4 Probability

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4.5 Independent Events

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Independent Events

The concept of independent events is necessary to continue our discussion on compound events.

Independent events Two events are *independent* if the occurrence (or nonoccurrence) of one gives us no information about the likeliness of occurrence of the other. In other words, if the probability of A remains unchanged after we know that B has happened (or has not happened), the events are independent.

Independent Events

In algebra: $P(\text{A}) = P(\text{A} \mid \text{B}) = P(\text{A} \mid \text{not B})$

In words: There are several equivalent ways to express the concept of independence:

1. The probability of event A is unaffected by knowledge that a second event, B, has occurred, knowledge that B has not occurred, or no knowledge about event B whatsoever.

Independent Events

2. The probability of event A is unaffected by knowledge, or no knowledge, about a second event, B, having occurred or not occurred.

3. The probability of event A (with no knowledge about event B) is the same as the probability of event A, knowing event B has occurred, and both are the same as the probability of event A, knowing event B has not occurred.

Not all events are independent.

Independent Events

Dependent events Events that are not independent. That is, the occurrence of one event does have an effect on the probability for occurrence of the other event.

Example 20 – *Understanding Independent Events*

A statewide poll of 750 registered Republicans and Democrats in 25 precincts from across New York State was taken.

Each voter was identified as being registered as a Republican or a Democrat and then asked, “Are you are in favor of or against the current budget proposal awaiting the governor’s signature?” The resulting tallies are shown here.

	Number in Favor	Number Against	Number of Voters
Republican	135	90	225
Democrat	315	210	525
Totals	450	300	750

Example 20 – *Understanding Independent Events* cont'd

Suppose one voter is to be selected at random from the 750 voters summarized in the preceding table. Let's consider the two events "The selected voter is in favor" and "The voter is a Republican." Are these two events independent?

To answer this, consider the following three probabilities:

(1) probability the selected voter is in favor;

(2) probability the selected voter is in favor, knowing the voter is a Republican; and

(3) probability the selected voter is in favor, knowing the voter is not a Republican.

Example 20 – *Understanding Independent Events* cont’d

Probability the selected voter is in favor = P(in favor)

= 450/750

= 0.60.

Probability the selected voter is in favor, knowing voter is a Republican = P(in favor | Republican)

= 135/225

= 0.60.

Probability the selected voter is in favor, knowing voter is not a Republican = Probability the selected voter is in favor, knowing voter is a Democrat

Example 20 – *Understanding Independent Events* cont'd

P(in favor | not Republican) = P(in favor | Democrat)

= 315/525

= 0.60.

Does knowing the voter's political affiliation have an influencing effect on the probability that the voter is in favor of the budget proposal?

With no information about political affiliation, the probability of being in favor is 0.60. Information about the event "Republican" does not alter the probability of "in favor." They are all the value 0.60. Therefore, these two events are said to be *independent events*.

Independent Events

When checking the three probabilities, P(A),P(A | B) ,and P(A | not B), we need to compare only two of them. If any two of the three probabilities are equal, the third will be the same value. Furthermore, if any two of the three probabilities are unequal, then all three will be different in value.

Note

Determine all three values, using the third as a check. All will be the same, or all will be different—there is no other possible outcome.

Special Multiplication Rule

Special Multiplication Rule

The multiplication rule simplifies when the events involved are independent. If we know two events are independent, then by applying the definition of independence, $P(B \mid A) = P(B)$, to the multiplication rule, it follows that:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

becomes $P(A \text{ and } B) = P(A) \cdot P(B)$

Special Multiplication Rule

Special Multiplication Rule

Let A and B be two independent events defined in a sample space S.

In words:

probability of A *and* B = *probability of* A × *probability of* B

In algebra: $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$ **(4.7)**

This formula can be expanded to consider more than two independent events:

$$P(\text{A and B and C and} \ldots \text{and E}) = P(\text{A}) \cdot P(\text{B}) \cdot P(\text{C}) \cdot \ldots \cdot P(\text{E})$$

Special Multiplication Rule

This equation is often convenient for calculating probabilities, but it does not help us understand the independence relationship between the events A and B.

It is *the definition* that tells us how we should think about independent events. Students who understand independence this way gain insight into what independence is all about.

Special Multiplication Rule

This should lead you to think more clearly about situations dealing with independent events, thereby making you less likely to confuse the concept of independent events with mutually exclusive events or to make other common mistakes regarding independence.

Note

Do not use $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$ as the definition of independence. It is a property that results from the definition. It can be used as a test for independence, but as a statement, it shows no meaning or insight into the concept of independent events.