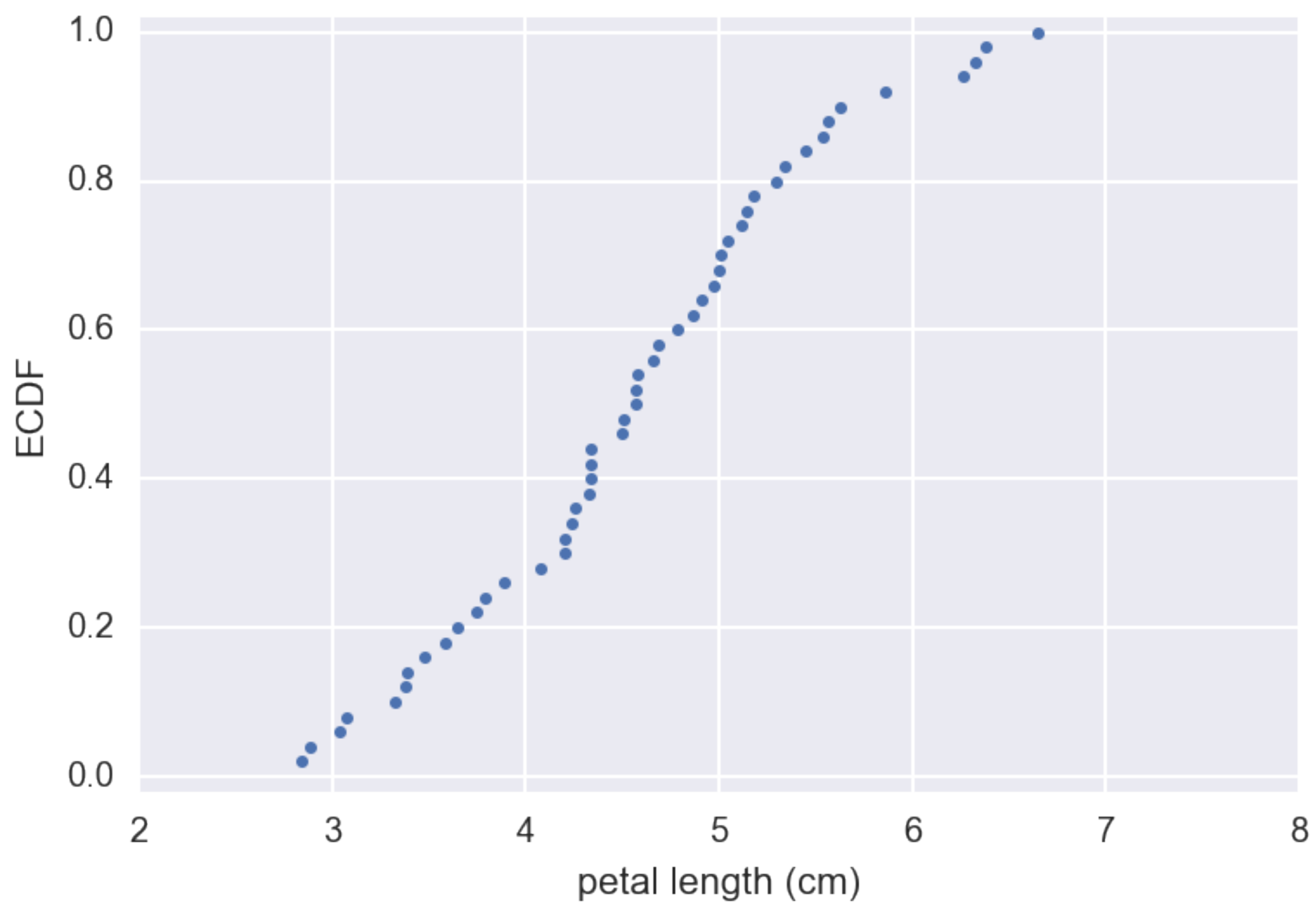
Probabilistic logic and statistical inference

50 measurements of petal length



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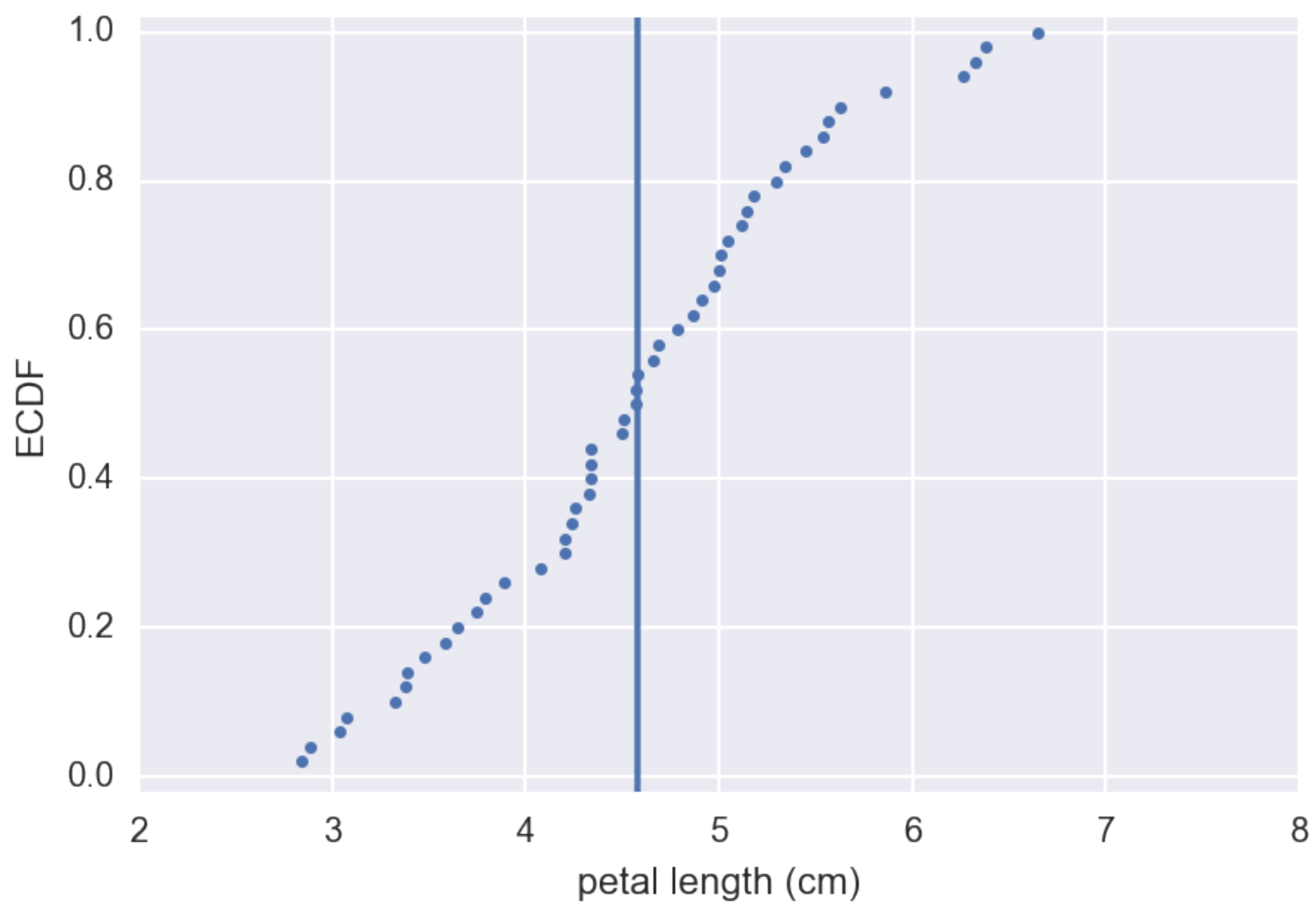

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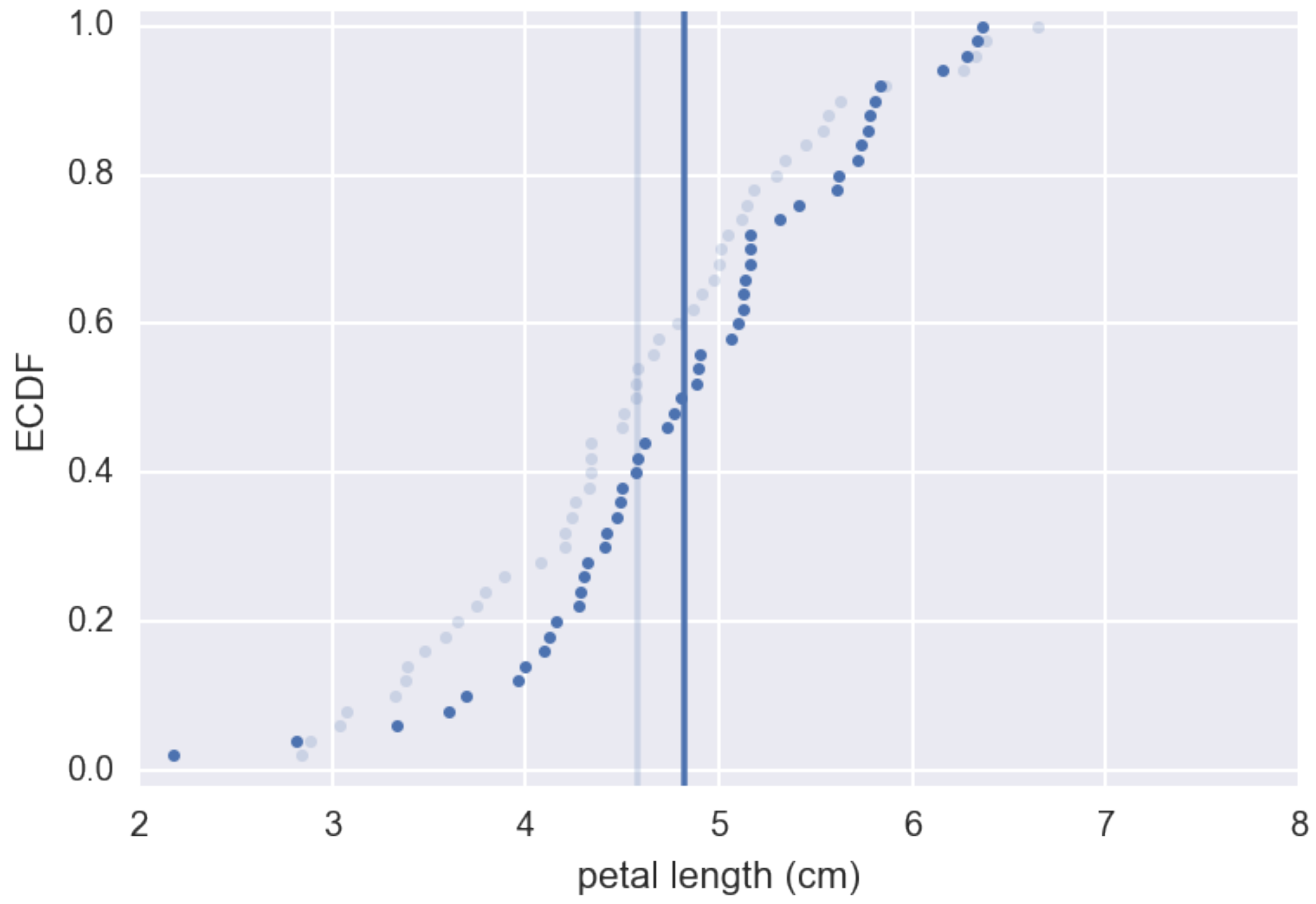

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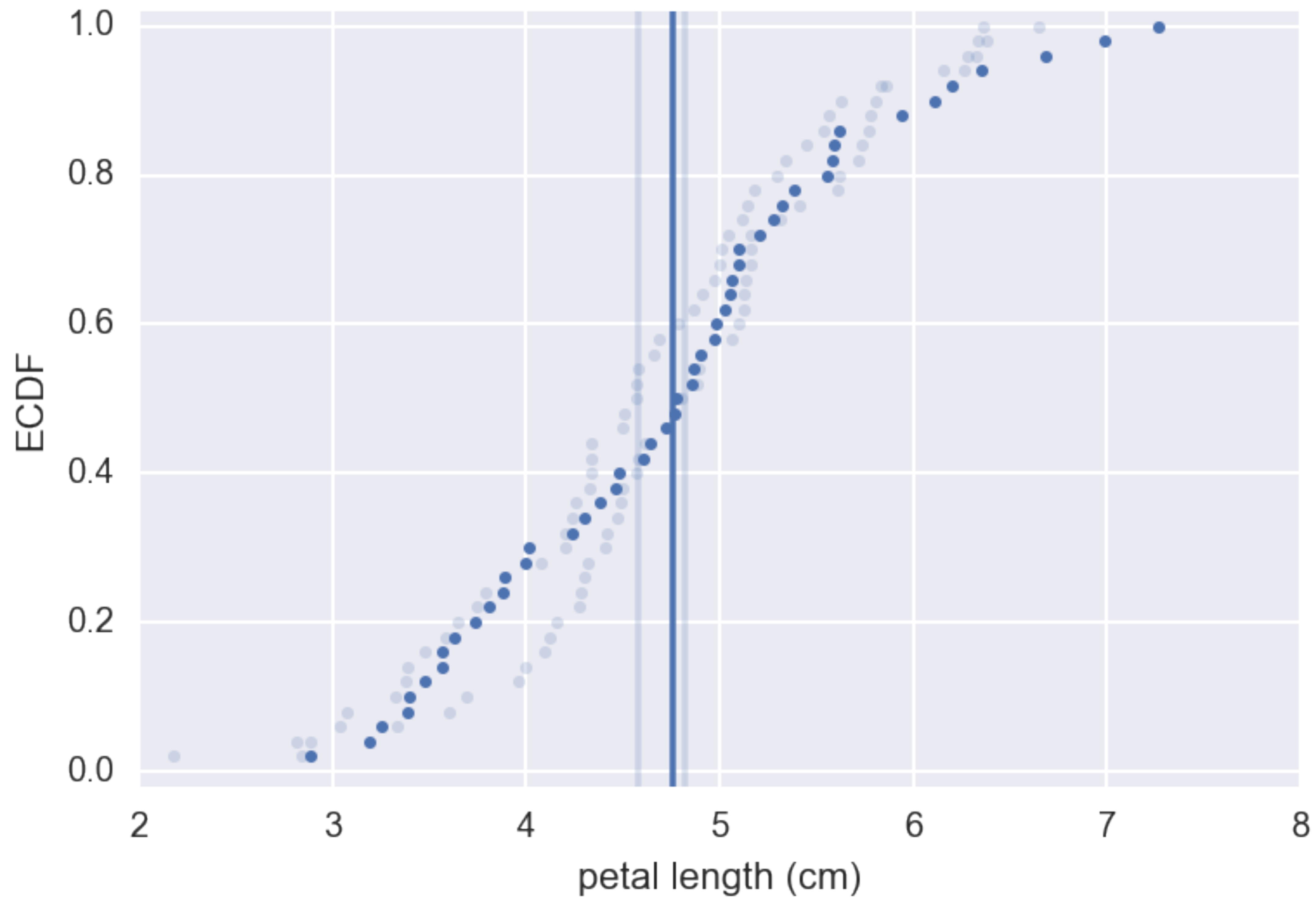

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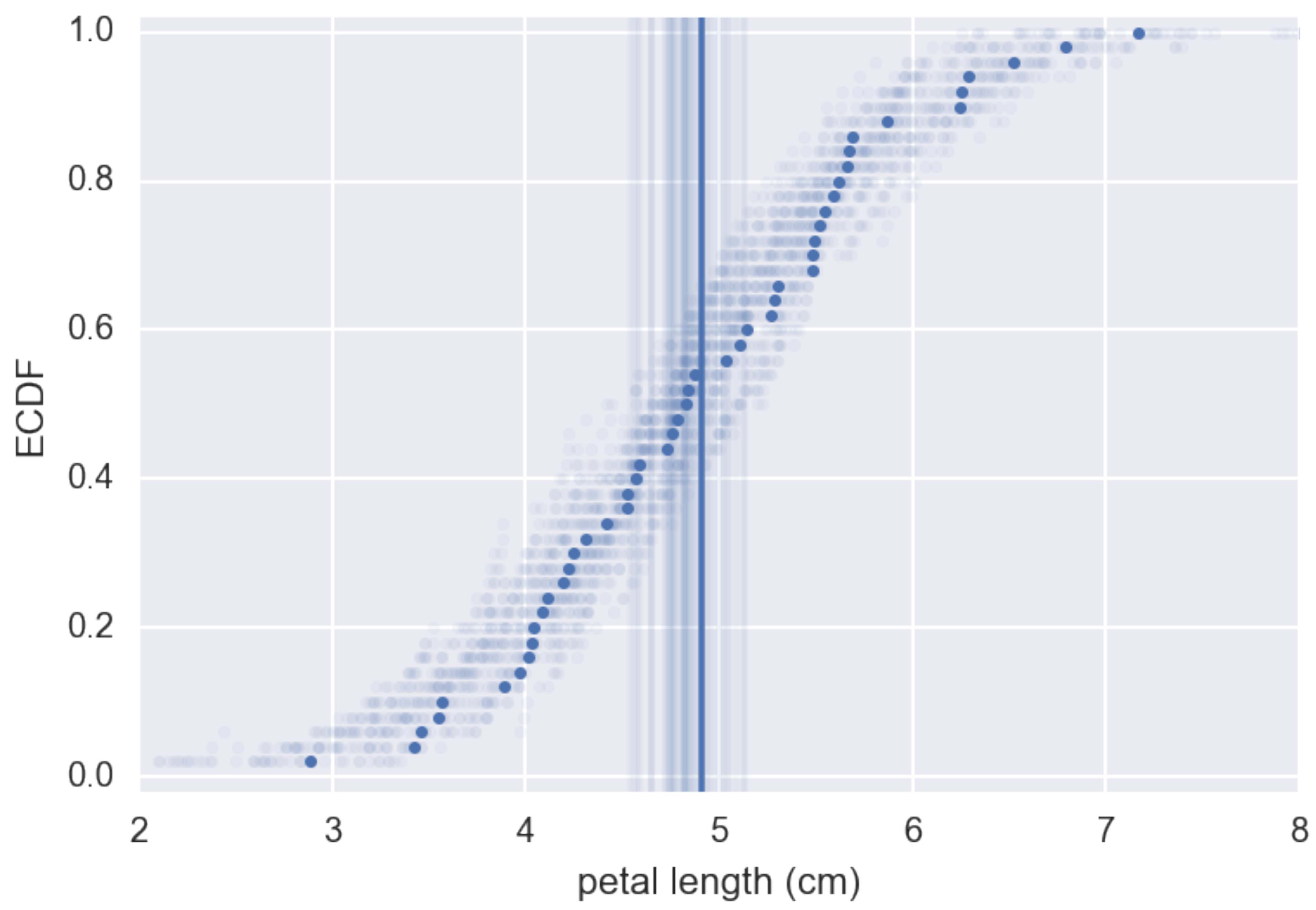
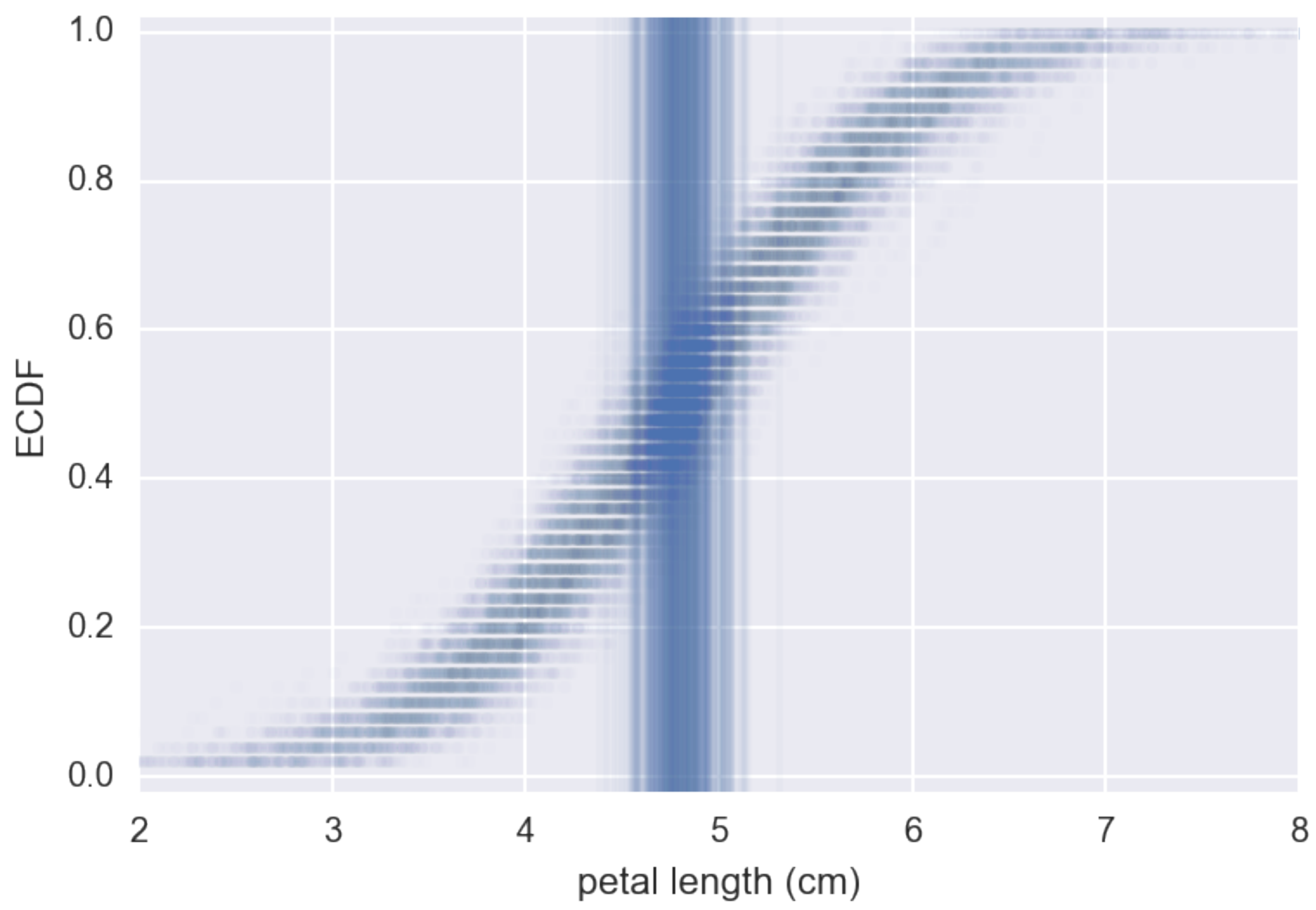


Repeats of 50 measurements of petal length



Let’s practice!

Random number generators and hacker statistics

Hacker statistics

- Uses simulated repeated measurements to compute probabilities.


Blaise Pascal

Image: artist unknown

Image: Heritage Auction

The `np.random` module

- Suite of functions based on *random number generation*
- `np.random.random()`:

  draw a number between 0 and 1

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< 0.5 ⟶ 

≥ 0.5 ⟶ 

Images: Heritage Auction

Bernoulli trial

- An experiment that has two options, "success" (True) and "failure" (False).

Random number seed

- Integer fed into random number generating algorithm
- Manually seed random number generator if you need reproducibility
- Specified using `np.random.seed()`

Simulating 4 coin flips

```
In [1]: import numpy as np

In [2]: np.random.seed(42)

In [3]: random_numbers = np.random.random(size=4)

In [4]: random_numbers
Out[4]: array([ 0.37454012,  0.95071431,  0.73199394,
0.59865848])

In [5]: heads = random_numbers < 0.5

In [6]: heads
Out[6]: array([ True, False, False, False], dtype=bool)

In [7]: np.sum(heads)
Out[7]: 1
```

Simulating 4 coin flips

```
In [1]: n_all_heads = 0  # Initialize number of 4-heads trials

In [2]: for _ in range(10000):
   ...:     heads = np.random.random(size=4) < 0.5
   ...:     n_heads = np.sum(heads)
   ...:     if n_heads == 4:
   ...:         n_all_heads += 1
   ...:
   ...:

In [3]: n_all_heads / 10000
Out[3]: 0.0621
```

Hacker stats probabilities

- Determine how to simulate data
- Simulate many many times
- Probability is approximately fraction of trials with the outcome of interest

Let’s practice!

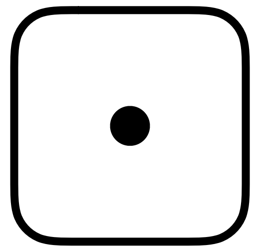
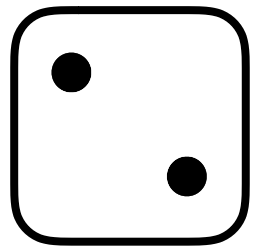
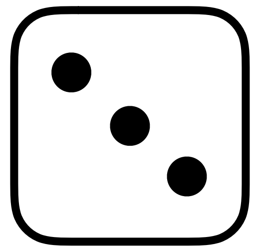
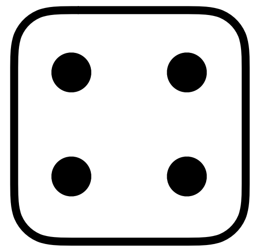
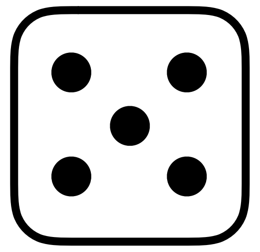
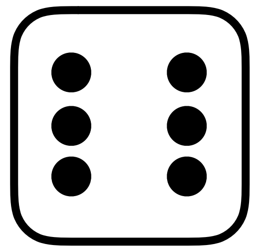
Probability distributions and stories: The Binomial distribution

Probability mass function (PMF)

- The set of probabilities of discrete outcomes

Discrete Uniform PMF

Tabular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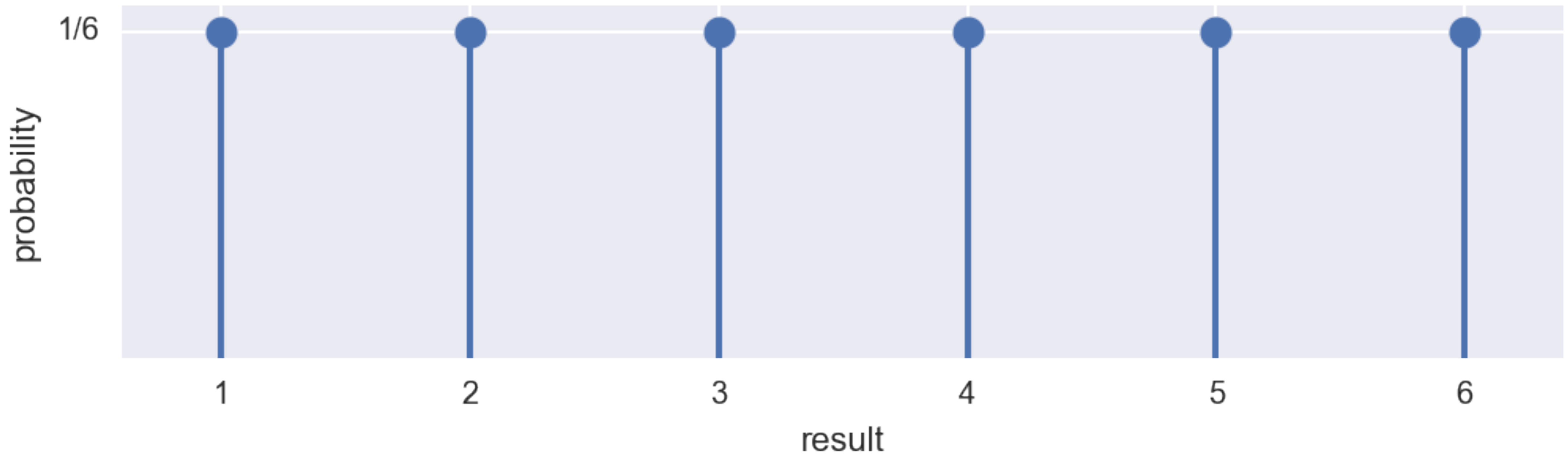
Graphical



Probability distribution

- A mathematical description of outcomes

Discrete Uniform distribution: the story

- The outcome of rolling a single fair die is Discrete Uniformly distributed.

Binomial distribution: the story

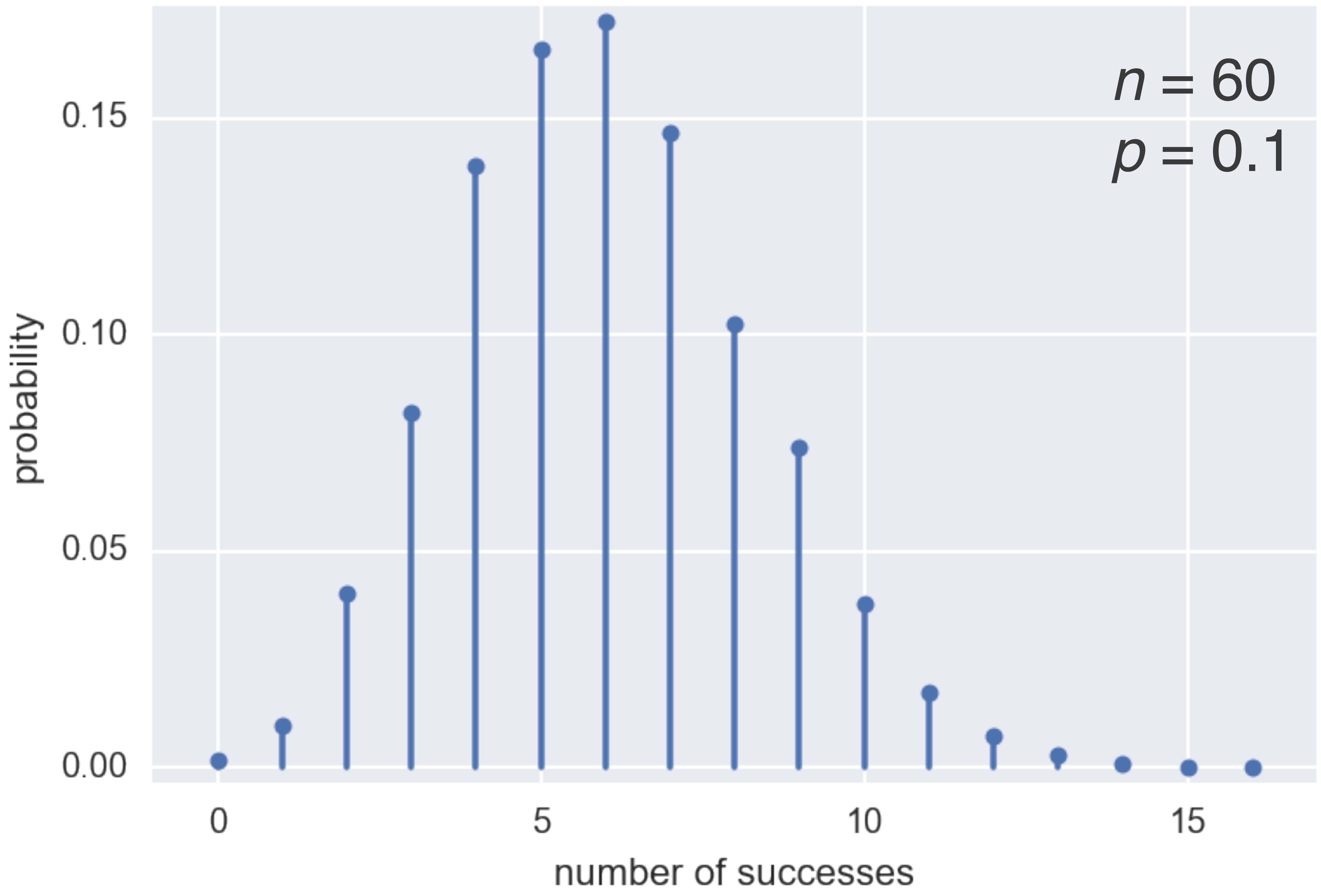
- The number r of successes in n Bernoulli trials with probability p of success, is Binomially distributed
- The number r of heads in 4 coin flips with probability 0.5 of heads, is Binomially distributed

Sampling from the Binomial distribution

```
In [1]: np.random.binomial(4, 0.5)
Out[1]: 2

In [2]: np.random.binomial(4, 0.5, size=10)
Out[2]: array([4, 3, 2, 1, 1, 0, 3, 2, 3, 0])
```

The Binomial PMF

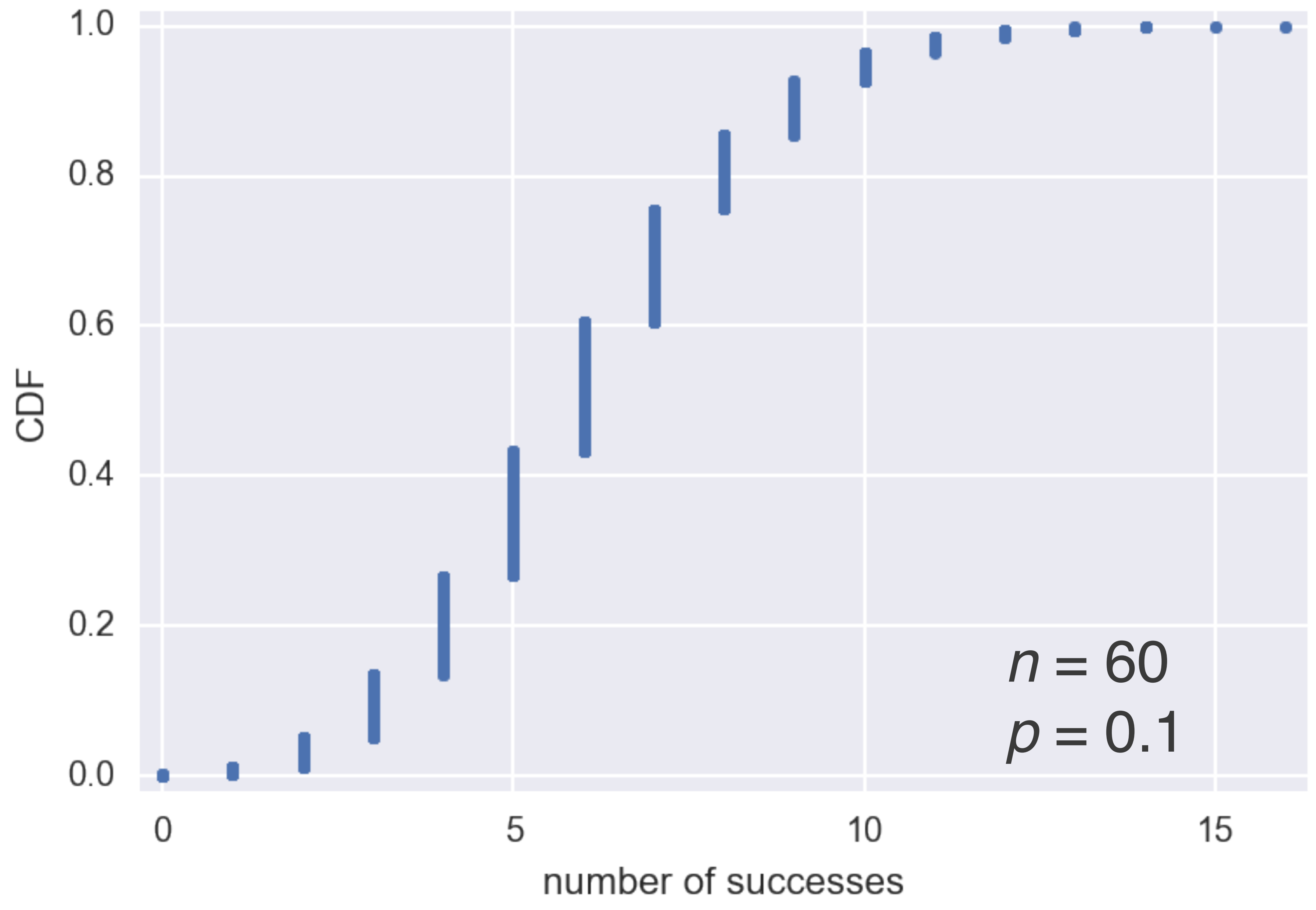
```
In [1]: samples = np.random.binomial(60, 0.1, size=10000)
```



The Binomial CDF

```
In [1]: import matplotlib.pyplot as plt

In [2]: import seaborn as sns

In [3]: sns.set()

In [4]: x, y = ecdf(samples)

In [5]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [6]: plt.margins(0.02)

In [7]: _ = plt.xlabel('number of successes')

In [8]: _ = plt.ylabel('CDF')

In [9]: plt.show()
```

The Binomial CDF



Let’s practice!

Poisson processes and the Poisson distribution

Poisson process

- The timing of the next event is completely independent of when the previous event happened

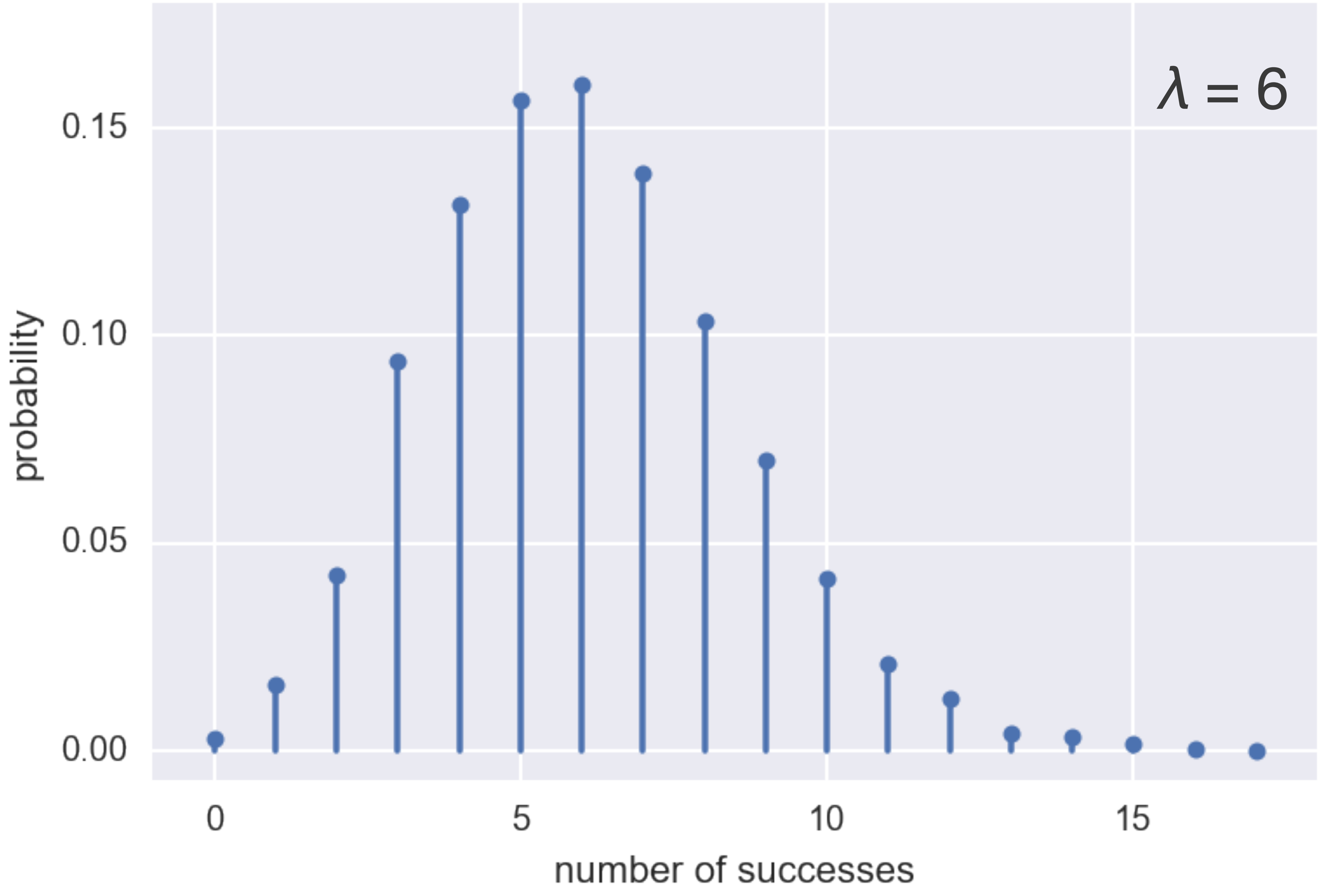
Examples of Poisson processes

- Natural births in a given hospital
- Hit on a website during a given hour
- Meteor strikes
- Molecular collisions in a gas
- Aviation incidents
- Buses in Poissonville

Poisson distribution

- The number r of arrivals of a Poisson process in a given time interval with average rate of λ arrivals per interval is Poisson distributed.
- The number r of hits on a website in one hour with an average hit rate of 6 hits per hour is Poisson distributed.

Poisson PMF



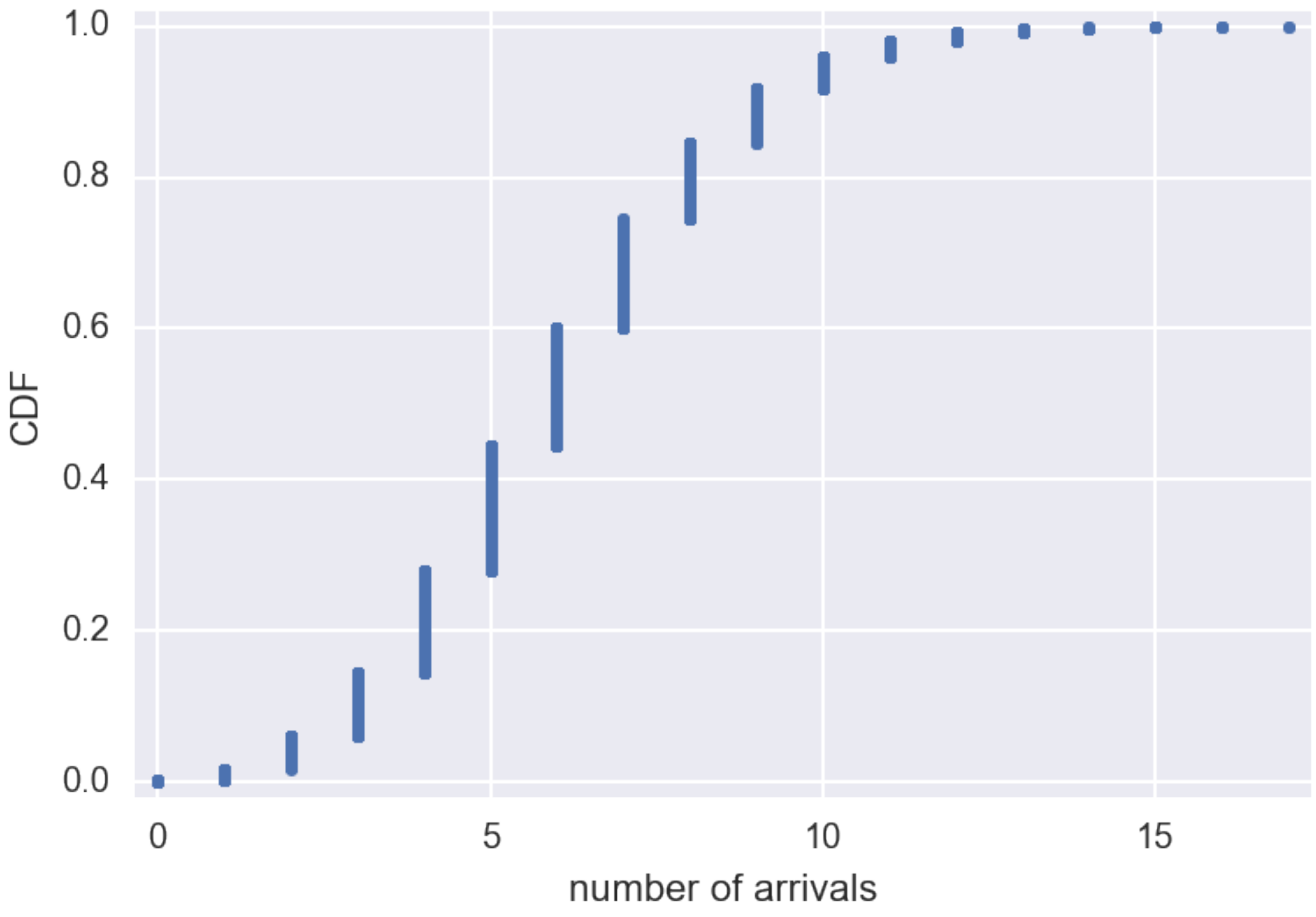
Poisson Distribution

- Limit of the Binomial distribution for low probability of success and large number of trials.
- That is, for rare events.

The Poisson CDF

```
In [1]: samples = np.random.poisson(6, size=10000)

In [2]: x, y = ecdf(samples)

In [3]: _ = plt.plot(x, y, marker='.', linestyle='none')

In [4]: plt.margins(0.02)

In [5]: _ = plt.xlabel('number of successes')

In [6]: _ = plt.ylabel('CDF')

In [7]: plt.show()
```

The Poisson CDF



Let’s practice!