Introduction to Scientific Python programming -Apdapted to TKT4140 Numerical Methods with Computer Laboratory

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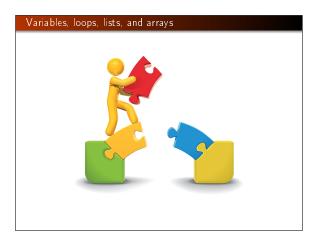
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This is a very quick intro to Python programming

- variables for numbers, lists, and arrays
- while loops and for loops
- functions
- if tests
- plotting
- files
- classes

Method: show program code through math examples



Do you have access to Python?

Many methods:

- Mac and Windows: Anaconda
- Ubuntu: sudo apt-get install
- Web browser (Wakari or SageMathCloud)

See How to access Python for doing scientific computing for more

Mathematical example

Most examples will involve this formula:

$$s = v_0 t + \frac{1}{2} a t^2 \tag{1}$$

We may view s as a function of t: s(t), and also include the parameters in the notation: $s(t; v_0, a)$.

A program for evaluating a formula Compute s for t = 0.5, $v_0 = 2$, and a = 0.2. Python code t = 0.5 v0 = 2 a = 0.2 s = v0*t + 0.5*a*t**2 print s Execution Terminal> python distance.py 1.025

Assignment statements assign a name to an object t = 0.5 v0 = 2 a = 0.2 s = v0*t + 0.5*a*t**2 Rule: e evaluate right-hand side; it results in an object eleft-hand side is a name for that object

```
    Formatted output with text and numbers
    Task: write out text with a number (3 decimals): s=1.025
    Method: printf syntax
    print 's=%g' % s  # g: compact notation print 's=%.2f' % s  # f: decimal notation, .2f: 2 decimals
    Modern alternative: format string syntax
    print 's=(s:.2f)' .format(s=s)
```

```
Programming with a while loop

• Task: write out a table of t and s(t) values (two columns), for
    t ∈ [0,2] in steps of 0.1
• Method: while loop

v0 = 2
a = 0.2
dt = 0.1 # Increment
t = 0 # Start value
vhile t <= 2:
s = v0*t + 0.5*a*t**2
print t, s
t = t + dt</pre>
```

```
Terminal> python while.py
0 0.0
0.1 0.201
0.2 0.404
0.3 0.609
0.4 0.816
0.5 1.025
0.6 1.236
0.7 1.449
0.8 1.664
0.9 1.881
1.0 2.1
1.1 2.321
1.2 2.544
1.3 2.769
1.4 2.996
1.5 3.225
1.6 3.456
1.7 3.689
1.8 3.924
1.9 4.161
```

Python Online Tutor: step through the program and examine variables a = 0 da = 0.4 while a <= 1.2: print a a = a + da (Visualize execution)

```
Rule: never a == b for real a and b! Always use a tolerance!

a = 1.2
b = 0.4 + 0.4 + 0.4
boolean_condition1 = a == b  # may be False

# This is the way to do it
tol = 1E-14
boolean_condition2 = abs(a - b) < tol # True
```

```
A list collects several objects in a given sequence

A list of numbers:

L = [-1, 1, 8.0]

A list can contain any type of objects, e.g.,

L = ['mydata.txt', 3.14, 10]  # string, float, int

Some basic list operations:

>>> L = ['mydata.txt', 3.14, 10]
>>> print L[0]  # print first element (index 0)
mydata.txt
>>> print L[1]  # print second element (index 1)
3.14
>>> del L[0]  # delete the first element
>>> print L[3.14, 10]
>>> print L[3.14, 10, -1]
```

```
v0 = 2
a = 0.2
dt = 0.1  # Increment
t = 0
t values = []
s.values = []
while t <= 2:
    s = v0·t + 0.5*a*t**2
    t.values append(t)
    s.values append(s)
    t = t + dt
print s_values # Just take a look at a created list

# Print a nicely formatted table
i = 0
while i <= len(t.values)-1:
    print '%.2f '%.4f' % (t_values[i], s_values[i])
    i += 1  # Same as i = i + f</pre>
```

```
For loops

A for loop is used for visiting elements in a list, one by one:

>>> L = [1, 4, 8, 9]
>>> for e in L:
... print e
...

1
4
8
9

Demo in the Python Online Tutor:

list1 = [0, 0.1, 0.2]
list2 = []
for element in list1:
    p = element + 2
    list2.append(p)
    print list2

(Visualize execution)
```

```
Traditional for loop: integer counter over list/array indices

somelist = ['file1.dat', 22, -1.5]

for i in range(len(somelist)):
    # access list element through index
    print somelist[i]

Note:

• range returns a list of integers
• range(a, b, s) returns the integers a, a+s, a+2*s, ...
    up to but not including (!!) b
• range(b) implies a=0 and s=1
• range(len(somelist)) returns [0, 1, 2]
```

```
v0 = 2
a = 0.2
dt = 0.1 # Increment
t_values = []
s_values = []
n = int(round(2/dt)) + 1 # No of t values
for i in range(n):
    t = i **edt
    s = v0**t + 0.5**a****2
    t_values.append(t)
    s_values.append(s)
    print s_values # Just take a look at a created list
# Nake nicely formatted table
for t, sin zip(t_values, s_values):
    print '%.2f %.4f' % (t, s)
# Ilternative implemention
for i in range(len(t_values)):
    print '%.2f %.4f' % (t_values[i], s_values[i])
```

Arrays are computationally efficient lists of numbers

Lists collect a set of objects in a single variable
Lists are very flexible (can grow, can contain "anything")

• Arrays require the numpy module

Array: computationally efficient and convenient list

• Arrays must have fixed length and can only contain numbers

of the same type (integers, real numbers, complex numbers)

```
for e1, e2, e3, ... in zip(list1, list2, list3, ...):

Alternative: loop over a common index for the lists

for i in range(len(list1)):
    e1 = list1[i]
    e2 = list2[i]
    e3 = list3[i]
    ...
```

Let's use arrays in our previous program

Note: no explicit loop for computing s_values!

Standard mathematical functions are found in the math module

```
>>> import math
>>> print math.sin(math.pi)
1.2246467991473632e-16 # Note: only approximate value

Get rid of the math prefix:

from math import sin, pi
print sin(pi)
# Or import everything from math
from math import *
print sin(pi), log(e), tanh(0.5)
```

Use the numpy module for standard mathematical functions applied to arrays

```
Matlab users can do
```

```
from numpy import *
x = linspace(0, 1, 101)
y = exp(-x)*sin(pi*x)
```

The Python community likes

```
import numpy as np
x = np.linspace(0, 1, 101)
y = np.exp(-x)*np.sin(np.pi*x)
```

Our convention: use np prefix, but not in formulas involving math functions

```
import numpy as np
x = np.linspace(0, 1, 101)
from numpy import sin, exp, pi
y = exp(-x)*sin(pi*x)
```

Array assignment gives view (no copy!) of array data

Consider array assignment b=a:

```
a = np.linspace(1, 5, 5)
b = a
```

Here, b is a just view or a pointer to the data of a - no copying of data!

See the following example how changes in b inflict changes in a

Copying array data requires special action via the copy method

Note: b has still the values from the previous example

Construction of tridiagonal and sparse matrices

- SciPy offers a sparse matrix package scipy.sparse
- The spdiags function may be used to construct a sparse matrix from diagonals
- Note that all the diagonals must have the same length as the dimension of their sparse matrix - consequently some elements of the diagonals are not used
- The first k elements are not used of the k super-diagonal
- The last k elements are not used of the -k sub-diagonal

We can solve Ax = b with tridiagonal system We can solve Ax = b with tridiagonal matrix A: choose some x, compute b = Ax (sparse/tridiagonal matrix product!), solve Ax = b, and check that x is the desired solution: >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # choose solution >>> x = np.1inspace(-1, 1, N) # corresponding dense matrix | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, 1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # standard matrix vector product | x = np.1inspace(-1, N) # stan

```
Plotting

Plotting is done with matplotlib:

import numpy as np
import matplotlib.pyplot as plt

v0 = 0.2
a = 2
n = 21 # No of t values for plotting

t = np.linspace(0, 2, n+i)
s = v0+t + 0.5*a*t**2

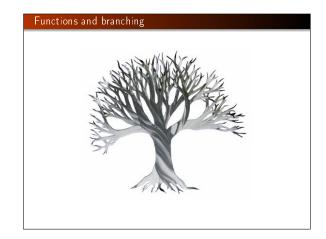
plt.plot(t, s)
plt.savefig('myplot.png')
plt.savefig('myplot.png')
plt.savefig('myplot.png')
plt.savefig('myplot.png')
plt.savefig('myplot.png')
```

```
import numpy as np import matplotlib.pyplot as plt

v0 = 0.2

n = 21  # No of t values for plotting

t = np.linspace(0, 2, n+1)
a = 2
s0 = v0*t + 0.5*a*t**2
a = 3
s1 = v0*t + 0.5*a*t**2
plt.plot(t, s0, 'r-',  # Plot s0 curve with red line
t, s1, 'bo')  # Plot s1 curve with blue circles
plt.xlabel('t')
plt.ylabel('t')
plt.title('Distance plot')
plt.legend('% s(; v_0-2, a=0.2)*', '%s(t; v_0-2, a=0.8)*'],
loc='upper left')
plt.savefig('myplot.png')
plt.show()
```



```
Functions

• s(t) = v_0 t + \frac{1}{2} a t^2 is a mathematical function
• Can implement s(t) as a Python function s(t)

def s(t):
	return v0*t + 0.5*a*t**2

v0 = 0.2
value = s(3)

**Call the function

Note:

• functions start with the keyword def
• statements belonging to the function must be indented
• function input is represented by arguments (separated by comma if more than one)
• function output is returned to the calling code
• v0 and a are global variables, which must be initialized before value = va
```

Functions can have multiple arguments

v0 and a as function arguments instead of global variables:

```
def s(t, v0, a):
return v0*t + 0.5*a*t**2
value = s(3, 0.2, 4) # Call the function
# More readable call
value = s(t=3, v0=0.2, a=4)
```

Keyword arguments are arguments with default values

```
def s(t, v0=1, a=1):
return v0*t + 0.5*a*t**2
```

- Arguments without the argument name are called positional
- Positional arguments must always be listed before the keyword arguments in the function and in any call
- The sequence of the keyword arguments can be arbitrary

Vectorization speeds up the code

Scalar code (work with one number at a time):

```
def s(t, v0, a):
return v0*t + 0.5*a*t**2
for i in range(len(t)):
    s_values[i] = s(t_values[i], v0, a)
```

Vectorized code: apply s to the entire array

```
s_values = s(t_values, v0, a)
```

How can this work?

- Expression: v0*t + 0.5*a*t**2 with array t
- r1 = v0*t (scalar times arrav)
- r2 = t**2 (square each element)
- r3 = 0.5*a*r2 (scalar times array)
- r1 + r3 (add each element)

Python functions written for scalars normally work for arrays

True if computations involve arithmetic operations and math functions:

```
from math import exp, sin
    return 2*x + x**2*exp(-x)*sin(x)
v = f(4) # f(x) works with scalar x
# Redefine exp and sin with their vectorized versions
from numpy import exp, sin, linspace x = linspace(0, 4, 100001)
v = f(x) # f(x) works with array x
```

Python functions can return multiple values

```
Return s(t) = v_0 t + \frac{1}{2} a t^2 and s'(t) = v_0 + a t:
```

```
def movement(t, v0, a):
    s = v0*t + 0.5*a*t**2
v = v0 + a*t
s_value, v_value = movement(t=0.2, v0=2, a=4)
```

return s, v means that we return a tuple (\approx list):

```
>>> def f(x):
        return x+1, x+2, x+3
>>> r = f(3) # Store all three return values in one object r
>>> print r
(4, 5, 6)
>>> type(r)
<type 'tuple'>
>>> print r[1]
                 # What type of object is r?
```

Tuples are constant lists (cannot be changed)

A more general mathematical formula (part 1)

Equations from basic kinematics:

$$v = \frac{ds}{dt}, \quad s(0) = s_0$$
$$a = \frac{dv}{dt}, \quad v(0) = v_0$$

Integrate to find v(t):

$$\int_0^t a(t)dt = \int_0^t \frac{dv}{dt}dt$$

$$v(t) = v_0 + \int_0^t a(t)dt$$

which gives

$$v(t) = v_0 + \int_0^t a(t)dt$$

A more general mathematical formula (part II)

Integrate again over [0, t] to find s(t):

$$s(t) = s_0 + v_0 t + \int_0^t \left(\int_0^t a(t) dt \right) dt$$

Example: $a(t) = a_0$ for $t \in [0, t_1]$, then a(t) = 0 for $t > t_1$:

$$s(t) = \begin{cases} s_0 + v_0 t + \frac{1}{2} a_0 t^2, & t \le t_1 \\ s_0 + v_0 t_1 + \frac{1}{2} a_0 t_1^2 + a_0 t_1 (t - t_1), & t > t_1 \end{cases}$$

Need if test to implement this!

Basic if-else tests

An if test has the structure

```
if condition:
    <statements when condition is True>
else:
          <statements when condition is False>
```

Here,

• condition is a boolean expression with value True or False.

```
 \begin{aligned} &\text{if } t <= t1; \\ &s = v0*t + 0.5*a0*t**2 \\ &\text{else}; \\ &s = v0*t + 0.5*a0*t1**2 + a0*t1*(t-t1) \end{aligned}
```

Multi-branch if tests

Just if, no else:

Implementation of a piecewisely defined function with if

A Python function implementing the mathematical function

$$s(t) = \begin{cases} s_0 + v_0 t + \frac{1}{2} a_0 t^2, & t \leq t_1 \\ s_0 + v_0 t_1 + \frac{1}{2} a_0 t_1^2 + a_0 t_1 (t - t_1), & t > t_1 \end{cases}$$

reads

```
def s_func(t, v0, a0, t1):
    if t <= t1:
        s = v0*t + 0.5*a0*t**2
else:
        s = v0*t + 0.5*a0*t1**2 + a0*t1*(t-t1)
    return s</pre>
```

Python functions containing if will not accept array arguments

```
>>> def f(x): return x if x < 1 else 2*x
...
>>> import numpy as np
>>> x = np.1Lnspace(0, 2, 5)
>>> f(x)
Traceback (most recent call last):
...
ValueError: The truth value of an array with more than one element is ambiguous. Use a any() or a all()
```

Problem: x < 1 evaluates to a boolean array, not just a boolean

Remedy 1: Call the function with scalar arguments

```
n = 201  # No of t values for plotting
ti = 1.5

t = np.linspace(0, 2, n+1)
s = np.zeros(n+1)
for i in range(len(t)):
    s[i] = s_func(t=t[i], v0=0.2, a0=20, t1=t1)

Can now easily plot:

plt.plot(t, s, 'b-')
plt.plot(t[t, ti], [0, s_func(t=t1, v0=0.2, a0=20, t1=ti)], 'r--')
plt.tlabel('t')
plt.ylabel('t')
plt.savefig('myplot.png')
plt.show()
```

Remedy 2: Vectorize the if test with where

Functions with if tests require a complete rewrite to work with arrays.

s = np.where(condition, s1, s2)

Explanation:

- condition: array of boolean values
- s[i] = s1[i] if condition[i] is True
- s[i] = s2[i] if condition[i] is False

Our example then becomes

```
s = np.where(t \le t1, v0*t + 0.5*a0*t**2, v0*t + 0.5*a0*t1**2 + a0*t1*(t-t1))
```

Note that t <= t1 with array t and scalar t1 results in a boolean array b where b[i] = t[i] <= t1.

Remedy 3: Vectorize the if test with array indexing

- Let b be a boolean array (e.g., b = t <= t1)
- s[b] selects all elements s[i] where b[i] is True
- Can assign some array expression expr of length len(s[b]) to s[b]: s[b] = (expr)[b]

Our example can utilize this technique with b as t <= t1 and t > t1:



File reading

Put input data in a text file:

```
v0 = 2
a = 0.2
dt = 0.1
interval = [0, 2]
```

How can we read this file into variables v = 0, a, dt, and interval?

Code for reading files with lines variable = value

Splitting lines into words is a frequent operation

```
>>> line = 'v0 = 5.3'
>>> variable, value = line.split('=')
>>> variable
'v0'
>>> value
'5.3'
>>> variable.strip() # strip away blanks
'v0'
```

Note: must convert value to float before we can compute with the value!

The magic eval function

eval(s) executes a string s as a Python expression and creates the corresponding Python object

```
>>> obj1 = eval('1+2')  # Same as obj1 = 1+2
>>> obj1, type(obj1)
(3, <type 'int'>)
>>> obj2, type(obj2)
([-1, 8, 10, 11], <type 'list'>)
>>> trye math import sin, pi
>>> x = 1
>>> obj3 = eval('sin(pi*x)')
>>> obj3, type(obj3)
(1.224667991473532e-16, <type 'float'>)
```

Why is this so great? We can read formulas, lists, expressions as text from file and with eval turn them into live Python objects!

Implementing a calculator in Python

Demo:

```
Terminal> python calc.py "1 + 0.5*2"
2.0
Terminal> python calc.py "sin(pi*2.5) + exp(-4)"
1.0183156388887342
```

Just 5 lines of code:

```
import sys
command_line_expression = sys.argv[1]
from math import * # Define sin, cos, exp, pi, etc.
result = eval(command_line_expression)
print result
```

Modern Python often applies the with statement for file handling

```
with open('.input.dat', 'r') as infile:
    for line in infile:
```

No need to close the file when using with

ile writing

- We have t and s(t) values in two lists, t_values and s_values
- Task: write these lists as a nicely formatted table in a file

Code

```
outfile = open('table1.dat', 'w')
outfile.write('#t s(t)\n') # write table header
for t, s in zip(t_values, s_values):
   outfile.write('%.2f %.4f\n' % (t, s))
```

Simplified writing of tabular data to file via numpy.savetxt

Simplified reading of tabular data from file via numpy.loadtxt

```
data = np.loadtxt('table2.dat', comments='#')
```

Note:

- Lines beginning with the comment character # are skipped in the reading
- data is a two-dimensional array: data[i,0] holds the t value and data[i,1] the s(t) value in the i-th row

Classes

- All objects in Python are made from a class
- You don't need to know about classes to use Python
- But class programming is powerful
- Class = functions + variables packed together
- A class is a logical unit in a program
- A large program as a combination of appropriate units

A very simple class

- One variable: a
- One function: dump for printing a

```
class Trivial:
    def __init__(self, a):
        self.a = a

def dump(self):
        print self.a
```

Class terminology: Functions are called methods and variables are called attributes.

How can we use this class?

First, make an instance (object) of the class:

Note:

- The syntax Trivial(a=4) actually means Trivial.__init__(t, 4)
- self is an argument in __init__ and dump, but not used in the calls
- __init__ is called constructor and is used to construct an object (instance) if the class
- t.dump() actually means Trivial.dump(t) (self is t)

The self argument is a difficult thing for newcomers...

It takes time and experience to understand the self argument in class methods!

- self must always be the first argument
- self is never used in calls
- self is used to access attributes and methods inside methods

We refer to a more comprehensive text on classes for better explanation of self.

self is confusing in the beginning, but later it greatly helps the understanding of how classes work!

A class for representing a mathematical function

Function with one independent variable t and two parameters v_0 and a:

$$s(t; v_0, a) = v_0 t + \frac{1}{2} a t^2$$

Class representation of this function:

- v0 and a are variables (data)
- ullet A method to evaluate s(t), but just as a function of t

Usage:

```
s = Distance(v0=2, a=0.5) # create instance

v = s(t=0.2) # compute formula
```

The class code

```
Class implementation of f(x, y, z; \rho_1, \rho_2, \dots, \rho_n)

• The n parameters p_1, p_2, \dots, p_n are attributes

• __call__(self, x, y, z) is used to compute f(x, y, z)

class F:

def __init__(self, p_1, p_2, ...):
    self p_1 = p_1
    self, p_2 = p_2
    ...

def __call__(self, x, y, z):
    # return formula involving x, y, z and self.p_1, self.p_2 = p_2

f = F(p_1 = \dots, p_2 = \dots, \dots) # create instance with parameters print f(1, 4, -1) # evaluate f(x, y, z) function
```