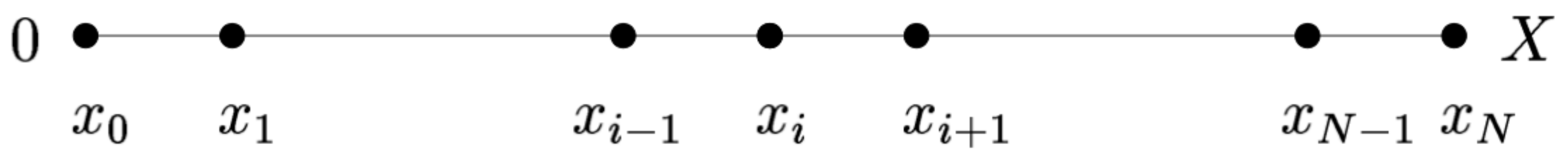
Finite difference methods

Principle: derivatives in the partial differential equation are approximated by linear combinations of function values at the grid points

1D: $\Omega = (0, X), \quad u_i \approx u(x_i), \quad i = 0, 1, \ldots, N$

grid points $x_i = i\Delta x$ mesh size $\Delta x = \frac{X}{N}$

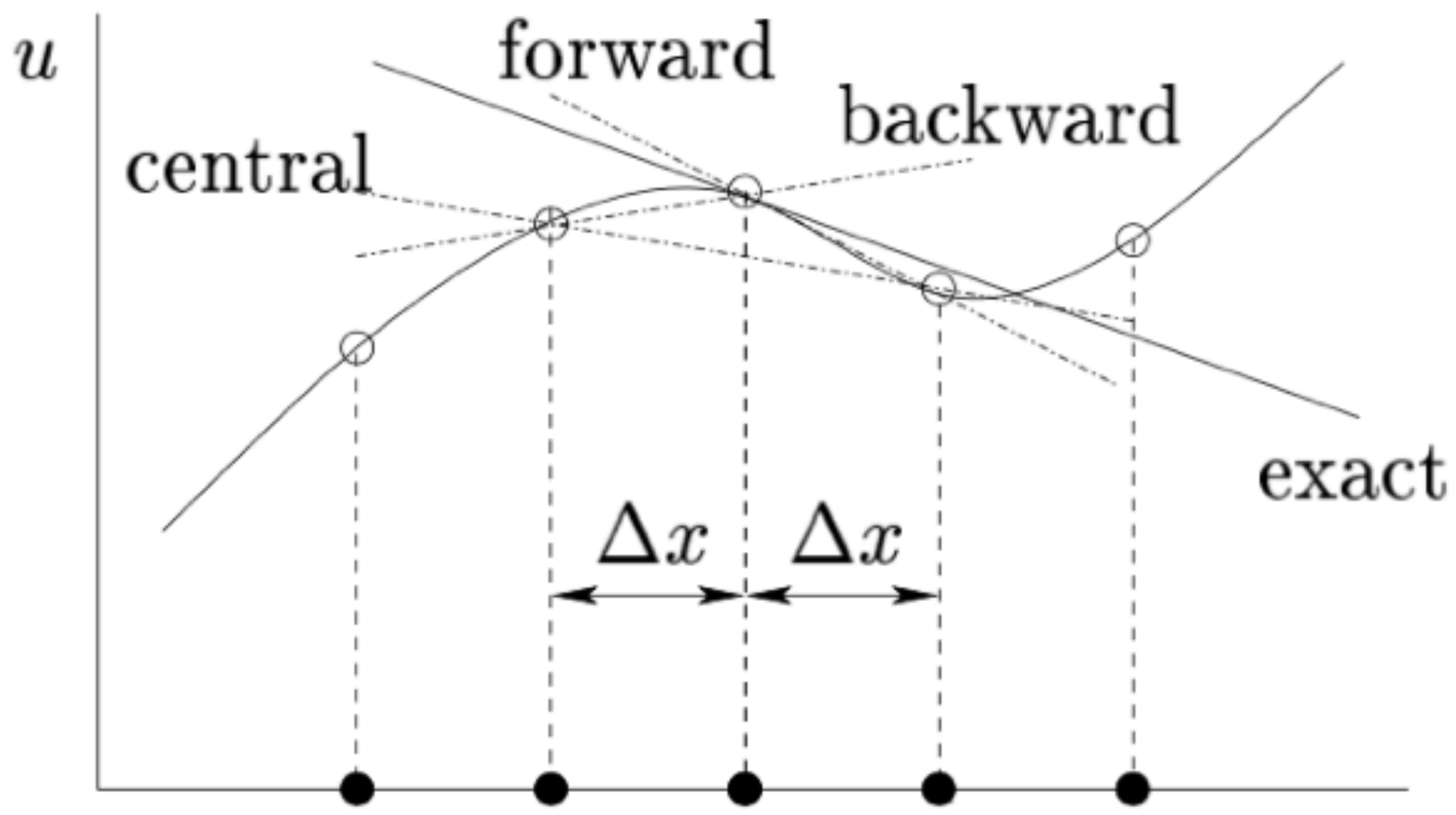


First-order derivatives

$$
\begin{aligned}
\frac{\partial u}{\partial x}(\bar{x}) &= \lim_{\Delta x \to 0} \frac{u(\bar{x} + \Delta x) - u(\bar{x})}{\Delta x} = \lim_{\Delta x \to 0} \frac{u(\bar{x}) - u(\bar{x} - \Delta x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{u(\bar{x} + \Delta x) - u(\bar{x} - \Delta x)}{2\Delta x} \qquad \text{(by definition)}
\end{aligned}
$$

Approximation of first-order derivatives

Geometric interpretation



$$\left(\frac{\partial u}{\partial x}\right)_i \approx \frac{u_{i+1}-u_i}{\Delta x}$$ forward difference

$$\left(\frac{\partial u}{\partial x}\right)_i \approx \frac{u_i-u_{i-1}}{\Delta x}$$ backward difference

$$\left(\frac{\partial u}{\partial x}\right)_i \approx \frac{u_{i+1}-u_{i-1}}{2\Delta x}$$ central difference