4 Probability

4.4 Mutually Exclusive Events

Mutually Exclusive Events

To further our discussion of compound events, the concept of “mutually exclusive” must be introduced.

Mutually exclusive events Nonempty events defined on the same sample space with each event excluding the occurrence of the other. In other words, they are events that share no common elements.

Mutually Exclusive Events

In algebra: $P(\text{A and B}) = 0$

In words: There are several equivalent ways to express the concept of mutually exclusive:

1. If you know that either one of the events has occurred, then the other event is excluded or cannot have occurred.

2. If you are looking at the lists of the elements making up each event, none of the elements listed for either event will appear on the other event's list; there are "no shared elements."

Mutually Exclusive Events

3. If you are looking at a Venn diagram, the closed areas representing each event “do not intersect”—that is, there are “no shared elements,” or as another way to say it, “they are disjoint.”

4. The equation says, “the **intersection** of the two events has a probability of zero,” meaning “the intersection is an empty set” or “there is no intersection.”

Mutually Exclusive Events

Note

The concept of mutually exclusive events is based on the relationship between the sets of elements that satisfy the events. Mutually exclusive is not a probability concept by definition; it just happens to be easy to express the concept using a probability statement.

Example 17 – *Mutually Exclusive Card Events*

Consider a regular deck of playing cards and the two events “card drawn is a queen” and “card drawn is an ace.” The deck is to be shuffled and one card randomly drawn.

In order for the event “card drawn is a queen” to occur, the card drawn must be one of the four queens: queen of hearts, queen of diamonds, queen of spades, or queen of clubs.

In order for the event “card drawn is an ace” to occur, the card drawn must be one of the four aces: ace of hearts, ace of diamonds, ace of spades, or ace of clubs.

Example 17 – *Mutually Exclusive Card Events* cont'd

Notice that there is no card that is both a queen and an ace. Therefore, these two events, "card drawn is a queen" and "card drawn is an ace," are mutually exclusive events.

In equation form: P(queen and ace) = 0.

Special Addition Rule

Special Addition Rule

The addition rule simplifies when the events involved are mutually exclusive.

If we know two events are mutually exclusive, then by applying $P(\text{A and B}) = 0$ to the addition rule for probabilities, it follows that

$P(\text{A or B}) = P(\text{A}) + P(\text{B}) – P(\text{A and B})$ becomes

$P(\text{A or B}) = P(\text{A}) + P(\text{B})$.

Special Addition Rule

Special Addition Rule

Let A and B be two mutually exclusive events defined in a sample space *S*.

In words:
Probability of A *or* B = *probability of* A + *probability of* B

In algebra: $P(\text{A or B}) = P(\text{A}) + P(\text{B})$ **(4.6)**

This formula can be expanded to consider more than two mutually exclusive events:

$P(\text{A or B or C or ... or E}) = P(\text{A}) + P(\text{B}) + P(\text{C}) + \ldots + P(\text{E})$

Special Addition Rule

This equation is often convenient for calculating probabilities, but it does not help us understand the relationship between the events A and B. It is the *definition* that tells us how we should think about mutually exclusive events.

Students who understand mutual exclusiveness this way gain insight into what mutual exclusiveness is all about.

Special Addition Rule

This should lead you to think more clearly about situations dealing with mutually exclusive events, thereby making you less likely to confuse the concept of mutually exclusive events with independent events or to make other common mistakes regarding the concept of mutually exclusive.

Notes

1. Define mutually exclusive events in terms of the sets of elements satisfying the events and test for mutual exclusiveness in that manner.

Special Addition Rule

2. Do not use P(A and B) = 0 as the definition of mutually exclusive events. It is a property that results from the definition. It can be used as a test for mutually exclusive events; however, as a statement, it shows no meaning or insight into the concept of mutually exclusive events.

3. In equation form, the *definition* of mutually exclusive events states:

   P(A and B) = 0 (Both cannot happen at same time.)
   P(A | B) = 0 and P(B | A) = 0
   (If one is known to have occurred, then the other has not.)

Special Addition Rule

Reconsider Example 17, with the two events “card drawn is a queen” and “card drawn is an ace” when drawing exactly one card from a deck of regular playing cards.

The one card drawn is a queen, or the one card drawn is an ace. That one card cannot be both a queen and an ace at the same time, thereby making these two events mutually exclusive. The special addition rule therefore applies to the situation of finding *P*(queen or ace).

$$P(\text{queen or ace}) = P(\text{queen}) + P(\text{ace}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$