4 Probability

4.3 Rules of Probability

Rules of Probability

Often, one wants to know the probability of a **compound event** but the only data available are the probabilities of the related simple events. (Compound events are combinations of more than one simple event.)

In the next few paragraphs, the relationship between these probabilities is summarized.

Finding the Probability of “Not A”

Finding the Probability of “Not A”

The concept of complementary events is fundamental to finding the probability of “not A.”

Complementary events The *complement of an event* A, $\overline{A}$, is the set of all sample points in the sample space that do not belong to event A.

Note

The complement of event A is denoted by $\overline{A}$ (read “A complement”).

Finding the Probability of “Not A”

A few examples of complementary events are

(1) the complement of the event “success” is “failure,”

(2) the complement of “selected voter is Republican” is “selected voter is not Republican,” and

(3) the complement of “no heads” on 10 tosses of a coin is “at least one head.”

Finding the Probability of “Not A”

By combining the information in the definition of complement with Property 2, we can say that

$P(\text{A}) + P(\overline{\text{A}}) = 1.0$ for any event A

As a result of this relationship, we have the complement rule:

Complement Rule

In words:

probability of A complement = one – probability of A

In algebra:

$P(\overline{\text{A}}) = 1 - P(\text{A})$ **(4.3)**

Finding the Probability of “Not A”

Note

Every event A has a complementary event $\overline{A}$. Complementary probabilities are very useful when the question asks for the probability of “at least one.”

Generally, this represents a combination of several events, but the complementary event “none” is a single outcome.

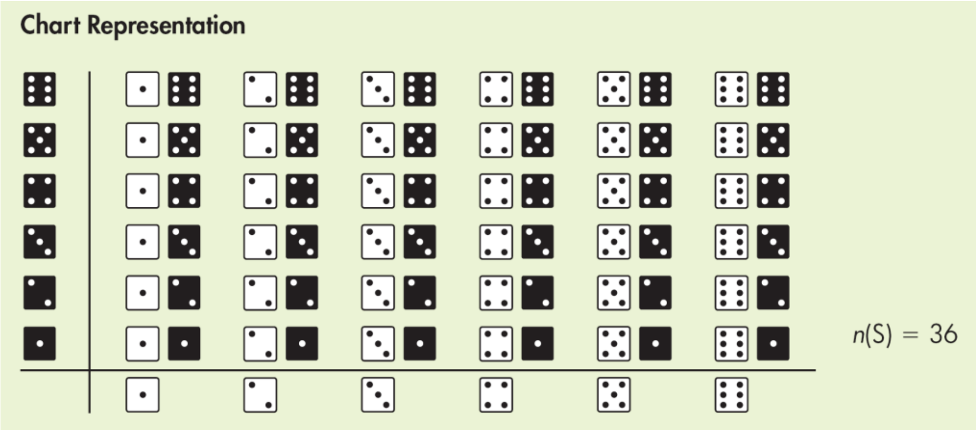
It is easier to solve for the complementary event and get the answer by using formula (4.3).

Example 11 – *Using Complements to Find Probabilities*

Two dice are rolled. What is the probability that the sum is at least 3 (that is, 3, 4, 5, . . . , 12)?

Solution:

Suppose one of the dice is black and the other is white. (See the chart; it shows all 36 possible pairs of results when rolling a pair of dice.)



Example 11 – *Solution*

cont'd

Rather than finding the probability for each of the sums 3, 4, 5, . . . , 12 separately and adding, it is much simpler to find the probability that the sum is 2 ("less than 3") and then use formula (4.3) to find the probability of "at least 3," because "less than 3" and "at least 3" are complementary events.

$$P(\text{sum of 2}) = P(\text{A})$$

$$= \frac{1}{36}$$ ("2" occurs only once in the 36-point sample space)

Example 11 – *Solution*

cont'd

P(sum is at least 3) $= P(\overline{A})$ using formula (4.3)

$$= 1 - P(A)$$

$$= 1 - \frac{1}{36}$$

$$= \frac{35}{36}$$

Finding the Probability of “A or B”

Finding the Probability of "A or B"

An hourly wage earner wants to estimate the chances of "receiving a promotion or getting a pay raise." The worker would be happy with either outcome.

Historical information is available that will allow the worker to estimate the probability of "receiving a promotion" and "getting a pay raise" separately.

In this section we will learn how to apply the **addition rule** to find the compound probability of interest.

Finding the Probability of “A or B”

General Addition Rule

Let A and B be two events defined in a sample space, *S*.

In words:

probability of A or B =

probability of A + probability of B – probability of A and B

In algebra:

$$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B}) \qquad \textbf{(4.4)}$$

Example 12 – *Understanding the Addition Rule*

A statewide poll of 800 registered voters in 25 precincts from across New York State was taken. Each voter was identified as being registered as Republican, Democrat, or other and then asked, “Are you in favor of or against the current budget proposal awaiting the governor’s signature?” The resulting tallies are shown here.

	Number in Favor	Number Against	Number of Voters
Republican	136	88	224
Democrat	314	212	526
Other	14	36	50
Totals	464	336	800

Example 12 – *Understanding the Addition Rule*

Suppose one voter is to be selected at random from the 800 voters summarized in the preceding table.

Let's consider the two events "The voter selected is in favor" and "The voter is a Republican." Find the four probabilities:

P(in favor), P(Republican), P(in favor or Republican), and P(in favor and Republican).

Then use the results to check the truth of the addition rule.

Example 12 – *Solution*

Probability the voter selected is "in favor" = P(in favor)

= 464/800

= <u>0.58.</u>

Probability the voter selected is "Republican"

= P(Republican)

= 224/800

= <u>0.28.</u>

Example 12 – *Solution*

cont'd

Probability the voter selected is "in favor or Republican"

$= P(\text{in favor or Republican})$

$= (136 + 314 + 14 + 88)/800$

$= 552/800$

$= \underline{0.69.}$

Example 12 – *Solution*

cont'd

Probability the voter selected is "in favor" and "Republican"

$= P(\text{in favor and Republican})$

$= 136/800$

$= \underline{0.17}$.

Notes about finding the preceding probabilities:

1. The connective "or" means "one or the other or both"; thus, "in favor or Republican" means all voters who satisfy either event.

Example 12 – *Solution*

cont'd

2. The connective "and" means "both" or "in common"; thus, "in favor and Republican" means all voters who satisfy both events.

Now let's use the preceding probabilities to demonstrate the truth of the addition rule.

Let A = "in favor" and B = "Republican." The general addition rule then becomes:

$$P(\text{in favor or Republican}) = P(\text{in favor}) + P(\text{Republican}) - P(\text{in favor and Republican})$$

Example 12 – *Solution*

cont'd

Remember: Previously we found:

P(in favor or Republican) = <u>0.69</u>.

Using the other three probabilities, we see:

P(in favor) + P(Republican) – P(in favor and Republican)

= 0.58 + 0.28 – 0.17

= <u>0.69</u>

Finding the Probability of “A and B”

Finding the Probability of “A and B”

Suppose a criminal justice professor wants his class to determine the likeliness of the event “a driver is ticketed for a speeding violation and the driver had previously attended a defensive driving class.”

The students are confident they can find the probabilities of “a driver being ticketed for speeding” and “a driver who has attended a defensive driving class” separately.

In this section we will learn how to apply the **multiplication rule** to find the compound probability of interest.

Finding the Probability of “A and B”

General Multiplication Rule

Let A and B be two events defined in a sample space, S.

In words:
probability of A *and* B = *probability of* A × *probability of* B, *knowing* A

In algebra:
P(A and B) = P(A) • P(B|A) **(4.5)**

Finding the Probability of “A and B”

Note

When two events are involved, either event can be identified as A, with the other identified as B. The general multiplication rule could also be written as

$$P(\text{B and A}) = P(\text{B}) \cdot P(\text{A} \mid \text{B})$$

Example 13 – *Understanding the Multiplication Rule*

A statewide poll of 800 registered voters in 25 precincts from across New York State was taken.

Each voter was identified as being registered as Republican, Democrat, or other and then asked, "Are you in favor of or against the current budget proposal awaiting the governor's signature?" The resulting tallies are shown here.

	Number in Favor	Number Against	Number of Voters
Republican	136	88	224
Democrat	314	212	526
Other	14	36	50
Totals	464	336	800

Example 13 – *Understanding the Multiplication Rule*

cont'd

Suppose one voter is to be selected at random from the 800 voters summarized in the preceding table.

Let's consider the two events: "The voter selected is in favor" and "The voter is a Republican." Find the three probabilities: P(in favor), P(Republican | in favor), and P(in favor and Republican). Then use the results to check the truth of the multiplication rule.

Example 13 – *Solution*

Probability the voter selected is "in favor" = P(in favor)

= 464/800

= <u>0.58</u>.

Probability the voter selected is "Republican, given in favor"

= P (Republican | in favor)

= 136/464

= <u>0.29</u>.

Example 13 – *Solution*

Probability the voter selected is in "favor" and "Republican"

= P (in favor and Republican)

= 136/800

$$= \frac{136}{800}$$

= 0.17.

Example 13 – *Solution*

cont'd

Notes about finding the preceding probabilities:

1. The conditional "given" means there is a restriction; thus, "Republican in favor" means we start with only those voters who are "in favor." In this case, this means we are looking only at 464 voters when determining this probability.

2. The connective "and" means "both" or "in common"; thus, "in favor and Republican" means all voters who satisfy both events.

Example 13 – *Solution*

cont’d

Now let’s use the previous probabilities to demonstrate the truth of the multiplication rule.

Let A = ”in favor” and B = ”Republican.” The general multiplication rule then becomes:

P(in favor and Republican) = *P*(in favor) • *P*(Republican | in favor)

Example 13 – *Solution*

cont'd

Previously we found: P(in favor and Republican)

$$= \frac{136}{800}$$

$$= \underline{0.17}$$

$$P(\text{in favor}) \cdot P(\text{Republican} \mid \text{in favor}) = \frac{464}{800} \cdot \frac{136}{464}$$

$$= \frac{136}{800}$$

$$= \underline{0.17}$$

Finding the Probability of “A and B”

You typically do not have the option of finding P(A and B) two ways, as we did here. When you are asked to find P(A and B), you will often be given P(A) and P(B).

However, you will not always get the correct answer by just multiplying those two probabilities together.

You will need a third piece of information: the conditional probability of one of the two events or information that will allow you to find it.