# Numerical methods

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- PDEs are necessary to describe “**rates of change**” which describes the **conservation principles**.

- One way of solving PDEs using computers is by numerical **discretisation** techniques:
  - transform each differential term into an approximate algebraic equation

# School of Geosciences

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# Navier-Stokes equations: fluid dynamics

## 150-year old physics

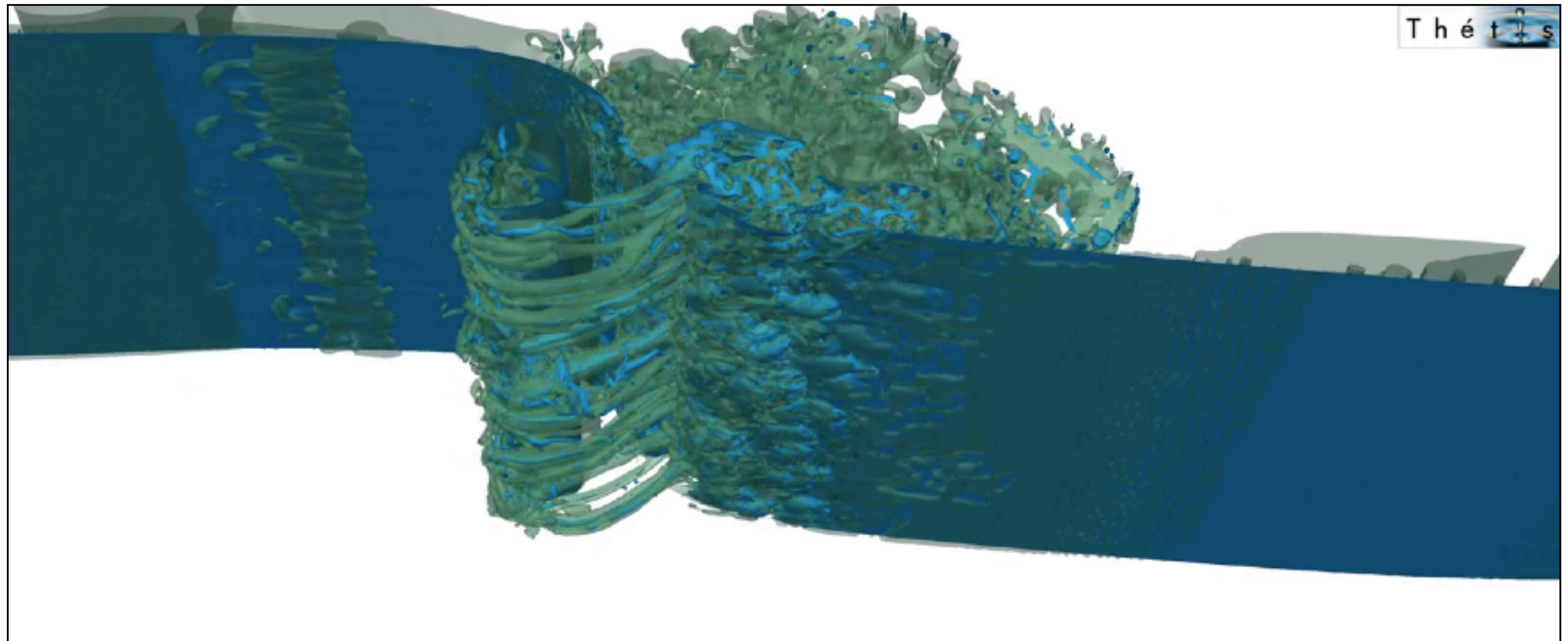
Georges G. Stokes (1819-1903)

$$\overbrace{\rho\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\text{Unsteady acceleration}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{Convective acceleration}}\Big)}^{\text{Inertia (per volume)}} = \overbrace{\underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}} + \underbrace{\mathbf{f}}_{\text{Other body forces}}.$$

$$\nabla\cdot\vec{v} = 0$$

$$\frac{\partial C_p T}{\partial t} + \vec{v}\cdot\nabla C_p T = \frac{1}{\rho}\nabla\cdot k\nabla T + H$$

$$\boxed{\rho = \rho_o(1-\alpha(T-T_o))} \quad \boxed{\sigma_{ij} = \eta\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) - p\delta_{ij}}$$




Massively Parallel Navier-Stokes Solver for Breaking Waves

Movie: S. Glockner, P. Lubin, Institut de Mécanique et d'Ingénierie - Bordeaux.

Movie: NASA Jet Propulsion Laboratory and MIT.