

PY 5021 MAGNETIC SENSORS

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Ireland.

1. Introduction,
2. Principles
3. Sensors
4. Applications
5. Future prospects



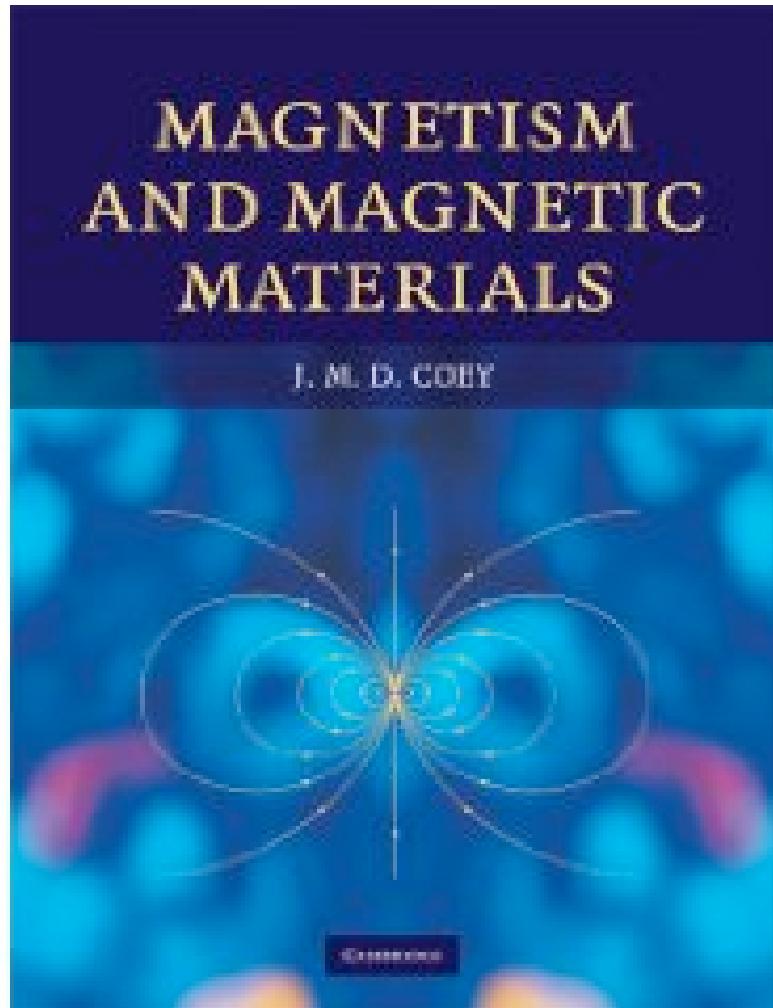
Comments and corrections please: jcoey@tcd.ie

www.tcd.ie/Physics/Magnetism

Billions of magnetic sensors are manufactured every year for hundreds of different applications. These lectures provide an up-to-date account of magnetic sensing and its applications. Sources of magnetic field are discussed, and the operating principles of the main sensor types will be described. There is a focus on thin-film devices, which offer the prospect of highly-sensitive, low cost integrated sensors and sensor arrays for applications ranging from nondestructive testing and biochips to advanced magnetic recording and magnetic imaging for clinical and security purposes

There is no up-to-date text devoted to magnetic sensors, but some useful references include:

- J. M. D. Coey; *Magnetism and Magnetic Materials*. Cambridge University Press (2010) 63-pp *An up to date general text on magnetism, which includes treatment of sensor principles and some applications.*
- P. Ripka (editor) *Magnetic Sensors and Magnetometers*, Artec House, Norwood MA 2001, 480 pp
- C. S. Roumenin *Solid State Magnetic Sensors, Handbook of Sensors and Actuators vol 2*, Elsevier, Amsterdam, 1994, 425 pp
- A new text by Panntier, Fermon and Coey is in preparation.



614 pages. Published March 2010

Available from amazon.co.uk for < £50

www.cambridge.org/9780521816144

- 1 Introduction
- 2 Magnetostatics
- 3 Magnetism of the electron
- 4 The many-electron atom
- 5 Ferromagnetism
- 6 Antiferromagnetism and other magnetic order
- 7 Micromagnetism
- 8 Nanoscale magnetism
- 9 Magnetic resonance
- 10 Experimental methods
- 11 Magnetic materials
- 12 Soft magnets
- 13 Hard magnets
- 14 Spin electronics and magnetic recording
- 15 Other topics
- Appendices, conversion tables.

1. Introduction

1.1 Overview of magnetic sensors.

1.2 Historical background



2. Principles of magnetic sensing

2.1 Basic concepts in magnetism

2.2 Sensor principles

2.2 Signal and Noise



3. Sensor types

3.1 Induction sensors

3.2 Semiconductor sensors

3.3 Thin film magnetic sensors

3.4 Superconductor sensors

3.5 Resonance sensors

3.6 Optical sensors



4. Sensor applications

4.1 Proximity sensing

4.2 Magnetic field and electric current measurement

4.3 Testing

4.4 Bioassy

4.5 Magnetic recording



5. Conclusions

5.1 Future prospects



Lecture 1: Introduction

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- I. Overview of magnetic sensors
2. Historical background



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1.1 Overview

Advantages of magnetic sensing. Units. Sources of magnetic field. Field magnitudes. A few applications - proximity sensors, field mapping, magnetic recording.



Magnetic sensors are passive devices that sense magnetic field. These sensors

- Offer contactless sensing
- Deliver an electrical response that is monotonic and often linear in field
- Detect one component of the field, or sometimes the scalar magnitude
- Can measure fields in the range fT to 100 T
- Sense magnetic field, electric current or proximity (with an external magnet, which may be the Earth)
- Operate on one of several quite distinct physical principles, detecting \mathbf{B} , \mathbf{H} , Φ or $d\Phi/dt$.
- Are manufactured in huge quantities, a few billion sensors every year in a market worth $\sim \text{€}7\text{B}$

A note on units:

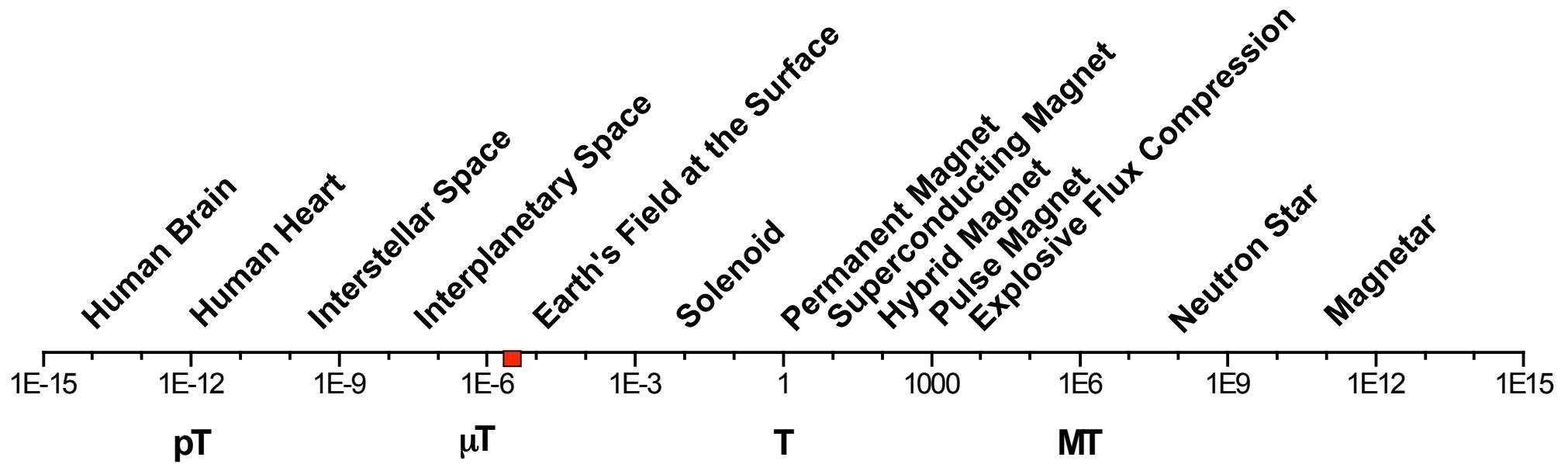
Magnetism is an experimental science, and it is important to be able to calculate numerical values of the physical quantities involved. There is a strong case to use SI consistently

- SI units relate to the practical units of electricity measured on the multimeter and the oscilloscope
- It is possible to check the dimensions of any expression by inspection.
- They are almost universally used in teaching
- Units of \mathbf{B} , \mathbf{H} , Φ or $d\Phi/dt$ will be introduced in §1.2

Range of magnetic fields in Nature

Magnetic field sources are

- distributions of electric current
- permanently magnetized material



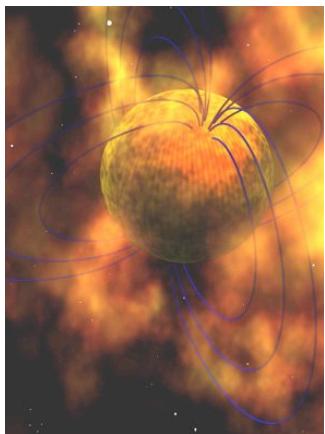
Typical values of B



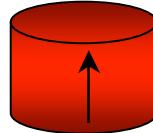
Earth $50 \mu\text{T}$



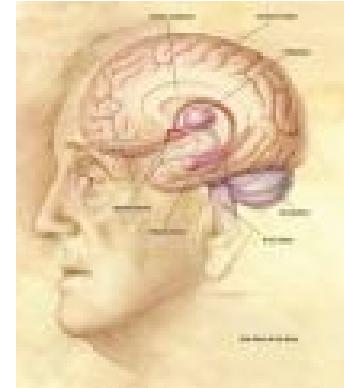
Helmholtz coils 10 mT



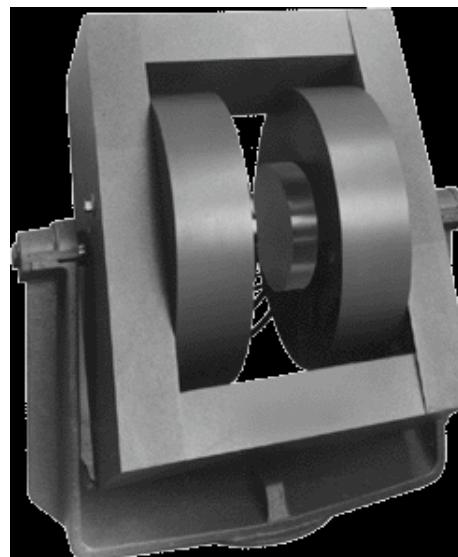
Magnetar 10^{12} T



Permanent magnet 1 T



Human brain 1 fT



Electromagnet 1 T



Superconducting magnet 10 T

Magnetic fields to be detected can be classified;

- Fields much greater than the Earth's field
- Fields of order the Earth's field $B_e \sim 40\mu T$
- Fields much less than the Earth's field

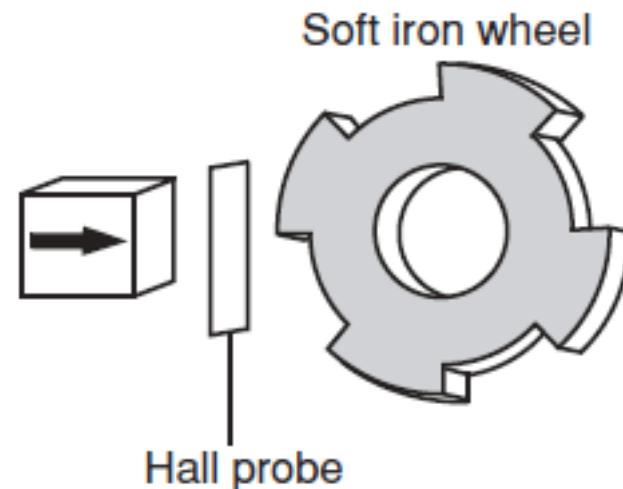


Magnetic sensor applications.

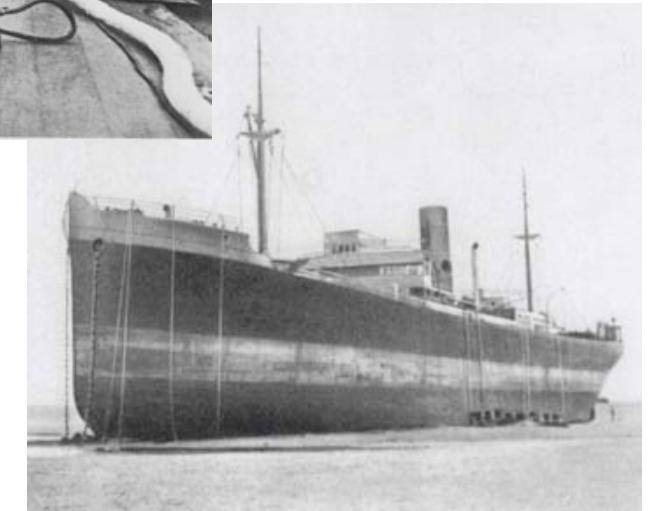
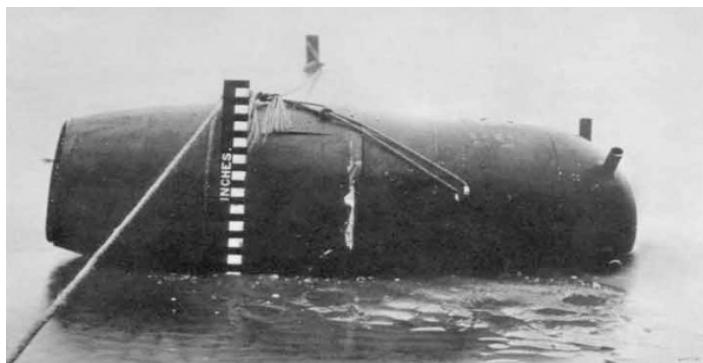
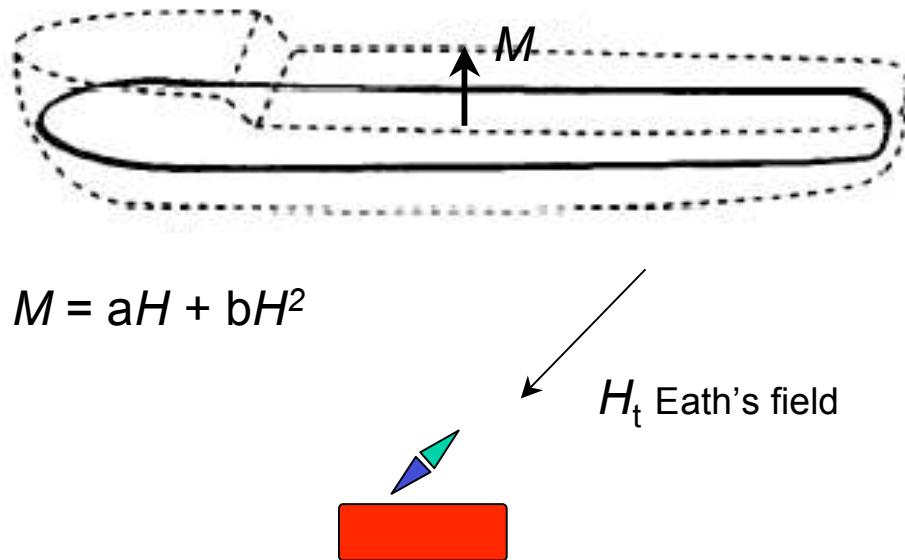
Proximity sensing.

Proximity of a magnet or magnetized object is detected via its stray field, usually in a 0/1 threshold mode.

- Simple proximity sensors
- Position sensors in brushless motors
- Angular position sensors
- Magnetic mines



The degaussing of ships. 1940



September 1939; War breaks out

Novembre 1939; First German mine recovered by the British

Janvier 1940; First demonstration of degaussing at Toulon

Avril 1940; Mission to London

Mai 1940 Dunkirk

Juin 1940; Armistice. 520 ships demagnetized (6 par jour à Toulon et Cherbourg)

Juillet 1940; Destruction of the French fleet at Mers el Kabir

Field mapping.

The magnetic field is mapped, quantitatively in space, usually in magnitude and direction.

- Geophysical surveys
- Planetary exploration
- Clinical applications (MEG, MCG)
- Nondestructive testing

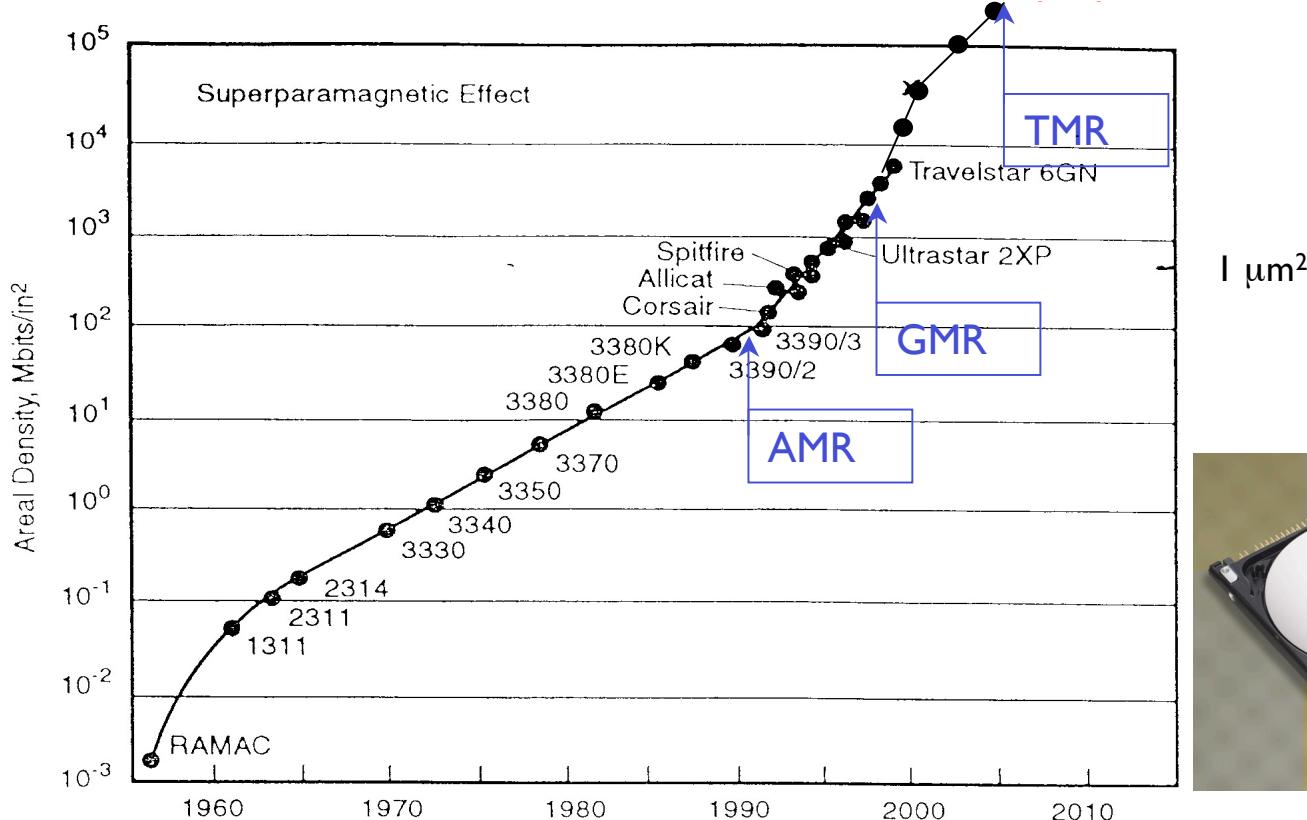


Data recording.

The stray field produced by records in a magnetic medium is detected, often at a very high data rate.

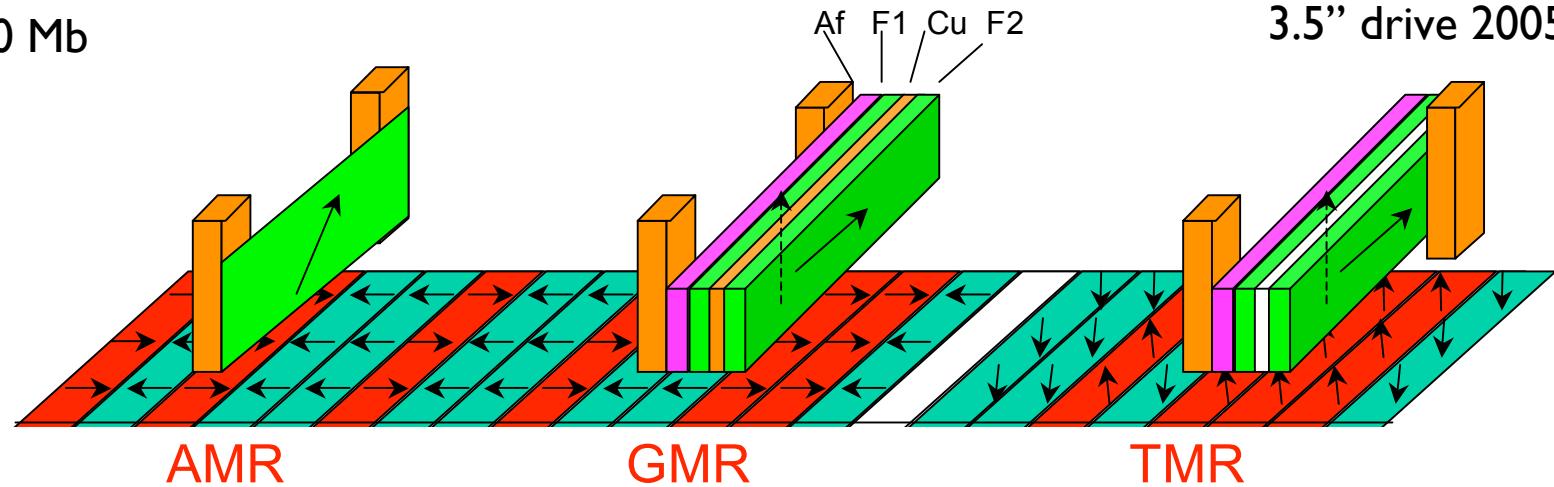
- Magnetic hard disc and tape recording
- Magnetic records on cards, tickets
- Magnetic barcode labels





RAMAC 1955 40 Mb

3.5" drive 2005 0.2 Tb



1.2 Historical Background

1.1.1 Historical note (Olmecs, Chinese South pointer, navigational compass, early images of the field of a magnet, dip circle, magnetic charts, the Earth as a magnet, Gauss's *Magnetverein* - international project to map the Earth's field in space and time which led to the development of spherical harmonic analysis - , electromagnetic induction, magnetoresistance, Hall effect, SQUIDS



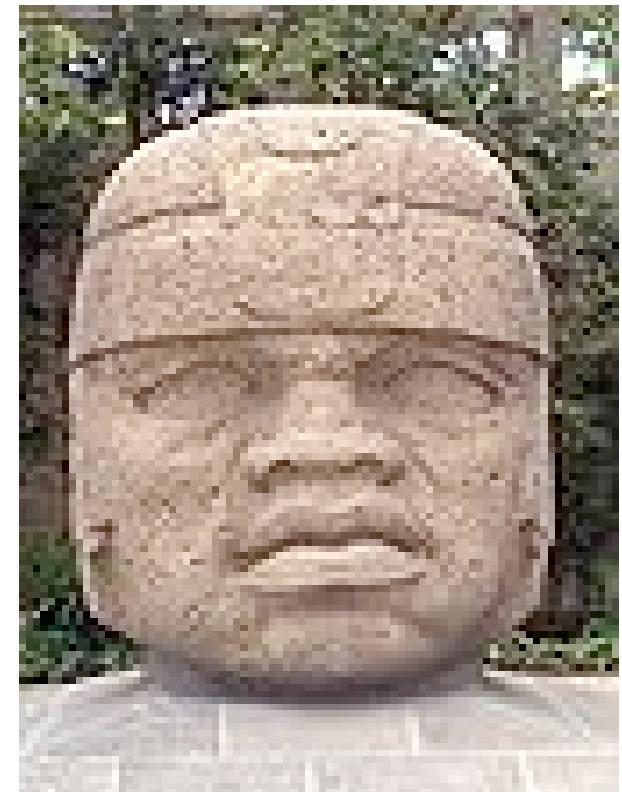
Olmechs and Sumerians



Sea turtle with a magnetic snout



Polished hematite bar



The Olmec heartland; 1400 - 400 BC



Chinese

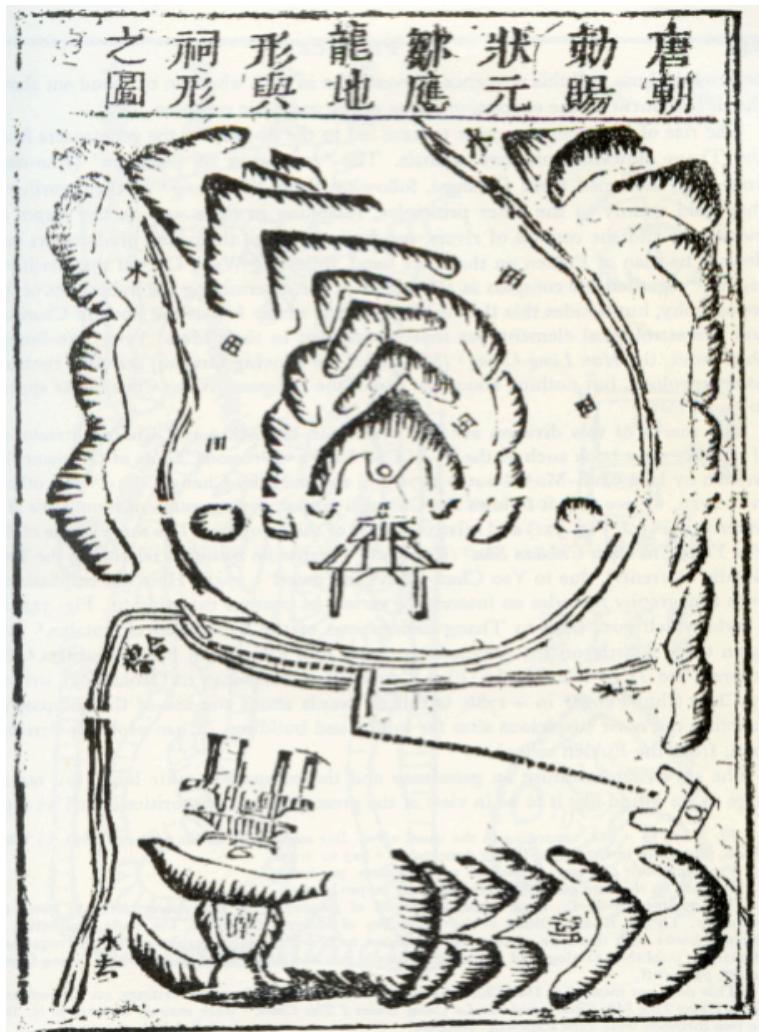
Geomancy

風水



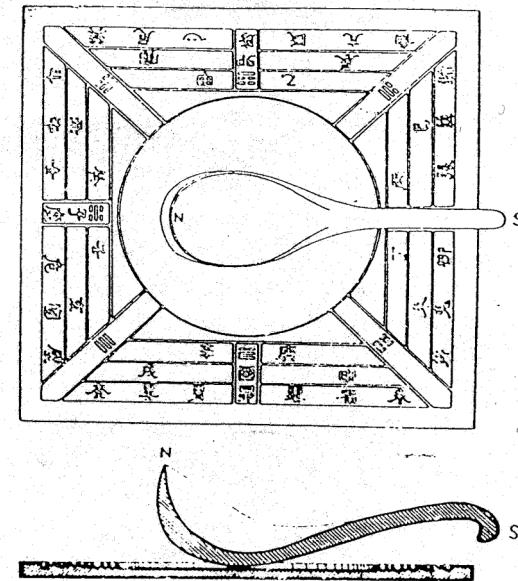
Geomancy

風水



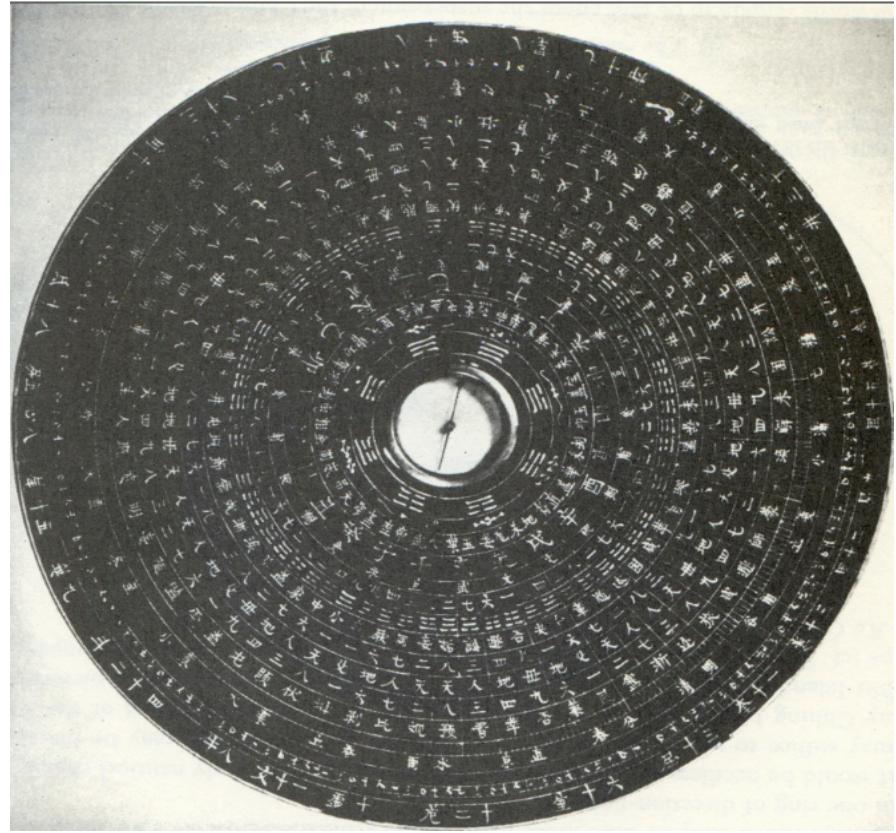
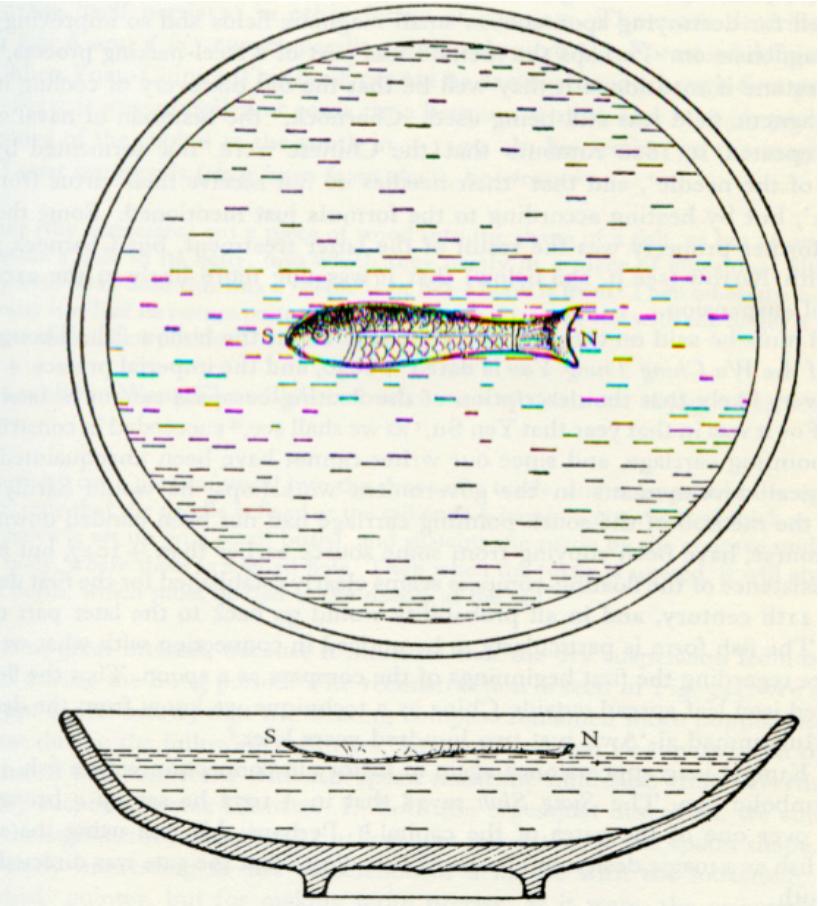
South pointer

Ganchow school
Fukien school





Evidence of declination in the street plan of Shantan, Kansu.



The Han Emperor Wu Ti had put to death his chief magician, the perfected-learning general, but he regretted he had not seen everything he was capable of.

In spring 113 BC, the Marquis Lo-Cheng recommended the eunuch Luan Ta, the magician and pharmacist to Prince Chiao Tung to the Emperor.

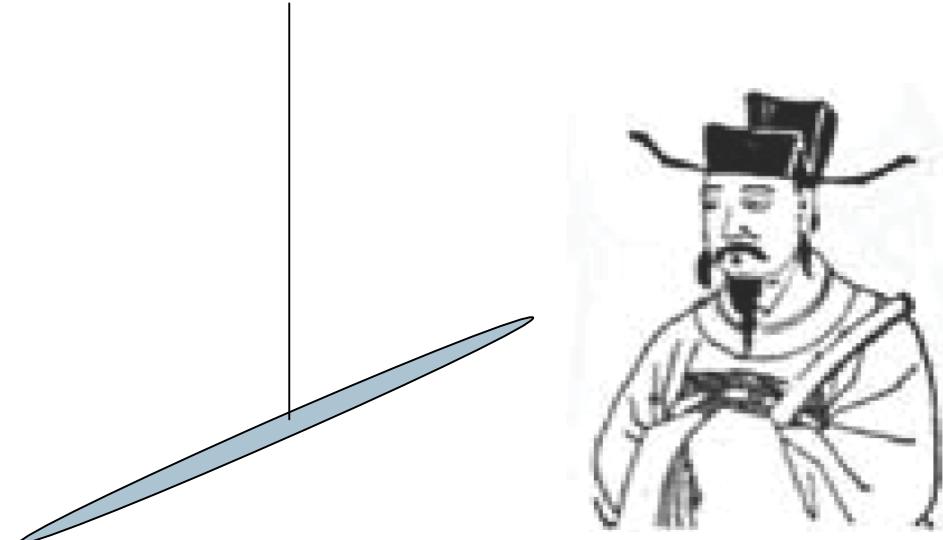
He demonstrated the ‘fighting chessmen’ to the Emperor, and a small example of his skill.

A commentary to the story explains the feat.

Take the blood of a cock and mix it with iron filings from the grinding of needles, pounding it with lodestone powder. In the day time put the paste on the heads of the chessmen, and let it dry in the Sun. Then put them on the board, and they will continually bounce against and repel one another.

Magnetic chessmen were used in divination.

Suspended compass



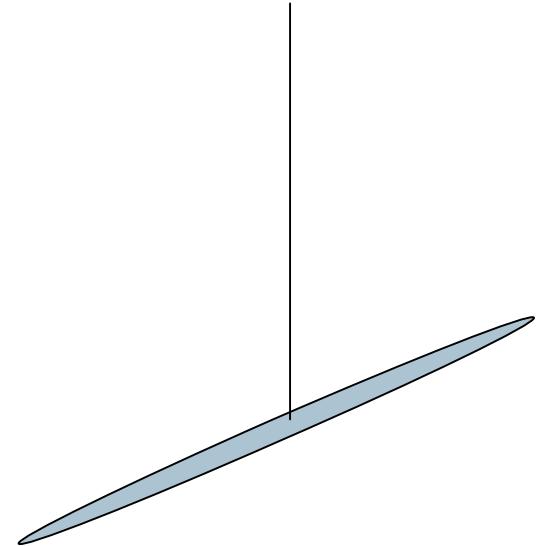
Shen Kua 1031-1095

Shen Kua (沈括) discovered how to make magnetized iron needles in 1060, and described the suspended needle compass in 1088. *Thermoremanence and induced magnetization* were discovered in China.

Suspended compass

Magicians rub the point of a needle with a lodestone; then it is able to point South. But it always inclines slightly to the East, and does not point directly South. It may be balanced on a fingernail or on the rim of a cup, where it can be made to turn easily, but these supports being hard and smooth, it is liable to fall off. It is best to suspend it by a single cocoon fibre of new silk attached to the centre of the needle with a piece of wax the size of a mustard seed – then hanging in a windless place it will always point to the South.

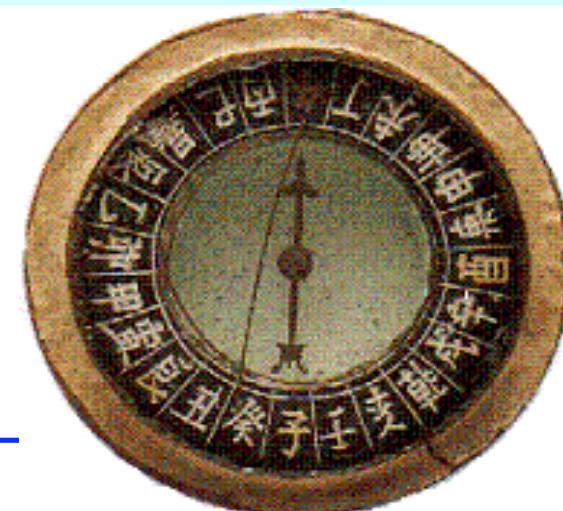
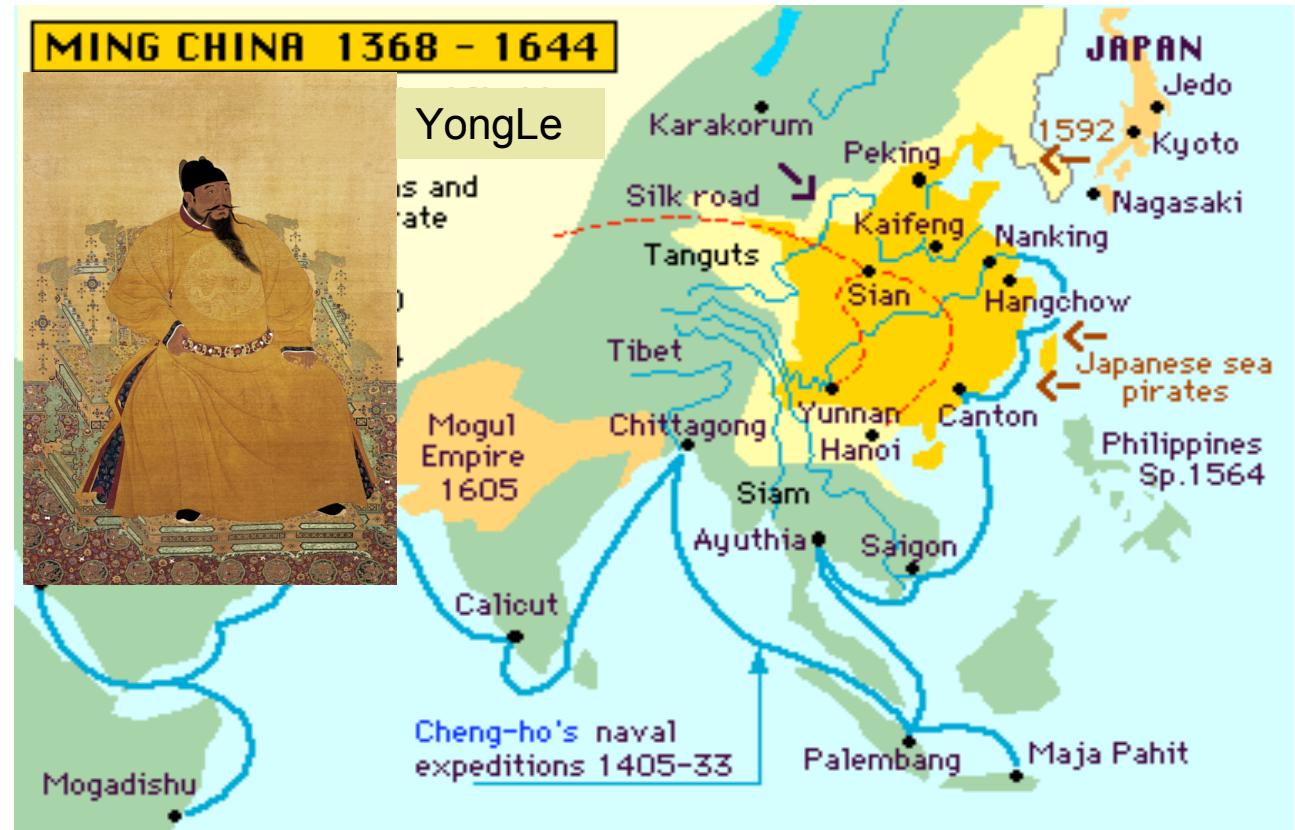
Shen Kwa: Dream pool essays 1088



The English monk Alexander Neckham gives the first European description of the compass at the University of Paris in 1190.



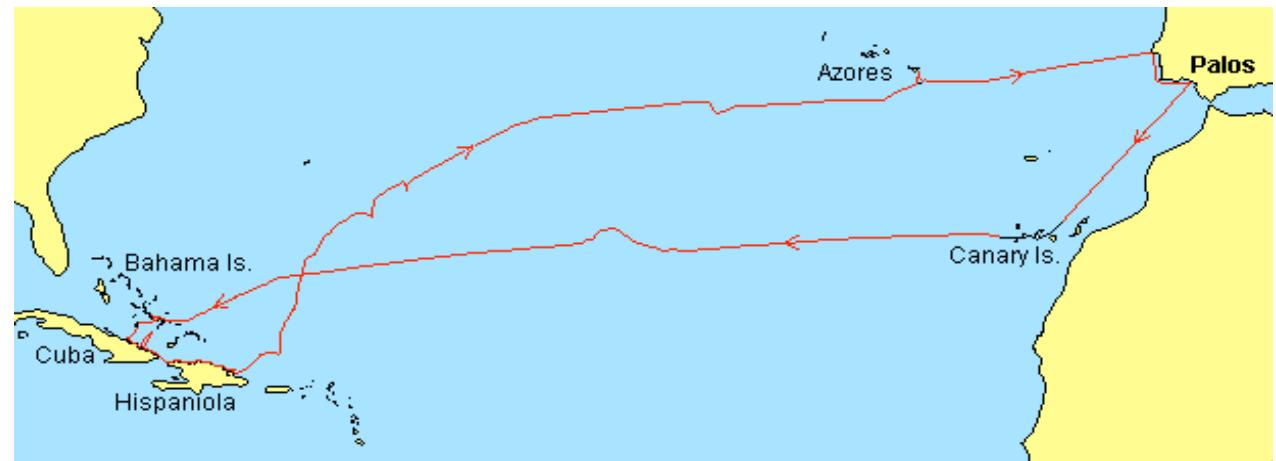
Cheng Ho 1371-1428



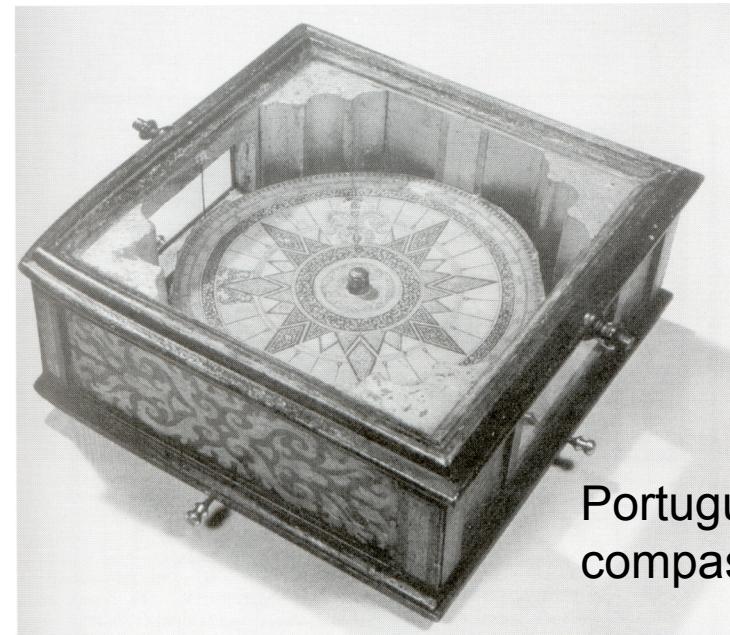
Chinese
compass



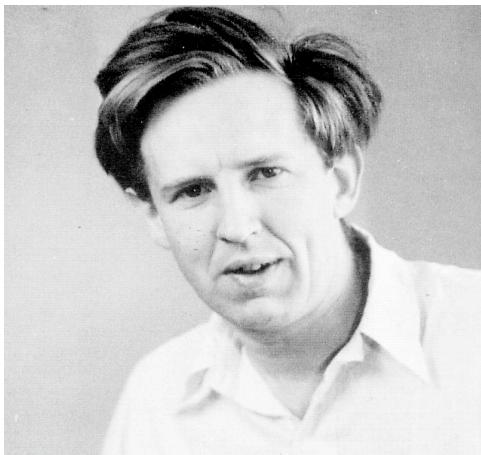
Christopher Columbus 1452 - 1506



Niña Pinta, Santa Maria,



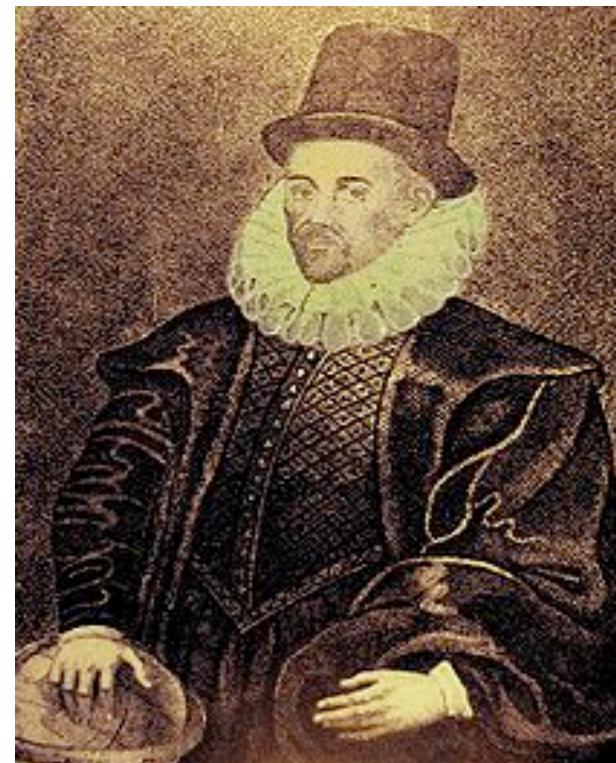
Portuguese
compass



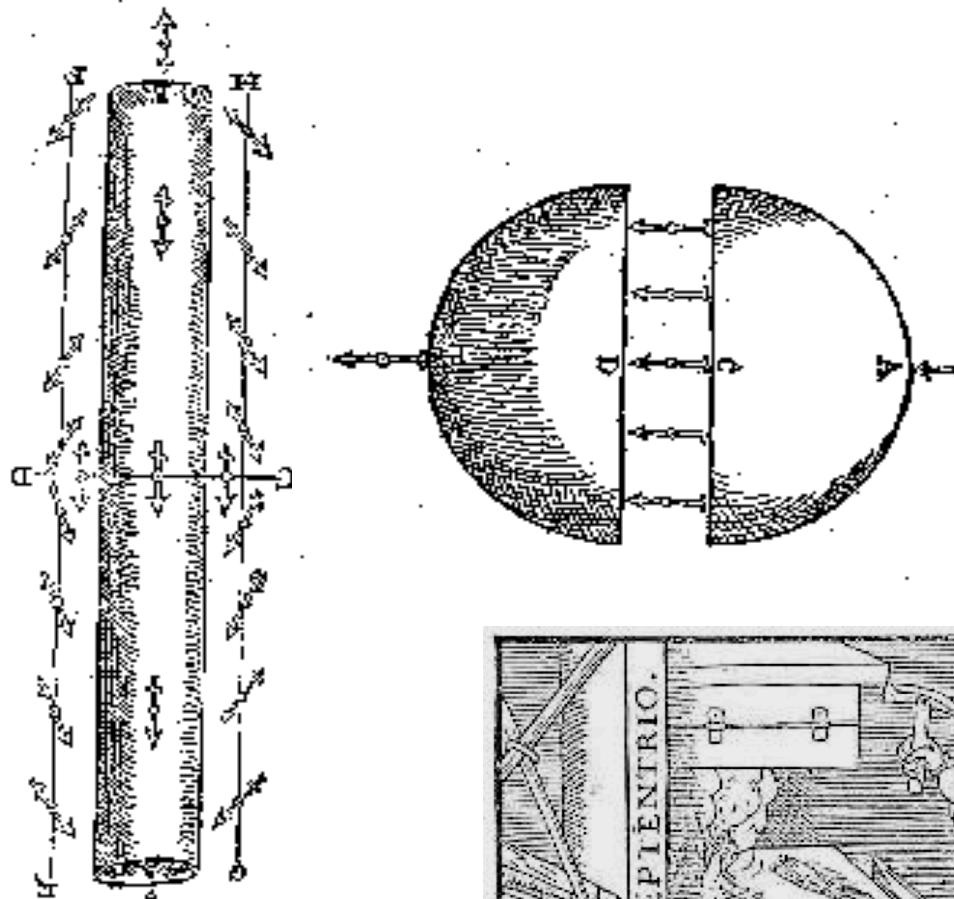
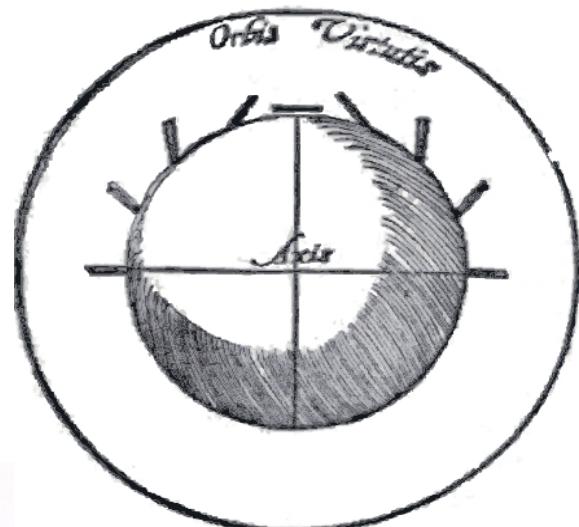
The Greeks knew the magnet attracted iron, and the attracted iron would attract other bits, and so on. But what they did not know was that there was any relation between the magnet and direction. This is one of the greatest discoveries: in fact *I would say without doubt, that it is the greatest discovery, in view of the difficulty in making it, in the whole of physics*. Because it was one of those things that could not be predicted. No one was in a position to say that if you take a magnet and suspend it freely, it will point north and south. First of all, why should you suspend a magnet freely? There was absolutely no reason to do so. Then why should it point north and south? What is there north and south that concerns a magnet? There is no *a priori* connection between the two.

J. D. Bernal; The Extension of Man

De Magnete The first scientific text 1600



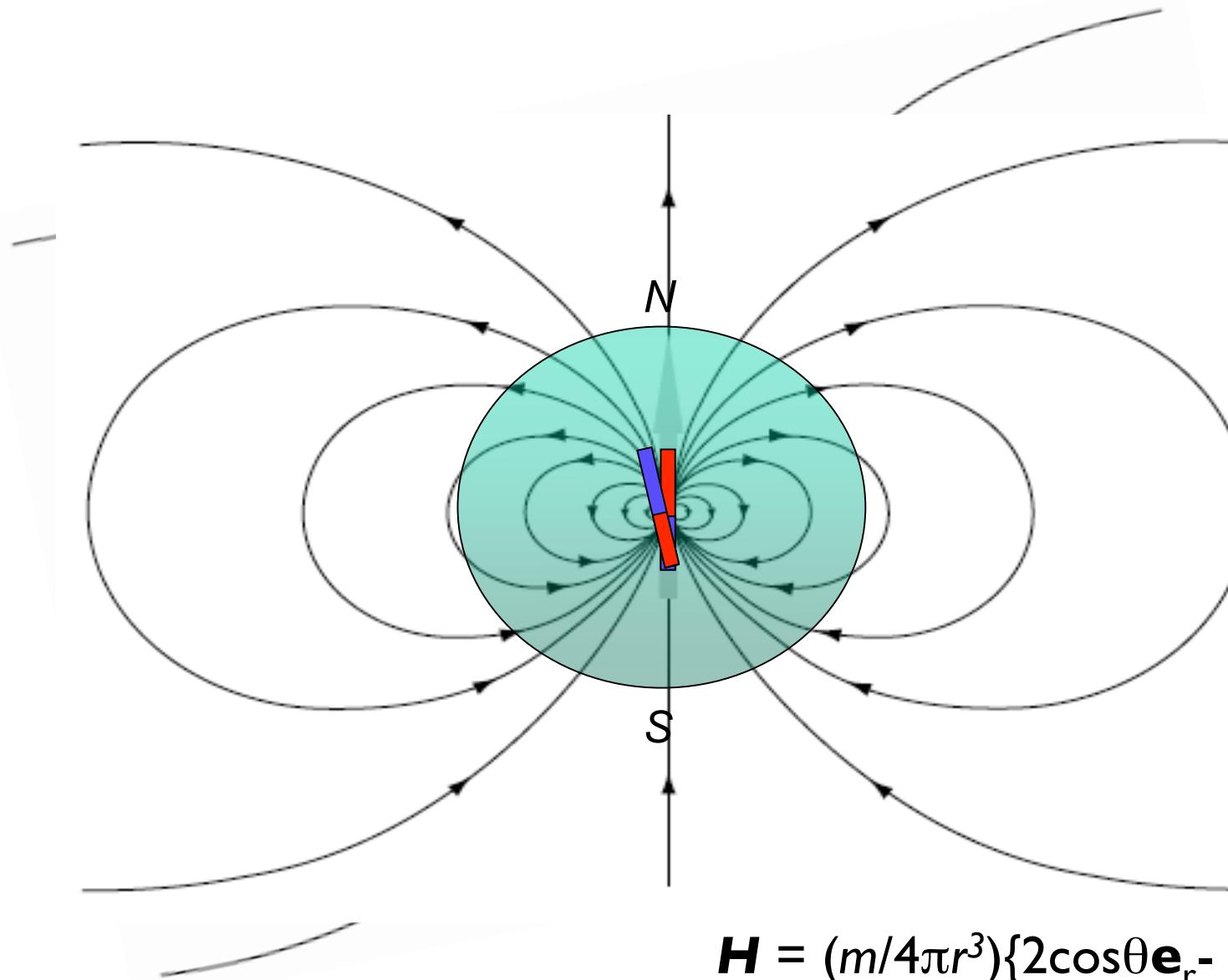
William Gilbert 1544 - 1603



Lodestone

Experiments on lodestone terellae led William Gilbert to conclude in 1600 that **Magnus magnes ipse est globus terrestris**





$$\mathbf{H} = (m/4\pi r^3) \{2\cos\theta \mathbf{e}_r - \sin\theta \mathbf{e}_\theta\}$$



Francis Bacon 1561-1628

"Printing, gunpowder and the compass: These three have changed the whole face and state of things throughout the world; the first in literature, the second in warfare, the third in navigation; whence have followed innumerable changes, in so much that no empire, no sect, no star seems to have exerted greater power and influence in human affairs than these mechanical discoveries."



Many will travel and knowledge will be increased

The Earth's magnetic field

Magnetic exploration of the Earth for much of the 18th century was motivated by the Navy's desire to solve the longitude problem



Edmond Halley 1656-1742



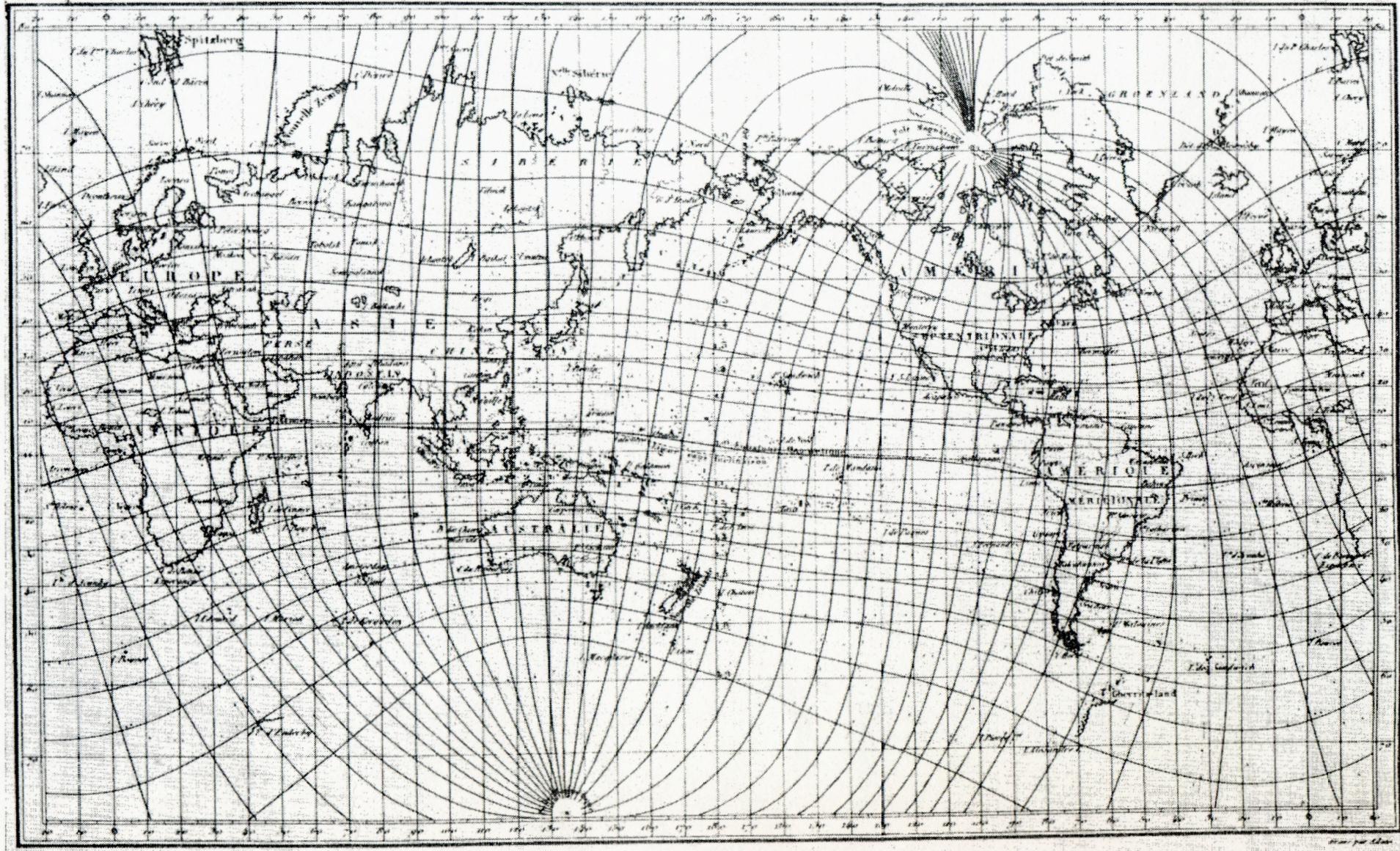
Edmond Halley led three research voyages from 1698 - 1701 to map the Earth's magnetic field.

CARTE DES MÉRIDIENS ET DES PARALLÈLES MAGNÉTIQUES DU GLOBE TERRESTRE.

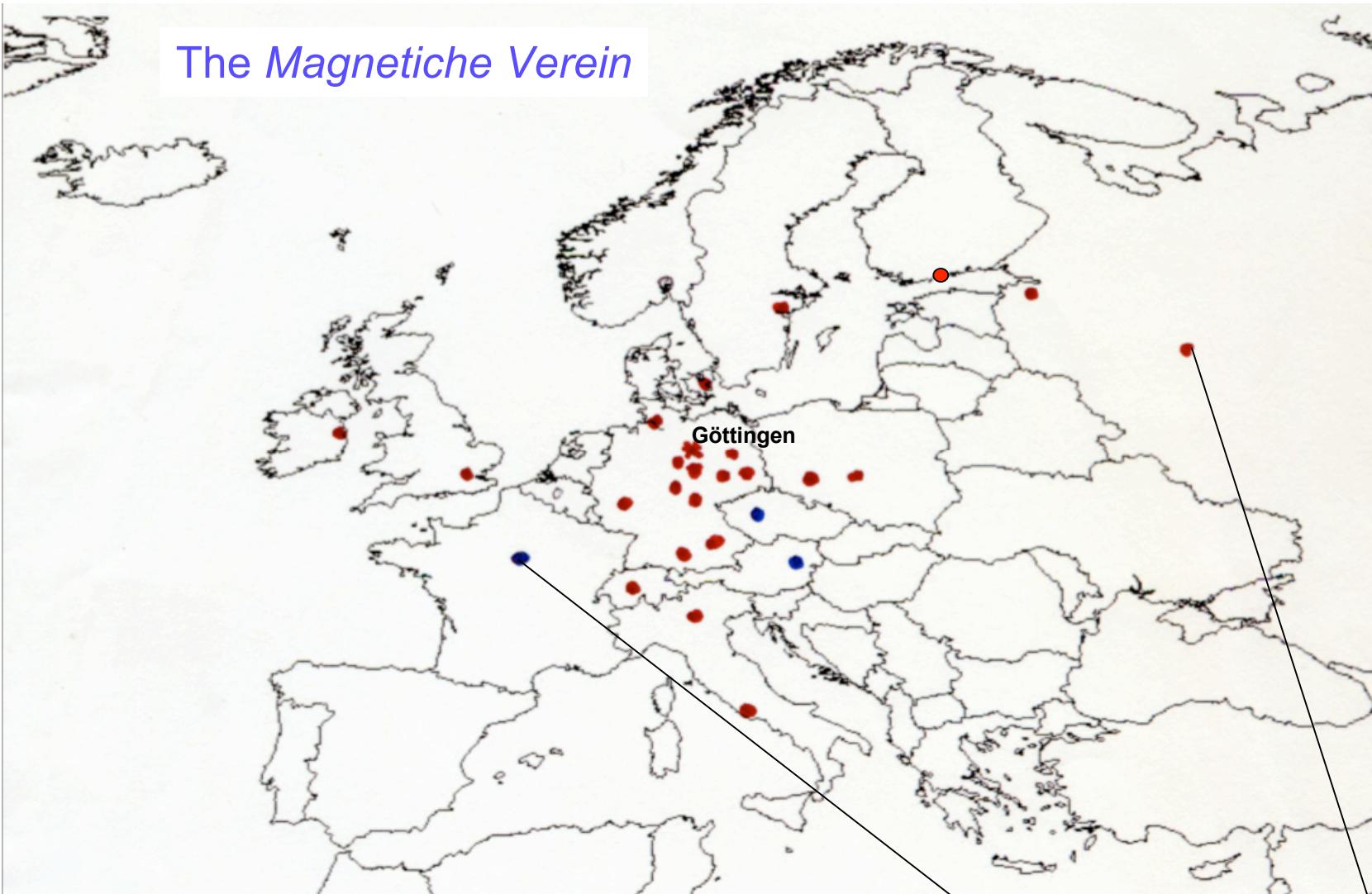
Tels qu'ils résultent des Observations de la Déclinaison de l'Aiguille aimantée. Par J. L. L. DUPERREY Cap^t de Frégate.

Planche XV.

TAV. 2^e part.



The Magnetische Verein

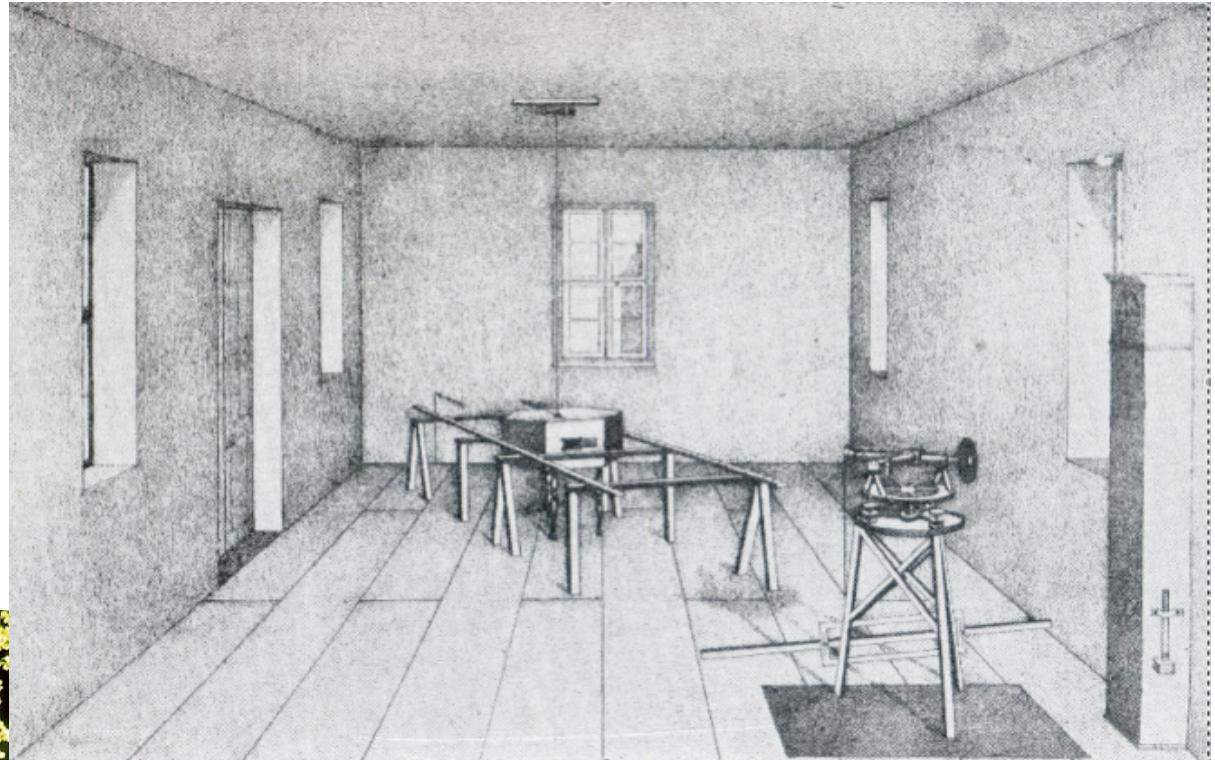


Network of Magnetic observatories \approx 1836

In 1830 simultaneous measurements were made of the fluctuations of the Earth's magnetic field during a 24 hour period in Paris and in Kazan



Carl Friedrich Gauss,
1775–1855.



Gauss's magnetic observatory in
Göttingen 1830.





The magnetical observatory
established at Trinity
College, Dublin, in 1835.



Carl Friedrich Gauss,
1775–1855.

$$\nabla^2 \varphi_m = 0$$
$$\varphi_m = \sum_{l=1}^{\infty} \sum_{m=0}^l [A_l^m r^l + B_l^m r^{-(l+1)}] Y_l^m(\theta, \phi)$$

About 99% of the Earth's magnetic field has an *internal* origin. It changes slowly

About 1% has an external origin. It fluctuates rapidly, on a daily basis

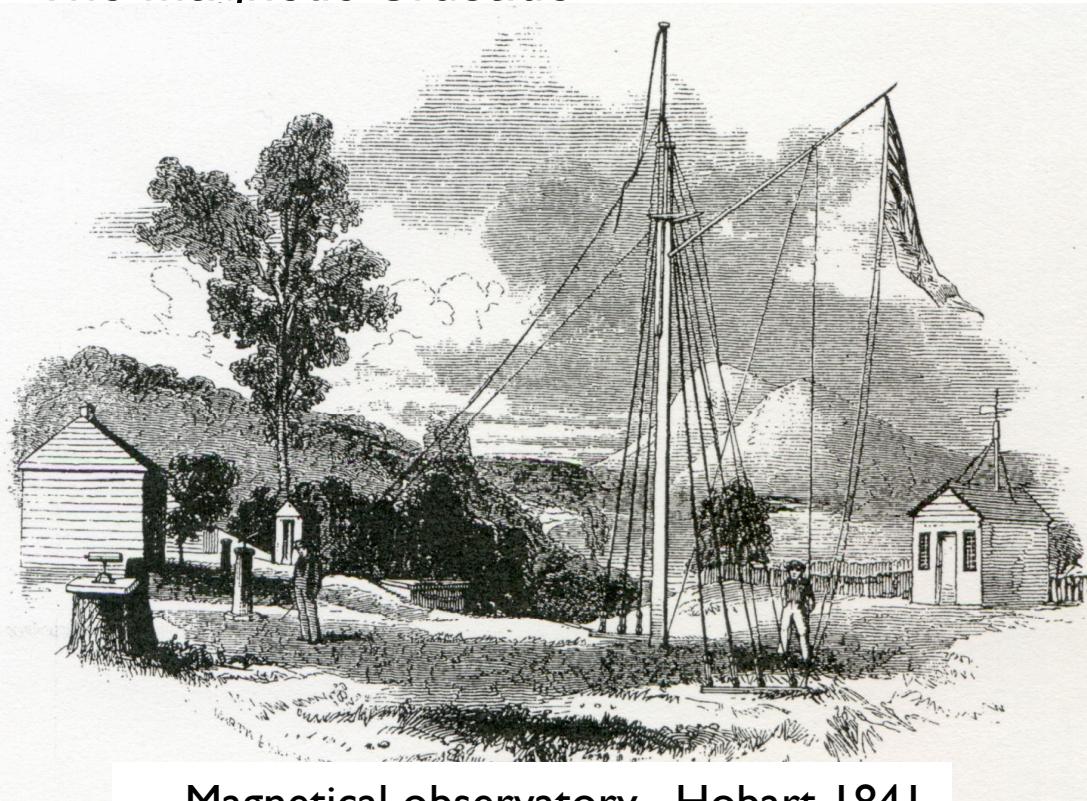
The humblest student of astronomy, or of any other physical science if he is to profit at all by his study must in some degree go over for himself, in his own mind, if not in part with the aid of his own observation and experiment, that process of induction which leads from familiar facts to obvious laws, then to the observation of facts that are more remote and to the discovery of laws of higher orders. And even if this study be a personal act, much more must that discovery have been individual. Individual energy, individual patience, individual genius have all been needed to tear fold after fold away which hung before the shrine of nature; to penetrate gloom after gloom into those Delphic depths, and force the reluctant Sibyl to utter her oracular responses.



William Rowan Hamilton

\mathcal{H}

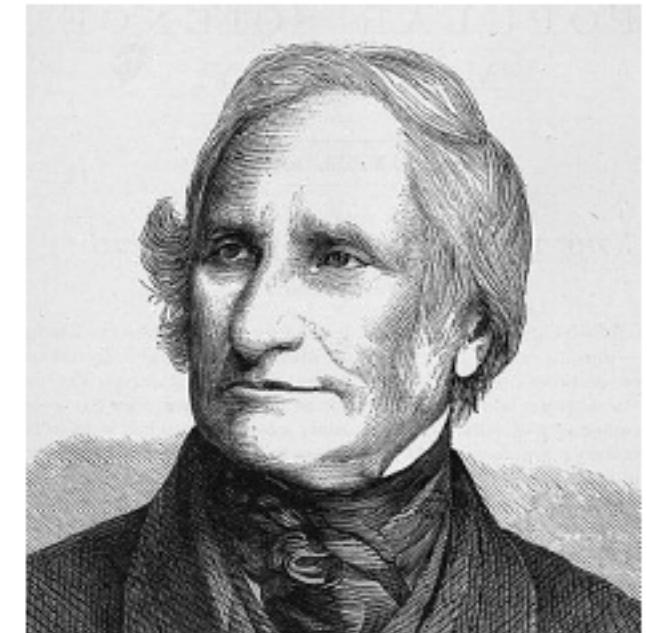
The Magnetic Crusade



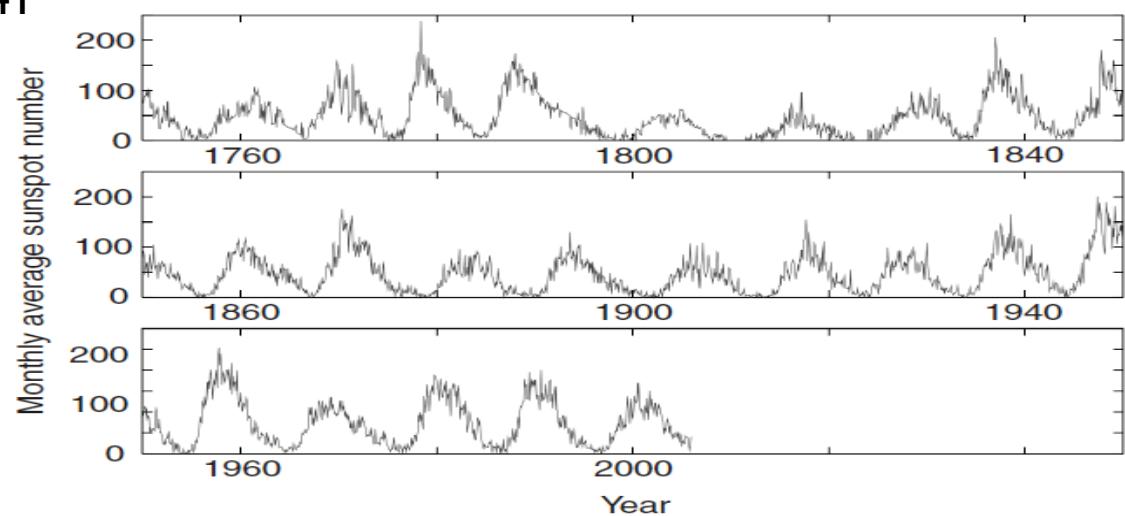
Magnetical observatory, Hobart 1841

The 11-year sunspot cycle
from 1760–2000.

The data from the observatories
showed short-term fluctuation that
reflect the 11-year sunspot cycle!

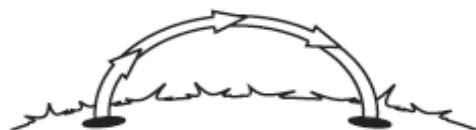


Edward Sabine, 1788–1883.

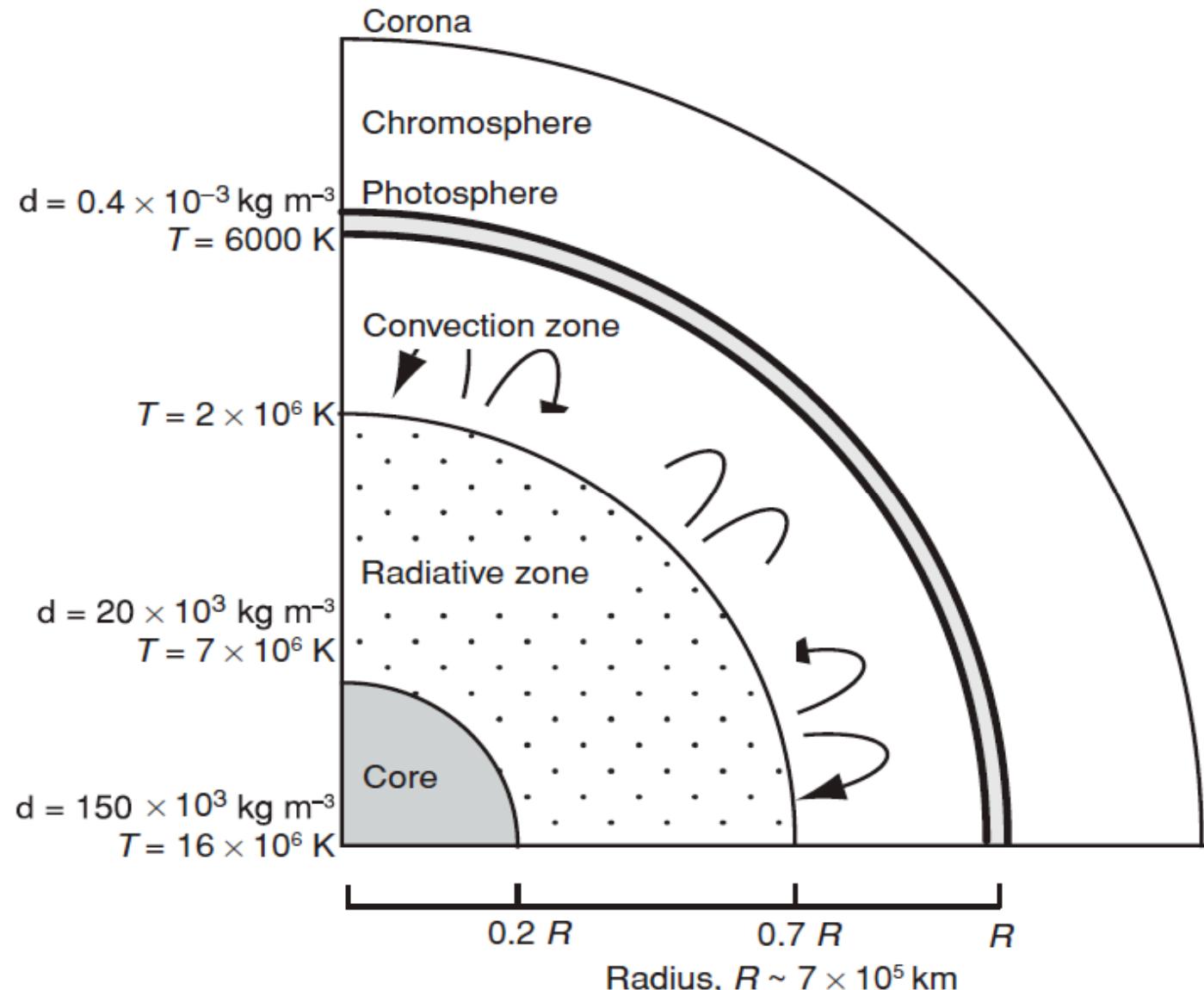


The internal structure of the Sun. Radii of the main structures, mean densities and temperatures at the centre and surface are given.

The Sun's field reverses every 11 years.

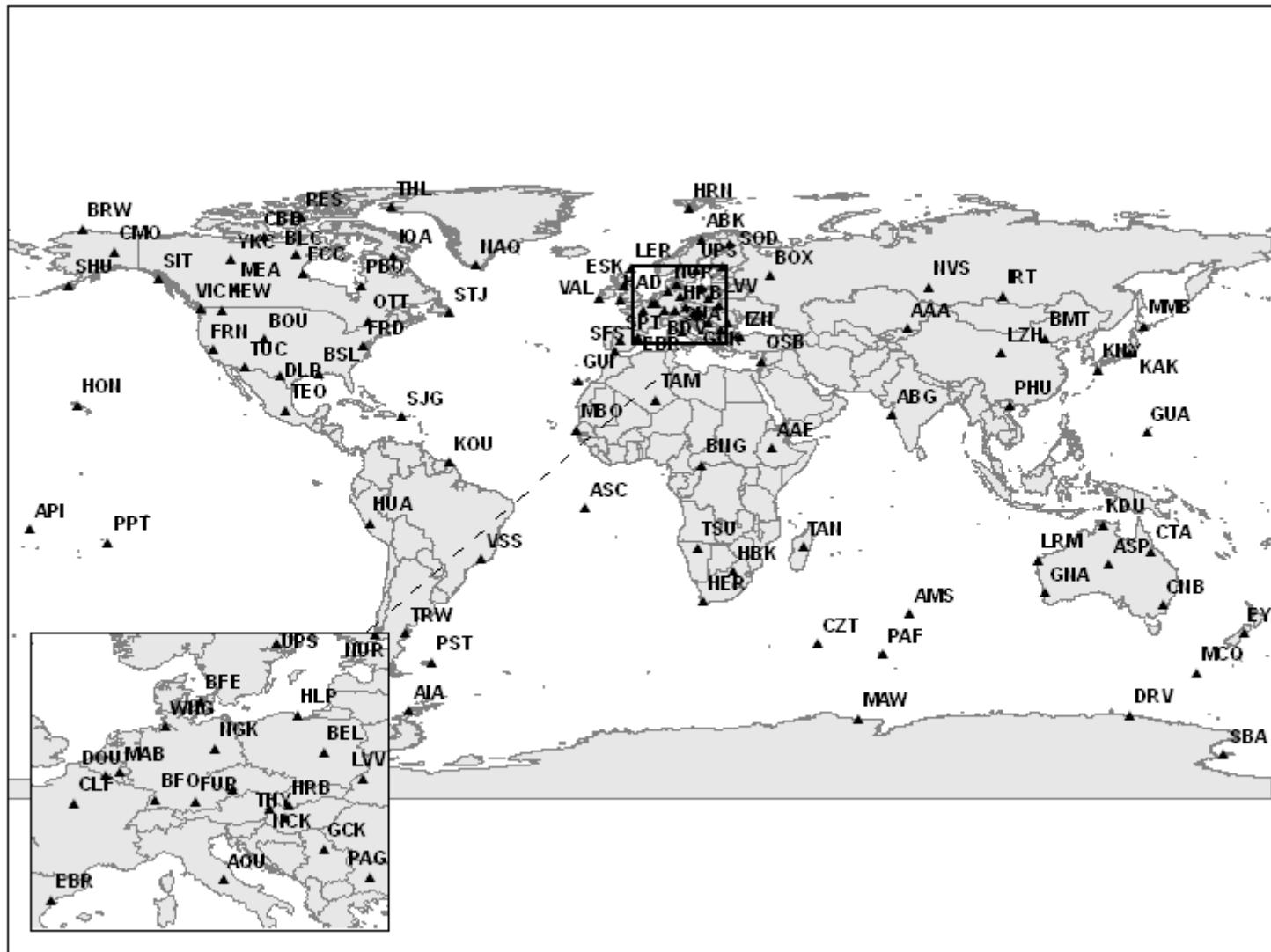


A flux tube which has been pushed out through the surface of the Sun, forming two sunspots.

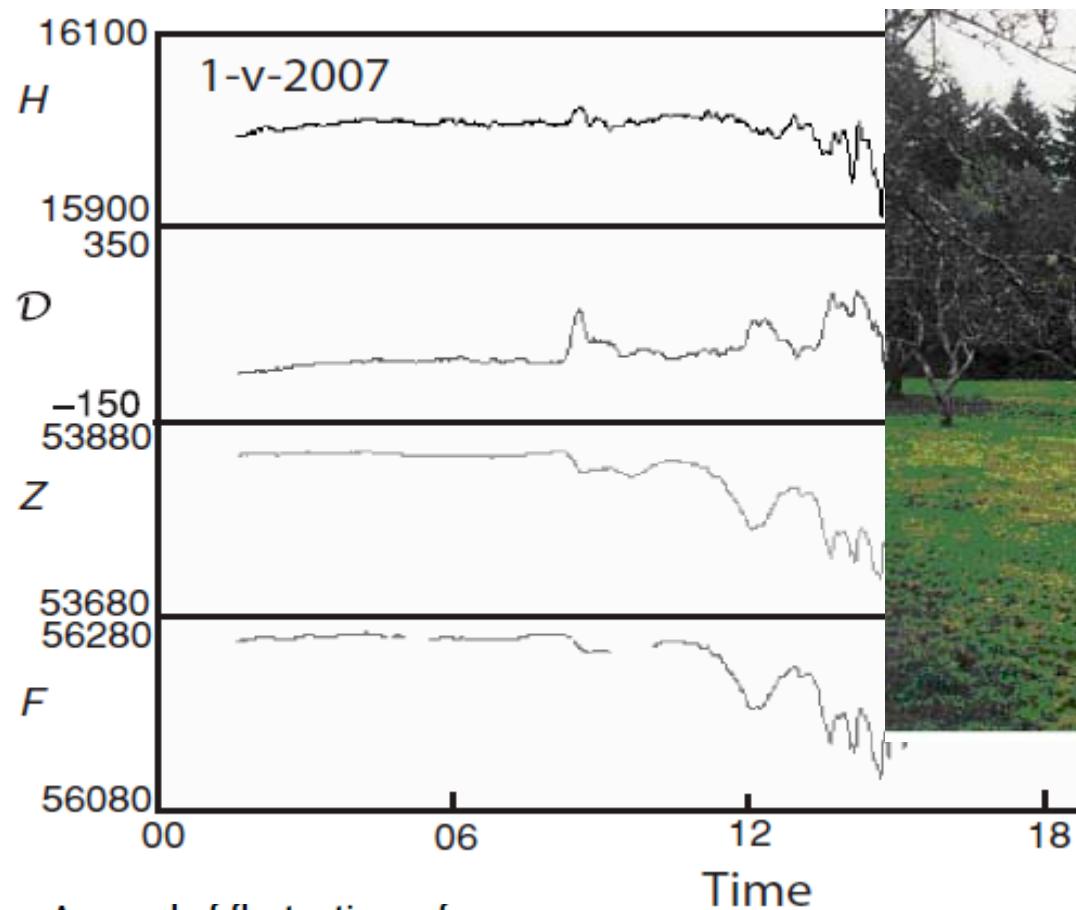


$$\mathcal{R}_m = v l \sigma \mu$$

INTERMAGNET - A world-wide network of magnetic observatorys



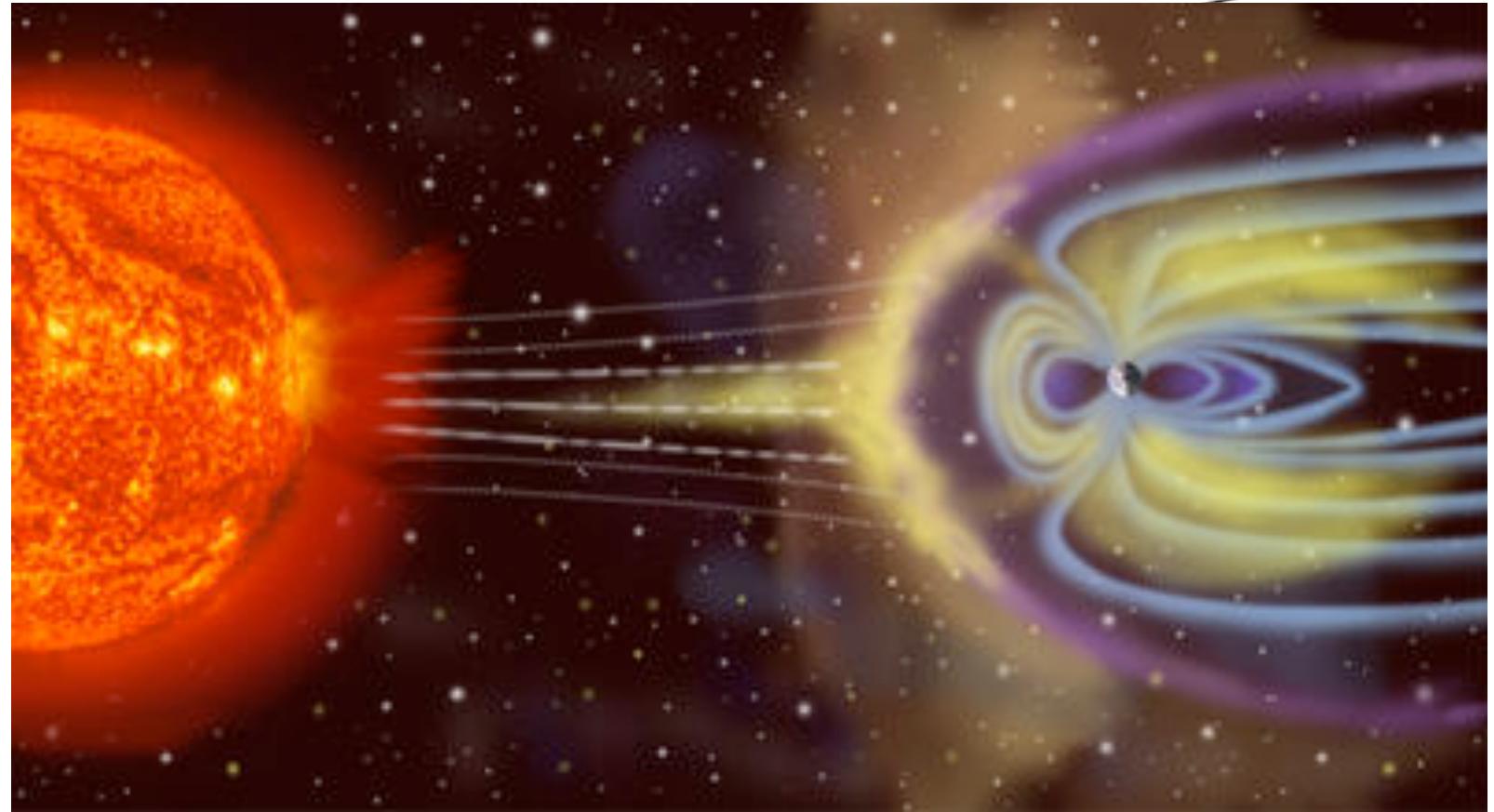
INTERMAGNET - A world-wide network



A record of fluctuations of the Earth's magnetic field, taken at Sitka in Alaska on 1 May 2007. A magnetic observatory has existed on this site since 1842. Units of H , Z and F are nanoteslas, the units of D are degrees.



Solar flares, transit
time from the Sun
~ 3 days.



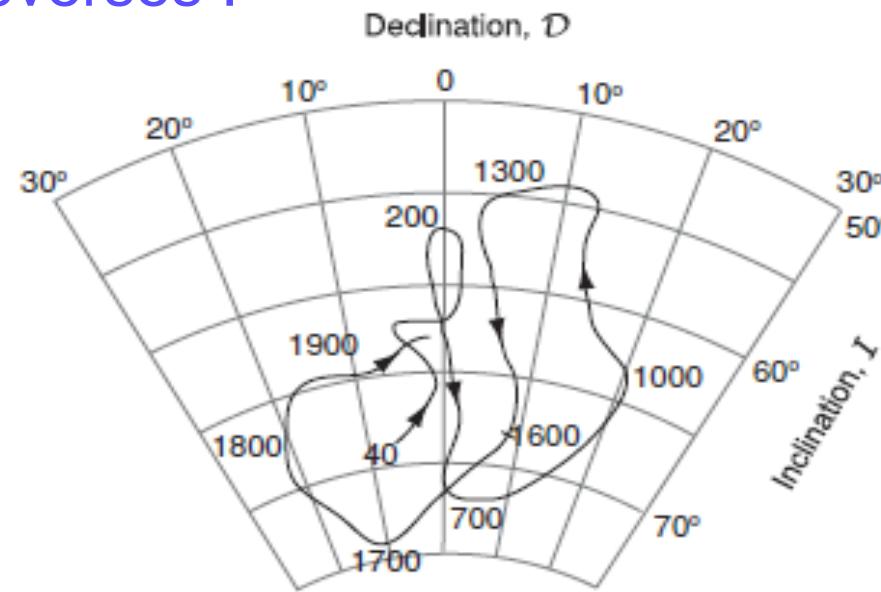
Shock wave

Space weather forecasting
can be critical.

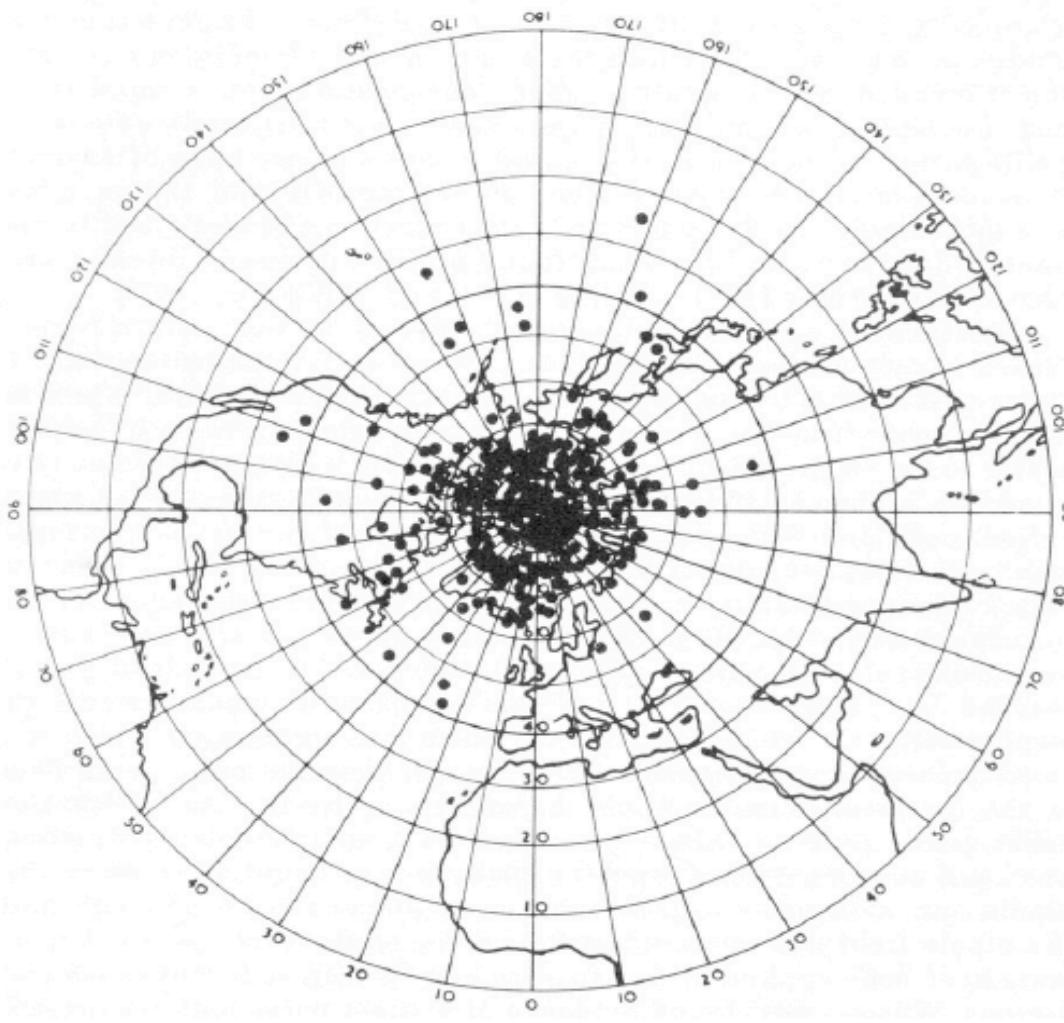
1989 blackout in Quebec

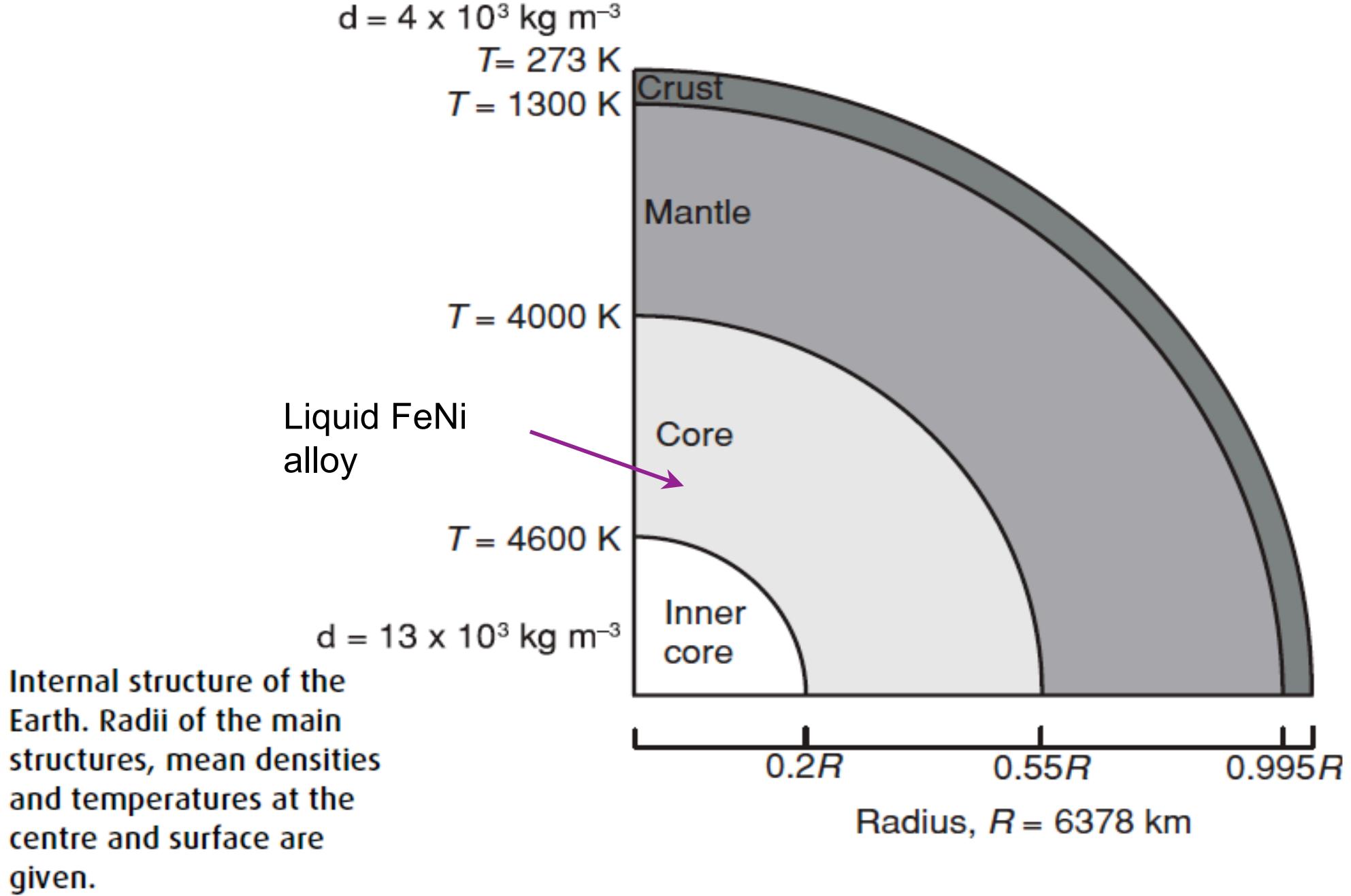
Chaos - It wanders and reverses !

The scalar variation of the Earth's field deduced by combining observations in Paris (>1600) with measurements of the remanence of baked clay (<1600).



Position of the Earth's magnetic pole deduced from measurements of recently formed igneous rocks. Half of the points have the present polarity, while the other half are reversed. On average the magnetic field is that of a geocentric axial dipole.



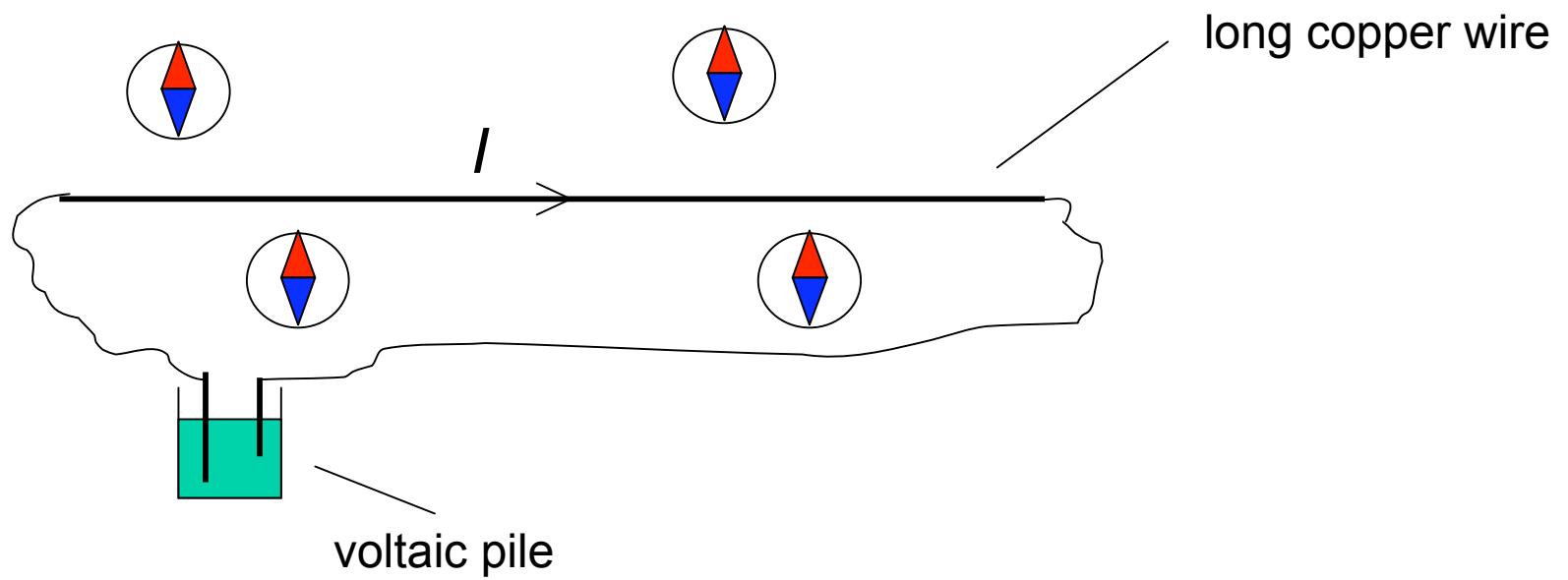


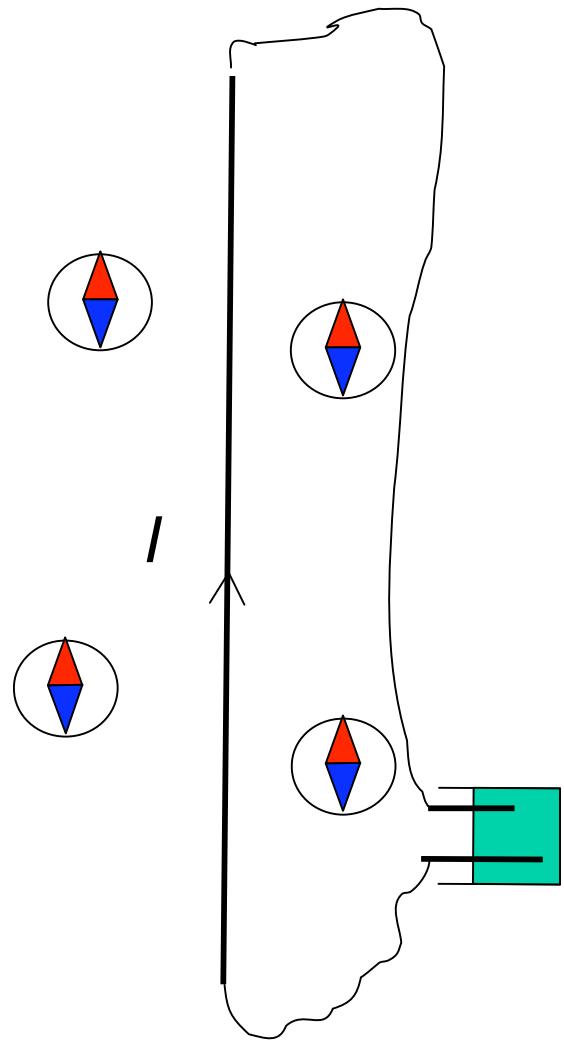
Oersted's great discovery

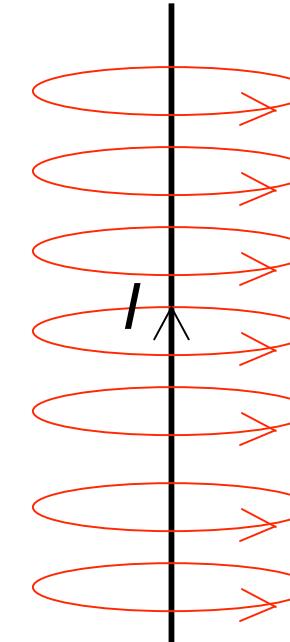
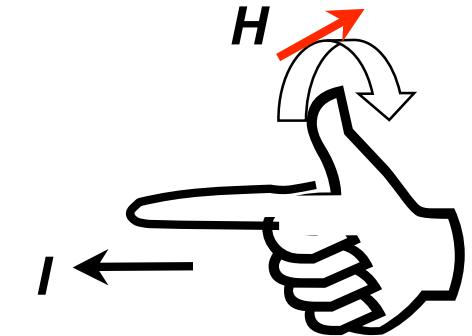
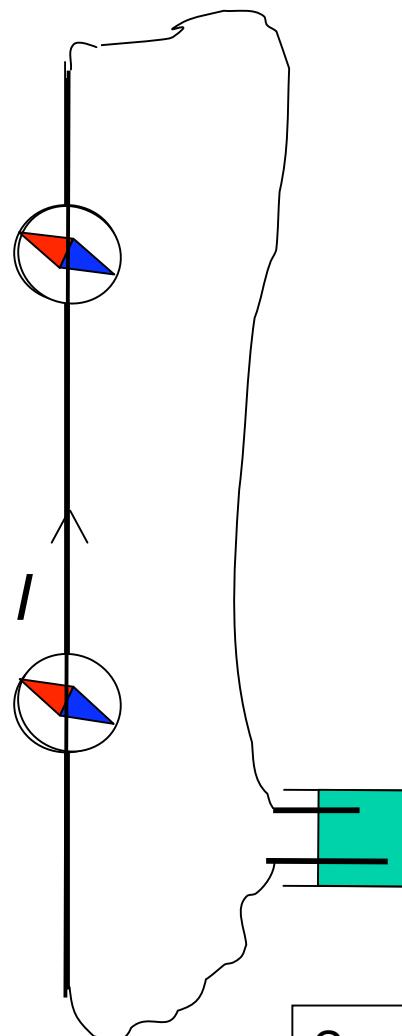


Hans-Christian Oersted stumbles on the truth









$$H = I/2\pi r$$

If $I = 12 \text{ A}$

$r = 0.1 \text{ m}$

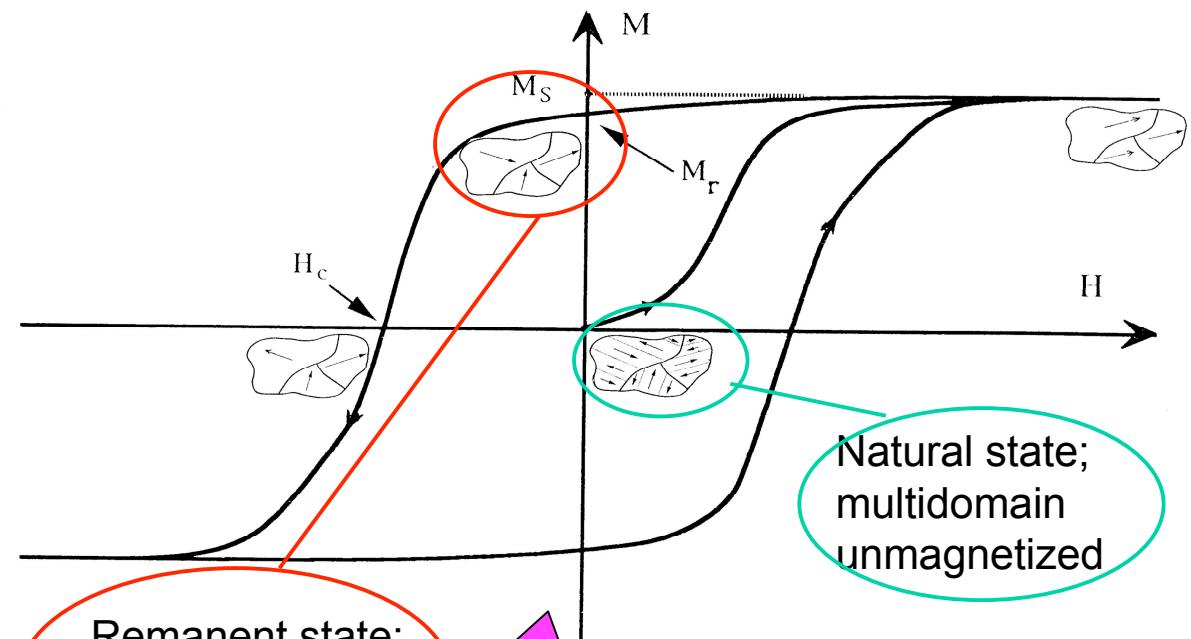
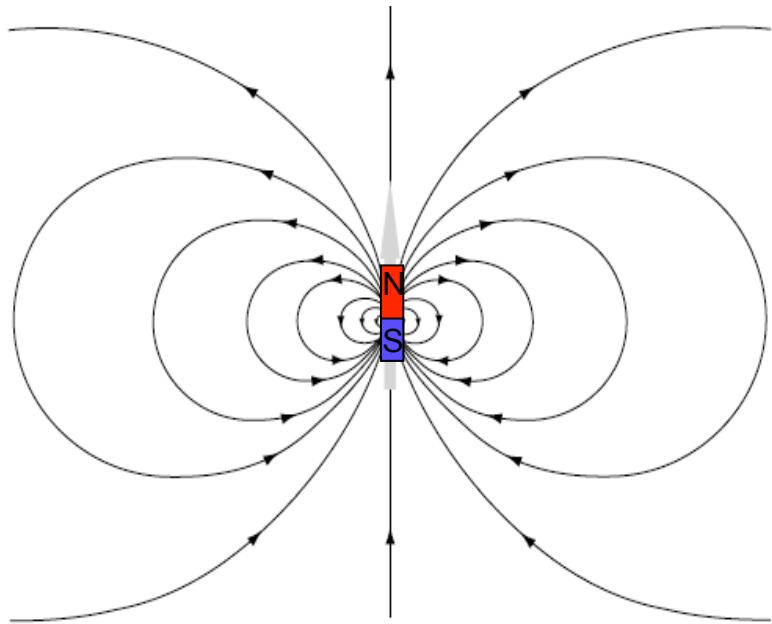
$H \approx 20 \text{ A/m}$

Field is comparable to the Earth's

Oersted's famous experiment showed that a current-carrying conductor created a field everywhere *perpendicular* to the wire

How is the lodestone magnetized ?

Hysteresis loop

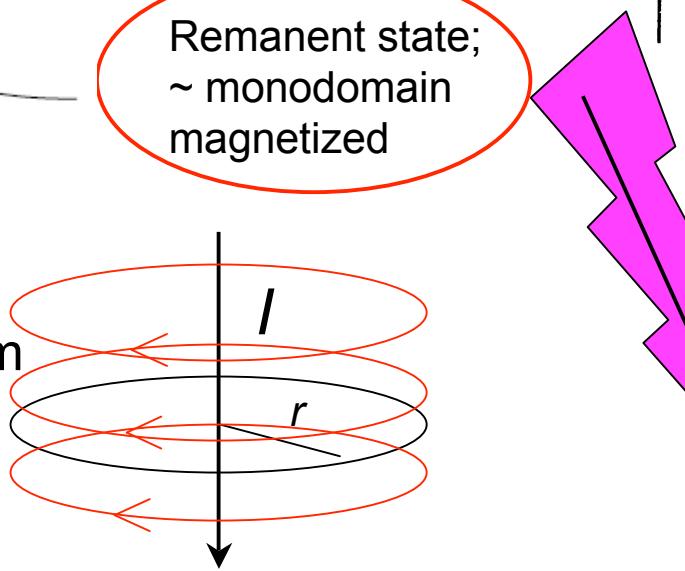


$$\oint \mathbf{H} \cdot d\mathbf{s} = I$$

$$2\pi r H = I; \text{ if } I = 10^6 \text{ A}, r = 0.5 \text{ m}$$

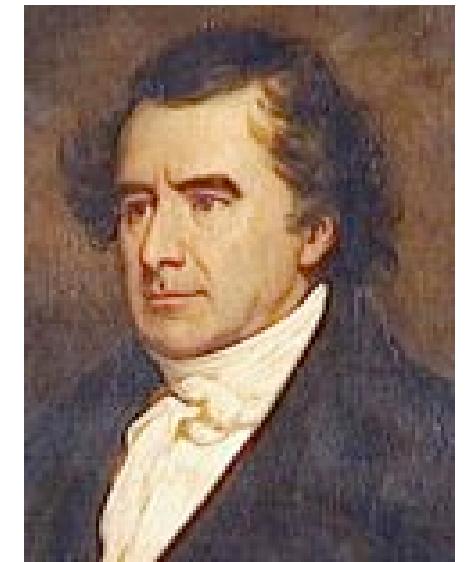
$$H = I/2\pi r$$

$$H = 10^6/\pi = 318 \text{ kA m}^{-1}$$



Within a week of the news reaching Paris, Ampère and Arago showed that a current loop acts as a magnet, especially when wound into a solenoid. Ampère measured the force between conductors, and proposed that huge internal electric currents were responsible for the magnetism of iron.

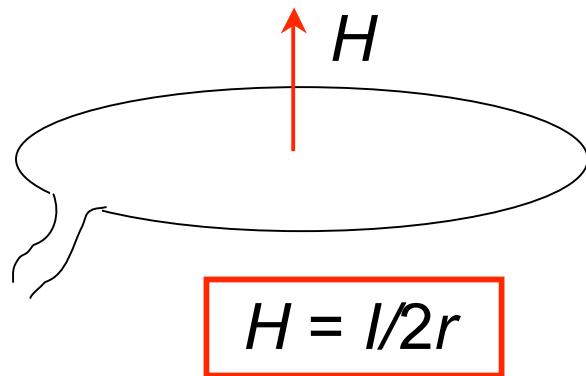
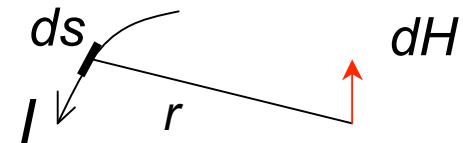
Laplace
Poisson
Fresnel
Fourier
Biot
Savart



Vive la révolution électromagnétique !

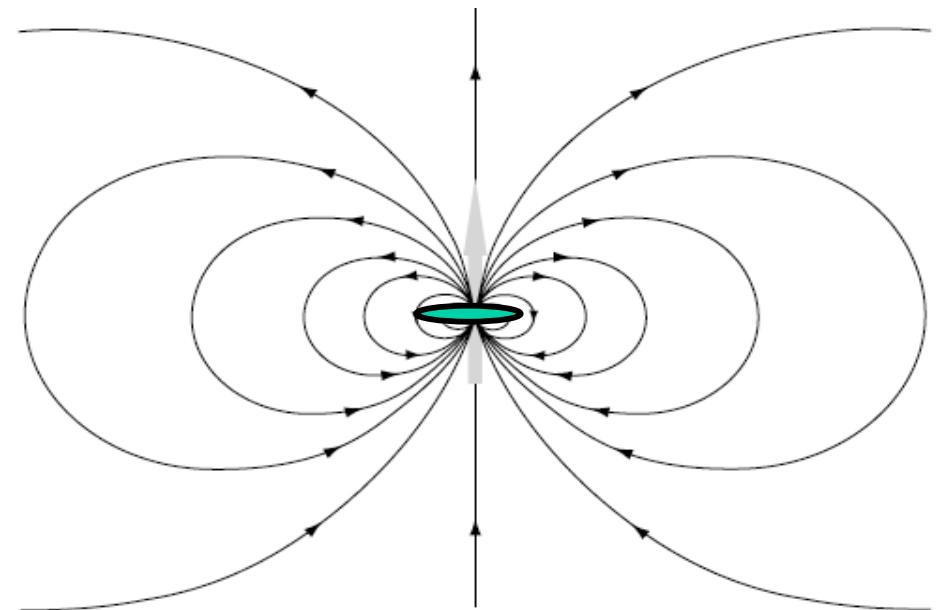
Biot-Savart law

$$d\mathbf{H} = I(d\mathbf{s} \times \mathbf{r})/4\pi r^3$$



$$\text{If } I = 12 \text{ A}$$
$$r = 0.1 \text{ m}$$

$$H \approx 60 \text{ A/m}$$

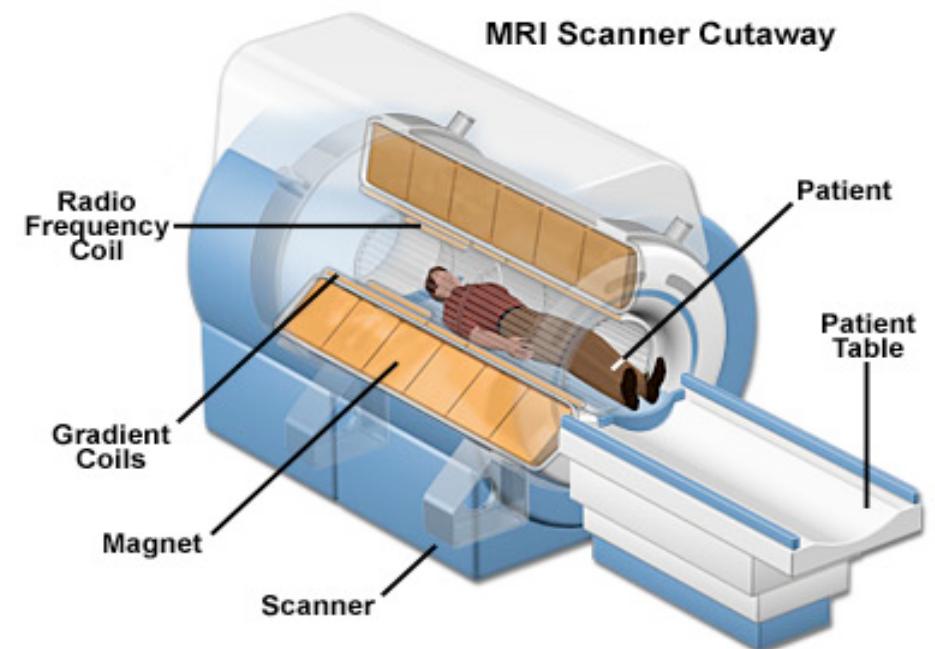
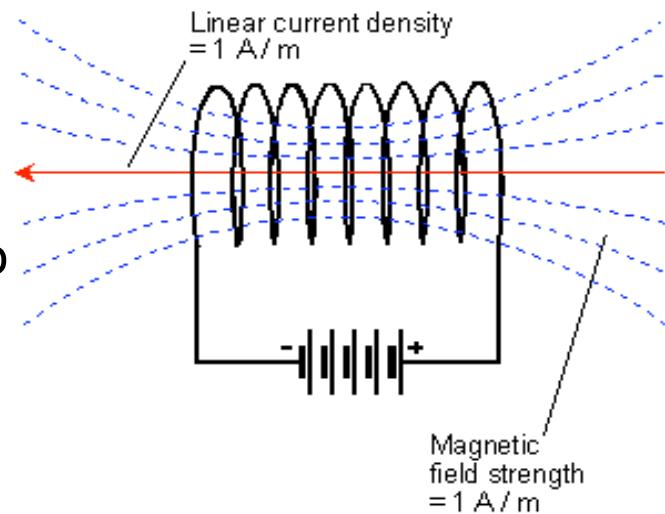


A current-carrying loop of wire is exactly equivalent to a magnet

For a solenoid, the formula is $H = nl$

n is the number of turns/m

If a superconducting magnet is to produce a field of 6 Tesla (1 T = 800 kA/m) and it can carry 120 A, then we need $n = 6 \times 800,000 \div 120 = 40,000$ turns/m.



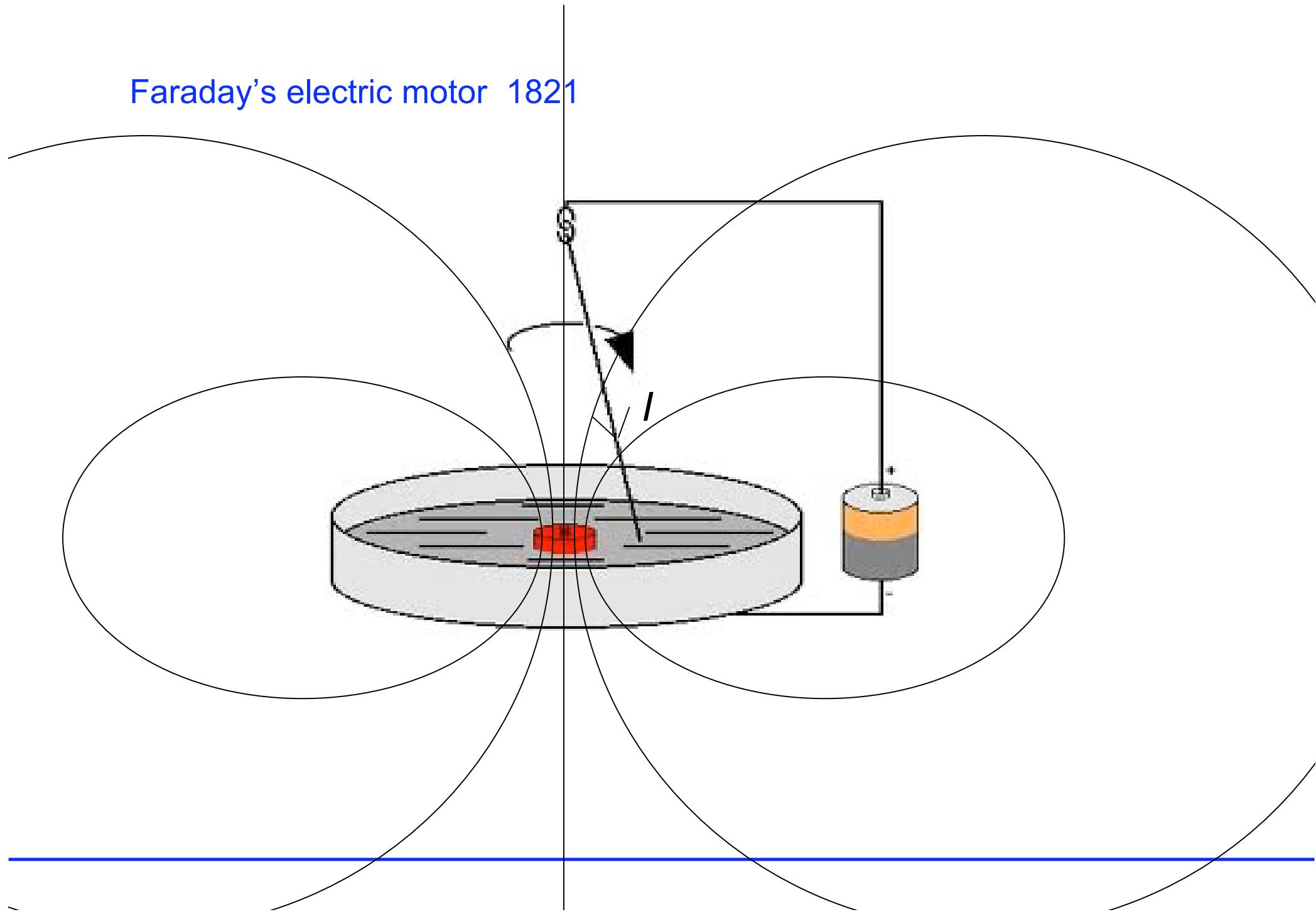
The electromagnetic revolution

It was a field day for experimentalists. The most intuitive and talented of them all was Michael Faraday, who made a simple motor, discovered electromagnetic induction in 1831 and found a connection between magnetism and light in 1845.

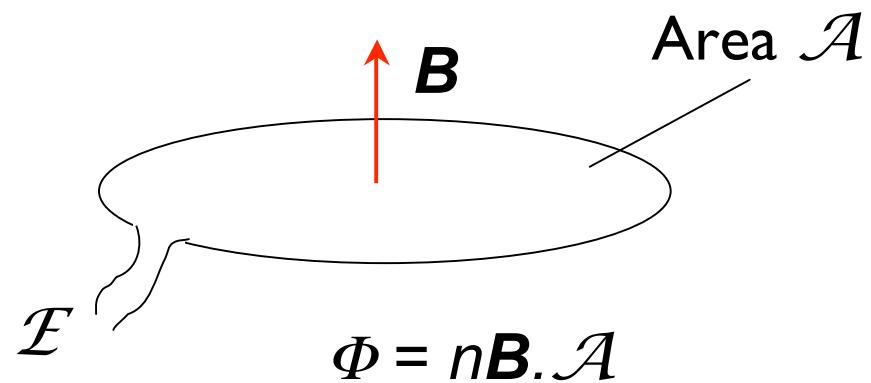


Faraday's electromagnet

Faraday's electric motor 1821

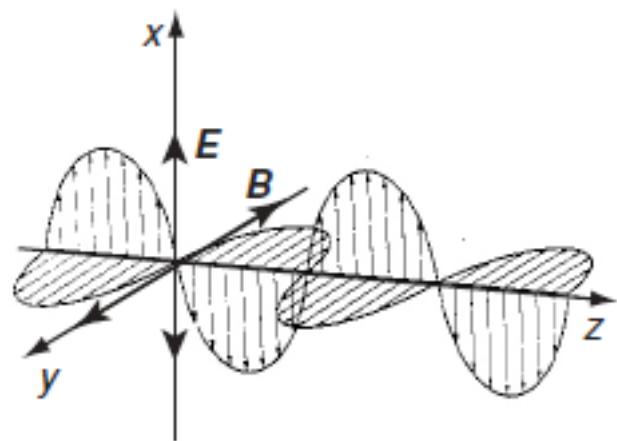


Electromagnetic induction 1831

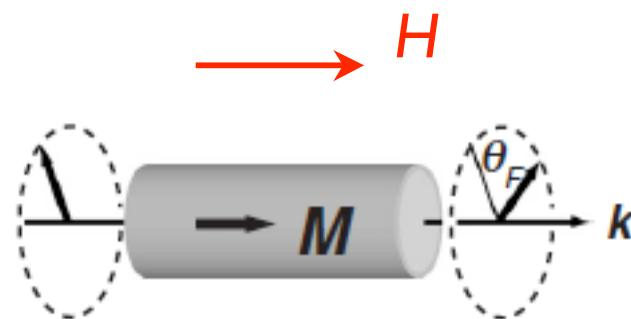


$$\mathcal{E} = -d\Phi/dt$$

Magneto-optic Faraday effect 1845



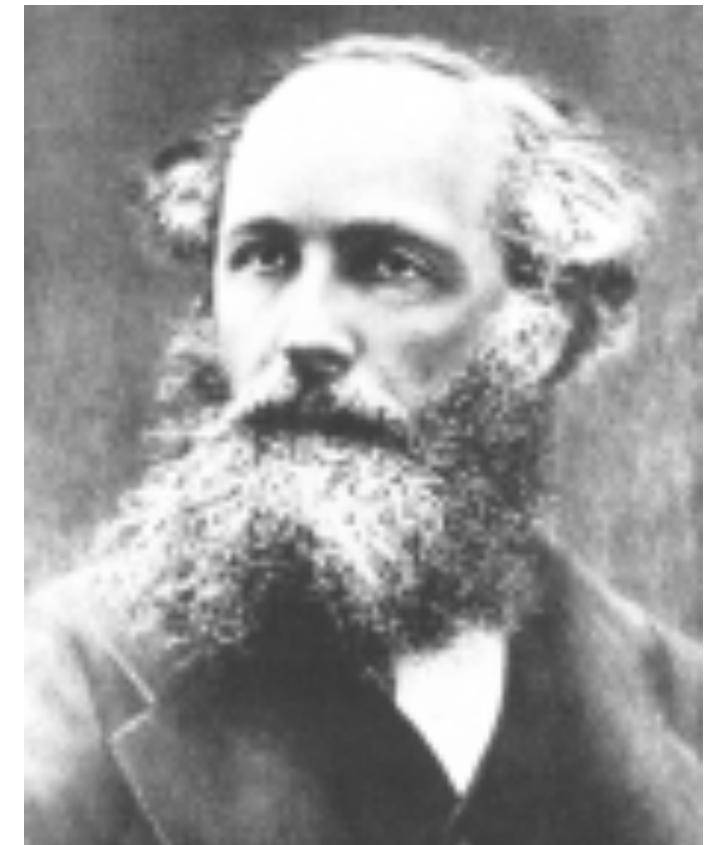
An electromagnetic wave.



The magneto-optic Faraday effect. The plane of polarization of incident light is rotated through an angle θ_F . In a magneto-optic isolator, $\theta_F = \pi/4$, and polarizers offset by this angle are placed at each end.

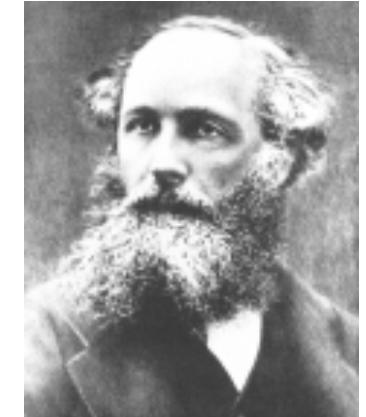
Before I began the study of electricity I resolved to read no mathematics on the subject till I had first read through Faraday's *Experimental Researches in Electricity*. I was aware that there was supposed to be a difference between Faraday's way of conceiving phenomena and that of the mathematicians, so that neither he nor they were satisfied with each other's language. I had also the conviction that this discrepancy did not arise from either party being wrong...As I proceeded with the study of Faraday, I perceived that his method of conceiving the phenomena was also a mathematical one, though not exhibited in the conventional form of mathematical symbols. I also found that these methods were capable of being expressed in the ordinary mathematical forms For instance, Faraday, in his mind's eye, saw lines of force traversing all space where the mathematicians saw centres of force attracting at a distance: Faraday saw a medium where they saw nothing but distance: Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids.

When I had translated what I considered to be Faraday's ideas into a mathematical form, I found that in general the results of the two methods coincided, so that the same phenomena were accounted for, and the same laws of action deduced by both methods, but that Faraday's methods resembled those in which we begin with the whole and arrive at the parts by analysis, while the ordinary mathematical methods were founded on the principle of beginning with the parts and building up the whole by synthesis.I also found that several of the most fertile methods of research discovered by the mathematicians could be expressed much better in terms of ideas derived from Faraday than in their original form.



James Clerk Maxwell 1831-1879

Maxwell's equations; the first great unification



$$\begin{aligned}\nabla \cdot \mathbf{H} &= 0 \\ \epsilon_0 \nabla \cdot \mathbf{E} &= \rho \\ \nabla \times \mathbf{H} &= \mathbf{j} + \epsilon_0 \partial \mathbf{E} / \partial t \\ \nabla \times \mathbf{E} &= -\mu_0 \partial \mathbf{H} / \partial t\end{aligned}$$

From a long view of the history of mankind, there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. Richard Feynmann

Written in terms of two fields \mathbf{H} (A m^{-1}) and \mathbf{E} (V m^{-1}), they are valid in free space.

They relate these fields to the charge density ρ (C m^{-3}) and the current density \mathbf{j} (A m^{-2}) at a point.

$$c = (\epsilon_0 \mu_0)^{1/2}$$

$$c = 2.998 \times 10^8 \text{ m s}^{-1}$$

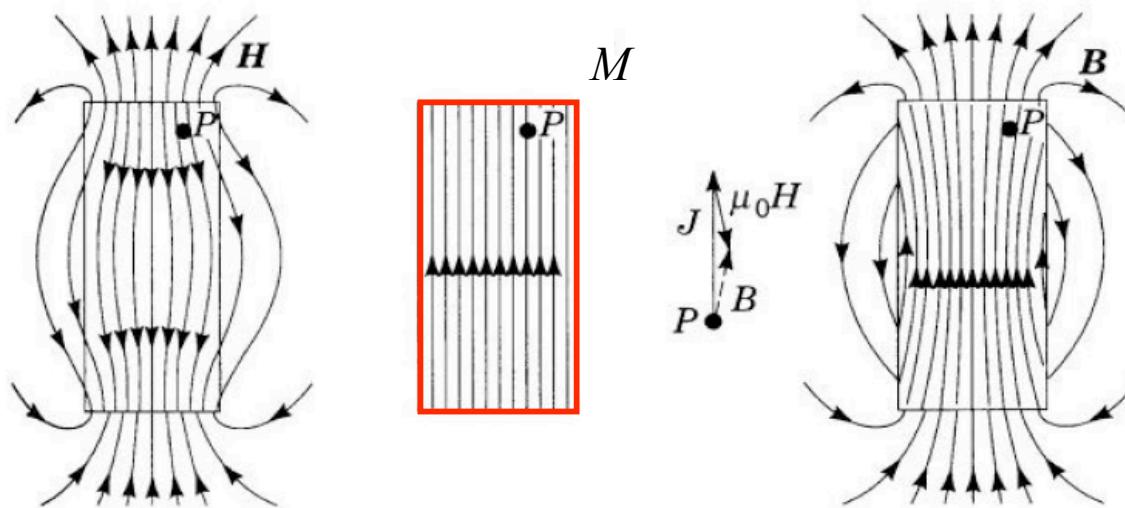
Also, the force on a moving charge q , velocity \mathbf{v}

$$\mathbf{F} = q(\mathbf{E} + \mu_0 \mathbf{v} \times \mathbf{H})$$

B and **H** fields

In free space, they are simply related $\mathbf{B} = \mu_0 \mathbf{H}$

μ_0 is a constant equal to $4\pi \cdot 10^{-7}$. 1 tesla $\approx 800,000 \text{ Am}^{-1}$



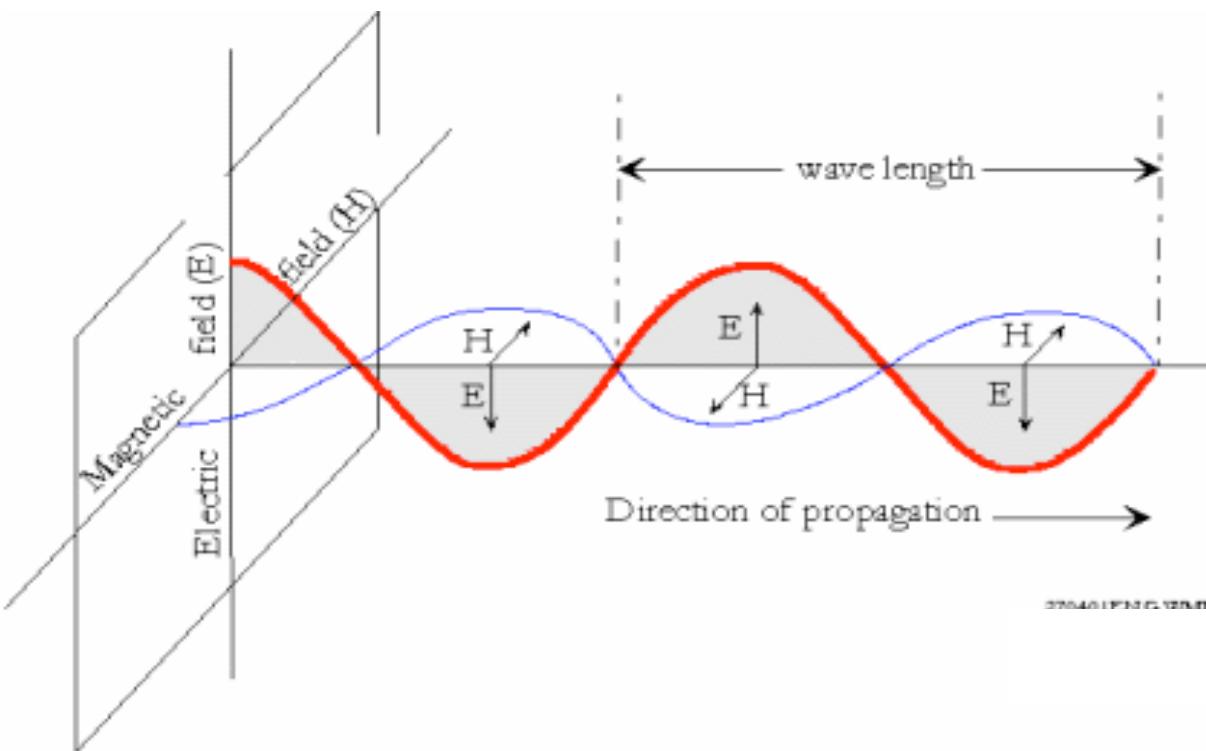
A solution of Maxwell's equations are electromagnetic waves. The electric and magnetic fields (E and H) follow coupled wave equations.

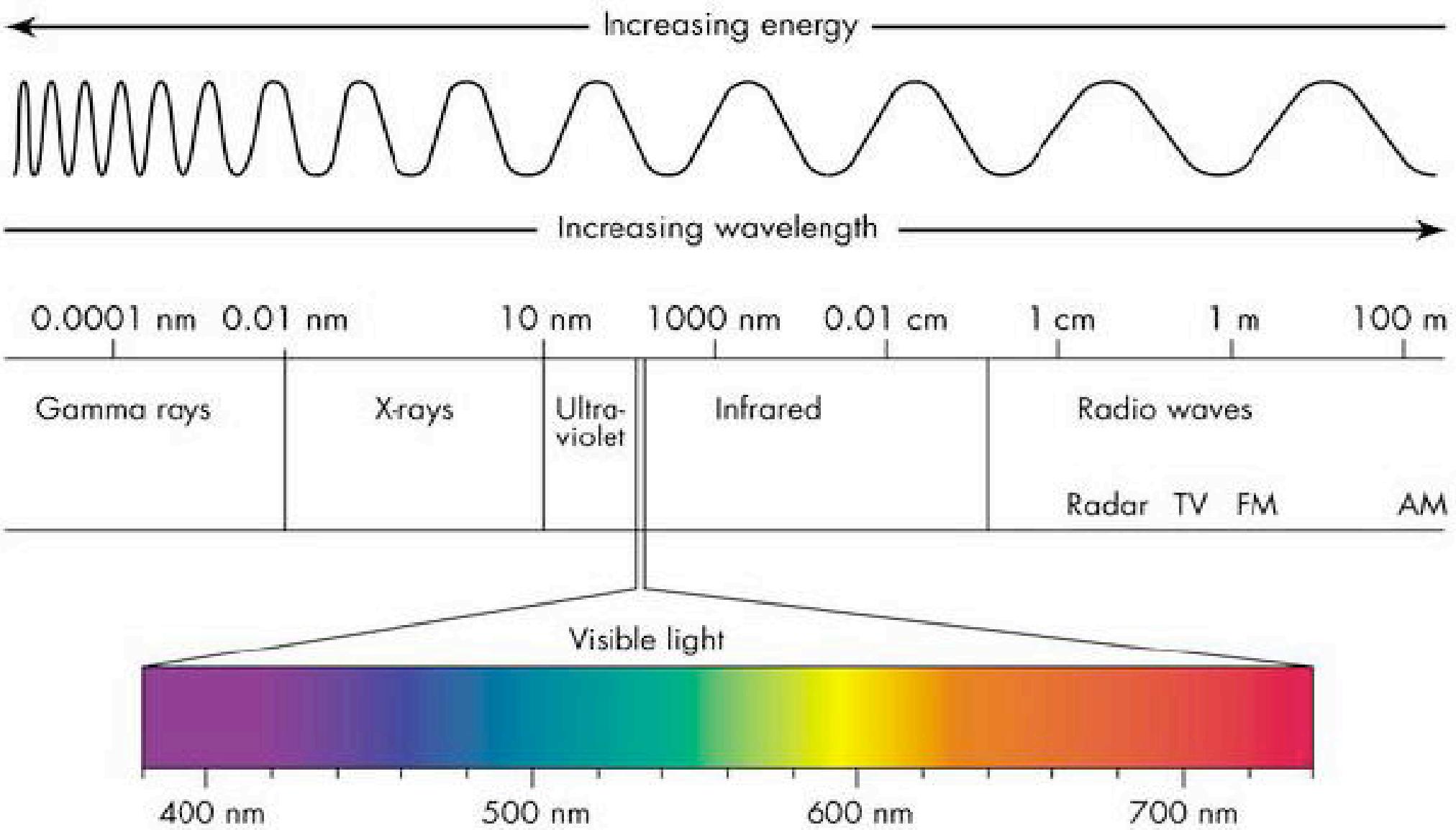
$$\nabla^2 E - \mu_0 \epsilon_0 \partial^2 E / \partial t^2 = 0$$

$$\nabla^2 H - \mu_0 \epsilon_0 \partial^2 H / \partial t^2 = 0$$

where $\mu_0 \epsilon_0 = c^2$

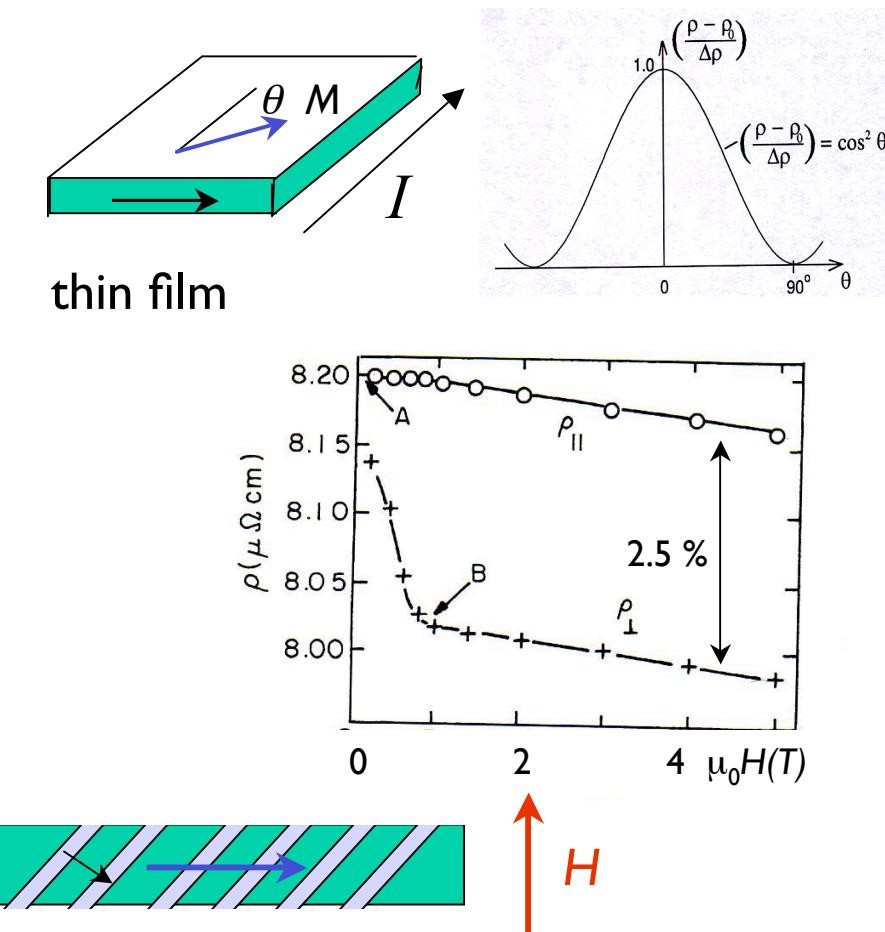
$$c = \lambda \nu$$





- 1820 Oersted discovers the magnetic effect of electric currents
- 1821 Ampere attributes the magnetism of matter to 'molecular' currents
- 1821 Faraday builds a primitive electric motor
- 1825 Sturgeon invents the first practical electromagnet
- 1831 Faraday discovers electromagnetic induction
- 1833 Gauss and Weber build a telegraph more than 1 km long, with a galvanometer as the receiver
- 1845 Faraday discovers paramagnetism and diamagnetism, also magneo-optic Faraday effect
- 1847 Helmholtz states the conservation of energy in a general form
- 1858 The first transatlantic telegraph cable
- 1864-73 Maxwell formulates the theory of electromagnetism**
- 1869 Gramme invents a practical dynamo
- 1879 Swan invents a practical incandescent bulb
- 1881 First public electric railway in Berlin
- 1882 First hydroelectric power station
- 1887 Hertz generates and detects radio waves
- 1887 Michelson and Morely *fail* to detect the motion of the aether
- 1888 Tesla invents a practical AC motor
- 1890 Ewing describes hysteresis
- 1895 Curie describes the temperature variation of paramagnetic susceptibility
- 1896 Marconi patents the radio; transmits radio signals across the Atlantic in 1901
- 1898 Valdemar Poulsen invents magnetic recording

(Anisotropic) magnetoresistance 1857



Discovered by William Thompson

$$\rho = \rho_0 + \Delta\rho \cos^2 \theta$$

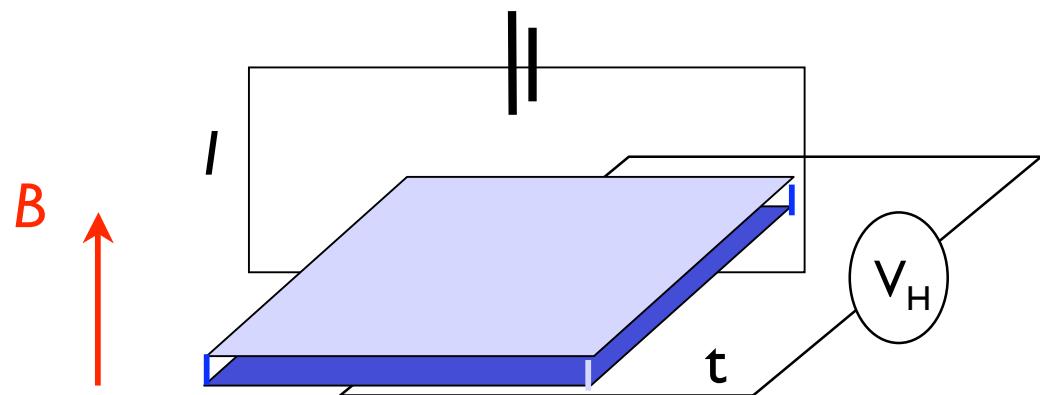
Magnitude of the effect $\Delta\rho/\rho < 3\%$
The effect is usually positive; $\rho_{||} > \rho_{\perp}$

Maximum sensitivity $d\rho/d\theta$ occurs
when $\theta = 45^\circ$. Hence the 'barber-pole'
configuration used for devices.

AMR is due to spin-orbit s-d scattering

Hall effect 1879

Effect discovered by Edwin Hall in 1879



$$V_H/t = R_0 j B \quad R_0 = (I/ne)$$

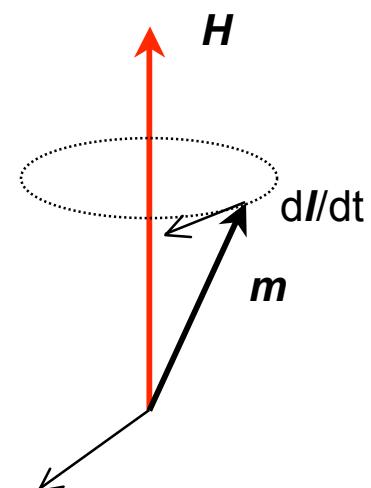
$$E/j = \rho_{xy} = R_0 B$$

$$\mathbf{E} = -R_0(\mathbf{j} \times \mathbf{B})$$

Magnetic resonance



Joseph Larmor 1857-1942



$$\Gamma = \mu_0 m \times H$$

classical

$$v_L = \mu_0 e H / 2\pi m_e$$

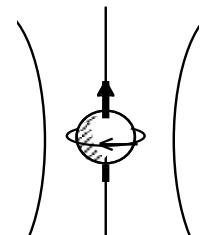
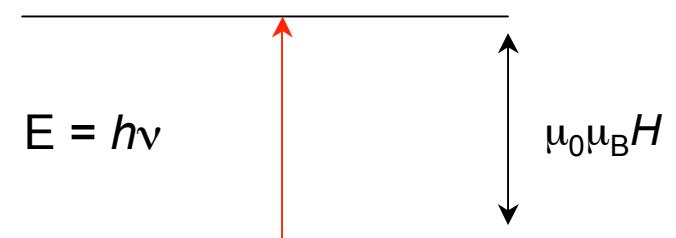
quantum

$$v_L = 35 \text{ kHz/(A m}^{-1}\text{)}$$

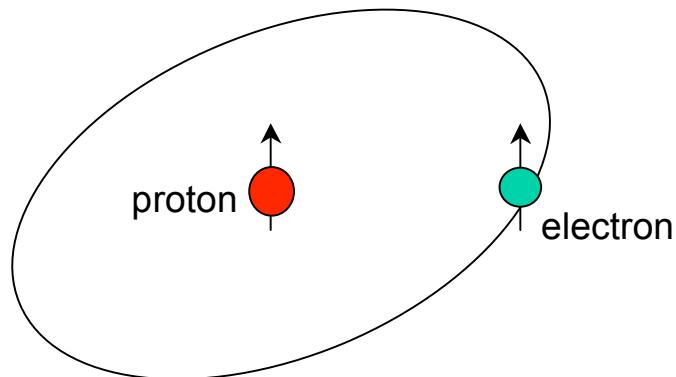
$$v_L = 28 \text{ GHz/T}$$

Resonance for an electron:

m is $e\hbar/2m_e = 1$ Bohr magneton



Nuclear and electron spin resonance



	moment Am ²
proton	$1.41 \cdot 10^{-26}$
electron	$9.27 \cdot 10^{-24}$

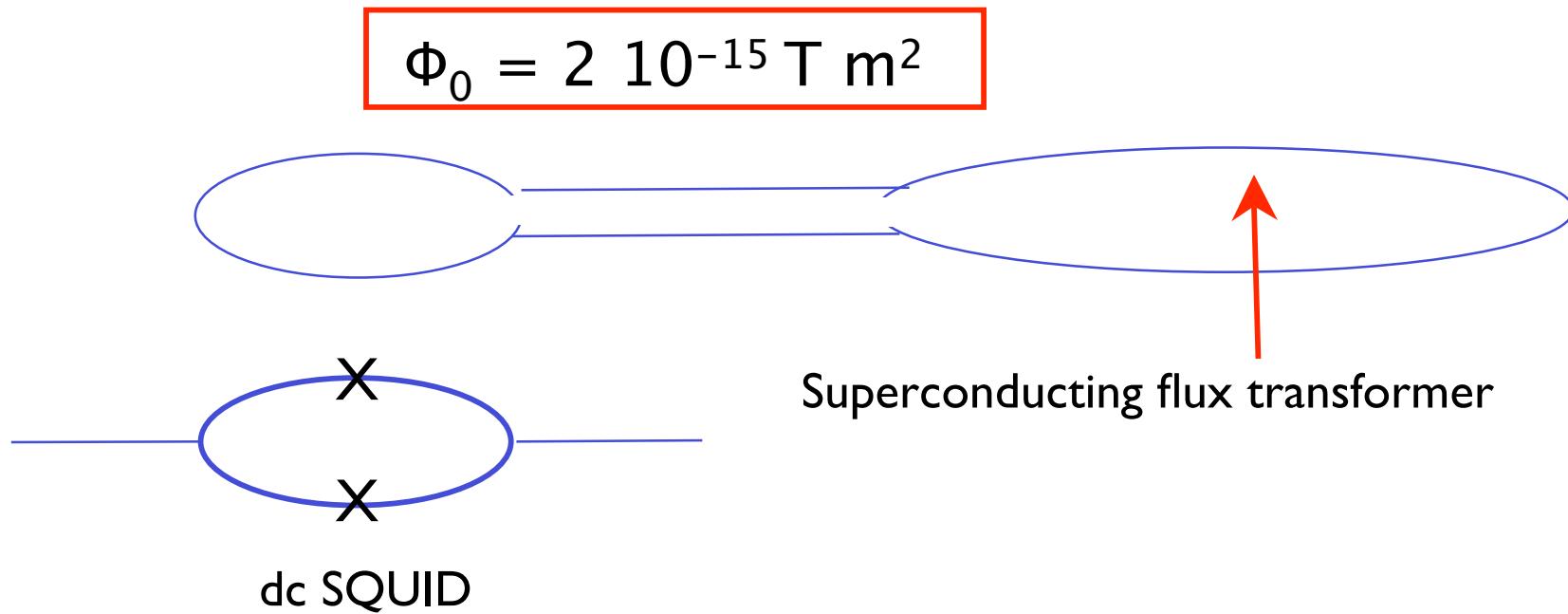


Radar was developed during WWII



Superconducting quantum interference devices (SQUIDs)

SQUIDs detect the change of flux threading a flux-locked loop. The flux is generally coupled to the SQUID via a superconducting flux transformer. The device is sensitive to a small fraction of a flux quantum. SQUIDSs offer ultimate field sensitivity. They generally operate with a flux-locked loop.

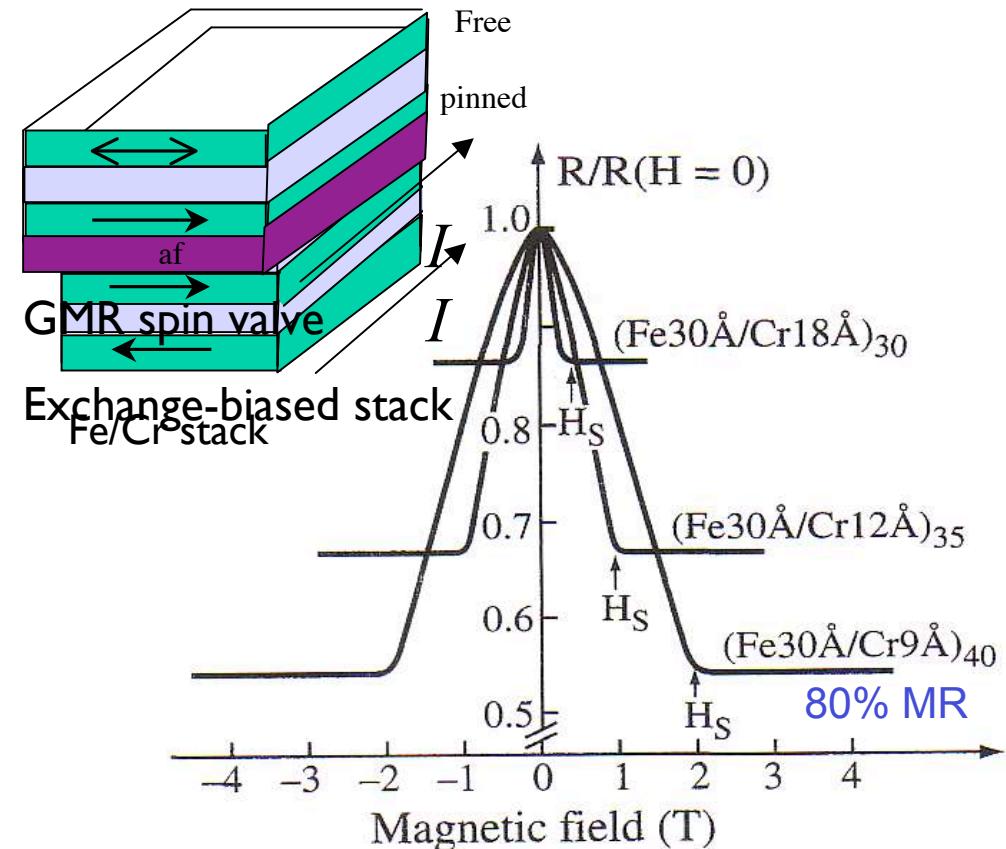


Giant magnetoresistance.



Peter Grunberg and Albert Fert;

10^9 GMR
sensors
per year



Discovery of GMR 1988

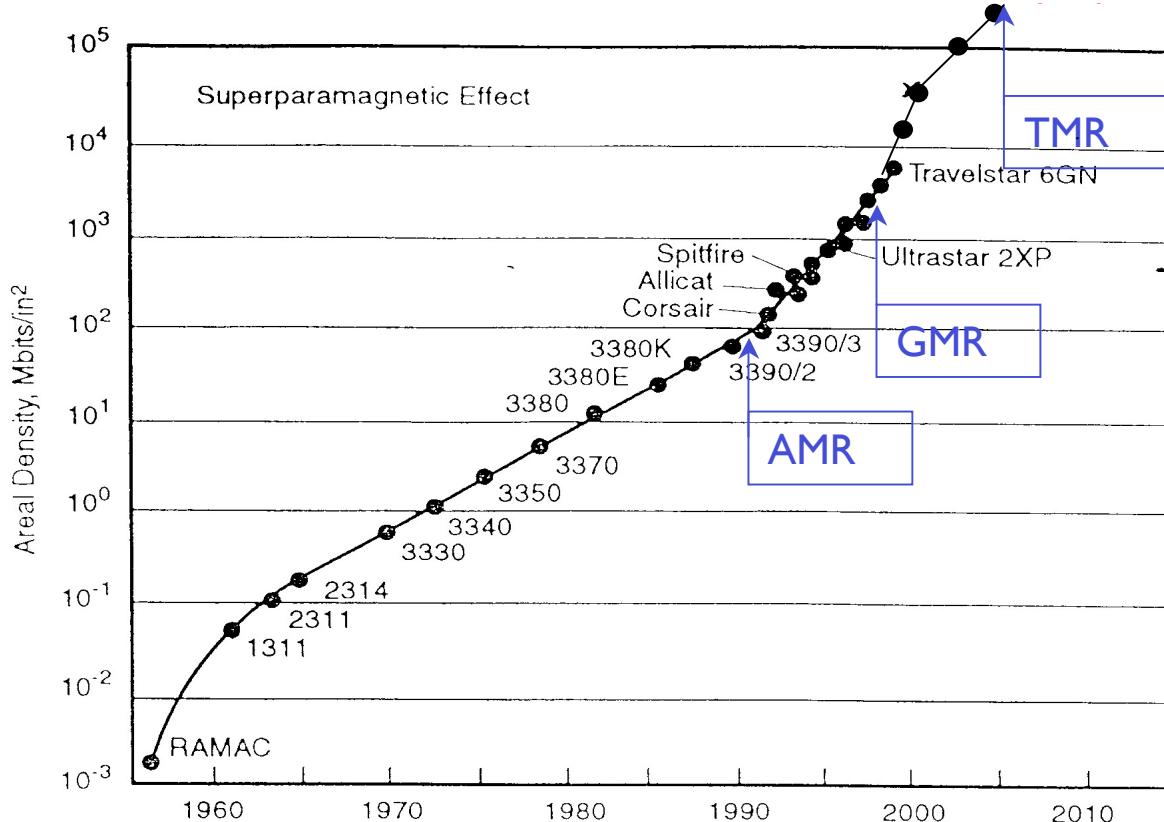
Implementation in hard disk drives 1998

Nobel Prize 2007

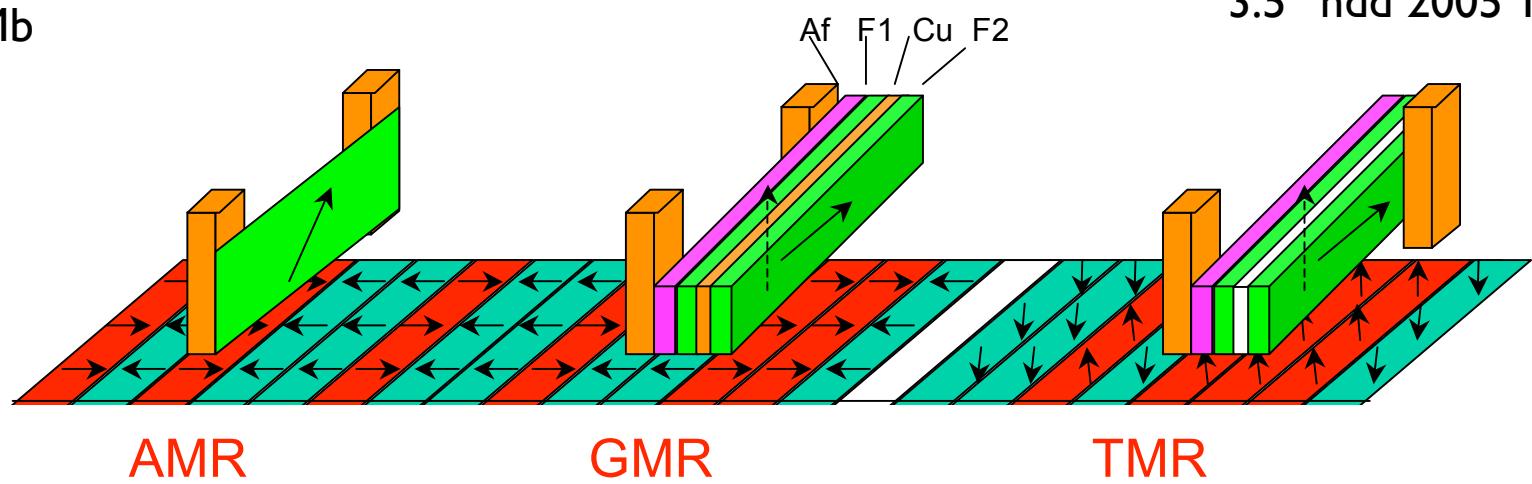
$\sim 10^{10}$ bytes/year

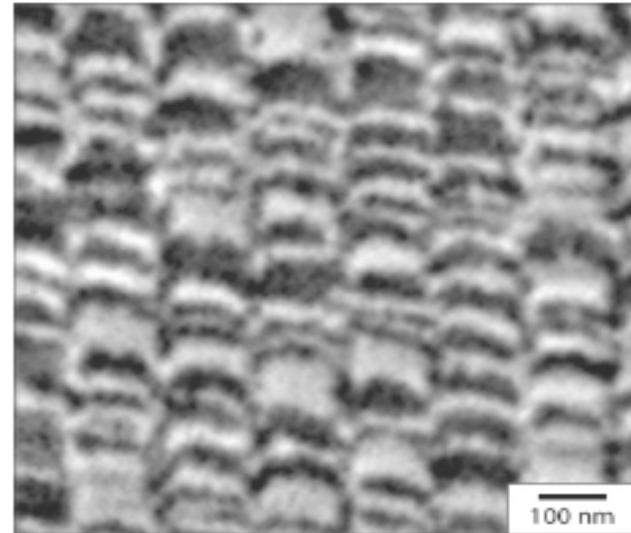
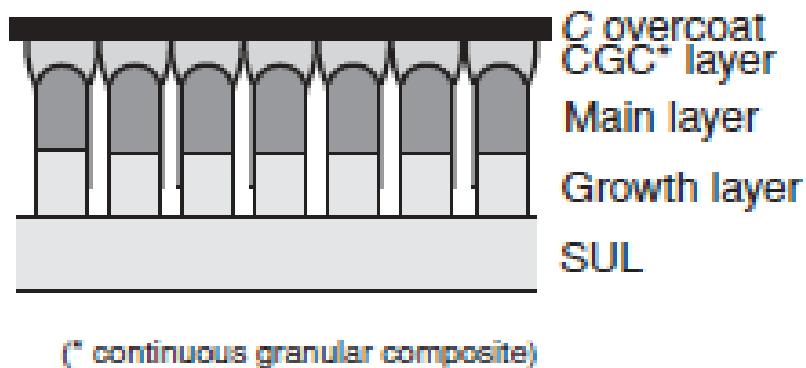
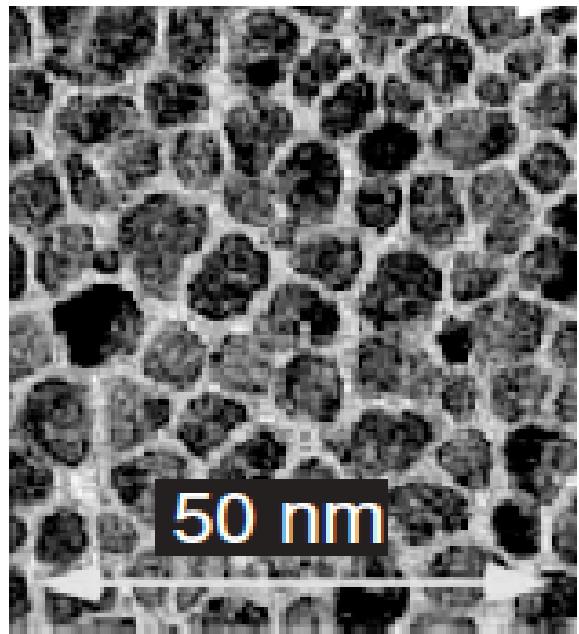


RAMAC 1955 40 Mb



3.5" hdd 2005 100 Gb

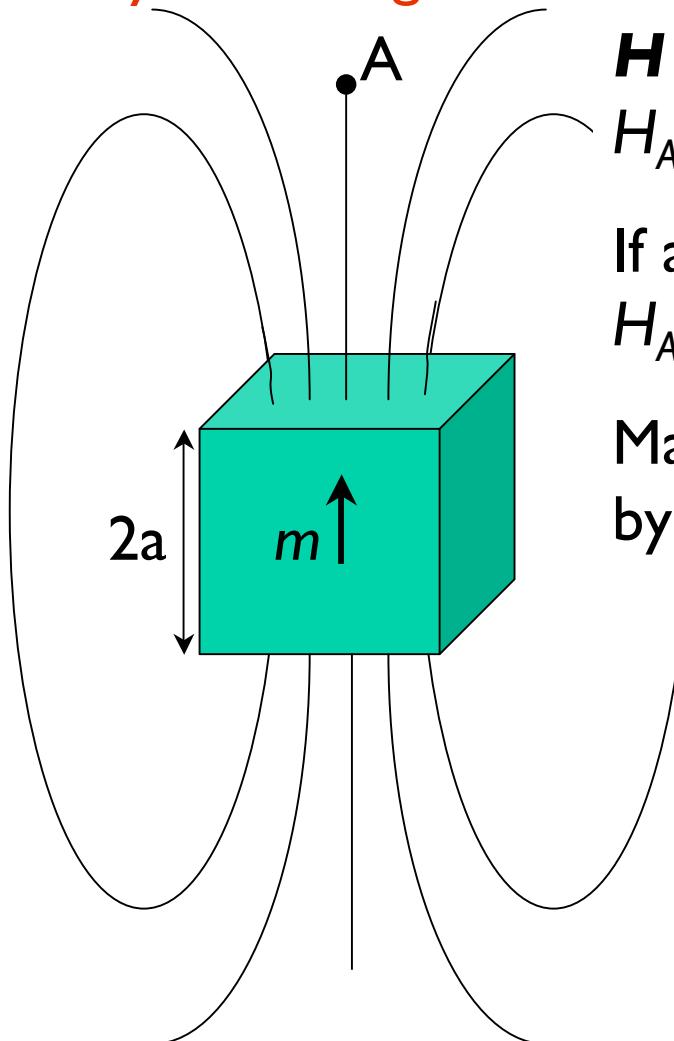




Perpendicular tracks on a hard disc imaged by high-resolution magnetic force microscopy. The width of the tracts is determined by the width of the write head. The recording density here is 300 bits μm^{-2} or 250 Gbit/square inch. (Courtesy of Nanoscan AG.)

Scaling

Why does magnetism lend itself to miniaturization ?



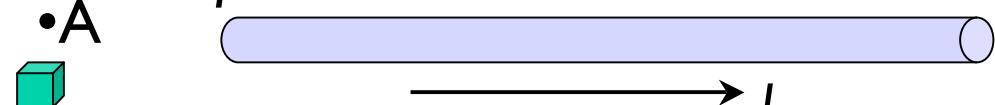
$$\mathbf{H} = (m/4\pi r^3)[2\cos\theta \mathbf{e}_r + \sin\theta \mathbf{e}_\theta]$$

$$H_A = 2M a^3 / 4\pi r^3;$$

If $a = 0.1\text{m}$, $r = 4a$, $M = 1 \text{ MAm}^{-1}$

$$H_A = 2M/16\pi = 40 \text{ kAm}^{-1} (50 \text{ mT})$$

Magnet-generated fields are limited
by M . Scale-independent



$$H = I/2\pi r = 8\mu_0 I r \quad H \sim r$$

Current-generated fields are
limited by j . Scaling is poor