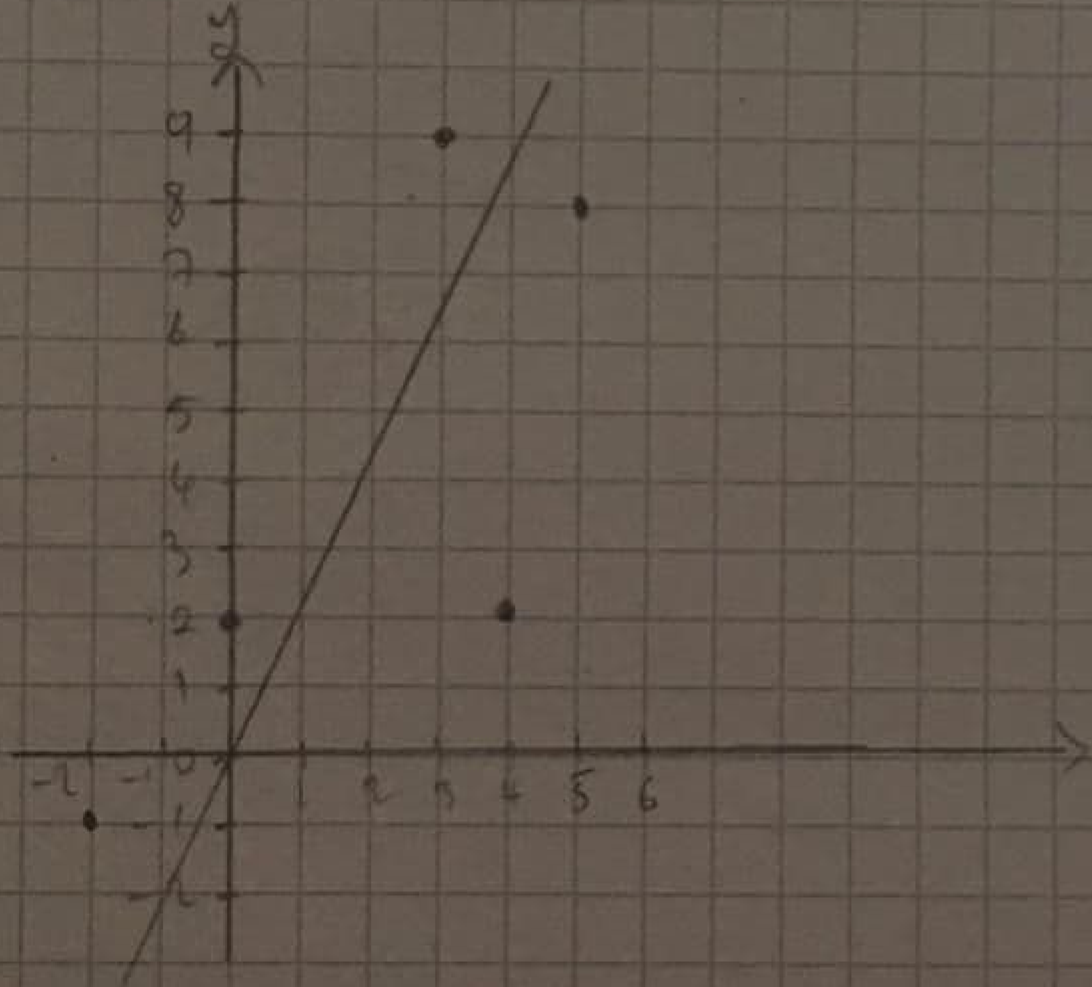
3.2 b)



Nein, die beidn Variabeln sind nicht stark korreliert, weil die einzeln Punkte unsere gerade nicht sehr nah sind.

Hausaufgabe 3

3.2

$(x_1, y_1), \ldots, (x_5, y_5) = (0,2), (5,8), (3,9), (-2,-1), (4,2)$

Mittelwert $x = 0+5+3+(-2)+4 = 2$

Mittelwert $y = 2+8+9+(-1)+2 = 4$

a) Kovarianzmatrix $= \begin{pmatrix} s_{xx} & s_{xy} \\ s_{yx} & s_{yy} \end{pmatrix} = s_{xx} \cdot s_{yy} - s_{xy} \cdot s_{yx}$

$s_{xy} = \frac{1}{n} \sum_i (x_i - \bar{x})(y_i - \bar{y})$

$\frac{1}{5} (0-2)(2-4) + (5-2)(8-4) + (3-2)(9-4) + (-2-2)(-1-4) + (4-2)(2-4)$

$= \frac{1}{5} (4+12+5+20-4)$

$s_{xy} = \frac{37}{5} = \underline{7{,}4}$

$s_x^2 = \frac{1}{n} \sum_i (x_i - \bar{x})^2$

$= \frac{1}{5} (0-2)^2 + (5-2)^2 + (3-2)^2 + (-2-2)^2 + (4-2)^2$

$= \frac{1}{5} (4+9+1+16+4) = \underline{\underline{6{,}8}}$

$s_x = \underline{2{,}6}$

$s_y^2 = \frac{1}{5} (2-4)^2 + (8-4)^2 + (9-4)^2 + (-1-4)^2 + (2-4)^2$

$= \underline{\underline{14{,}8}}$

$s_y = \underline{\underline{3{,}8}}$

Kovarianzmatrix $= s_{xx} \cdot s_{yy} - s_{xy} \cdot s_{yx} = 6{,}8 \cdot 14{,}8 - 7{,}4 \cdot 7{,}4 = \underline{\underline{45{,}88}}$

b) Korrelation $r_{xy} = \frac{s_{xy}}{s_x \cdot s_y} = \frac{7{,}4}{(2{,}6)(3{,}8)} = \underline{\underline{0{,}75}}$

$\Downarrow$

d) $x_i' := 2 \cdot x_i$ für alle $i = 1, \dots, n$

$(x_1, y_1), \dots, (x_5, y_5) = (0,2), (10,8), (6,9), (-4,-1), (8,2)$

$$\text{Kovarianzmatrix} = \begin{pmatrix} s_{xx} & s_{xy} \\ s_{yx} & s_{yy} \end{pmatrix} = s_{xx} \cdot s_{yy} - s_{xy} \cdot s_{yx}$$

Mittelwert $x = 0 + 10 + 6 + (-4) + 8 = \underline{\underline{20}}$

Mittelwert $y = \underline{4}$

$$s_x^2 = \frac{1}{n} \sum_i (x_i - \bar{x})^2$$

$$= \frac{1}{5} \overset{400}{(0-20)^2} + \overset{100}{(10-20)^2} + \overset{196}{(6-20)^2} + \overset{576}{(-4-20)^2} + \overset{144}{(8-20)^2}$$

$$= \frac{1.416}{5} = \underline{\underline{283,2}}$$

$$s_y^2 = \frac{1}{n} \sum_i (y_i - \bar{y})^2$$

$$= \frac{1}{5} \qquad = \underline{\underline{14,8}}$$

$$s_{xy} = \frac{1}{n} \sum_i (x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{5} (0-20)(2-4) + (10-20)(8-4) + (6-20)(9-4) + (-4-20)(-1-4) + (8-20)(2-4)$$

$$= \frac{1}{5} (40 - 40 - 70 + 120 + 24)$$

$$= 74/5 = \underline{\underline{14,8}}$$

$$\text{Kovarianzmatrix} = 283,2 \cdot 14,8 - 14,8 \cdot 14,8$$

$$= \underline{\underline{3972,32}}$$

c) $(x_i', y_i') := (x_i, y_i) + (-1, 2)$ für alle $i = 1, \dots n$

$$(x_1, y_1), \dots, (x_5, y_5) = (-1,4), (4,10), (2,11), (-3,1), (3,4)$$

Mittelwert $x = (-1) + 4 + 2 + (-3) + 3 = 5$

Mittelwert $y = 4 + 10 + 11 + 1 + 4 = 30$

Kovarianzmatrix $= \begin{pmatrix} s_{xx} & s_{xy} \\ s_{yx} & s_{yy} \end{pmatrix} = s_{xx} \cdot s_{yy} - s_{xy} \cdot s_{yx}$

$$s_x^2 = \frac{1}{n} \sum_i (x_i - \bar{x})^2$$

$$= \frac{1}{5} \overset{36}{(-1-5)^2} + \overset{1}{(4-5)^2} + \overset{9}{(2-5)^2} + \overset{64}{(-3-5)^2} + \overset{4}{(3-5)^2}$$

$$= \frac{114}{5} = \underline{22{,}8}$$

$$s_y^2 = \frac{1}{n} \sum_i (y_i - \bar{y})^2$$

$$= \frac{1}{5} \overset{676}{(4-30)^2} + \overset{400}{(10-30)^2} + \overset{361}{(11-30)^2} + \overset{841}{(1-30)^2} + \overset{676}{(4-30)^2}$$

$$= \frac{2954}{5} = \underline{590{,}8}$$

$$s_{xy} = \frac{1}{n} \sum_i (x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{5} (-1-5)(4-30) + (4-5)(10-30) + (2-5)(11-30) + (-3-5)(1-30) + (3-5)(4-30)$$

$$= \frac{1}{5} (-32 + 20 + 57 + 232 + 52)$$

$$= \underline{65{,}8}$$

Kovarianzmatrix $= 22{,}8 \cdot 590{,}8 - 65{,}8^2 = \underline{91406}$

Aufgabe 3.3)

A)

$$\bar{x} = 1 \; ; \; \bar{y} = \frac{7}{4}$$

$$s_x^2 = \frac{1}{4} \sum_i (x_i - \bar{x})^2 = 2.5$$

$$s_x = \cancel{1{,}5811} \sqrt{2.5}$$

$$s_{xy} = \frac{1}{4} \sum_i (x_i - \bar{x})(y_i - \bar{y}) = 2.75$$

$$a = \frac{s_{xy}}{s_x^2} = 1.1 \qquad b = \bar{y} - a\bar{x} = 0.65$$