Parameters of Type III Distribution given mean and variance

Let the scale, location and shape factors of an EV Type III Distribution be a, u and k respectively.

Then, for a variate X,

$$F_X(x) = \exp\{-[1 - k(x-u)/a]^{1/k}\} \tag{1}$$

Alternatively the c.d.f. can be written [1],

$$F_X(x) = \exp\left[-\left(\frac{w-x}{w-u}\right)^{1/k}\right] \tag{2}$$

where w is the upper limit of x equal to u + (a/k)

It can be shown [1] that the mean, m, and variance σ^2 of the distribution are:

$$m = w - (w-u)\Gamma(1+k) = u + (a/k)[1 - \Gamma(1+k)]$$

$$\sigma^2 = (w-u)^2\,[\Gamma(1+2k) - \Gamma^2(1+k)] = (a/k)^2\,[\Gamma(1+2k) - \Gamma^2(1+k)]$$

where $\Gamma(\,)$ is the Gamma Function.

These can be written as:

$$m = u + aA, \quad \text{where } A = (1/k)[1 - \Gamma(1+k)] \tag{3}$$

$$\sigma^2 = a^2 B^2 \quad \text{where } B^2 = (1/k)^2\,[\Gamma(1+2k) - \Gamma^2(1+k)] \tag{4}$$

Thus, A and B are only functions of the shape factor k. $(\Gamma(1+k))^2$

Hence the coefficient of variation, $V = \sigma/m = aB/m$ (5)

from (5), $a = mV/B$ (6)

from (3), $u = m - aA = m\,[1 - (A/B)V]$ (7)

Knowing k, m and V, Equations (6) and (7) can be used to evaluate a and u, and hence the cumulative probability distribution is defined by Equation (1) (or (2)).

Reference

1. A.H-S. Ang and W. Tang. "Probability concepts in engineering planning and design. Volume II. Decision, risk and reliability". published by the authors, 1990.