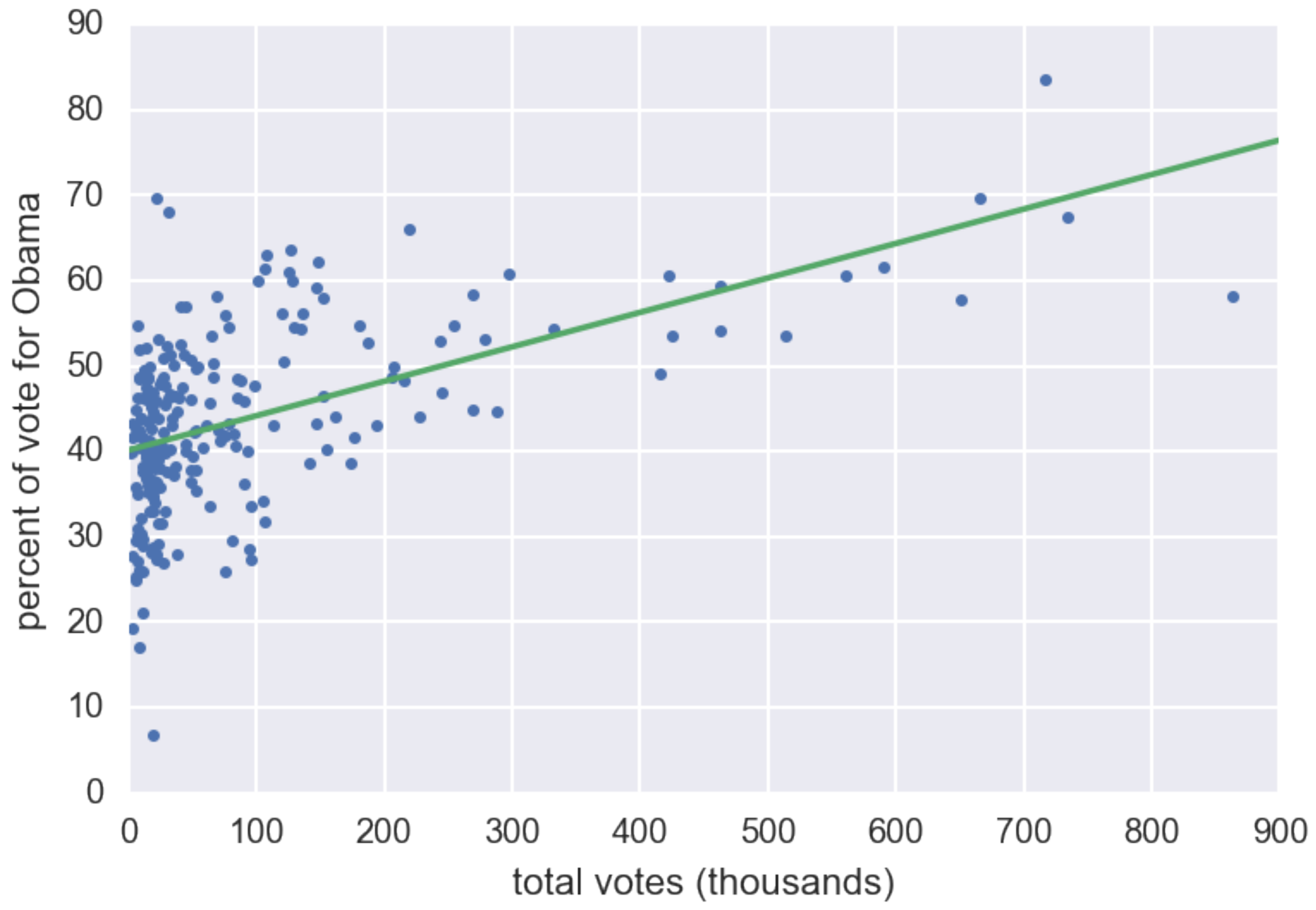
Formulating and simulating hypotheses

2008 US swing state election results



Data retrieved from Data.gov (https://www.data.gov/)

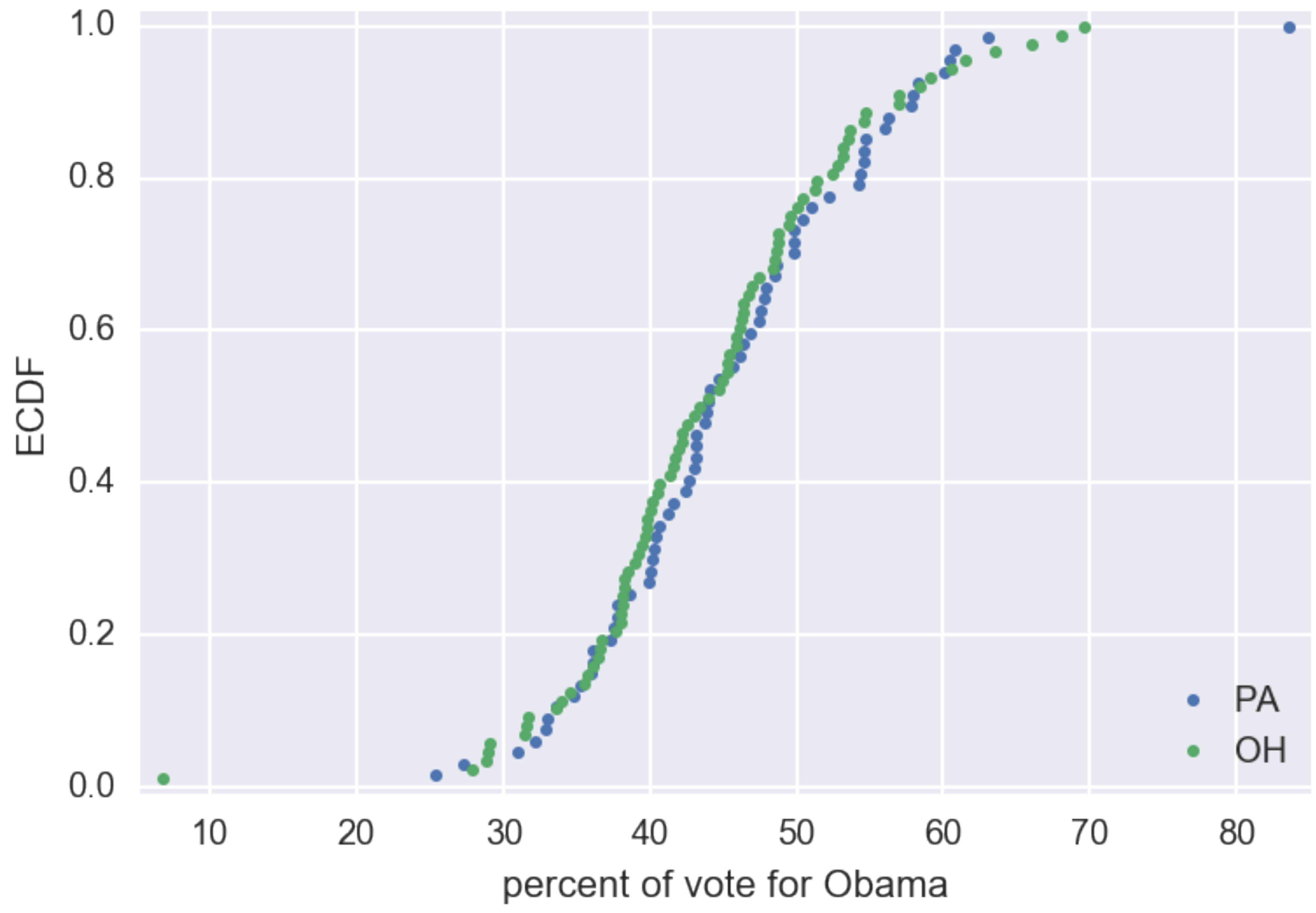
Hypothesis testing

- Assessment of how reasonable the observed data are assuming a hypothesis is true

Null hypothesis

- Another name for the hypothesis you are testing

ECDFs of swing state election results



Data retrieved from Data.gov (https://www.data.gov/)

Percent vote for Obama

	PA	OH	PA — OH difference
mean	45.5%	44.3%	1.2%
median	44.0%	43.7%	0.4%
standard deviation	9.8%	9.9%	—0.1%

Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis

```
60.08,  40.64,  36.07,  41.21,  31.04,  43.78,  44.08,  46.85,
44.71,  46.15,  63.10,  52.20,  43.18,  40.24,  39.92,  47.87,
37.77,  40.11,  49.85,  48.61,  38.62,  54.25,  34.84,  47.75,
43.82,  55.97,  58.23,  42.97,  42.38,  36.11,  37.53,  42.65,
50.96,  47.43,  56.24,  45.60,  46.39,  35.22,  48.56,  32.97,
57.88,  36.05,  37.72,  50.36,  32.12,  41.55,  54.66,  57.81,
54.58,  32.88,  54.37,  40.45,  47.61,  60.49,  43.11,  27.32,
44.03,  33.56,  37.26,  54.64,  43.12,  25.34,  49.79,  83.56,
40.09,  60.81,  49.81,  56.94,  50.46,  65.99,  45.88,  42.23,
45.26,  57.01,  53.61,  59.10,  61.48,  43.43,  44.69,  54.59,
48.36,  45.89,  48.62,  43.92,  38.23,  28.79,  63.57,  38.07,
40.18,  43.05,  41.56,  42.49,  36.06,  52.76,  46.07,  39.43,
39.26,  47.47,  27.92,  38.01,  45.45,  29.07,  28.94,  51.28,
50.10,  39.84,  36.43,  35.71,  31.47,  47.01,  40.10,  48.76,
31.56,  39.86,  45.31,  35.47,  51.38,  46.33,  48.73,  41.77,
41.32,  48.46,  53.14,  34.01,  54.74,  40.67,  38.96,  46.29,
38.25,   6.80,  31.75,  46.33,  44.90,  33.57,  38.10,  39.67,
40.47,  49.44,  37.62,  36.71,  46.73,  42.20,  53.16,  52.40,
58.36,  68.02,  38.53,  34.58,  69.64,  60.50,  53.53,  36.54,
49.58,  41.97,  38.11
```

Pennsylvania

Ohio

Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis

```
60.08,  40.64,  36.07,  41.21,  31.04,  43.78,  44.08,  46.85,
44.71,  46.15,  63.10,  52.20,  43.18,  40.24,  39.92,  47.87,
37.77,  40.11,  49.85,  48.61,  38.62,  54.25,  34.84,  47.75,
43.82,  55.97,  58.23,  42.97,  42.38,  36.11,  37.53,  42.65,
50.96,  47.43,  56.24,  45.60,  46.39,  35.22,  48.56,  32.97,
57.88,  36.05,  37.72,  50.36,  32.12,  41.55,  54.66,  57.81,
54.58,  32.88,  54.37,  40.45,  47.61,  60.49,  43.11,  27.32,
44.03,  33.56,  37.26,  54.64,  43.12,  25.34,  49.79,  83.56,
40.09,  60.81,  49.81,  56.94,  50.46,  65.99,  45.88,  42.23,
45.26,  57.01,  53.61,  59.10,  61.48,  43.43,  44.69,  54.59,
48.36,  45.89,  48.62,  43.92,  38.23,  28.79,  63.57,  38.07,
40.18,  43.05,  41.56,  42.49,  36.06,  52.76,  46.07,  39.43,
39.26,  47.47,  27.92,  38.01,  45.45,  29.07,  28.94,  51.28,
50.10,  39.84,  36.43,  35.71,  31.47,  47.01,  40.10,  48.76,
31.56,  39.86,  45.31,  35.47,  51.38,  46.33,  48.73,  41.77,
41.32,  48.46,  53.14,  34.01,  54.74,  40.67,  38.96,  46.29,
38.25,   6.80,  31.75,  46.33,  44.90,  33.57,  38.10,  39.67,
40.47,  49.44,  37.62,  36.71,  46.73,  42.20,  53.16,  52.40,
58.36,  68.02,  38.53,  34.58,  69.64,  60.50,  53.53,  36.54,
49.58,  41.97,  38.11
```

Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis

```
59.10,  38.62,  51.38,  60.49,   6.80,  41.97,  48.56,  37.77,
48.36,  54.59,  40.11,  57.81,  45.89,  83.56,  40.64,  46.07,
28.79,  55.97,  33.57,  42.23,  48.61,  44.69,  39.67,  57.88,
48.62,  54.66,  54.74,  48.46,  36.07,  43.92,  49.85,  53.53,
48.76,  41.77,  36.54,  47.01,  52.76,  49.44,  34.58,  40.24,
44.08,  46.29,  49.81,  69.64,  60.50,  27.32,  45.60,  63.10,
35.71,  39.86,  40.67,  65.99,  50.46,  37.72,  50.96,  42.49,
31.56,  38.23,  37.26,  41.21,  37.53,  46.85,  44.03,  41.32,
45.88,  40.45,  32.12,  35.22,  49.79,  43.12,  43.18,  45.45,
25.34,  46.73,  44.90,  56.94,  58.23,  39.84,  36.05,  43.05,
38.25,  40.47,  31.04,  54.25,  46.15,  57.01,  52.20,  47.75,
36.06,  47.61,  51.28,  43.43,  42.97,  38.01,  54.64,  45.26,
47.47,  34.84,  49.58,  48.73,  29.07,  54.58,  27.92,  34.01,
38.07,  31.47,  36.11,  39.26,  41.56,  52.40,  40.18,  47.87,
46.33,  46.39,  43.11,  38.53,  33.56,  42.65,  68.02,  35.47,
40.09,  36.43,  36.71,  60.08,  50.36,  39.43,  28.94,  58.36,
42.20,  47.43,  44.71,  43.78,  39.92,  37.62,  63.57,  53.61,
40.10,  46.33,  53.16,  32.88,  38.96,  41.55,  56.24,  38.11,
42.38,  38.10,  43.82,  45.31,  60.81,  54.37,  53.14,  32.97,
61.48,  50.10,  31.75
```

Data retrieved from Data.gov (https://www.data.gov/)

Simulating the hypothesis

```
59.10, 38.62, 51.38, 60.49,  6.80, 41.97, 48.56, 37.77,
48.36, 54.59, 40.11, 57.81, 45.89, 83.56, 40.64, 46.07,
28.79, 55.97, 33.57, 42.23, 48.61, 44.69, 39.67, 57.88,
48.62, 54.66, 54.74, 48.46, 36.07, 43.92, 49.85, 53.53,
48.76, 41.77, 36.54, 47.01, 52.76, 49.44, 34.58, 40.24,
44.08, 46.29, 49.81, 69.64, 60.50, 27.32, 45.60, 63.10,
35.71, 39.86, 40.67, 65.99, 50.46, 37.72, 50.96, 42.49,
31.56, 38.23, 37.26, 41.21, 37.53, 46.85, 44.03, 41.32,
45.88, 40.45, 32.12, 35.22, 49.79, 43.12, 43.18, 45.45,
25.34, 46.73, 44.90, 56.94, 58.23, 39.84, 36.05, 43.05,
38.25, 40.47, 31.04, 54.25, 46.15, 57.01, 52.20, 47.75,
36.06, 47.61, 51.28, 43.43, 42.97, 38.01, 54.64, 45.26,
47.47, 34.84, 49.58, 48.73, 29.07, 54.58, 27.92, 34.01,
38.07, 31.47, 36.11, 39.26, 41.56, 52.40, 40.18, 47.87,
46.33, 46.39, 43.11, 38.53, 33.56, 42.65, 68.02, 35.47,
40.09, 36.43, 36.71, 60.08, 50.36, 39.43, 28.94, 58.36,
42.20, 47.43, 44.71, 43.78, 39.92, 37.62, 63.57, 53.61,
40.10, 46.33, 53.16, 32.88, 38.96, 41.55, 56.24, 38.11,
42.38, 38.10, 43.82, 45.31, 60.81, 54.37, 53.14, 32.97,
61.48, 50.10, 31.75
```

"Pennsylvania"

"Ohio"

Data retrieved from Data.gov (https://www.data.gov/)

Permutation

- Random reordering of entries in an array

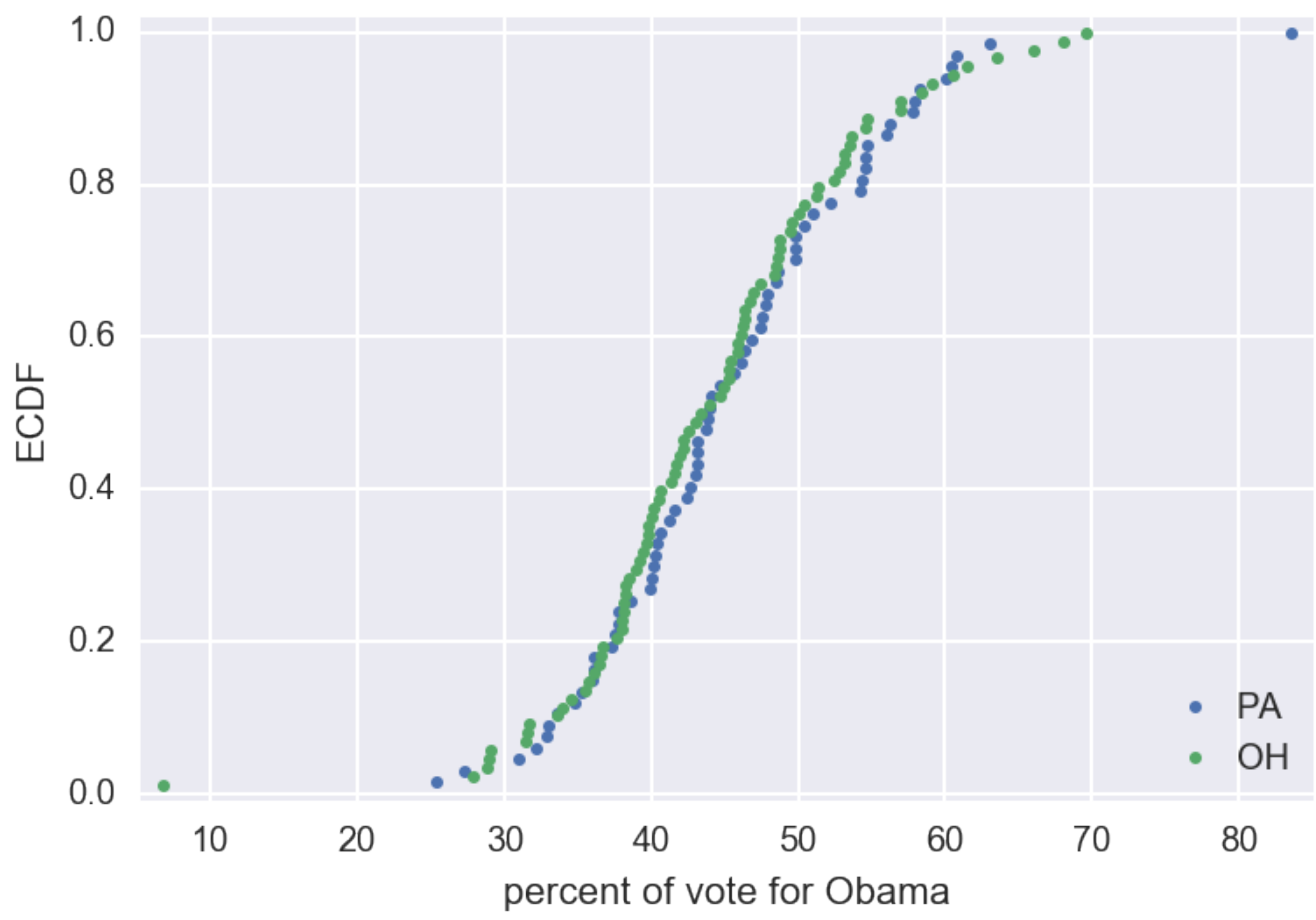
Generating a permutation sample

```
In [1]: import numpy as np

In [2]: dem_share_both = np.concatenate(
   ...:                         (dem_share_PA, dem_share_OH))

In [3]: dem_share_perm = np.random.permutation(dem_share_both)

In [4]: perm_sample_PA = dem_share_perm[:len(dem_share_PA)]

In [5]: perm_sample_OH = dem_share_perm[len(dem_share_PA):]
```

Let’s practice!

Test statistics and p-values

Are OH and PA different?



Data retrieved from Data.gov (https://www.data.gov/)

Hypothesis testing

- Assessment of how reasonable the observed data are assuming a hypothesis is true

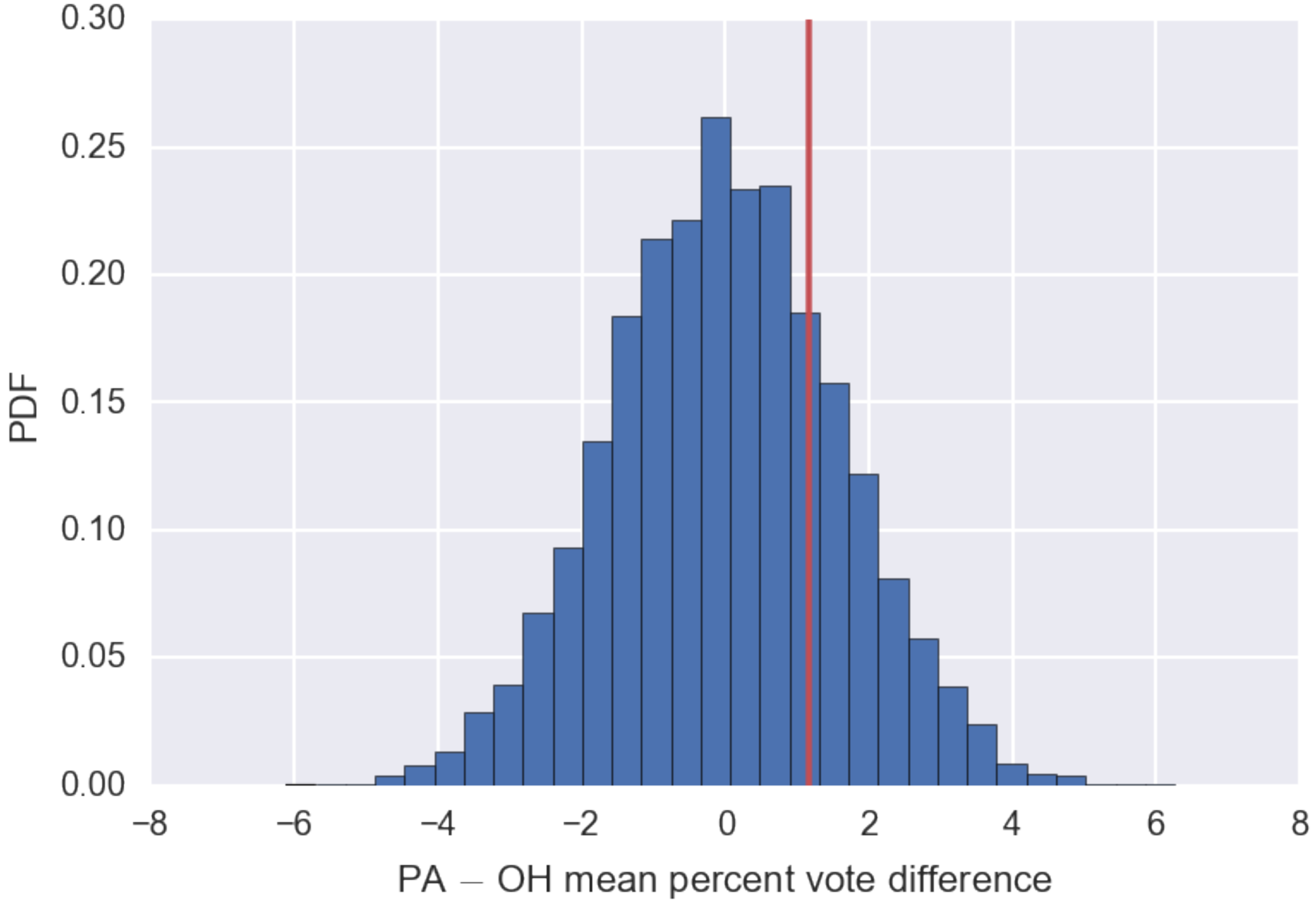
Test statistic

- A single number that can be computed from observed data and from data you simulate under the null hypothesis
- It serves as a basis of comparison between the two

Permutation replicate

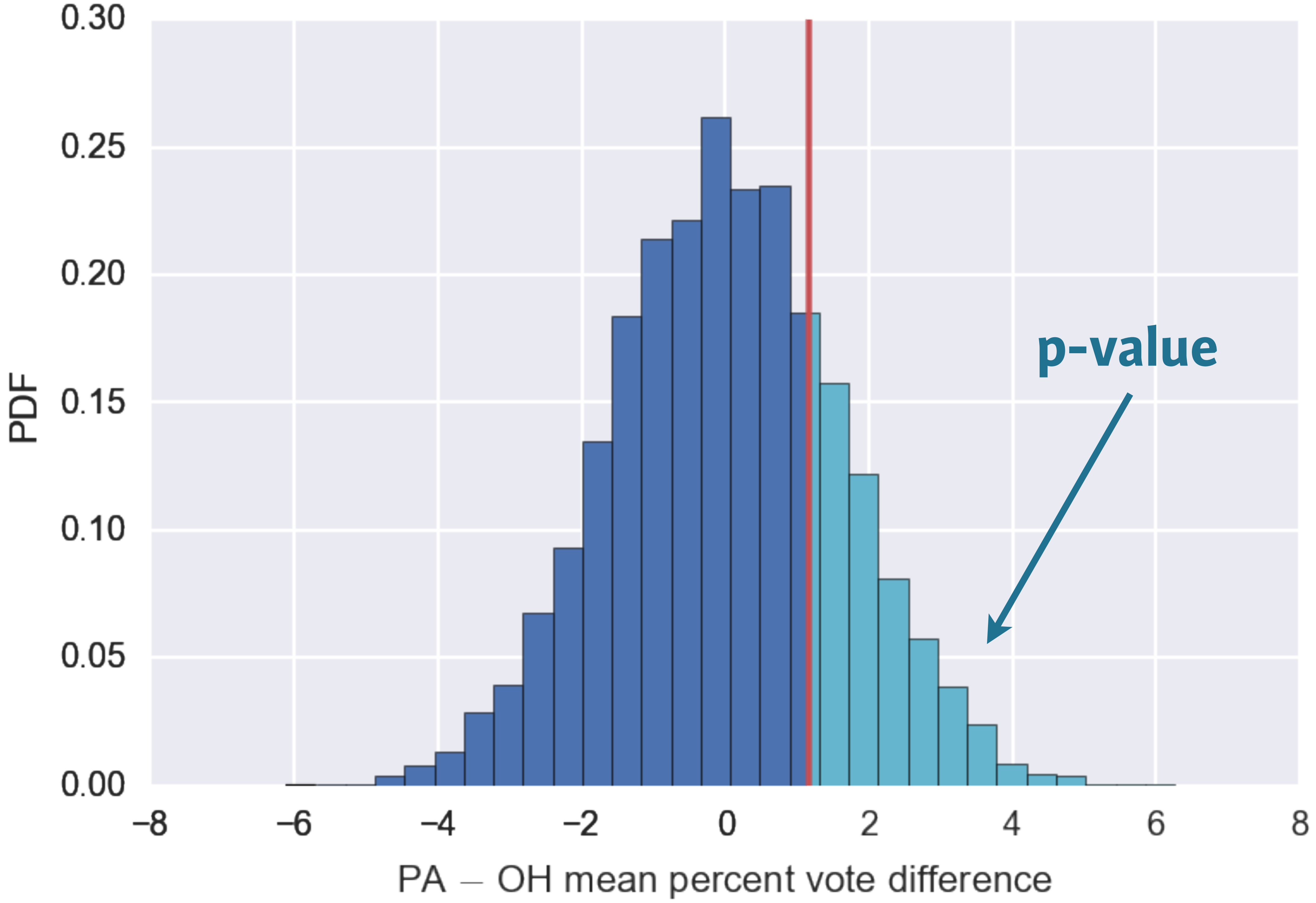
```
In [1]: np.mean(perm_sample_PA) - np.mean(perm_sample_OH)
Out[1]: 1.122220149253728

In [2]: np.mean(dem_share_PA) - np.mean(dem_share_OH) # orig. data
Out[2]: 1.1582360922659518
```

Mean vote difference under null hypothesis



Data retrieved from Data.gov (https://www.data.gov/)

Mean vote difference under null hypothesis



Data retrieved from Data.gov (https://www.data.gov/)

p-value

- The probability of obtaining a value of your test statistic that is at least as extreme as what was observed, under the assumption the null hypothesis is true
- **NOT** the probability that the null hypothesis is true

Statistical significance

- Determined by the smallness of a p-value

Null hypothesis significance testing (NHST)

- Another name for what we are doing in this chapter statistical significance ≠ practical significance

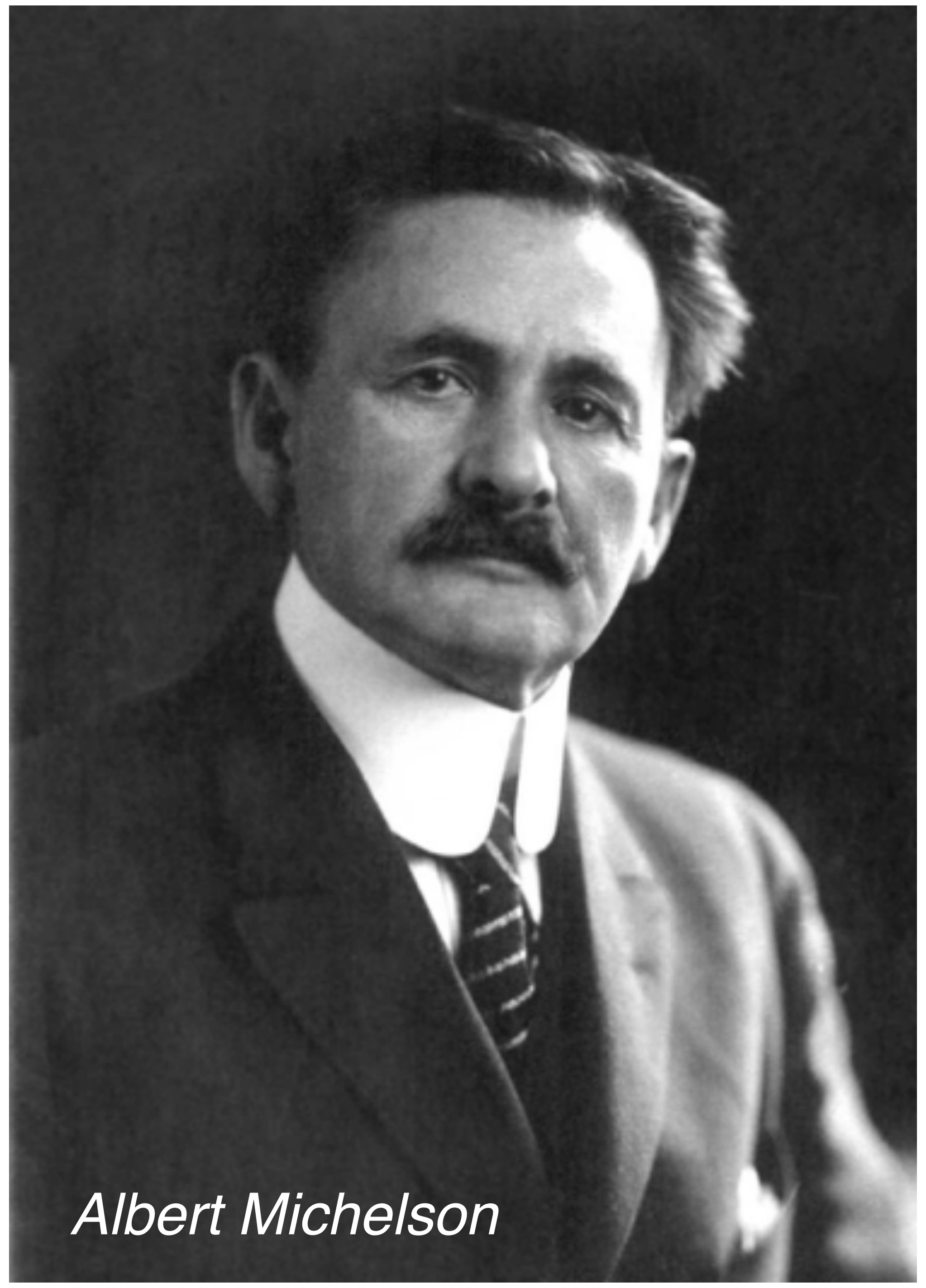
Let's practice!

Bootstrap hypothesis tests

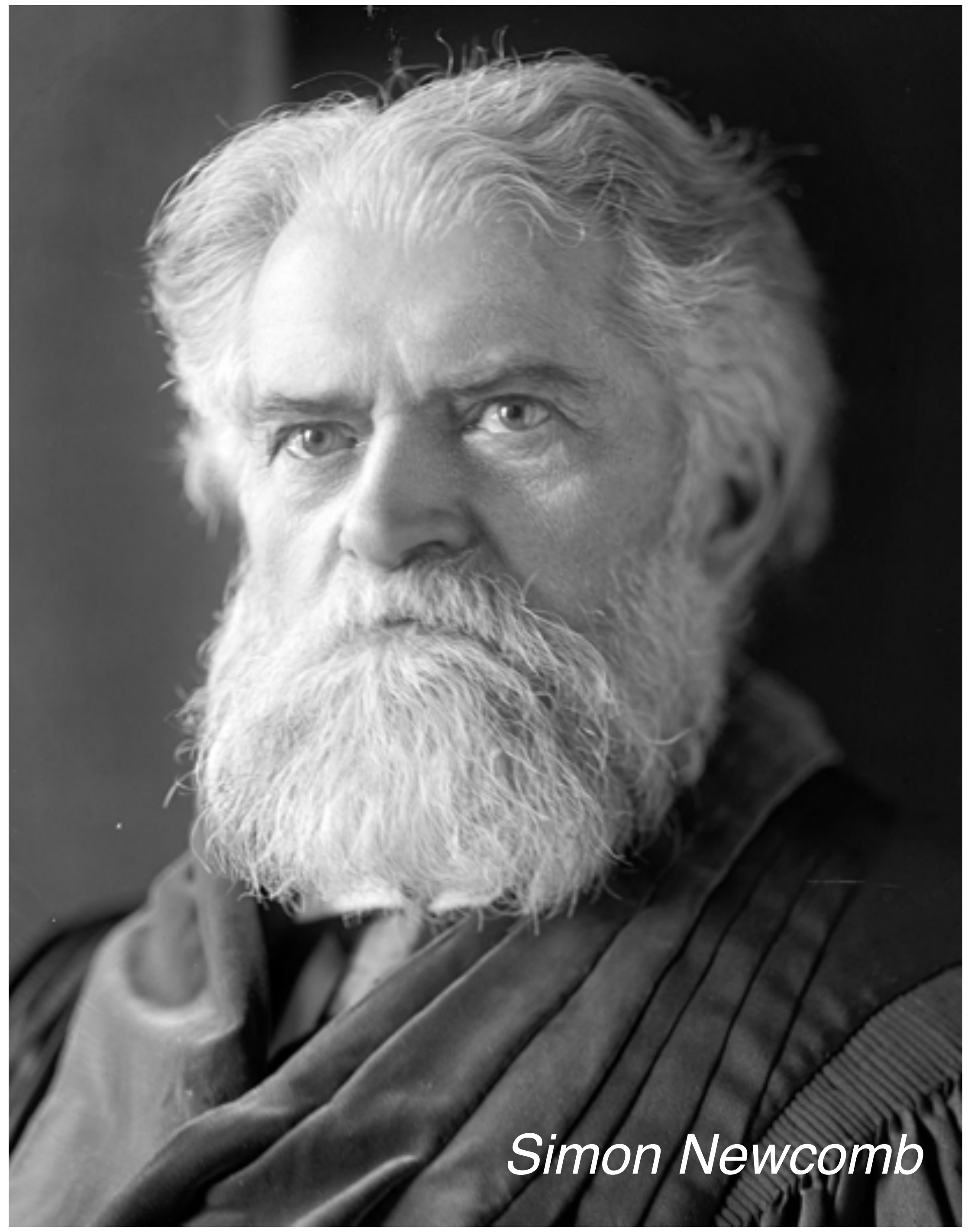
Pipeline for hypothesis testing

- Clearly state the null hypothesis
- Define your test statistic
- Generate many sets of simulated data assuming the null hypothesis is true
- Compute the test statistic for each simulated data set
- The p-value is the fraction of your simulated data sets for which the test statistic is at least as extreme as for the real data

Michelson and Newcomb: speed of light pioneers



299,852 km/s



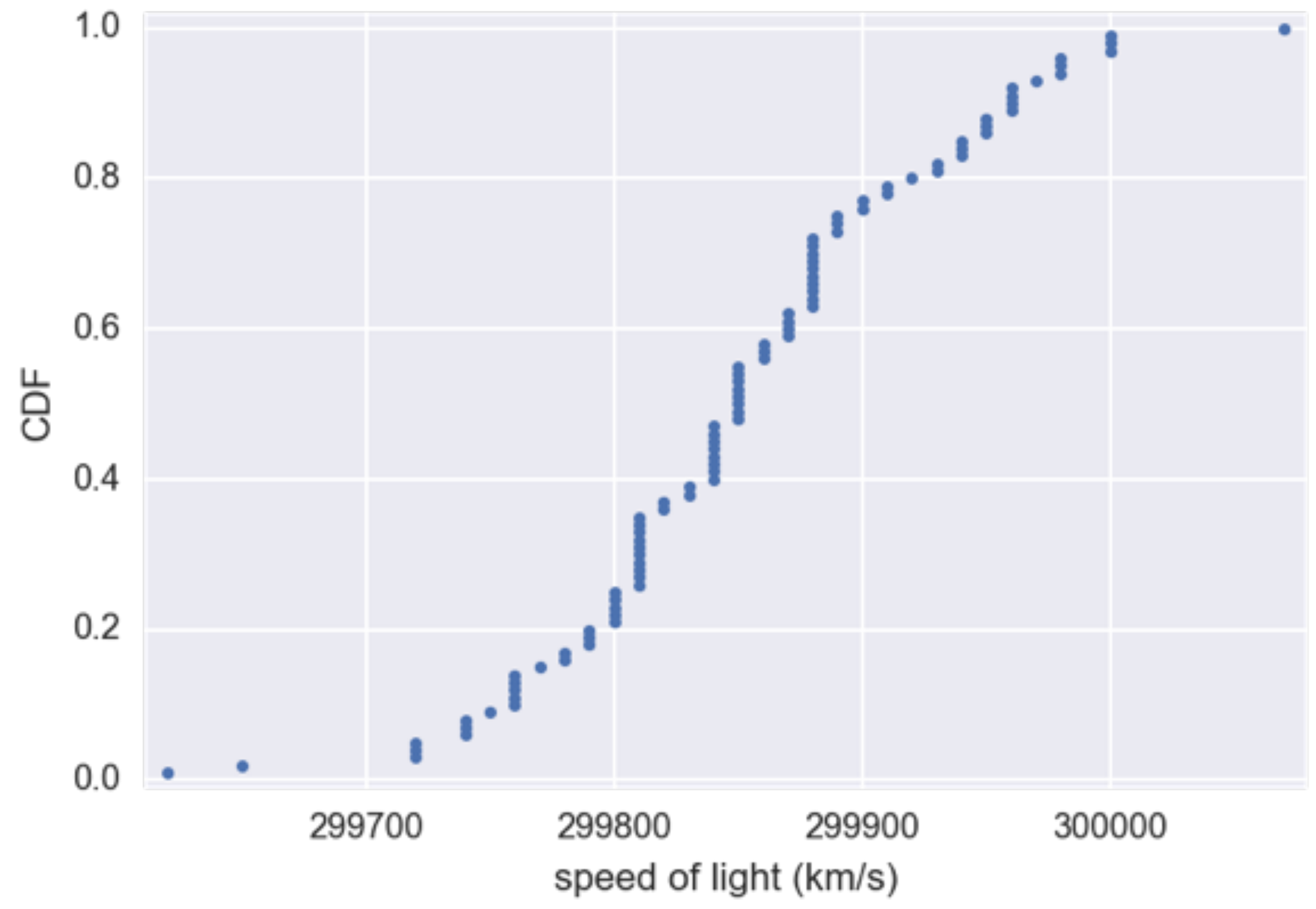
299,860 km/s

Michelson image: public domain, Smithsonian

Newcomb image: US Library of Congress

The data we have

Michelson:



Newcomb:

mean = 299,860 km/s

Data: Michelson, 1880

Null hypothesis

- The true mean speed of light in Michelson’s experiments was actually Newcomb's reported value

Shifting the Michelson data

```
In [1]: newcomb_value = 299860  # km/s

In [2]: michelson_shifted = michelson_speed_of_light \
   ...:         - np.mean(michelson_speed_of_light) + newcomb_value
```

Shifting the Michelson data

```
In [1]: newcomb_value = 299860  # km/s

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   ...:         - np.mean(michelson_speed_of_light) + newcomb_value
```

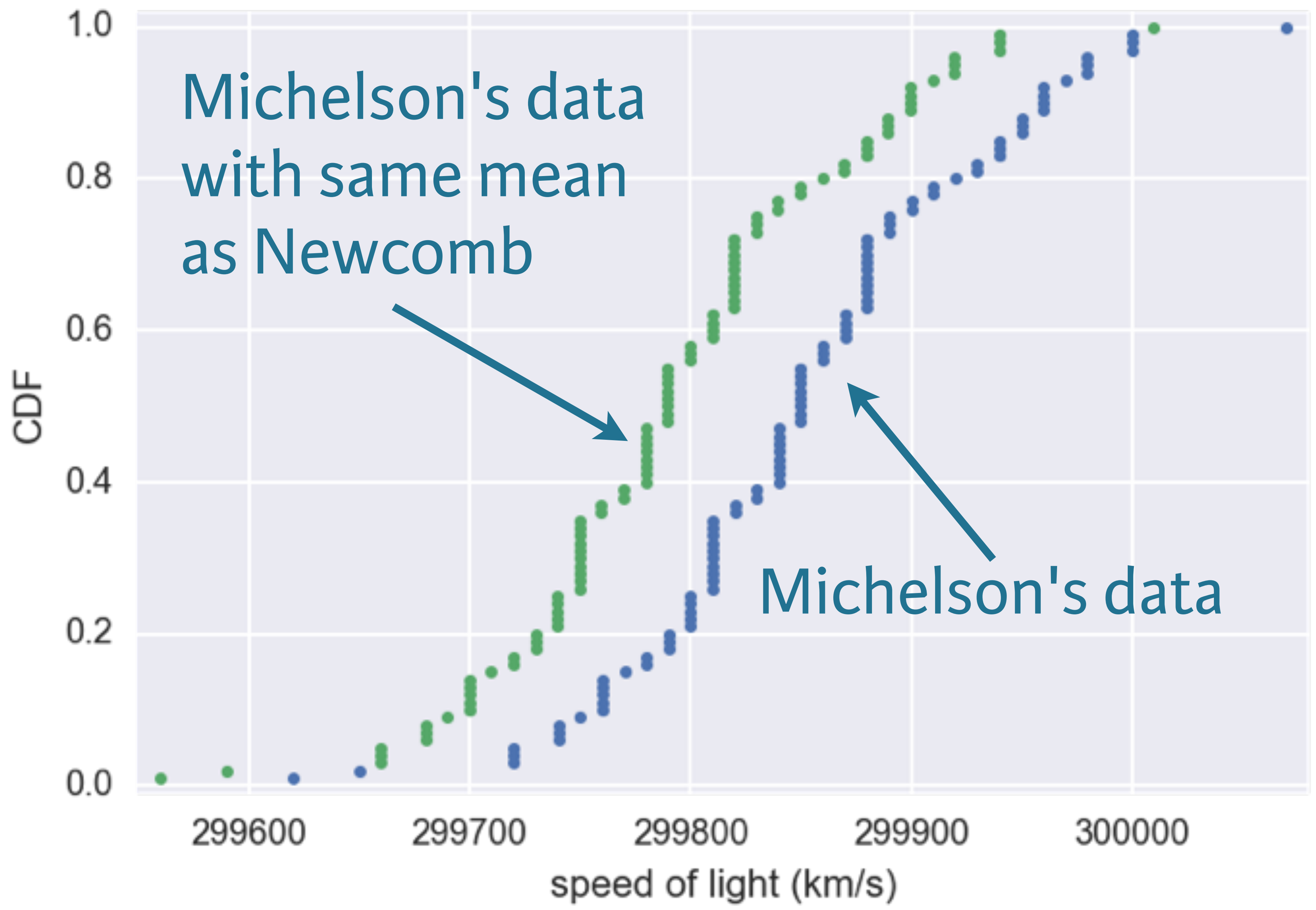

Calculating the test statistic

```
In [1]: def diff_from_newcomb(data, newcomb_value=299860):
   ...:     return np.mean(data) - newcomb_value
   ...:

In [2]: diff_obs = diff_from_newcomb(michelson_speed_of_light)

In [3]: diff_obs
Out[3]: -7.5999999999767169
```

Computing the p-value

```
In [1]: bs_replicates = draw_bs_reps(michelson_shifted,
   ...:                              diff_from_newcomb, 10000)

In [2]: p_value = np.sum(bs_replicates <= diff_observed) / 10000

In [3]: p_value
Out[3]: 0.16039999999999999
```

One sample test

- Compare one set of data to a single number

Two sample test

- Compare two sets of data

Let's practice!