Georges G. Stokes (1819-1903)

150-year old physics

$$\nabla \cdot \vec{v} = 0$$

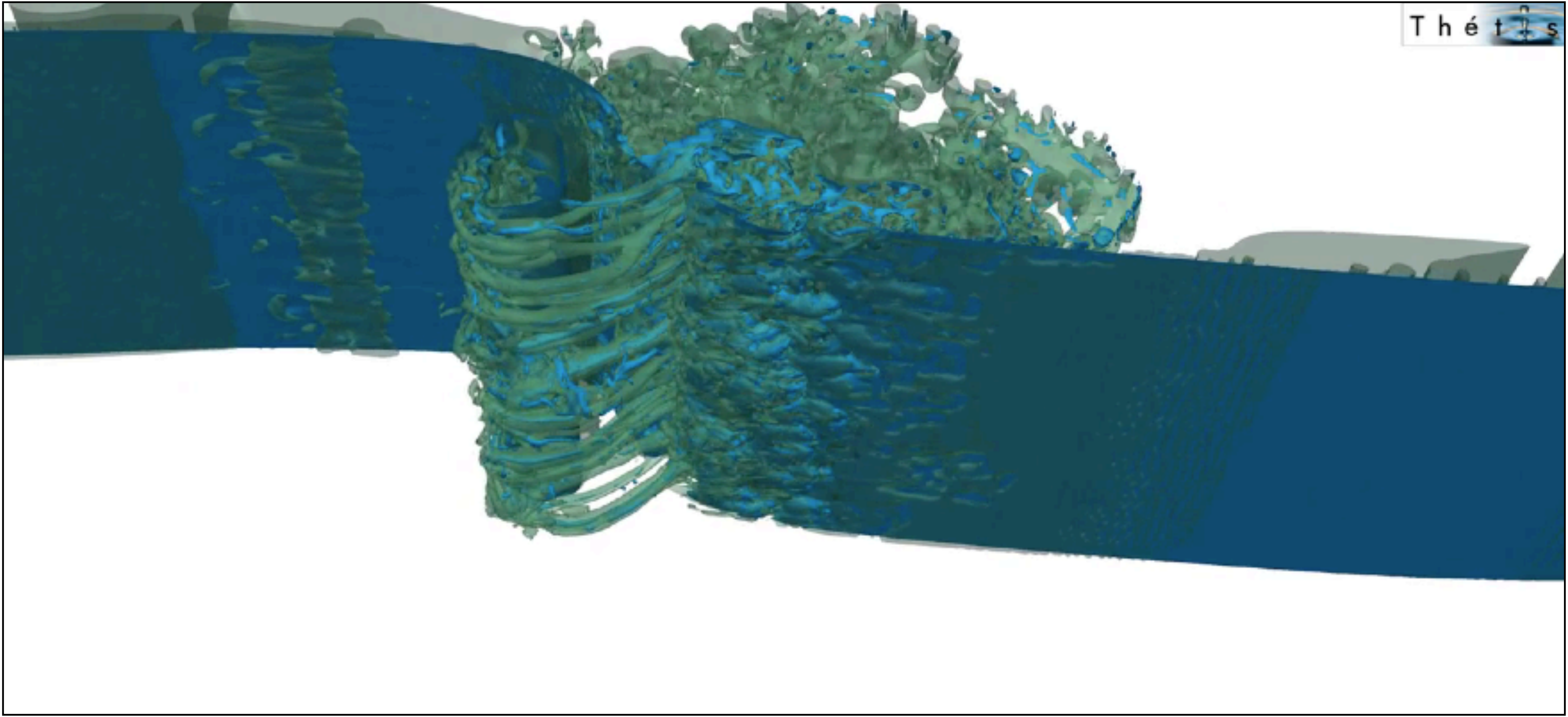
$$\overbrace{\rho\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\substack{\text{Unsteady}\\\text{acceleration}}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\substack{\text{Convective}\\\text{acceleration}}}\Big)}^{\text{Inertia (per volume)}} = \overbrace{\underbrace{-\nabla p}_{\substack{\text{Pressure}\\\text{gradient}}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}} + \underbrace{\mathbf{f}}_{\substack{\text{Other}\\\text{body}\\\text{forces}}}.$$

$$\frac{\partial C_p T}{\partial t} + \vec{v} \cdot \nabla C_p T = \frac{1}{\rho} \nabla \cdot k \nabla T + H$$

$$\boxed{\rho = \rho_o(1 - \alpha(T - T_o))}$$

$$\boxed{\sigma_{ij} = \eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - p\delta_{ij}}$$

Navier-Stokes equations: fluid dynamics



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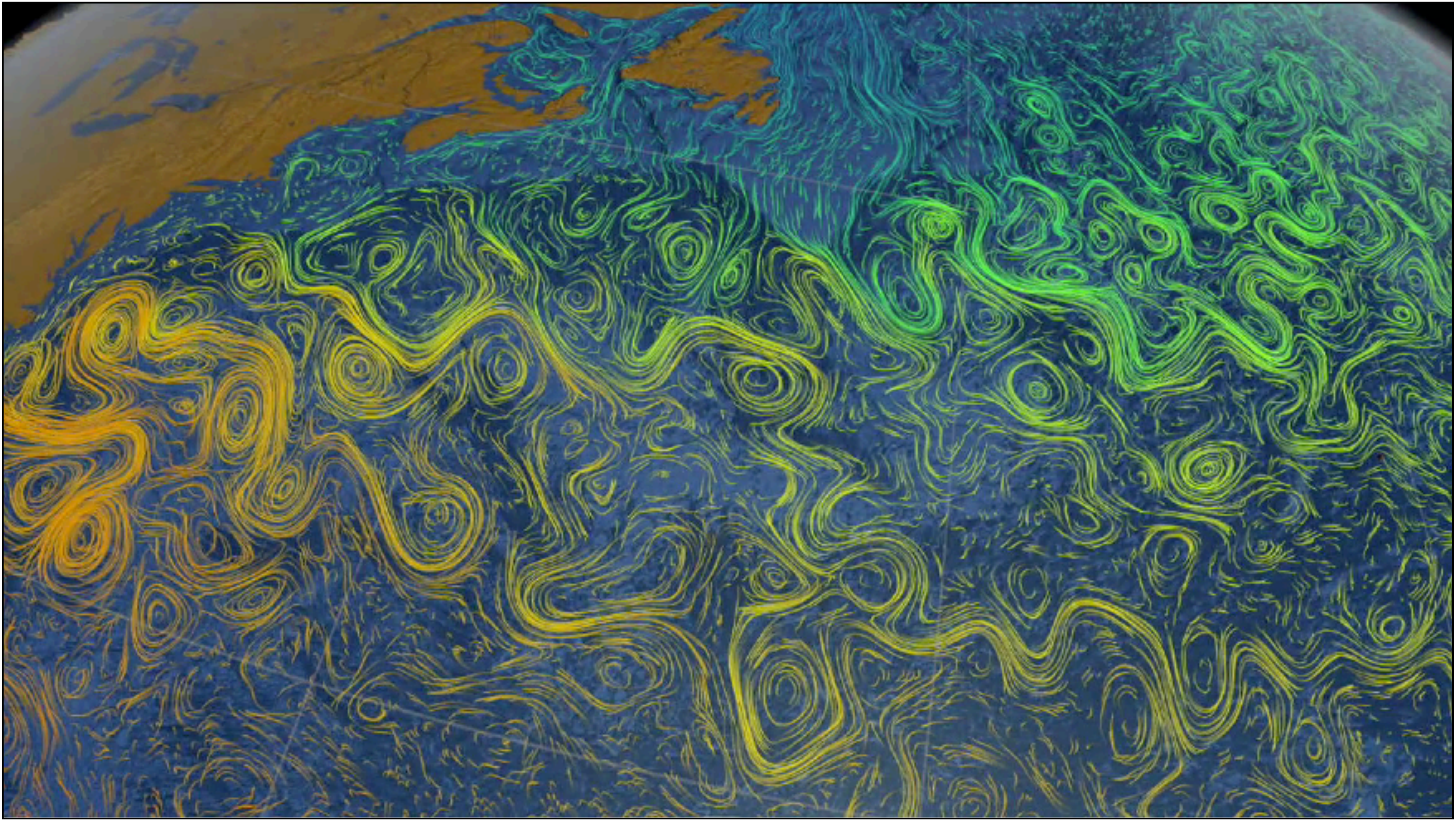
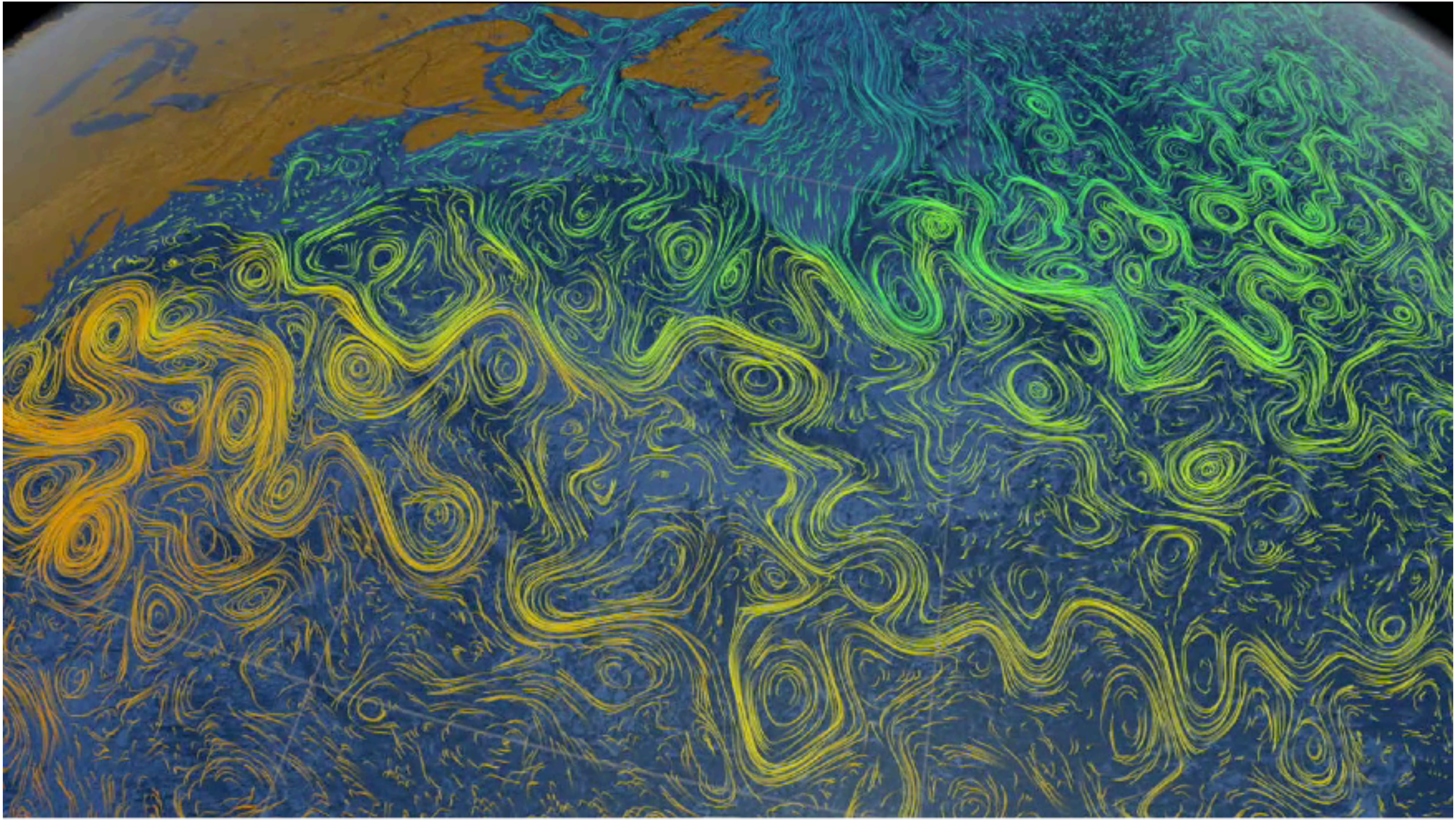
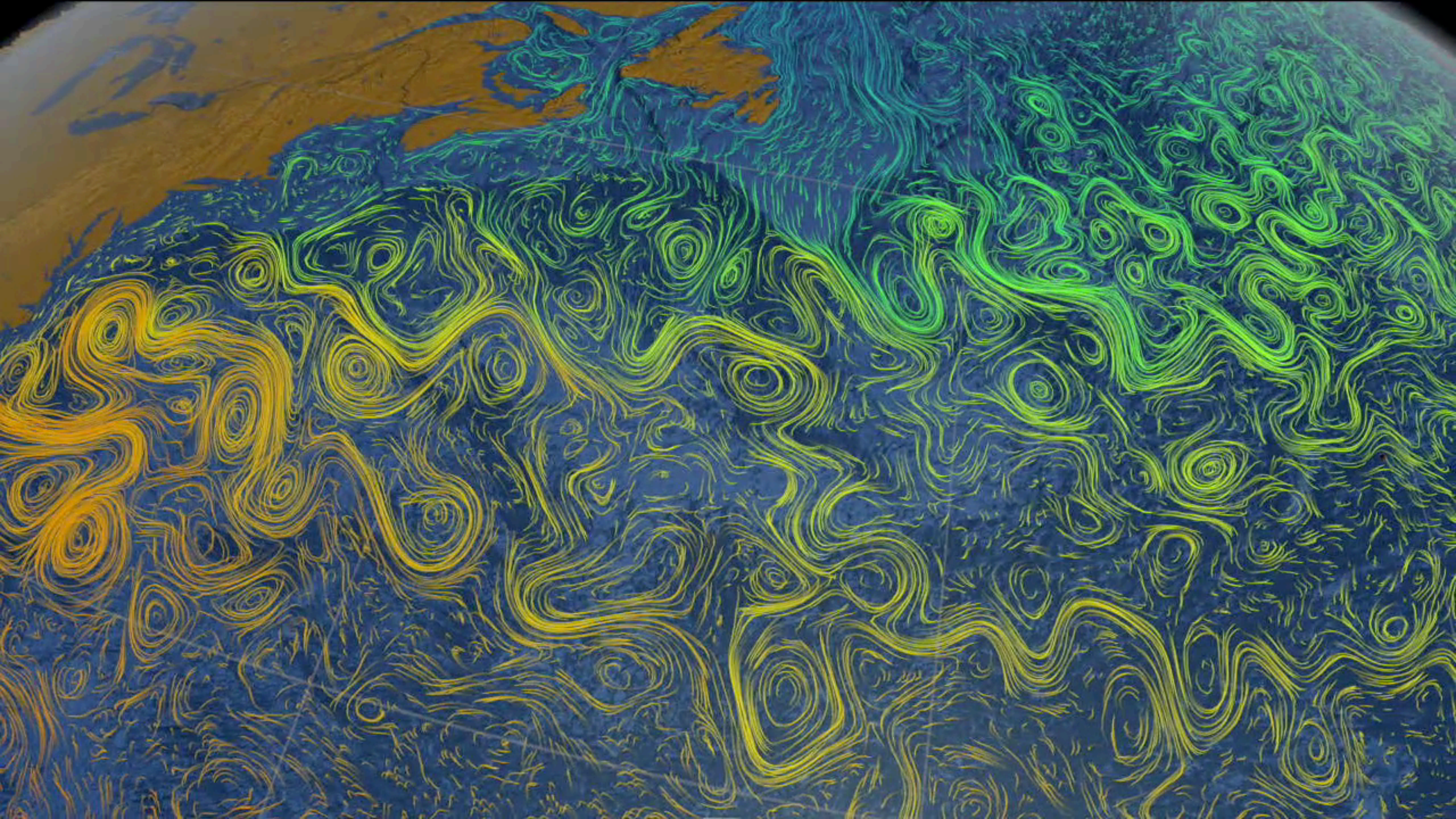


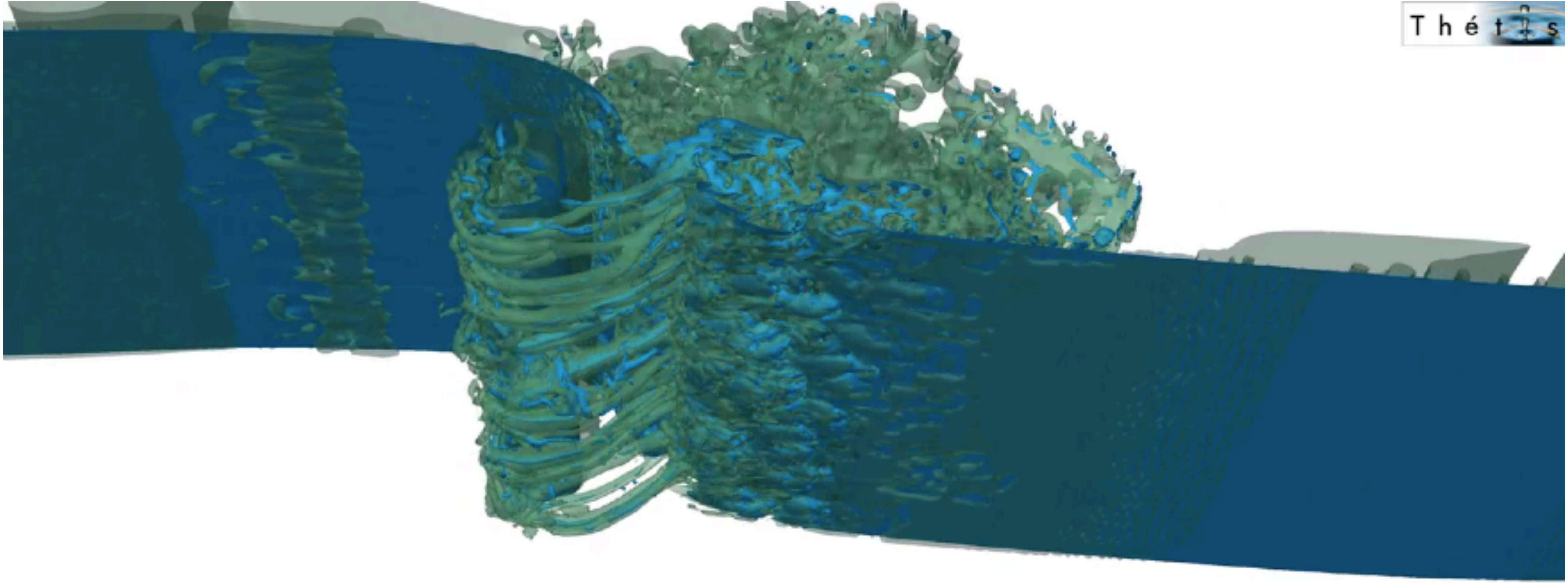
Massively Parallel Navier-Stokes Solver for Breaking Waves

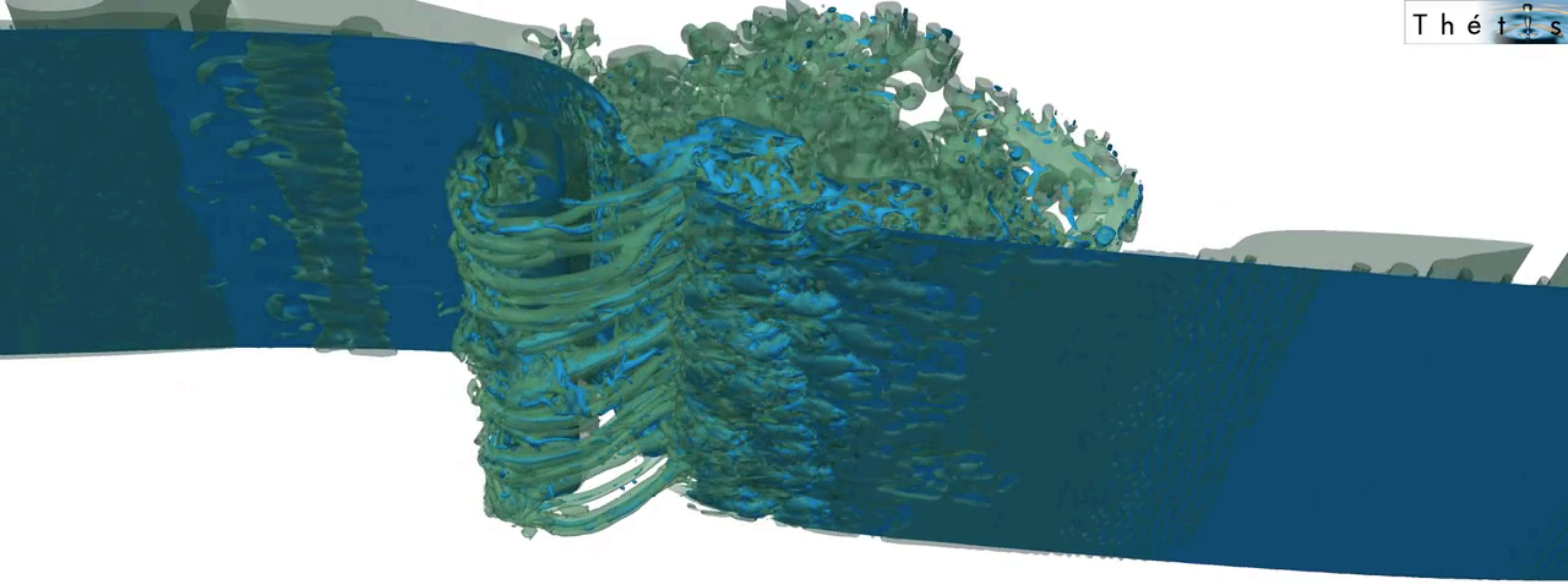
Movie: S. Glockner, P. Lubin, Institut de Mécanique et d'Ingénierie - Bordeaux.

Movie: NASA Jet Propulsion Laboratory and MIT.









Navier-Stokes equations: fluid dynamics

150-year old physics

Georges G. Stokes (1819-1903)

$$\rho\Big(\underbrace{\overbrace{\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\text{Unsteady acceleration}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{Convective acceleration}}}^{\text{Inertia (per volume)}}}\Big) = \overbrace{\underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}} + \underbrace{\mathbf{f}}_{\text{Other body forces}}.$$

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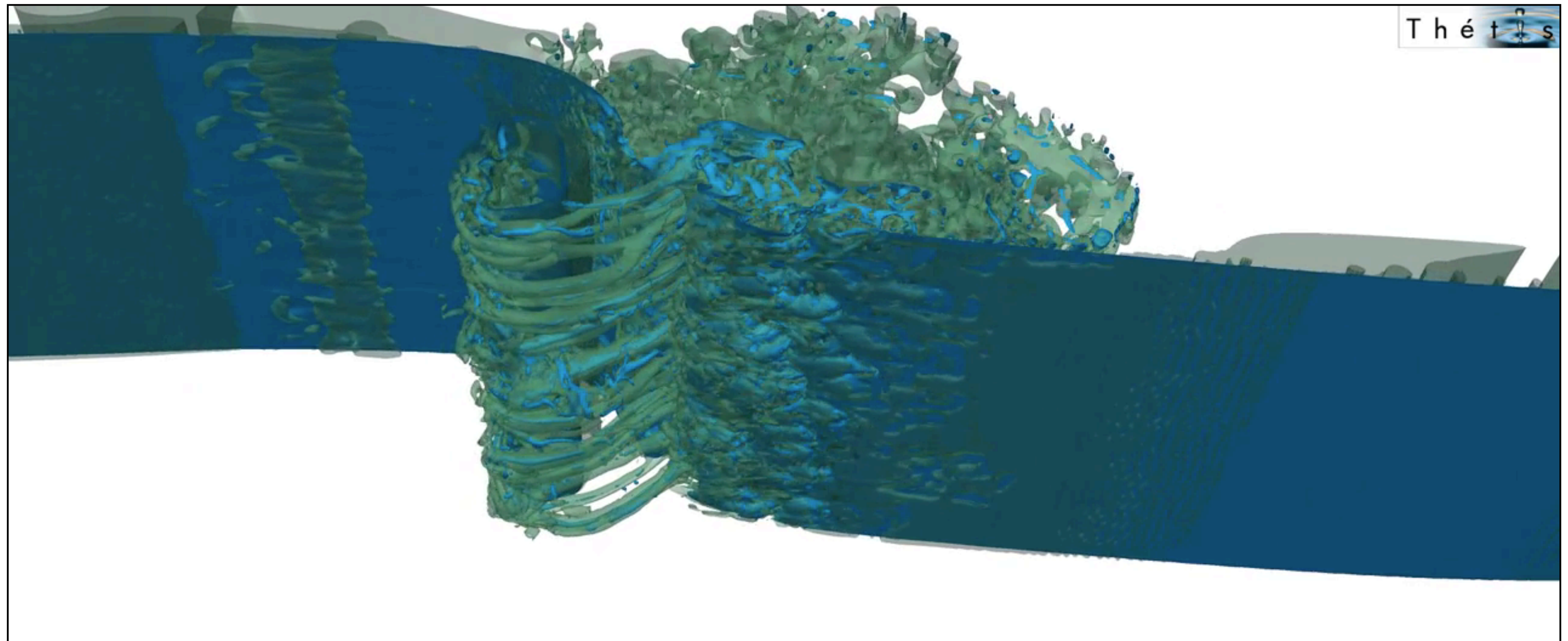
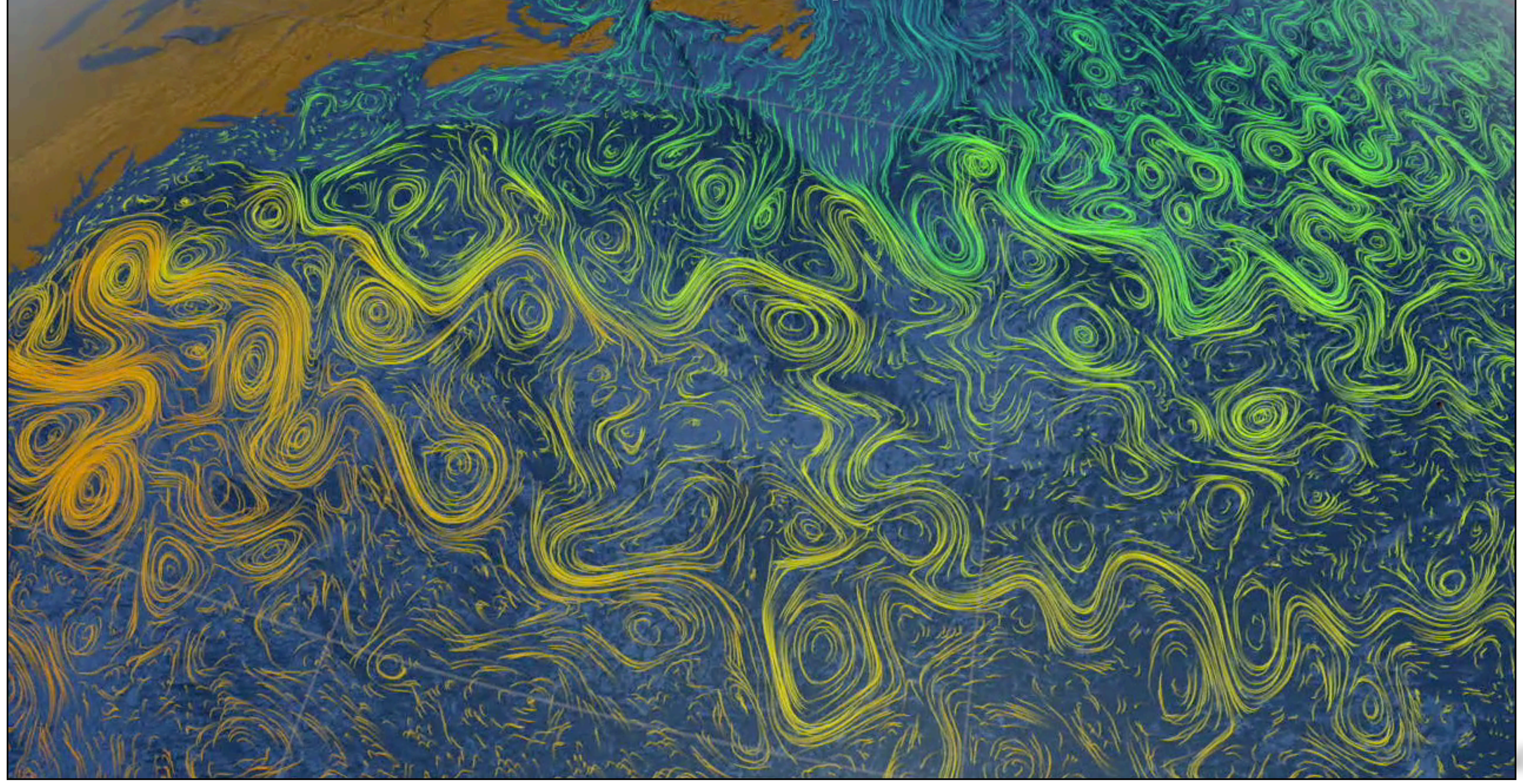


Massively Parallel Navier-Stokes Solver for Breaking Waves

Movie: S. Glockner, P. Lubin, Institut de Mécanique et d'Ingénierie - Bordeaux.



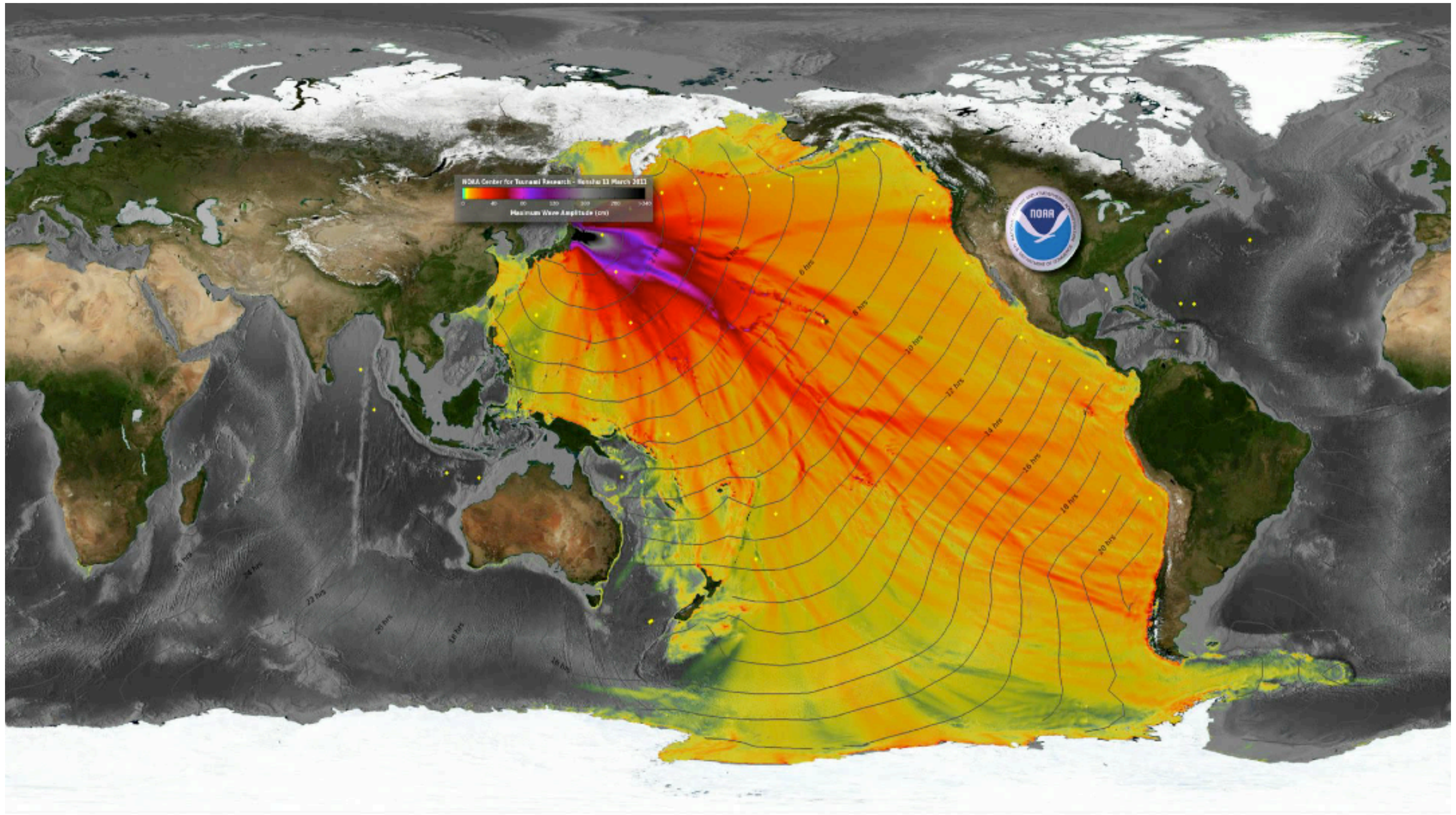
Data assimilation & Navier-Stokes equations

Movie: NASA Jet Propulsion Laboratory and MIT.

of Geosciences

Saint-Venant equations: fluid dynamics

- depth-integrating the Navier–Stokes equations considering the horizontal length scale is much greater than the vertical length scale.
- conservation of mass implies that the vertical velocity of the fluid is small.
- shallow water equation models have only one vertical level, so they cannot directly encompass any factor that varies with height.



$$\frac{\partial}{\partial t} h + \frac{\partial}{\partial x}(uh) + \frac{\partial}{\partial y}(vh) = 0,$$

$$\frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}\left(u^2 + \frac{1}{2}gh^2\right) + \frac{\partial}{\partial y}(uvh) = 0,$$

$$\frac{\partial}{\partial t}(vh) + \frac{\partial}{\partial t}(uvh) + \frac{\partial}{\partial y}\left(v^2 h + \frac{1}{2}gh^2\right) = 0,$$