1) a) $\vec{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $g_1: \vec{x} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + r \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

b) $g_2: \vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + r \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$

c) $g_3: \vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + r \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

2) a)

$$r \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix} + s \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + u \begin{pmatrix} 4 \\ 0 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -3 & 0 \\ -1 & 1 & 1 & 0 & 0 \end{array}\right) \begin{array}{l} \cdot(-2) \\ \\ + \\ \\ \end{array} \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 0 \\ 0 & 1 & -1 & -8 & 0 \\ 0 & 1 & 1 & -3 & 0 \\ 0 & 1 & 2 & 4 & 0 \end{array}\right) \begin{array}{l} \cdot(-1) \\ \\ \cdot(-1) \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 0 \\ 0 & 1 & -1 & -8 & 0 \\ 0 & 0 & 2 & 5 & 0 \\ 0 & 0 & 3 & 12 & 0 \end{array}\right) \begin{array}{l} \\ \\ \cdot(-1{,}5) \\ \\ \end{array} \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 0 \\ 0 & 1 & -1 & -8 & 0 \\ 0 & 0 & 2 & 5 & 0 \\ 0 & 0 & 0 & 4{,}5 & 0 \end{array}\right)$$

IV $4{,}5\,u = 0 \Rightarrow u = 0$ III $2t + 5u = 0 \Rightarrow t = 0$

II $s - t - 8u \Rightarrow s = 0$ I $r + t + 4u = 0 \Rightarrow r = 0$

$L = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ die 4. Vektoren sind linear unabhängig

2)b)

$$r\begin{pmatrix}1\\2\\3\\3\end{pmatrix}+s\begin{pmatrix}-2\\1\\4\\-4\end{pmatrix}+t\begin{pmatrix}5\\-3\\-8\\10\end{pmatrix}+u\begin{pmatrix}3\\3\\6\\8\end{pmatrix}=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

$$\left(\begin{array}{cccc|c}1&-2&5&3&0\\2&1&-3&3&0\\3&4&-8&6&0\\3&-4&10&8&0\end{array}\right)\cdot(-2)\ \cdot(-3)\ \cdot(-3)\Rightarrow\left(\begin{array}{cccc|c}1&-2&5&3&0\\0&5&-13&-3&0\\0&10&-23&-3&0\\0&2&-5&-1&0\end{array}\right)\cdot(-2)\ \cdot(-2{,}5)$$

$$\left(\begin{array}{cccc|c}1&-2&5&3&0\\0&5&-13&-3&0\\0&0&3&3&0\\0&0&0{,}2&0{,}2&0\end{array}\right)\cdot\left(-\frac{1}{15}\right)\Rightarrow\left(\begin{array}{cccc|c}1&-2&5&3&0\\0&5&-13&-3&0\\0&0&3&3&0\\0&0&0&0&0\end{array}\right)\Rightarrow \text{Nullzeile}$$

$u=u'$

III $3t+3u=0 \Rightarrow t=-u'$

II $5s-13t-3u=0 \Rightarrow s=-2u'$

I $r-2s+5t+3u=0 \Rightarrow r=-2u'$

$$L=\begin{pmatrix}-2u'\\-2u'\\-u'\\u'\end{pmatrix}$$

Die 4 Vektoren sind linear abhängig, denn es gibt unendlich viele Lösungen

a) $\overrightarrow{AB} = \begin{pmatrix}10-4\\-8-2\\9-(-1)\end{pmatrix} = \begin{pmatrix}6\\-10\\10\end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix}4-4\\0-2\\1-(-1)\end{pmatrix} = \begin{pmatrix}0\\-2\\2\end{pmatrix}$

$\overrightarrow{OS} = \frac{1}{3}\overrightarrow{AB} + \frac{1}{3}\overrightarrow{AC} = \begin{pmatrix}2\\-4\\4\end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix}-6\\8\\-8\end{pmatrix}$

$\overrightarrow{AM_a} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} = \begin{pmatrix}3\\-6\\6\end{pmatrix}$ $|\overrightarrow{AM_a}| = \sqrt{(3)^2+(-6)^2+(6)^2} = 9\ LE$

$\overrightarrow{BM_b} = -\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC} = \begin{pmatrix}-6\\9\\-9\end{pmatrix}$ $|\overrightarrow{BM_b}| = \sqrt{(-6)^2+(9)^2+(-9)^2} = 3\sqrt{22}\ LE$

$\overrightarrow{CM_c} = \frac{3}{2}(\overrightarrow{OS} - \overrightarrow{OC}) = \begin{pmatrix}-3\\-6\\4,5\end{pmatrix}$ $|\overrightarrow{CM_c}| = \frac{3\sqrt{29}}{2}$

b) $\frac{1}{3}\left(\begin{pmatrix}4\\2\\-1\end{pmatrix} + \begin{pmatrix}10\\-8\\9\end{pmatrix} + \begin{pmatrix}4\\0\\1\end{pmatrix}\right) = \begin{pmatrix}6\\-2\\3\end{pmatrix}$

c) $\overrightarrow{OS} - \frac{1}{3}\overrightarrow{OA} - \frac{1}{3}\overrightarrow{OB} = \frac{1}{3}\overrightarrow{OC} = \begin{pmatrix}5\\6\\3\end{pmatrix} - \frac{1}{3}\begin{pmatrix}4\\9\\2\end{pmatrix} - \frac{1}{3}\begin{pmatrix}15\\18\\9\end{pmatrix} = \begin{pmatrix}-\frac{4}{3}\\9\\-\frac{2}{3}\end{pmatrix}$

$\frac{1}{3}\overrightarrow{OC} = \begin{pmatrix}-\frac{4}{3}\\9\\-\frac{2}{3}\end{pmatrix} \Rightarrow \overrightarrow{OC} = \begin{pmatrix}-4\\27\\-2\end{pmatrix}$ $C(-4; 27; -2)$