Adversarial Games

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October 29, 2024

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Game theory

There are three stances we can take towards multi-agent environbments:

- When there is a very large number of agents, consider them in the aggregate as an economy, e.g., predicting that increasing demand will cause prices to rise
- We would consider adversarial agents as just a part of the environment, like rain sometimes falls and sometimes does not. In this case, we miss the idea that adversaries are actively trying to defeat us.
- Explicitly model the adversarial agents with the techniques of adversarial game-tree search

Game Theory

Definition: Game theory is the study of mathematical models of strategic interaction between rational decision-makers.

- Focuses on decision-making in competitive environments.
- Players aim to maximize their payoff by choosing optimal strategies.
- Types of games:
  - Zero-sum games: One player's gain is another player's loss.
  - Non-zero-sum games: Both players can win or lose together.
- Common solution concepts:
  - Nash equilibrium
  - Dominant strategies
  - Minimax strategy

Nash Equilibrium

Definition: A Nash equilibrium is a situation in a game where no player can benefit by changing their strategy while the other players keep their strategies unchanged.

- At Nash equilibrium, all players are playing optimally given the strategies of others.
- No player has an incentive to deviate from their strategy.
- A game can have one, multiple, or no Nash equilibria.

Example: Prisoner's Dilemma

- The dominant strategy for both prisoners is to "confess," resulting in a Nash equilibrium where both confess.
- Even though mutual silence would have been better for both, the rational choice leads them to confess.

Dominant Strategies

Definition: A strategy is dominant if, regardless of what the other players do, the strategy earns a player a higher payoff than any other.

- A dominant strategy is the best strategy for a player, no matter what the opponents decide.
- If all players in a game have a dominant strategy, the outcome of the game is determined.
- Not all games have dominant strategies for all players.

Key Points:

- A player with a dominant strategy will always choose it.
- The existence of a dominant strategy simplifies the decision-making process for that player.

Example:

- In the Prisoner's Dilemma, the dominant strategy for each player is to "confess," as it leads to a better outcome for them regardless of the opponent's choice.

Optimal Decisions in Games

Goal: Find the best possible move in a game given the current state.

- **Minimax Algorithm:** A decision rule for minimizing the possible loss in a worst-case scenario.
  - Each player selects the move that maximizes their payoff, assuming the opponent will minimize it.
  - Typically applied in two-player, zero-sum games.
- **Minimax Principle:** Choose the action that maximizes the minimum gain (for the max player) or minimizes the maximum loss (for the min player).
- **Optimality:** The minimax algorithm leads to optimal decisions in deterministic games where all information is available.

Games in General

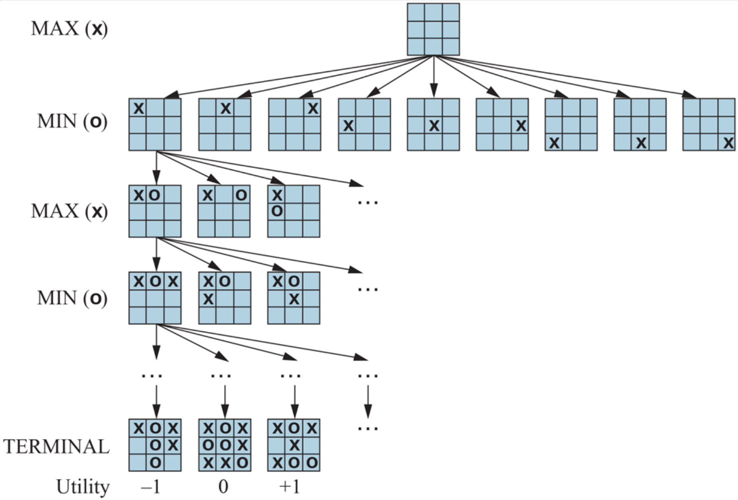
- In multiagent environments, each agent considers the actions of the other agents
- In competitive environments, agents' goals are in conflict. This competition results in adversarial search
- A game of perfect information happens in a deterministic and fully observable environments
- In a game of imperfect information, agents don't see the actions of the other agents but they may know the possibilities
- Pruning allows us to ignore the portions of the search tree that make no difference to the final choice.
- The heuristic evaluation functions allow us to approximate the true utility of a state without doing a complete search.

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

Two-Player Zero-Sum Games

- zero-sum means what is good for one player is just as bad for the other
- move is a synonym for "action", and position is a synonym for "state"
- We call our two players 'MAX' and 'MIN'
- S_0: The initial state, which specifies how the game is set up at the start
- $TO - MOVE(s)$: The player whose turn it is to move in state s
- $ACTIONS(s)$: The set of legal moves in state s
- $RESULT(s, a)$: The transition model defines the state resulting from taking action a in state s
- $IS - TERMINAL(s)$: A terminal test, which is true when the game is over and false otherwise
- $UTILITY(s, p)$: A utility function (objective function or payoff function) defines the final numeric value to player p when the game ends in terminal state s
- game tree is a search tree that follows every sequence of moves all the way to a terminal state

Example: Game tree (2-player, deterministic, turns)

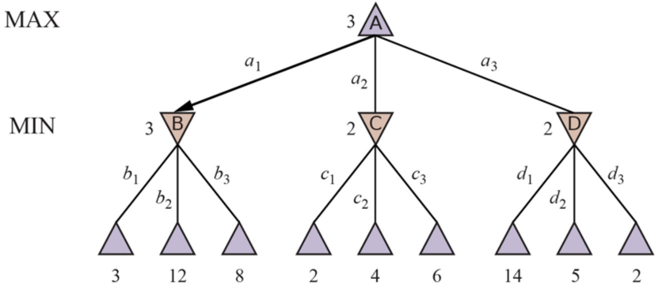


Minimax

- Perfect play for deterministic, perfect-information games
- MAX wants to find a sequence of actions to a win, but MIN has something to say about it; MAX's strategy must be a conditional plan
- Idea: choose move to position with highest

  minimax value= best achievable payoff against best play

E.g., 2-ply game:



Minimax Algorithm

function MINIMAX_SEARCH(*game, state*) **returns** *an action*
  inputs: *state*, current state in game

  player $\leftarrow$ *game.TO_MOVE(state)*
  value, *move* $\leftarrow$ *MAX_VALUE(game, state)*
  return *move*

---

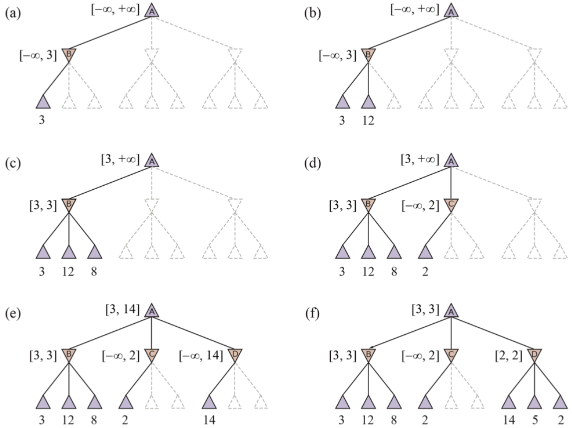
function MAX_VALUE(*game, state*) **returns** a *(utility, move)* pair
  if game.IS_TERMINAL(*state*) **then return** game.UTILITY(*(state, player), null*)
  $v \leftarrow -\infty$
  for each *a* **in** game.ACTIONS(*state*) **do**
    v2, a2 $\leftarrow$ MIN_VALUE(*game, game.*RESULT(*state,a*))
    if $v2 > v$ **then**
      v,move $\leftarrow$ *v2,a*
  return *v, move*

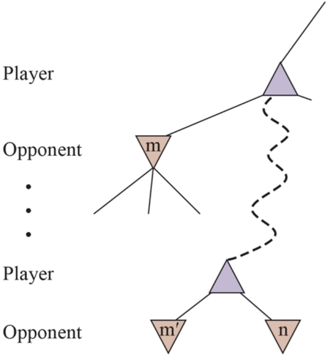
---

function MIN_VALUE(*game, state*) **returns** a *(utility, move)* pair
  if game.IS_TERMINAL(*state*) **then return** game.UTILITY(*(state, player), null*)
  $v \leftarrow +\infty$
  for each *a* **in** game.ACTIONS(*state*) **do**
    v2, a2 $\leftarrow$ MAX_VALUE(*game, game.*RESULT(*state,a*))
    if $v2 < v$ **then**
      v,move $\leftarrow$ *v2,a*
  return *v, move*

α–β Pruning

- The number of game states is exponential in the depth of the tree.
- Pruning large parts of the tree that make no difference to the outcome will reduce the number of states





- If m or m' is better than n for Player, we will never get to n in play.
- α: The value of the best (highest value) choice we have found so far at any choice along the path for *MAX*. Think: $\alpha =$ '*at_least*'
- β: The value of the best (lowest value) choice we have found so far at any choice along the path for *MIN*. Think: $\beta =$ '*at_most*'

Alpha-Beta Algorithm

```
function ALPHA_BETA_SEARCH(game, state) returns an action
   inputs: state, current state in game

    player ← game.TO_MOVE(state)
    value, move ← MAX_VALUE(game, state, −∞, +∞ )
   return move
```

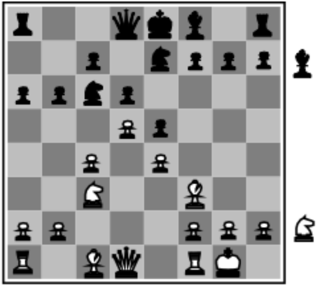
```
function MAX_VALUE(game, state, α, β) returns a (utility, move)
pair
    if game.IS_TERMINAL(state) then return game.UTILITY((state,
player), null)
   v ← −∞
   for each a in game.ACTIONS(state) do
      v2, a2 ← MIN_VALUE(game, game.RESULT(state,a), α, β)
      if v2>v then
         v,move ← v2,a
         α ← (MAX)(α, v)
      if v≥β then  return v, move
   return v, move
```

```
function MIN_VALUE(game, state, α, β) returns a (utility, move) pair
    if game.IS_TERMINAL(state) then return game.UTILITY((state,
player), null)
   v ← +∞
   for each a in game.ACTIONS(state) do
      v2, a2 ← MAX_VALUE(game, game.RESULT(state,a), α, β)
      if v2>v then
         v,move ← v2,a
         β ← (MIN)(β, v)
      if v≤α then  return v, move
   return v, move
```

Evaluation Functions

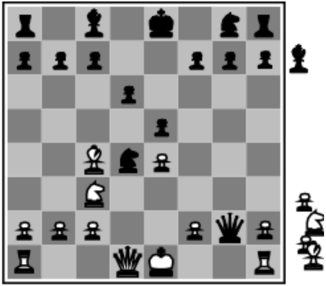
- A heuristic evaluation function $EVAL(s, p)$ returns an estimate of the expected utility of state s to player p
- For terminal states, $EVAL(s, p) = UTILITY(s, p)$
- For non-terminal states, $UTILITY(loss, p) \leq EVAL(s, p) \leq UTILITY(win, p)$
- The computation must not take too long
- The evaluation function should be strongly correlated with the actual chances of winning
- Most evaluation functions work by calculating various features of the state
- Most evaluation functions compute separate numerical contributions from each feature and then combine them to find the total value

Evaluation Functions-Chess



Black to move

White slightly better



White to move

Black winning

For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens), etc.

Monte Carlo Tree Search (MCTS)

- The basic MCTS strategy does not use a heuristic evaluation function
- The value of a state is estimated as the average utility over a number of simulations of complete games starting from the state
- A simulation chooses moves first for one player, than for the other, repeating until a terminal position is reached
- The get useful information from the simulation, we need playout policy that biases the moves towards the good ones
- Selection is starting at the root of the search tree and choosing a move guided by the selection policy
- Expansion is growing the search tree by a new child of the selected node
- Simulation is performing a playout from the newly generated child node, choosing move for both players according to playout policy
- Back-propagation is using the result of the simulation to update all the search tree nodes going up to the root

MCTS Algorithm

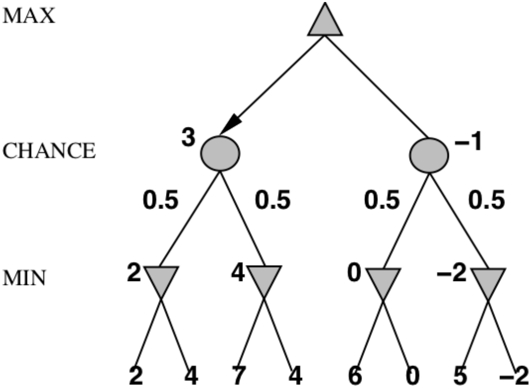
function MONTE_CARLO_TREE_SEARCH(*state*) **returns** *an action*

tree ← NODE(*state*)
while IS_TIME_REMAINING() **do**
  leaf ← SELECT(*tree*)
  child ← EXPAND(*leaf*)
  result ← SIMULATE(*child*)
  function BACK_PROPOGATE(*result, child*) **returns**
return the move in ACTION(*state*) whose node has the highest number of playouts

Stochastic Games in General

In nondeterministic games, chance introduced by dice, card-shuffling

Simplified example with coin-flipping:



Algorithm for Stochastic Games

We can only calculate expected value of a position: the avreage over all possible outcomes of the chance nodes

Expectiminimax gives perfect play

Just like Minimax, except we must also handle chance nodes:

EXPECTIMINIMAX(s)=
if IS_TERMINAL(s)
then UTILITY(s, MAX)
if TO_MOVE(s)=MAX
then $\max_a$EXPECTIMINIMAX($RESULT(s, a)$))
if TO_MOVE(s)=MIN
then $\min_a$EXPECTIMINIMAX($RESULT(s, a)$))
if TO_MOVE(s)=CHANCE
then $\sum_r P(r)$EXPECTIMINIMAX($RESULT(s, r)$))

Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI

- Game theory and types of games (zero-sum, non-zero-sum)
- Optimal decisions in games using the Minimax algorithm
- Heuristic improvements with Alpha-Beta pruning
- Monte Carlo Tree Search for large and complex search spaces
- Stochastic games and decision-making under uncertainty