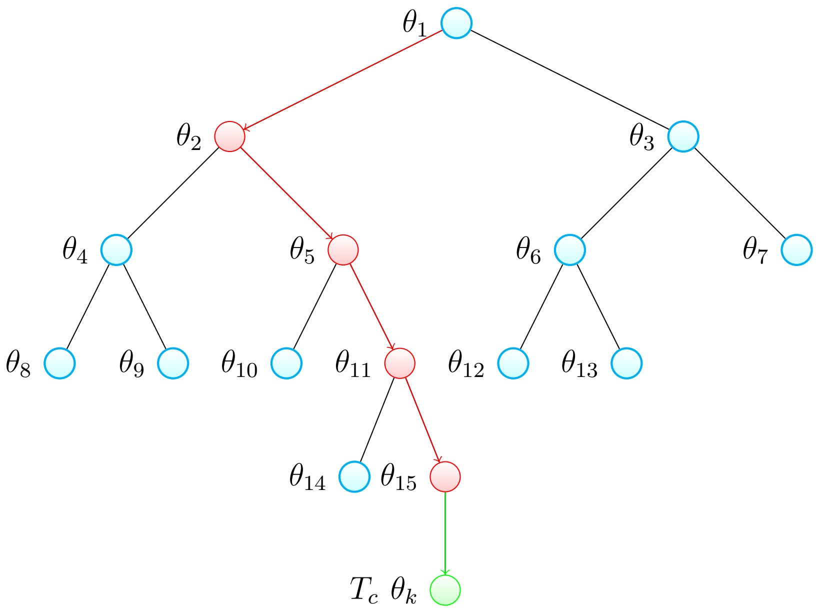


Figure 3.6: The critical round T_c at where the best response of an agent in that depth is to share without inspecting an event.

depth that the quantity w_{kT_c} will affect agents' belief that the story is fake, q_{kT_c} that the expected response is to share the story without inspection. In other words, agents' interpretation of their depth is an increasing possibility of an existing agent that checked the story and decided to share to the subsequent agents.

One question that emerges from the above analysis is the impact of depth over agents opinion that a story is false, namely q_{it}.

Corollary 3.3. *For an agent i with prior opinion θ and his posterior belief that $m = y$ is fake, q_{it}, it holds that:*

- *If agent i chooses action S in round t_0, $A_{it_0} = S$ then $\forall t > t_0$ subsequent rounds, $A_{it} = S$.*
- *If agent i chooses action B in round t_0, $A_{it_0} = B$ then $\forall t \in [1, t_0]$ previous rounds, $A_{it} = B$.*

Proof. We will prove the first bullet, since the second is proven in symmetrical manner with opposite monotonicity. Lets assume that agent chooses to share a story without inspecting it, at a given time t_0 which means that $A_{it_0} = S$. This choice optimal when his expected utility is maximized via action S, more specifically, whenever it holds that $1 - q_{it_0} > q_{it_0}$ and $1 - q_{it_0} > 1 - K$. Since q_{it} is decreasing in time, then the quantity

$1 - q_{it_0}$ is increasing in time and since it is upper bound for both q_{it} and $1 - K$, then it will remain an upper bound as t increases, which proves the claim. For the second bullet, the proof is symmetric since $q_{it_0} > 1 - q_{it_0}$ and $q_{it_0} > 1 - K$ when the response is $A_{it_0} = B$. Adding the fact that q_{it} is decreasing in time, from lemma 3.2, we have that the above inequalities hold, for each round $1 < t < t_0$. $\square$

Chapter 4

Platforms' Inspection Mechanisms

So far we created a setup for information exchange between a group of rational agents alongside with the rationalization of the assumptions we made for our model. The next step is to define the behaviour of our authoritative entity, in our case the platform which is the social medium where agents interact and share stories with each other. Given that the platform has an overview of the whole process, we utilize this knowledge under a number of assumptions, to develop appropriate tools for our platform in order to intervene and inspect information. Our goal is to consider the properties of our network that will specify the optimal time for inspection.

In this chapter we introduce platform's role in the sharing process model and the inspection problem. We modify the basic sequential model introduced by [13] in order to work for asynchronous propagation within tree structures, adding the necessary assumptions. We continue with an analysis of the properties that emerge from those assumptions. Finally, we leverage the structure features in order to find an approximate solution for intervention and inspection of a story at proper time.

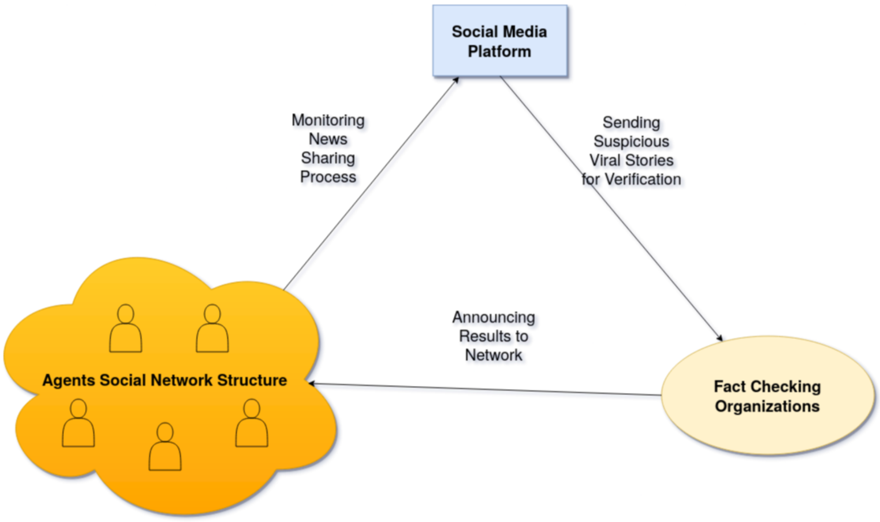


Figure 4.1: The fact checking model of a social media service. The fact-checking process is assumed to an external service for the platform, which of course comes with a given cost for the platform. In practical scenarios, the platforms cannot conduct a fact check over stories because this would affect their policy, given that fact checking comes with a cost.

4.1 Introducing Platform in the Sharing Process

So far we have an ecosystem where agents share messages called *stories*, as we mentioned in previous chapter, and their actions are under the assumption that are responsible individuals. This technically means that they act in order to maximize their expected utility by not sharing non trusted stories according to their judgment. Now we introduce the platform, where those individuals reside in. Examples of such entities are Facebook, Twitter, YouTube and many more, where millions of user interact and share information with each other. In our case study, platform is a *super* agent providing those online services in order for agents to interact with each other. In our study, the platform observes the evolution of the propagation tree, and infers the posterior probabilities of the involved users, so as to have its own posterior belief about the validity of a story.

In figure 4.1 we have our ecosystem with the introduction of platform. We see that our platform monitors the activity of agents network in order to maintain a trustworthy social network of information sharing. If platform suspects that a story is shared effortless (without any inspection), then there is a global check via fact checking organizations. Afterwards, the results are announced to our network and there are two options:

- If the story is validated as truthful, then this announcement leads to a sharing cascade. Additionally, the platform will receive a discounted reward for each share of the story in our network.
- If the story is fake, then the sharing process is terminated and the the platform will receive only a penalty for each (previous) share of the fake story, along with a fixed global-check cost.

In our case study, we assume that the inspection yields a perfect result. This assumption holds for both agents and the authoritative entities such as the platform or fact checking partners. We also remind that this setup is easily extensible for cases where the fact checking action occurs with error.

Another important part of our setup is the amount of privileges that such a platform possesses. We mentioned above that our platform is a super user with extra knowledge. We assume that our platform observes the creation of edges in our network without knowing if they are product of checking action or blindly sharing the story. In other words, platform observes reactions at given round t from agents without knowing the exact action, i.e. if it $A_{it} = S$ or $A_{it} = C$ in round t. Additionally, the platform is unaware if an agent discontinued the sharing process by blocking of checking the story. More specifically, platform cannot observe the round t where an agent decided to not share a story with either $A_{it} = B$ or via checking with $A_{it} = C$. This assumption reflects a real life application where a social media administrator cannot monitor if an individual user of such a service conducted a research or not in order to share information within a network.

This brings us to a stronger assumption that makes the building block of our model.

Assumption 4.1. *The exact value of random variables $\theta_i \sim \mathcal{N}(\theta, \sigma^2)$ for each agent i are hidden from platform. Platform only assumes a normal distribution $N(\theta, \sigma^2)$ for the independent sampling of prior beliefs for the agents.*

The reasoning behind this assumption, that strengthens our model in order to work under uncertainty, is that we cannot predict the exact value of a prior opinion of a random agent. For example, let's assume that we have an emerging topic. In such case, most probably, we do not have prior opinions formed on the topic or even worse,

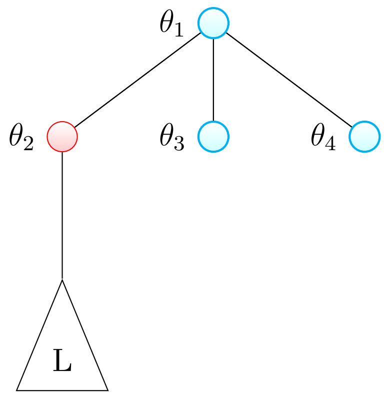


Figure 4.2: An example of a ternary sharing tree where each agent is picked uniformly at random to react. The right triangle L is a subtree of the sharing tree.

we formed wrong prior opinions. Another important property this assumption has is the fact that this model is more confidential since we can work without monitoring users of such services where there are issues of private information leaks.

As we mentioned above, the platform monitors reactions of agents, more specifically, those reactions that share information from parents to children. In other words, platform increases the round from t to $t+1$ whenever a node i decides to share and create edges to his *friends*, *followers* or children (from data structure perspective). We need to decide how those agents are picked to react and choose an action at round t. There are two approaches for that issue:

- Agents that are terminal leaves in the sharing tree are picked uniformly at random.
- Agents are picked with probability $(1/m)^l$ where m is the amount of children that each node have (for m-ary trees) and l is the height of node i in the sharing tree.

It is obvious that the first approach tends to create unbalanced trees in depth first manner. In figure 4.2 we see such a case. After the root decided to share the information, let's assume that node θ_2 is picked with probability $1/3$, since we have a ternary propagation tree, tree, to react at round 2. The set of leaves at that round, we call it frontier from now on, consists of nodes θ_3, θ_4 and the children of θ_2, D_{θ_2}. Because we assumed that agents are picked uniformly at random, hence the probability of picking a node on the frontier is now $1/(\mid \{\theta_3, \theta_4\} \cup D_{\theta_2}) \mid$, in case of our example it is $1/5$ since we have 3 children of θ_2 plus the nodes in level 1. This implies that θ_2 will probably will be picked, with probability $3/5$, rather than θ_3 or θ_4, with probability $2/5$. Moving forward in the next round, the probability of picking one node to react in the subtree L will increase once again, making more likely an expansion of the sharing tree towards L.

On the other hand we have the second approach where the level of each node matters, hence we have that node i is picked with probability $(1/m)^l$ where m is the amount of children each node is assumed to have[1] and l is the level where node i belongs. This approach is used to tackle down the issue that we have with unbalanced tree when we are picking uniformly at random and also it is reasonable to say that we expect from nodes who received the story earlier to react faster than freshly activated nodes in the sharing tree.

Now that have established the role of the platform, alongside with the appropriate assumptions, we proceed with the technical part and the introduction of the optimal inspection problem.

4.2 Platforms' Inspection Problem

Since we assumed that inspection yields a perfect outcome, it is sufficient to find one agent that inspected the element. If an agent that inspects the story and decides to share that means he found the story truthful. Because we assumed that the result is irrefutable, the announcement of that the story is truthful will trigger an information cascade in the sharing tree, and each sharing action afterwards will increase the discounted reward that platform receives, i.e. advertise revenue or another monetization strategy that is benefit from sharing content. Apart from the agents' private inspections (fact-checks) for the type of a story, the platform may also intervene by requesting a global fact-check (e.g., from an external third-party fact-checking service) and then communicating the result of this inspection to the entire community.

On the other hand, we have agents that are not reacting, and we cannot presume if the chose $A_{it} = C$ or B as an action, as we specified in section 4.1. An agent might block a story either with or without inspection and that is hidden from platform since it does not posses the exact value of θ for each agent nor it is clear if the

[1]We are using mean field analysis in our approach with an average degree for children in our tree structure. Thus we are developing this model over m-ary trees.

agent inspected or plainly blocked the story. Thus it is important for the platform to estimate the probability that some agent received the story and decided to inspect and block it. Once again, this event is sufficient enough since inspection is perfect. The platform can also utilize the probability of that event and dispute the validity of that story via fact checking organizations. We assume that the intervention of the platform in the evolution of the emergent story via fact-checking comes with a cost K_p. Now that we established those two warning mechanisms for global check, we provide two propositions in order to calculate the probabilities of those events.

Proposition 4.1. *Let T be a sharing tree and $m = y$ is the story that propagates in T. Then the platform's belief there exist an agent i and $A_{it} = C$ at some round t in T, under the assumption that platform observes independent random experiments over agents, is:*

$$r_T = 1 - \mathbb{P}_{pT}\{\forall i \in \mathcal{V}_T : A_{it} = S \mid H_i, m = y\} = 1 - \prod_{\substack{k=0 \\ i \in \mathcal{V}_T}}^{t-1} \mathbb{P}[A_{ik} = S^p_{ik}]$$

where $\mathcal{V}_T$ are the set of nodes of T, respectively.

Before we begin with the proof of proposition 4.1, it is important to mention that time is irrelevant in that equation since we can rearrange the sequence of shares and agents id's in order. We keep the annotation of A_{it} though to avoid *polluting* this thesis with unnecessary notations. That being said, we only use the next definition in order to specify the platform's belief over agents' i action:

Definition 4.1. For each action $A_{it} = \{B, C, S\}$ of an agent i in round t, we define the probabilities B^p_{it}, C^p_{it} , S^p_{it} that estimate the platform's perception over the probabilities B_{it}, C_{it} , S_{it} for each action respectively.

Proof. (Proposition 4.1) We need to calculate the probability of the event that at least one agent chose C given that we have a sharing T and a story claiming $m = y$. Let this probability be r_T. We have that $r_T = \mathbb{P}_{pT}\{\exists i \in \mathcal{V}_T : A_{it} = C \mid T, m = y\} = 1 - \mathbb{P}_{pT}\{\forall i \in \mathcal{V}_T : A_{it} = S \mid T, m = y\}$. The last equality is equivalent, since the worst case scenario is that none inspected the story and chose to share, in other words $A_{it} = S$, $\forall i \in \mathcal{V}_T$. Since we assumed that events are independent the probability r_T is the product of all those independent experiments, thus $r_T = 1 - \mathbb{P}_{pT}\{\forall i \in \mathcal{V}_T : A_{it} = S \mid T, m = y\} \approx 1 - S^p_{00} S^p_{11} S^p_{22} \ldots S^p_{i(t-1)}$ where we rearranged agents id's to match the round at where they reacted throughout the propagation process due to the fact that actions are calculated independently. Thus we have that $r_T = 1 - \prod_{\substack{k=0 \\ i \in \mathcal{V}_T}}^{t-1} \mathbb{P}[A_{ik} = S^p_{ik}]$, where S^p_{ik} is platform's perceived value that agent i chose S in round t. $\square$

This value is the platform's posterior belief, just before the t-th round of sharing, that at least one of the internal nodes actually conducted a check. Additionally, the above calculation provides us with a warning indicator that approximates the approach followed in [13], where platform has a normalized belief q_p that a story is fake, based on the evolution of a sharing process in a path. This approach cannot be used in the case where we have a tree structure since there are nodes that we are unsure of their reaction. To further understand this issue, let's assume that we have an m-ary tree and there is a node in first level that did not react after $m + t_0$ rounds. If the value of t_0 is large enough at a point where the process evolved in lower depths, then it is safe to assume that agent i either rejected the story by choosing B or inspected the story and disclosed it's validity with C. It is obvious that there is a challenge in order to calculate this probability since we need to take into account the fact that the agent's reaction concerning shares are partially hidden. The next proposition calculates the existence of such an agent, more specifically, the probability that at least one agent inspected the story and found it fake.

Proposition 4.2. *Let T be an k-ary sharing tree and $m = y$ is the story that propagates in T. Then the platform's belief that a terminal node i blocked the story m by inspecting it, $A_{it} = C$, in round t is:*

$$n_{it} = [1 - (1/k)^{l_i}]^{t-t_i} (1/k)^{l_i} \frac{C^p_{it}}{B^p_{it} + C^p_{it}}$$

where l_i is the level of node i in T and t_i is the round where i received the story.

Proof. Let i be a random node of T and $l_i > 0$ [2] its' depth in it. We will prove the claim by induction over $t - t_i$. For $t - t_i = 0$, which means after the first time that node i was candidate to react, there are to cases:

- With probability $1 - (1/k)^{l_i}$, node i it is not picked to react at current round.
- With probability $(1/k)^{l_i}$, node i reacts in that round.

[2] We do not bother proving the proposition for root since it holds trivially.

In case where the node is picked to react, the probability that this node will inspect the story and then decide to block it because it is fake, is equal to $\frac{C_{it}^p}{B_{it}^p+C_{it}^p}$. Thus we have that:

$$n_{i,t_i} = (1/k)^{l_i} \frac{C_{it}^p}{B_{it}^p + C_{it}^p}$$

after the first round that i was candidate to react and it was picked for that round. On the other hand, if we move at the next round where i is once again candidate to react, then the probability that he will react in $t - t_i = 1$ is:

$$n_{i,t_i+1} = P\{\text{did not react in previous round}\} \text{ P\{ reacts in t round with C\}} =$$

$$[1 - (1/k)^{l_i}](1/k)^{l_i} \frac{C_{it}^p}{B_{it}^p + C_{it}^p}$$

Assuming the claim holds for $t - t_i$, we can easily prove that it holds for $t - t_i + 1$ as well. □

4.3 Optimization Criterion for Inspection Time

In this section we develop an optimization criterion based on utility maximizing approach as in [13]. Recall that we developed a model where the agents are socially responsible, which means that they share only truthful stories and also aim to maximize their utility. We assume that it is to the interest of the platform to forbid the propagation of fake stories, i.e. such platforms try to maintain their reliability in order to form a profitable model from advertisement revenue. In each round t, the platform observes an activation of a node in the sharing tree. This means that rounds represent a counter for the amount of nodes that reacted throughout the sharing process. At each round, and according it's belief for the validity of the story, platform can perform a global check if it suspects that the story is fake and can cause damage to it's credibility. In case where platform believes that the story is truthful we do not have any interruption of the story.

In order to develop a criterion to calculated the existence of an optimal time to interrupt the process in order to avoid further damage that un checked shares will cause, we develop a utility maximizing scheme. We assume that if the story is fake, then for each successful share platform receives penalty P. For each successful share of a truthful story, the platform will receive a discounted reward R, with a discount factor $\delta < 1$ that is affected by the depth of the sharing process. The discount factor rationalization is that freshly shares are more relevant in order to form an opinion over the validity of our story. If platform decides to intervene in order to check the validity of a story, this action occurs with a cost K_p, and the this decision is once and for all. This means that after the announcement of the results, we have two cases. If the current story is fake, then the process ends with the appropriate penalties for each share, while if it is truthful, an information cascade triggers and platform will collect all future discounted rewards.

Platform determines its' policy by estimating the utility of the sharing tree and the option of inspecting or not is viable respectively. At each round t, the process already created a sharing tree T. Therefore, we have an *observed* utility that either we will receive penalty if the story is fake or collect reward otherwise. We define the utility that platform gains, for each agent i in round t, as:

$$g_{pT}(V) = \begin{cases} P, & \text{w.p. } S_{it} q_{pT} \textit{ if } V = F \\ R, & \text{w.p. } (S_{it} + C_{it}) q_{pT} \textit{ if } V = T \\ 0, & \text{else} \end{cases} \tag{4.1}$$

Assuming that platforms belief, that the story is fake, is q_{pT}, we have the following lemma for the observed utility of our platform:

Lemma 4.1. *The expected observed discounted utility that platform gains from a sharing tree T is:*

$$O_p(T) = \sum_{i \in \mathcal{V}_T} \delta^{l_i} \left[P(S^p_{it} q_{pT}) + R(S^p_{it} + C^p_{it}) q_{pT} \right]$$

where l_i is the depth that agent i belongs and $\mathcal{V}_T$ is the set of nodes/agents that belong in T.

Proof. Similarly to proposition 3.2, we have that:

$$O_p(T) = \mathbb{E}[g_{pT}(V)]_{\forall i \in \mathcal{V}_T, V \in \{T,F\}}$$

Thus, for each internal node that we already observed its reaction, we receive a discounted reward in case it successfully shared the story. In case an internal node shares a fake story, and assuming platform's belief that the story is fake equals to q_{pT}, it comes with penalty P. According to equation 4.1:

$$O_p(T) = \mathbb{E}[g_{pT}]_{\forall i} = \sum_{i \in \mathcal{V}_T} \delta^{l_i} \left[p(S^p_{it} q_{pT}) + r(S^p_{it} + C^p_{it}) q_{pT} \right]$$

□

Let's assume that we observe a sharing tree T and define the frontier $\mathcal{F}$, that is a subgraph of T, as those nodes that are leaves of T (candidates to react). Then we can calculate the expected utility that we will gain with the addition/s of nodes that belong in frontier. Let's assume that node j will react and enter in the sharing tree such $T \cup \{j\}$. The we can anticipate either from a subtree T', with j as root, if the story is truthful. In worst case scenario, where the story is fake, we expect from the same tree, a penalty for each descendant of j. We have two possible policies, to make a global check, annotated as $\mathcal{C}$, and collect reward/penalty or to let the process continue, annotated as $\mathcal{E}$, and reevaluate the expected utility once again. If the platforms' belief that the story is fake equals to q_{pT} then the expected discounted utility gained from subtree T' if platform chooses to global check, is:

$$(\delta^{l_i} R + \delta^{l_i+1} kR + \delta^{l_i+2} k^2 R + ...)(1 - q_{pT}) - K_p$$

where K_p is the negative utility gained because the cost of inspection. If $\delta k < 1$ we conclude that:

$$U_{\mathcal{C}} = \delta^{l_i}(R + \delta kR + \delta^2 k^2 R + ...)(1 - q_{pT}) - K_p = \delta^{l_i}(1 - q_{pT})\frac{R}{1 - \delta k} - K_p$$

Observe that this is the anticipated utility gained only by one agent $j \in \mathcal{F}$. If we want to in account every other agent j in frontier we sum up the anticipated costs and we have:

$$U_{\mathcal{C}} = \sum_{j \in \mathcal{F}} \delta^{l_j+t}(1 - q_{pT})\frac{R}{1 - \delta k} - K_p$$

Now let us discuss the utility that the platform will receive if it decides that it will not intercept the sharing process. According to that strategy, $\mathcal{E}$, we have the next probable outcomes:

- With probability C_{it}, agent i checks and decide to only share a truthful story and platform collects reward.
- With probability $(1 - q_{pT})S_{it}$, agent i shares a truthful story and platform collects reward.
- With probability $q_{pT}S_{it}$, agent i shares a fake story and platform receives penalty.

Then, the utility we gain from agent i, if the platform decides to $\mathcal{E}$, is given by the equation bellow:

$$C_{it}(1-q_{pt})R + S_{it}(1-q_{pT})R + S_{it}q_{pT}P$$

Observe that the reaction of agent i creates a subtree T' with node i as root. Then we have the recursive formula for the discounted anticipated utility that will grow exponentially and make the calculations complex. It is obvious that it is not feasible to calculate the utility that way for two main reasons. First reason is the complexity of calculations and the fact that is hard to find a closed form type for that expected utility in order to maximize it. Secondly, the platform's belief is changing while the depth increases and this formula should recalculate q_{pT} at each step which increases the complexity even more.

In order to deal with that problem, we propose the next approximation method. Suppose that platform decided already the validity of the story at round t. Then we have two probable cases where:

- The story is fake with probability q_{pT} and platform chooses strategy $\mathcal{E}$.
- The story is true, following the same strategy.

This modification simplifies the calculations by breaking down the problem in two different cases. Let us see what happens in the case where platform decides that the story is fake. There is only one way that fake stories will propagate after the platform decides to let the propagation evolve and it is only if an agent decides to share without check, namely S_{ih}. Notice that if he decided to check the story given that platform believes it is fake, implies that the agent will discontinue the propagation. So we have that the utility in that case is:

$$\sum_{t=0}^{\infty} \delta^{l_j+t}\left[S_{l_j+t}k^t P\right]$$

where l_j is the depth that agent j belongs, $\mathcal{T}_j$ is the subtree, rooted on agent j that belongs in frontier and δ is the discount factor. If the platform decides that a story is true over a propagation tree T, then the story propagates with two ways. A story can be sent to the next level in the tree with S or C action since it is true and agents even when fact-checking it, will share it. Therefore, the probability that a story is shared equals to $(1-B_{it})$. This observation simplifies things since it is easier to express the monotonicity of the expression bellow. For that policy, we have that the next equation that expresses that utility:

$$U_t^A = \sum_{t=0}^{\infty} \delta^{l_j+t} \left[(1 - B_{l_j+t}) k^t R \right]$$

where $\mathcal{T}_j$ is the subtree, rooted on agent j that belongs in frontier. Now we are ready to collect those expressions in the next proposition that gives a formula for the platforms utility if it lets the propagation evolve.

Proposition 4.3. *The anticipated utility that platform gains if it decided to let the news sharing process evolve, for an agent $j \in \mathcal{F}$ in frontier is:*

$$U_f^A = \sum_{t=0}^{\infty} \delta^{l_j+t} \left[S_{l_j+t} k^t P \right]$$

for fake stories and

$$U_t^A = \sum_{t=0}^{\infty} \delta^{l_j+t} \left[(1 - B_{l_j+t}) k^t R \right]$$

for truthful.

The above probabilities, S_{il_i} and $(1 - B_{il_i})$. Both of the above values will express the utility we gain for each agent for a sub-tree created in the frontier, rooted at agent j. This means we get the appropriate utility from agent j, discounted by δ^{l_j}, plus the corresponding discounted utility of his k descendants and so on. Before we find the boundaries, we collect the above quantities in order to form the final expression of our expected discounted utility:

$$U_p = O_p(T) + A_p(T)$$

where the anticipated utility of our propagation tree network T is:

$$A_p(T) = U_{\mathcal{E}} - U_{\mathcal{C}} \tag{4.2}$$

Now remains the issue of limits. As we fore mentioned, for each agent in the frontier, in order to calculate the anticipated utility we have to calculate the probabilities of S and B indefinitely. In order to deal with that, we need convergence for U_f^A and U_t^A. The next proposition uses the monotinicity of probabilities S and B in order to bound those values.

Proposition 4.4. *For the anticipated utilities over fake and truthful news respectively, it holds that:*

$$U_f^A < \delta^{l_j} S_{l_j} \frac{P}{1-\delta k} = \overline{U}_f^A$$

for fake stories and

$$U_t^A < \delta^{l_j}(1 - B_{l_j}) \frac{R}{1-\delta k} = \overline{U}_t^A$$

for truthful, where j is the corresponding root agent that belongs in frontier.

Proof. We have from corollary 3.3 that S_{l_j} probability is increasing in depth l_j, thus we have that $(S_{l_j} < S_{l_i}$ in every level l_i lower than l_j, since S increases in depth implies that $(1 - S_{l_j})$ decrease in depth. Thus we have that:

$$U_f^A = \delta^{l_j}(P(1 - S_{(l_j)}) + \delta k P(1 - S_{(l_j+1)}) + \delta^2 k^2 P(1 - S_{(l_j+2)}) + ...) <$$
$$\delta^{l_j}(P(1 - S_{(l_j)}) + \delta k P(1 - S_{(l_j)}) + \delta^2 k^2 P(1 - S_{j(l_j)}) + ...) = \delta^{l_j} S_{l_j} \frac{P}{1-\delta k}$$

Respectively, we have that B is decreasing in depth. Thus $(1 - B_{l_i}) > (1 - B_{l_j})$ for every level l_j lower than l_i. In similar manner we can restrict, the anticipated utility for the case a true story, such that:

$$U_t^A = \delta^{l_j}(R(1 - B_{(l_j)}) + \delta k R(1 - B_{j(l_j+1)}) + \delta^2 k^2 R(1 - B_{(l_j+2)}) + ...) <$$
$$\delta^{l_j}(R(1 - B_{(l_j)}) + \delta k R(1 - B_{(l_j)}) + \delta^2 k^2 R(1 - B_{j(l_j)}) + ...) = \delta^{l_j}(1 - B_{l_j}) \frac{R}{1-\delta k}$$

□

In the last proposition, we use the probabilities of root j in the frontier, in order to bound the geometric series. This is an important step, which will help in advance to find a closed type equation for the anticipated utility. This allows the platform to make a better prediction instead of letting the process continue up to the point that agents propagation process arrive at some depth T_c where they all choose to share.

The next proposition calculates upper and lower thresholds for the values U_f^A and U_t^A respectively. Those values and their existence depends on the starting state of the process and the perceived values of proposition 3.4. Additionally, those bounds reduce the complexity of the calculations, since we do not go further in each subtree, in order to calculate the probabilities B_{it}, C_{it}, S_{it}.

Proposition 4.5. *Depending the initial state of thresholds of proposition 3.4, it holds that:*

$$\delta^{l_j} B_{l_j} \frac{P}{1-\delta k} = \underline{U}_f^A < U_f^A < \overline{U}_f^A$$

for fake stories and

$$\delta^{l_j}(1 - S_{l_j}) \frac{R}{1-\delta k} = \underline{U}_t^A < U_t^A < \overline{U}_t^A$$

for truthful. The above thresholds hold only when $S_{i0} > B_{i0}$ at the initial state.

Proof. The proof of this claim is similar with the proposition 4.4. We we only bound the values $(1 - S_{l_i})$ and $(1 - B_{l_i})$ with the constraint we have in the proposition that $S_{i0} > B_{i0}$ and the limits of geometric series are derived with the same manner. First, we have that initial $S_{i0} > B_{i0}$, thus we have that $1 - S_{i0} < 1 - B_{i0}$. From the monotonicity of those probabilities we have that:

$$\begin{aligned} 1 - S_{i1} &< 1 - B_{i1} \\ 1 - S_{i2} &< 1 - B_{i2} \\ &\dots \\ 1 - S_{ih} &< 1 - B_{ih} \end{aligned} \tag{4.3}$$

This proves the claim since we can bound all the terms from each inequality, such that:

$$\begin{aligned} U_f^A = \delta^{l_j}(P(S_{(l_j)} + \delta k P S_{(l_j+1)} + \delta^2 k^2 P S_{(l_j+2)} + \dots) > \\ \delta^{l_j}(P B_{(l_j)} + \delta k P B_{(l_j+1)} + \delta^2 k^2 P B_{(l_j+2)} + \dots) > \\ \delta^{l_j}(P B_{(l_j)} + \delta k P B_{(l_j)} + \delta^2 k^2 P B_{(l_j)} + \dots) = \delta^{l_j} B_{l_j} \frac{P}{1-\delta k} \end{aligned}$$

and

$$\begin{aligned} U_t^A = \delta^{l_j}(R(1 - B_{(l_j)}) + \delta k R(1 - B_{j(l_j+1)}) + \delta^2 k^2 R(1 - B_{(l_j+2)}) + \dots) > \\ \delta^{l_j}(R(1 - S_{(l_j)}) + \delta k R(1 - S_{(l_j+1)}) + \delta^2 k^2 R(1 - S_{(l_j+2)}) + \dots) > \\ \delta^{l_j}(R(1 - S_{(l_j)}) + \delta k R(1 - S_{(l_j)}) + \delta^2 k^2 R(1 - S_{(l_j)}) + \dots) = \delta^{l_j}(1 - S_{l_j}) \frac{R}{1-\delta k} \end{aligned}$$

□

Notice that the above boundaries are not that tight in terms of how they enclose the actual values. They also depend on the initial state of the system which is not necessary useful from algorithmic perspective. Their significance is on the time complexity and the fact that include all the characteristics of the propagation process,

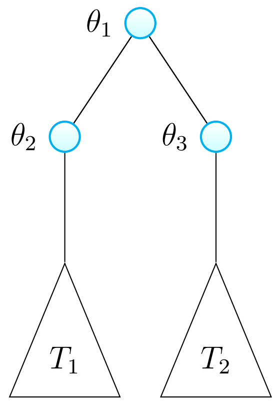


Figure 4.3: Evolution of platforms' utility on a binary propagation tree. On round 1, the utility of agent was already evaluated and the policy was chosen by platform. This is a greedy approach of finding the optimal policy.

the discount factor, the platforms belief, the depth of the tree and the breadth (the average neighborhood size which is k).

With the above propositions, and now that we establish the thresholds for the anticipated utility, we can prove that the observed utility does not affect the sign of the total expected utility. Therefore, it is sufficient to observe the anticipated utility, for both policies, in order to decide whether to terminate the process. If begin at round $t = 0$, where the first agent, 1 with θ_1, has already reacted and we have to calculate the anticipated utility that platform will gain from his children. We assume that the average neighbor size is $k = 2$. Then we have that:

$$U_p(T) = O_p(T) + A_p(T) = O_p(\{\theta_1\}) + A_p(\{\theta_1\})$$

where in that case the T consists only the first node θ_1 and $A_p(\{\theta_1\}) = U_{\mathcal{E}} - U_{\mathcal{C}}$.

Whenever a node reacts, it adds k new nodes in frontier and removes the node that reacted in the previous round as we see in figure 4.3. In our case, since $k = 2$, we get 2 new nodes in the first level. Since the observed cost is $\sum_{i \in \mathcal{V}_T} \delta^{l_i} \left[p(S^p_{it} q_{pT}) + r(S^p_{it} + C^p_{it}) q_{pT}\right]$, where in first round i is the root node with prior opinion θ_1, is a positive quantity. Observed utility is positive for each round t. Thus the sign of total utility is affected only from the new nodes added in the current round, 2 in the case of binary tree. This implies that if exists a round t where the difference of the anticipated value change, the observed value would not affect this change. By induction we can prove the claim for every round t. Also the proof holds for any k integer by induction.

To sum up, we established the mechanism under how platform monitors and reacts to the process. First, we observed that the tree structure is affected on how agents are picked react. In order to avoid unbalanced trees, we decided that the level at where agent belongs affects the reaction time (agents that receive the story earlier will probably react earlier as well). Secondly, we observed because the inspection is perfect, if a path reaches at some depth will imply that we have at least one checking action, which is sufficient to rely on him. We can use the last observation as a flag where in order to intervene and verify the story (using a third party fact checking organism). Lastly, we follow the same strategy as Papanastasiou in [13], using a utility maximization criterion. After we provide bounds for the expected anticipated utility, we use can observe where the penalty or cost will greater than the earning, and make an earlier decision before we reach critical depths, as we mentioned previously in this paragraph.

Chapter 5

Conclusions and Future Work

In this thesis we introduced a model for fake news based on the structural pattern of the propagation process. After research on current literature, we concluded that structural patterns are more appropriate for dealing with the problem and can be flexible in terms of application in real scenarios. After studying and adjusting the preexisting model, given by [13] which is a simplified version, we constructed a similar analysis for a tree propagation network. We achieve to retain the most important properties from the simplified, that we found useful in for the problem of fake news.

Specifically, we deal with the complexity we have on tree structure by assuming the correct independence assumption and the fact that the distribution is a Gaussian, which simplifies the model, for what agents presume for others actions. We also provide alternative thresholds for actions, that are simpler to compute. Finally, for agents propagation dynamics, we reform threshold values in simpler forms and we revise the point at where the process will trigger a cascade in trees structures.

In chapter 4, we introduced the platform's role in our model. In the case where the propagation network is a directed path, things are very simple and we have almost perfect values for cascade time, the optimal inspection time and the threshold values. We mention the challenges that arise in tree propagation networks and we find solutions in order to find optimal inspection time. We calculate threshold values for the optimal inspection time, with respect the assumptions and the observations we demonstrate and prove in that section.

Our thesis indicates that this approach is more appropriate in practice, since it includes details that related literature ignores, such as the perceived knowledge that entities have and the fact that beliefs are non homogeneous (the fact when news are spread with a certain way, we change our beliefs accordingly). The threshold we provide in our work is a first approach with the properties we prove in previous chapters. Given that we need an approximate solution for the utility maximization, we can focus on improving those thresholds. For future work, it is interesting to develop a model where agents and platform conduct imperfect inspection (i.e. we need certain paths to reach at some point where we most certainly we have *enough* checking agents). Also, we can go one step further, and study a model that work on directed acyclic graphs in general. This reflects a more realistic setup where agents are receiving the story from more than one agent, and react at some round t.

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Appendix A

Supplementary Material for Chapter 3

Proof of proposition 3.3.

Proof. Let B_{it}, S_{it}, C_{it} be the probabilities that agent i chooses action B, S, C respectively, at round t. Then, we have from 3.1 that:

$$q_{it} = \mathbb{P}_i\{V = F|m = y, H_i\} = \frac{\beta v w_{it}}{\beta v w_{it} + [a\theta_i + (1-a)(1-\theta_i)](1-v)} \tag{A.1}$$

where $w_{it} = \prod_{k=0}^{t-1} \frac{S_{ik}}{S_{ik} + C_{ik}}$ and $w_{i0} = 1$. Independently from the fact that we are using a Gaussian distribution, we have that:

$$B_{it} = \mathbb{P}(q_{it} > 1 - K)$$

$$S_{it} = \mathbb{P}(1 - q_{it} > 1 - K)$$

$$C_{it} = 1 - B_{it} - S_{it}$$

Using A.1 and the properties a Gaussian pdf, we have the equations of the propositions. □

Proofs of Lemmas 3.1 and 3.2.

Proof. We will use proposition 3.1 in order to prove those lemmas. It is obvious that:

$$q_{it} = \mathbb{P}_i\{V = F|m = y, H_i\} = \frac{\beta v w_{it}}{\beta v w_{it} + [a\theta_i + (1-a)(1-\theta_i)](1-v)}$$

is strictly decreasing while θ is decreasing since $1/2 < a < 1$.

For lemma 3.2 we have the likelihood function:

$$\frac{q_{it}}{1-q_{it}} = \frac{\beta v w_{it}}{[a\theta_i + (1-a)(1-\theta)](1-v)}$$

and since it is strictly increasing in w_{it} we have that q_{it} is strictly increasing in w_{it} which is non increasing since it is a product of fractions that are lower than 1. This concludes the claim that q_{it} is non increasing in t. $\square$

Short Biography

Georgiadis Ioannis was born in Kavala, Greece in 1991. He studied in Department of Mathematics, University of Ioannina, where he graduate in 2018. In the same year, he enrolled in Department of Computer Science and Engineering of Ioannina as MSc student. He recently start working as a software development. His research interests are in the areas of applied mathematics, more specifically in applied algebra as well in graph theory.