

Windborne debris risk assessment

Ning Lin*, Erik Vanmarcke

Department of Civil and Environmental Engineering, Princeton University, Princeton, NJ, USA

Received 31 July 2007; accepted 28 January 2008

Available online 4 March 2008

Abstract

A probabilistic model, based on Poisson random measure theory, is developed for predicting windborne debris damage in residential areas. It is intended for incorporation into improved methodology to estimate economic losses due to hurricanes to individual houses or entire residential developments. A sample application to a coastal housing development is provided.

© 2008 Elsevier Ltd. All rights reserved.

Keywords: Windborne debris; Hurricane risk; Poisson; Probability; Statistics; Wind tunnel

1. Introduction

Windborne debris is known to be a major source of damage in strong winds such as during hurricanes. Windborne debris modeling is logically a key component in evaluating hurricane risk to buildings (and, especially, clusters of buildings). However, there are few published articles on windborne debris compared to the literature on loads caused by wind pressure, and especially on the inherent probabilistic aspects of the debris-damage problem. Twisdale et al. [1] developed an integrated methodology to analyze debris risk in residential areas, combining numerical models of the hurricane wind field and debris generation, trajectory, and impact. They also produced reliability curves for typical residential environments, which lie at the basis of recommendations in ASTM [2] for debris impact risk analysis. However, their probabilistic model assumes that impact parameters (e.g., number of impacts, momentum at impact) are identically distributed for all the houses in the study area, and hence it is mainly suitable for estimating mean debris risk in an area. This paper proposes a methodology to predict the debris risk to individual buildings in a development, as well as the aggregate debris risk. First, a debris risk assessment methodology is established based on Poisson random measure theory (see, e.g., Cinlar [3]). Second, the approach is shown, with reasonable approximation, to be amenable to matrix formulation. Next, probability distributions

characterizing the stochastic debris trajectories are proposed and their parameters are obtained based on the experimental debris trajectory models developed by Lin et al. [4,5]. Finally, the way to incorporate the debris-damage model into hurricane risk analysis is discussed.

2. Debris risk model

The debris risk model relies on Poisson random measure theory to predict the impact damage to a residential area under hurricane wind conditions, due to debris generated from building sources. Other debris sources (e.g., street signs, trash cans) are, at this stage of model development, ignored, as they are mainly susceptible to vertical updrafts occurring during tornadoes. We first define a relatively isolated region, containing the structures of interest, such that the region is not likely to interact with the outside in terms of debris damage. Denote the residences by integers $1, 2, \dots, I$, where I is their total number within the region. Every house can generate debris and can be hit by debris generated from any house (itself included) in the region, depending on the wind conditions (e.g., mean wind speed and direction, and degree of turbulence). We use the subscript i ($i = 1, 2, \dots, I$) to identify the properties of a house when it is regarded as a debris source house, and j ($j = 1, 2, \dots, I$) when it is regarded as an impact target house. Let A_j denote the area occupied by house j and E the union of all A_j . Suppose there are S_i types of debris generated from house i , and denote $S = \max\{S_i, i = 1, 2, \dots, I\}$. In a residential area, for instance, the

* Corresponding author.

E-mail address: nlin@Princeton.EDU (N. Lin).

roof structure of some houses may become partially damaged and roofing materials may become windborne as debris. The common debris types that have been observed in hurricane damage surveys are roof covers, roof sheathings, and 2×4 timbers.

We focus first on the impacts due to one type of debris generated from one arbitrary house, under a prescribed wind condition. Let the number of items of type s debris ($s \leq S_i$) generated from house i be a random variable $L_{s,i}$, and assume it has the Poisson distribution, with mean value $\lambda_{s,i}$. Define the landing positions of these debris items as independent identically distributed random variables $X_{s,i}^1, \dots, X_{s,i}^{L_{s,i}}, \dots, X_{s,i}^{L_{s,i}}$, with $\mu_{s,i}(dx)$ as their common probability distribution. Define $H_{s,i}(A_j)$ as the number of debris items that will land on the area occupied by house j . Hence, the number of items of debris hitting house j may be expressed as:

$$H_{s,i}(A_j) = \sum_{l_{s,i}=1}^{L_{s,i}} \delta(X_{s,i}^{l_{s,i}}, A_j), \quad (1)$$

where δ is the identity kernel, so that $\delta(x, A) = 1$ if x belongs to A , and $\delta(x, A) = 0$ otherwise.

Under these conditions, it can be shown that $H_{s,i}$ is a Poisson random measure on E , with mean value $\nu_{s,i}(dx) = \lambda_{s,i} \mu_{s,i}(dx)$. This means that for every A_j ($A_j \subset E$, $j = 1, 2, \dots, I$), the random variable $H_{s,i}(A_j)$ has the Poisson distribution with the following mean value $\nu_{s,i}(A_j)$ (i.e., the mean number of hits on house j):

$$\nu_{s,i}(A_j) = \lambda_{s,i} \int_{A_j} \mu_{s,i}(dx). \quad (2)$$

Specifically, the probability that house j suffers exactly N impacts is:

$$P\{H_{s,i}(A_j) = N\} = \frac{e^{-\nu_{s,i}(A_j)} [\nu_{s,i}(A_j)]^N}{N!}. \quad (3)$$

Define the horizontal impact momentum of debris item $l_{s,i}$ (hitting on the ground or on a house) as a random variable $Y_{s,i}^{l_{s,i}}$ (As noted earlier, vertical momentum is of secondary importance in the hurricane wind condition; see Lin et al. [4]). We may write $Y_{s,i}^{l_{s,i}} = m_{s,i} u_{s,i}^{l_{s,i}}$, where $u_{s,i}^{l_{s,i}}$ is the horizontal impact speed of the debris item and $m_{s,i}$ is its mass (assumed to be a known constant for a given debris type). It has been found, from experiments and numerical simulations, that the horizontal impact speed of a debris item is primarily a function of the distance traveled (e.g., Lin et al. [4], Holmes et al. [6], and Lin et al. [5]). Hence, we define $\phi_{s,i}(dy|x)$ as the common conditional distribution, for every item of debris $l_{s,i} = 1, 2, \dots, L_{s,i}$, of $Y_{s,i}^{l_{s,i}}$ given $X_{s,i}^{l_{s,i}}$, the landing position of the debris item. Denoting $X_{s,i} = \{X_{s,i}^1, \dots, X_{s,i}^{L_{s,i}}, \dots, X_{s,i}^{L_{s,i}}\}$ and $Y_{s,i} = \{Y_{s,i}^1, \dots, Y_{s,i}^{L_{s,i}}, \dots, Y_{s,i}^{L_{s,i}}\}$, it is reasonable to assume that the different elements $Y_{s,i}^{l_{s,i}}$ are conditionally independent given $X_{s,i}$. Then let $M_{s,i}$ denote the random measure formed by $X_{s,i}$ and $Y_{s,i}$. It can be shown that $M_{s,i}$ is a Poisson random

measure on $E \times R^+$ (where R^+ stands for the positive real line), with mean measure $\nu_{s,i}(dx, dy) = \nu_{s,i}(dx) \phi_{s,i}(dy|x)$. Suppose the impact resistance of a particular (type of) vulnerable area (e.g., windows) of house j is ζ_j , a threshold expressed in terms of the horizontal impact momentum. Let $M_{s,i}(A_j, y > \zeta_j)$ denote the number of debris items that landed within the area occupied by house j and that have horizontal momentum exceeding the threshold ζ_j . Let $\alpha_{s,i}(j)$ denote the mean number of these over-threshold impacts on house j ,

$$\begin{aligned} \alpha_{s,i}(j) &= \nu_{s,i}(A_j, y > \zeta_j) = \lambda_{s,i} \int_{A_j} \mu_{s,i}(dx) \int_{\zeta_j}^{\infty} \phi_{s,i}(dy|x) \\ &= \lambda_{s,i} \int_{A_j} \mu_{s,i}(dx) \Phi_{s,i}(y > \zeta_j|x), \end{aligned} \quad (4)$$

where $\Phi_{s,i}(y > \zeta_j|x) = 1 - \Phi_{s,i}(y \leq \zeta_j|x)$ is the complementary cumulative distribution function evaluated at the threshold ζ_j . Then the probability that house j suffers N over-threshold impacts is:

$$P\{M_{s,i}(A_j, y > \zeta_j) = N\} = \frac{e^{-\alpha_{s,i}(j)} [\alpha_{s,i}(j)]^N}{N!}. \quad (5)$$

Consider now the behavior of arbitrary types of debris generated from all houses, under a prescribed wind condition, in the region. It can be assumed, to the first approximation, that debris items fly independently of each other, as debris items typically fly close to the mean wind direction so that the probability that debris items interact with each other in the air is relatively small. (Since the generation of an item of debris may depend on the generation of other debris items from the same house, a useful extension of the model might be to consider randomly-sized clusters of debris items, and interdependence in the generation of different types of debris.)

We construct $M_{s,i}$ for $s = 1, 2, \dots, S$ and $i = 1, 2, \dots, I$ as described above. ($M_{s,i} = 0$, in case $s > S_i$.) Then $M_{s,i}$ are independent Poisson random measures on $E \times R^+$, with mean measures $\nu_{s,i}$, respectively. Let M be the sum of $M_{s,i}$. Again, M is a Poisson random measure on $E \times R^+$, with mean measure ν given by:

$$\begin{aligned} \nu(dx, dy) &= \sum_{s=1}^S \sum_{i=1}^I \nu_{s,i}(dx, dy) \\ &= \sum_{s=1}^S \sum_{i=1}^I \lambda_{s,i} \mu_{s,i}(dx) \phi_{s,i}(dy|x). \end{aligned} \quad (6)$$

Denoting by $M(A_j, y > \zeta_j)$ the total number of over-threshold impacts on house j , we have

$$M(A_j, y > \zeta_j) = \sum_{s=1}^S \sum_{i=1}^I M_{s,i}(A_j, y > \zeta_j). \quad (7)$$

Similarly, $\nu(A_j)$, the mean total number of impacts on house j , is

$$\nu(A_j) = \sum_{s=1}^S \sum_{i=1}^I \nu_{s,i}(A_j) = \sum_{s=1}^S \sum_{i=1}^I \lambda_{s,i} \int_{A_j} \mu_{s,i}(dx), \quad (8)$$

and $\alpha(j)$, the mean total number of over-threshold impacts on house j , is

$$\begin{aligned}\alpha(j) &= \sum_{s=1}^S \sum_{i=1}^I \alpha_{s,i}(j) = \sum_{s=1}^S \sum_{i=1}^I v_{s,i}(A_j, y > \zeta_j) \\ &= \sum_{s=1}^S \sum_{i=1}^I \lambda_{s,i} \int_{A_j} \mu_{s,i}(dx) \Phi_{s,i}(y > \zeta_j | x). \quad (9)\end{aligned}$$

The probability $P(j, N)$ that house j suffers, in total, N over-threshold impacts is:

$$P(j, N) = P\{M(A_j, y > \zeta_j) = N\} = \frac{e^{-\alpha(j)} [\alpha(j)]^N}{N!}. \quad (10)$$

Furthermore, denoting by $P_\zeta(j, n)$ the probability of n over-threshold impacts on the vulnerable area of house j , we may write:

$$P_\zeta(j, n) = \sum_{N=n}^{\infty} P_\zeta(j, n|N) P(j, N), \quad (11)$$

where $P_\zeta(j, n|N)$ is the conditional probability of n impacts on the “vulnerable area” (e.g., the windows) of house j , given that there are N impacts on house j . Here, we define “debris damage” to a house as meaning that its vulnerable area suffers at least one over-threshold impact. Then the probability of debris damage to house j , denoted by $P_D(j)$, is:

$$P_D(j) = \sum_{n=1}^{\infty} P_\zeta(j, n) = \sum_{n=1}^{\infty} \sum_{N=n}^{\infty} P_\zeta(j, n|N) \frac{e^{-\alpha(j)} [\alpha(j)]^N}{N!}. \quad (12)$$

If we assume that the hits on the building envelope are uniformly distributed (as in [1]), and denoting by q_j the vulnerable fraction (the ratio of the vulnerable area to the area of the building envelope), then $P_\zeta(j, n|N)$ can be approximated by the binomial distribution,

$$P_\zeta(j, n|N) = \binom{N}{n} q_j^n (1 - q_j)^{N-n}. \quad (13)$$

Substituting Eq. (13) into Eq. (12), we obtain the following expression for the probability of debris damage to a house:

$$P_D(j) = 1 - e^{-q_j \alpha(j)}, \quad (14)$$

which depends on its vulnerable fraction and mean number of over-threshold impacts.

3. Discretization and matrix formulation

In practice, if the study region is relatively large compared to an individual building’s plan area and the continuous probability density of debris landing positions varies negligibly within the typical area size of the building, it can be assumed that the value of $\mu_{s,i}$ is constant within the area occupied by each house (A_j). This assumption generally holds because (1) the debris problem often involves a dense residential development, with the study region defined to include all

the buildings (so that it is, by definition, isolated from the outside in terms of debris effect), and (2) the distribution of debris landing locations is likely to be widely spread out in space, due to turbulence effects and other complex factors, e.g., imperfect debris-item shapes owing to highly variable pressure damage. Consequently, the calculation of $\alpha(j)$, the only quantity required to estimate the probability of debris damage using Eq. (14), can usually be simplified.

Denoting by $\mu_{s,i}(j)$ the value of the distribution $\mu_{s,i}$ at the center point (defined in terms of longitude and latitude) of house j and by $v_{s,i}(j)$ the mean number of hits, the integral in Eq. (8) may be approximated as a summation:

$$v(j) = \sum_{s=1}^S \sum_{i=1}^I v_{s,i}(j) = \sum_{s=1}^S \sum_{i=1}^I \lambda_{s,i} \mu_{s,i}(j) A_j. \quad (15)$$

Similarly, house locations may also be approximated by their center points so that the horizontal impact momentum is only conditioned on the distances between the center points of the source house and the target house. Let $\phi_{s,i}(dy|j)$ be the conditional distribution of the horizontal momentum of type s debris generated from house i , when hitting house j . Eq. (9) for the mean total number of over-threshold impacts becomes:

$$\begin{aligned}\alpha(j) &= \sum_{s=1}^S \sum_{i=1}^I \alpha_{s,i}(j) \\ &= \sum_{s=1}^S \sum_{i=1}^I \lambda_{s,i} \mu_{s,i}(j) A_j \Phi_{s,i}(y > \zeta_j | j). \quad (16)\end{aligned}$$

It is worth mentioning that the scale of debris flight distances in the relatively large study region is different in concept from the scale of debris impact on the vulnerable area of a particular building. A house may be approximated as a point when evaluating the probability of debris hits and the possible impact intensities, neglecting the effect of house dimensions in order to simplify the calculation. However, the specific debris impact location on the building envelope matters, since the debris can only cause damage when it hits the vulnerable area with an over-threshold momentum, as is specifically considered in Eq. (11).

Adopting the above approximations, a matrix presentation can be formulated. In particular, the square matrix $(O_s)_{I \times I}$ of the mean number of impacts by type s debris (i.e., the matrix element $(O_s)_{i,j} = v_{s,i}(j)$ is the mean number of hits on house j by type s debris generated from house i) may be expressed by matrix multiplication in terms of the diagonal matrix $(A_s)_{I \times I}$ with diagonal elements $(A_s)_{i,i} = \lambda_{s,i}$, the square matrix $(\Theta_s)_{I \times I}$ with elements $(\Theta_s)_{i,j} = \mu_{s,i}(j)$, and the diagonal matrix $(A)_{I \times I}$ with diagonal elements $(A)_{j,j} = A_j$, as follows:

$$O_s = A_s \Theta_s A. \quad (17)$$

Also, the square matrix $(\Delta_s)_{I \times I}$ of the mean number of over-threshold impacts by type s debris, with elements $(\Delta_s)_{i,j} = \alpha_{s,i}(j)$, may be obtained through element-by-element multiplication of $(O_s)_{I \times I}$ and $(\Phi_s)_{I \times I}$, where $(\Phi_s)_{I \times I}$

is the square matrix with elements $(\Phi_s)_{i,j} = \Phi_{s,i} (y > \zeta_j | j)$:

$$\Delta_s = O_s \cdot \Phi_s. \quad (18)$$

Summing up the matrices for all types of debris, we obtain the matrix of the mean total number of impacts, denoted by $(O)_{I \times I}$, and the matrix of the total number of over-threshold impacts, denoted as $(\Delta)_{I \times I}$, respectively:

$$O = \sum_{s=1}^S O_s, \quad (19)$$

and

$$\Delta = \sum_{s=1}^S \Delta_s. \quad (20)$$

Finally, the value of $v(j)$ is the sum of the elements in the j th column of the matrix O , and the value of $\alpha(j)$ equals the sum of the elements in the j th column of the matrix Δ .

4. Probabilistic modeling of debris trajectories

The aim of the stochastic debris trajectory model is to obtain the probability distribution of debris landing positions, $\mu_{s,i}$, and the conditional distribution of the horizontal momentum, $\phi_{s,i}$, for each type of debris generated from each house, under prescribed wind conditions (including mean wind speed and direction). Although investigations on building damage have been carried out after each significant storm, historical information on debris flight behavior is generally not available. Therefore, the probability distributions of debris trajectory parameters may have to be established based on experiments and/or numerical simulations, and with some help from intuition and common sense, considering the complexity of the problem in real situations.

We model the probabilistic distribution of the landing position $\mu_{s,i}$ as 2D Gaussian. This is motivated by the Tachikawa's [7] observation in wind-tunnel experiments on debris trajectories. He placed a catch-net perpendicular to the direction of the wind at various distances in front of the debris original position and found that debris impact locations were almost always (approximately) uniformly distributed within circles in the center of the net, suggesting the Gaussian distribution to be a good candidate, at least for an initial model. Given a mean wind speed and direction, we first rotate the geographical x - z coordinate system (x_j -longitude and z_j -latitude of house j , the effect of earth's curvature being neglected) to a new x' - z' system such that the wind direction is in the positive x' direction. Then assume that debris will most likely fly in (or close to) the mean wind direction and thus the mode (and mean) of the 2D Gaussian distribution of debris landing is in the positive x' direction at the most likely debris flight distance (at landing), denoted as $d_{s,i}$, from its original position (house i). Thus we express the Gaussian distribution of debris landing as

$$\mu_{s,i}(j) = \frac{1}{2\pi\sigma_{s,i}^{x'}\sigma_{s,i}^{z'}} e^{-\left[\frac{(x'_j - x'_i - d_{s,i})^2}{2(\sigma_{s,i}^{x'})^2} + \frac{(z'_j - z'_i)^2}{2(\sigma_{s,i}^{z'})^2}\right]}. \quad (21)$$

We may assume that the values of the standard deviations, $\sigma_{s,i}^{x'}$ and $\sigma_{s,i}^{z'}$, equal some fraction of $d_{s,i}$. For example, in case $\sigma_{s,i}^{x'} = (1/3)d_{s,i}$ and $\sigma_{s,i}^{z'} = (1/12)d_{s,i}$, the debris item is estimated to have the probability 0.9919 of landing within a rectangular area centered at its most likely landing point, and with the length of the longer side equal to $2d_{s,i}$ and the length of the shorter side equal to $(1/2)d_{s,i}$. Note that it is possible that a debris item will hit its source building or fly in the direction opposite to the mean wind, due to local turbulence effects. Although intuitively negligible, this possibility is allowed for in the Gaussian model (and has a probability less than 0.002 in this example). Note that, in reality, the target house that is located slightly closer than the distance $d_{s,i}$ to the source house may have the largest probability of its envelope being hit, due to the effects of house dimensions. In this context, we suggest replacing $d_{s,i}$ in Eq. (21) with $(d_{s,i} - h/2)$, where h is the typical height of house eaves in the region under study, so that $\mu_{s,i}$ will more realistically represent the probability distribution of houses being hit.

The Gaussian distribution could also be adopted, mainly for analytical convenience, for the conditional probability distribution $\phi_{s,i}$ of debris horizontal impact momentum, although the preferred stochastic model is the lognormal distribution (as lognormal random variables are non-negative). The random variable $Y_{s,i|j} = m_{s,i}u_{s,i|j}$ is the horizontal momentum of type s debris generated from house i when hitting house j ; its parameters are the mean $\bar{Y}_{s,i|j} = m_{s,i}\bar{u}_{s,i|j}$ and standard deviation $\sigma_{s,i|j}^Y = m_{s,i}\sigma_{s,i|j}^u$, where $\bar{u}_{s,i|j}$ and $\sigma_{s,i|j}^u$ are the respective mean and standard deviation of the conditional distribution of the impact speed $u_{s,i|j}$. (Owing to current lack of data, we may again assume the value of $\sigma_{s,i|j}^u$ to be some fraction of $\bar{u}_{s,i|j}$, like $(1/6)\bar{u}_{s,i|j}$.) Other distributions, e.g., Weibull and truncated Gaussian, will be considered in further study.

Although statistical data on debris trajectories in real storms are not available at present, experiments and numerical simulations serve to estimate the parameters of the distributions $\mu_{s,i}$ and $\phi_{s,i}$. Lin et al. [4], Holmes et al. [6] and Lin et al. [5] conducted extensive wind-tunnel experiments and numerical simulations to study debris flight behavior in straight-line winds (as approximations to the winds in traditional boundary layers as well as in hurricanes). In particular, an empirical model of the debris horizontal trajectory was proposed by Lin et al. [4,5] for the three generic debris types classified by Wills et al. [8], namely: 'compact-like' (e.g., roof gravel), 'plate-like' (e.g., roof covers and sheathings), and 'rod-like' (e.g., 2×4 timbers). Simple empirical expressions were derived to estimate the most likely flight distance and horizontal impact speed for each debris type. These expressions are used in our study to rapidly estimate $d_{s,i}$ and $\bar{u}_{s,i|j}$ in the study region, for a given uniform mean wind velocity. We assume, in the current study, that the same types of debris generated from different houses have similar flight trajectories, so that $d_{s,i}$ has the same statistical properties for different source houses generating the same types of debris. The effects of geometric characteristics of the source house (i.e., height and roof configuration) on

the debris trajectory were neglected. Modifications might be necessary if the model is used to study debris risk in a region with buildings of very different shapes. Key aspects of the empirical model for flight trajectories are summarized in the [Appendix](#).

Further experimental and numerical studies on debris trajectories may, of course, yield finer classifications and improved empirical expressions. The above-mentioned probability distributions (Gaussian and lognormal) of the debris landing position and impact strength also need further validation and likely modification, based on new data about debris trajectories. It is notable, however, that the Poisson-model-based approach developed in Sections 2 and 3 would apply regardless of the types of probability distributions adopted for the trajectory parameters.

5. Application to hurricane damage risk analysis

The proposed debris model may be incorporated into hurricane risk analyses. The wind condition can be characterized in terms of a mean (or gust) wind velocity around each source house, but it is generally thought sufficient, and common in practice, to assume a uniform wind velocity over the whole study region; the local turbulence effects may, after all, already have been partially incorporated into the probability distributions. The model parameters needed, as input, for each house are house longitude x and latitude z , house area A , the impact resistance threshold ζ , the vulnerable fraction q , and the mean number of each type of generated debris λ , for the given wind condition. Locations and areas of residential houses are usually available as deterministic data. The values of ζ and q may perhaps be treated as random variables, based on information about the building stock in the study area, if site-specific data are not available. The types and characteristics of house-source debris may likewise be assigned at random, based on information on local building material supplies. The damage to house components that could become debris may be estimated, according to the structural vulnerability studies in the literature. The mean percentage of damage for each component, and thus the value λ for each type of debris from each house, can then be derived. Monte Carlo simulation then may be carried out to obtain estimates of the debris-damage risk corresponding to a given wind velocity.

We demonstrate this approach by an example. The site chosen is in Brevard County, Florida, with 2200 houses clustered in a relatively separated and independent development. We studied the probability of damage on house windows by roof covers and sheathings; $S = S_i = 2$ in this case. The mean wind speed used was 49 m/s (U.S. 1-min average wind strength of a Saffir–Simpson Category 2 Hurricane), blowing from North and constant over the study region. Data on house locations and areas were obtained from Brevard County's Property Appraiser's Office. The type of cover materials (e.g., shingles or tiles) was randomly assigned to each house, based on the information about the local building stock. The characteristic data of 45 types of shingles, 31 types of tiles, and 11 types of sheathings were collected from local

construction material manufacturers. The types of cover and sheathing debris were then uniformly assigned to each house. The number of covers and sheathings on each house were estimated, based on the structural area and roof cover density. The mean percentage of damage to roof covers was estimated to be 5% and to roof sheathings 1%, for the wind speed in this example, based on the simulation results from a component-based pressure-induced damage model (IHRC and FIU [9]). Noting that some damaged materials may remain attached to the roofs and not fly, we assumed that half of the damaged roof covers and sheathings became windborne debris. (Debris generation will be the subject of future study). The impact resistance capacity of a typical glass window for each house was randomly assigned, assuming a Gaussian distribution with mean 0.025 kg m/s and standard deviation 0.0025 kg m/s. The window fraction for each house was uniformly assigned a number between 0.1 and 0.2. The most likely flight distance and impact speed were estimated using the empirical expressions in Lin et al. [4] (included in the [Appendix](#)) for plate-type debris. Monte Carlo simulation was carried out to estimate the debris risk for this region. On the 3D map (of a part of the study region) shown in [Fig. 1](#), the height of the bar at each house location represents the estimated number of debris impacts (1a) and the probability of window damage (1b) of the house, with the values varying from house to house. Although most houses are expected to be hit by less than 10 debris items, some houses that are located in the middle of a relatively dense area may suffer many more impacts than others. This can also be seen from the histogram, for all houses in the study region, of the mean number of impacts in [Fig. 2\(a\)](#). The histogram of the probability of damage in [Fig. 2\(b\)](#) is much more evenly distributed among the houses, since it also involves the effect of the impact momentum.

In strong and long-lasting hurricane winds, debris penetration not only damages building envelopes, but also induces internal pressurization, approximately doubling the net loading on roofs. Consequently, failed roofing structures become new debris sources, starting a 'chain reaction' of failures. The proposed debris-damage risk model can be used in conjunction with a pressure-induced damage model (see, e.g., IHRC and FIU [9] and Vickery et al. [10]) to estimate the cumulative structural damage in such situations. The model of pressure-induced damage, which generates debris, and the model of debris damage, which affects internal pressure (causing more damage and debris), may be interactively applied to a given residential area. Given a storm time-history (expressed in terms of evolving temporal-mean wind speeds, e.g., 3 s gust, and directions), the probability of debris-induced damage can be computed for each house at every time step (e.g., at 15 min intervals), preceded by calculations of pressure damage, and the effect of the debris damage can be simulated by updating the building characteristics (as well as the internal pressure) at the next time step. The debris model presented herein lends itself to fast and efficient computation, since it involves only explicit calculations and matrix operations, and the house parameters (e.g., impact resistant capacity, vulnerable fraction, and roof types) need to

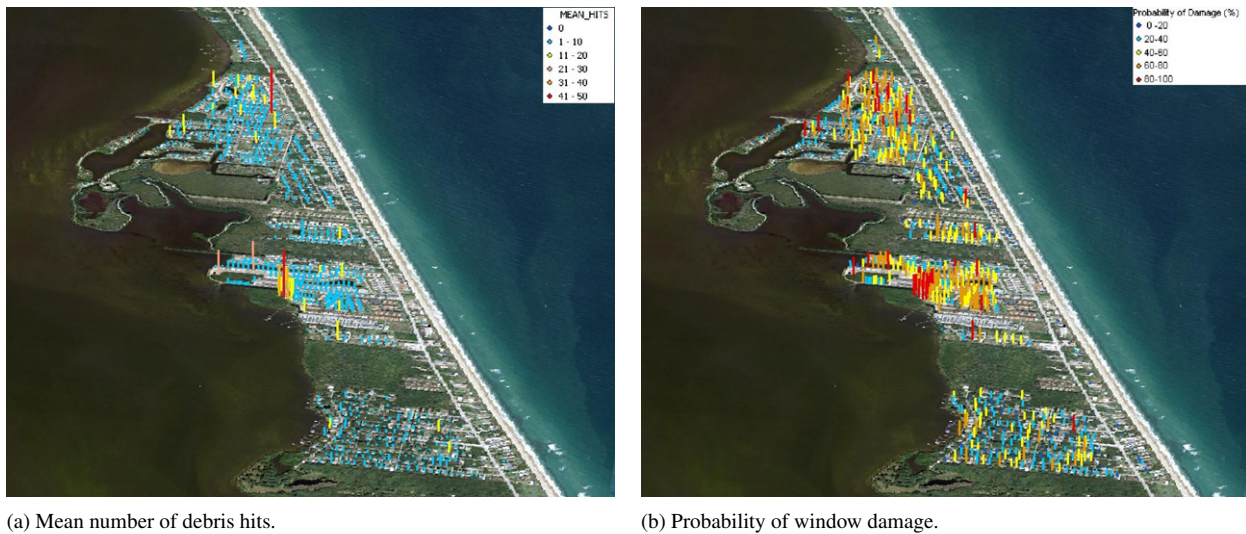


Fig. 1. Analysis of debris-damage risk for a residential area in Brevard County, Florida.

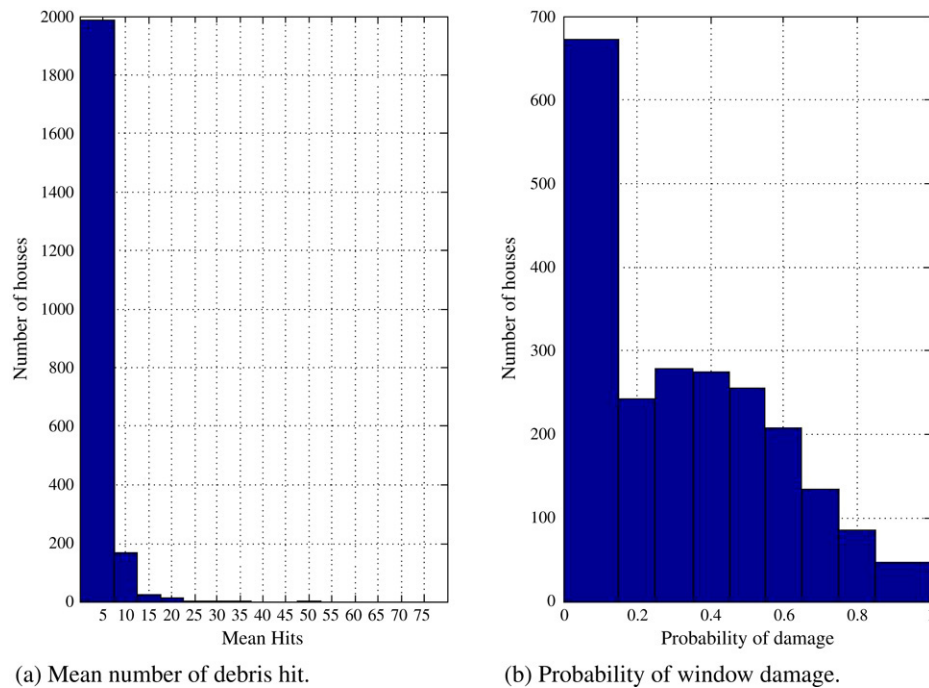


Fig. 2. Histograms of debris-damage risk for a residential area in Brevard County, Florida.

be assigned only once (by means of Monte Carlo simulation) for a storm time-history.

6. Conclusions and comments on further work

A debris-damage model, intended for incorporation into more comprehensive hurricane risk analysis, is proposed, based on Poisson random measure theory. Combined with new-technology tools such as GIS and Google Earth, this model can be applied to the study of debris-damage risk (under strong wind conditions typifying a hurricane) to a particular house, a residential development, a postal-code region or some other jurisdictional area.

Further research is needed to validate and further improve the stochastic debris trajectory model. Whereas mutual statistical independence of the random variables involved has been assumed, alternate treatments of statistical dependence are under investigation. Also, a stochastic model of clustered debris generation is expected to be developed as an extension of this study.

Acknowledgments

The research reported herein is supported by a grant from Baseline Management Corporation (BMC) to Princeton University. The first author gratefully acknowledges the instruction and advice on Poisson modeling from Professor

Erhan Cinlar of the Department of Operations Research and Financial Engineering at Princeton University.

Appendix

This appendix concerns the empirical model of horizontal debris trajectory developed by Lin et al. [4,5], and its use in stochastic debris trajectory modeling. Extensive simulations of trajectories of three typical types of debris were carried out in the Texas Tech University wind tunnel, under a wide range of wind speeds and experimental settings. Debris type is mainly defined by the debris shape and characteristic dimension, as these govern the debris aerodynamics. Although the trajectories of a certain type of debris showed great variation in the vertical direction, their horizontal components showed definite patterns, dependent mainly on a non-dimensional parameter, later called the Tachikawa Number by Holmes et al. [11]. Based on this observation, Lin et al. [4] established empirical expressions, in dimensionless form, to describe the horizontal trajectory of plate-type debris, and Lin et al. [5] developed similar expressions for compact-type (i.e., cubes and spheres) and rod-type debris. Numerical simulations by Holmes et al. [6] and Lin et al. [5] showed, in general, satisfactory agreement with the experimental data and this explicit model.

The basic dimensionless variables used in the expressions given below are:

$$\begin{aligned} x^* &= \frac{gx}{U^2} \quad (\text{dimensionless horizontal displacement}), \\ t^* &= \frac{gt}{U} \quad (\text{dimensionless time}), \\ u^* &= \frac{u}{U} \quad (\text{dimensionless horizontal debris velocity}), \\ K &= \frac{\rho_a U^2}{2gh_m \rho_m} \quad (\text{Tachikawa Number}), \end{aligned} \quad (\text{A.1})$$

where U is the mean wind speed, u is the debris horizontal flight speed, x is the flight distance, t is time, h_m is the debris thickness, ρ_m is the debris density, ρ_a is the air density, and g is the acceleration of gravity.

The horizontal flight distance was expressed as a function of flight time as follows:

$$Kx^* = \frac{1}{2}C(Kt^*)^2 + a(Kt^*)^3 + b(Kt^*)^4 + c(Kt^*)^5 + \dots \quad (\text{A.2})$$

where the values of the coefficients (C, a, b, c, \dots) depend on the shape of the debris. Least-square fitting of large experimental data sets yields these empirical equations, for plates,

$$\begin{aligned} Kx^* &\approx 0.456(Kt^*)^2 - 0.148(Kt^*)^3 \\ &\quad + 0.024(Kt^*)^4 - 0.0014(Kt^*)^5; \end{aligned} \quad (\text{A.3})$$

for cubes,

$$\begin{aligned} Kx^* &\approx 0.405(Kt^*)^2 - 0.036(Kt^*)^3 \\ &\quad - 0.052(Kt^*)^4 + 0.008(Kt^*)^5; \end{aligned} \quad (\text{A.4})$$

for spheres,

$$\begin{aligned} Kx^* &\approx 0.248(Kt^*)^2 + 0.084(Kt^*)^3 \\ &\quad - 0.1(Kt^*)^4 + 0.006(Kt^*)^5; \end{aligned} \quad (\text{A.5})$$

and for rods,

$$\begin{aligned} Kx^* &\approx 0.4005(Kt^*)^2 - 0.16(Kt^*)^3 \\ &\quad + 0.036(Kt^*)^4 - 0.0032(Kt^*)^5. \end{aligned} \quad (\text{A.6})$$

The horizontal velocity is expressed as follows as a function of flight distance:

$$u^* = 1 - e^{-\sqrt{2CKx^*}}, \quad (\text{A.7})$$

with the value of coefficient C the same as in Eq. (A.2), namely, $C = 0.911$ for plates, 0.809 for cubes, 0.496 for spheres, and 0.801 for rods.

In modeling stochastic debris trajectories (Section 4) in residential areas, the most likely flight distance ($d_{s,i}$) may be estimated from Eq. (A.3) for roof tiles, shingles and sheathings; from Eq. (A.4) or (A.5) for roof gravel; and from Eq. (A.6) for 2×4 timbers. The flight time of an item of debris (until it hits the ground) is observed in the experiments to be a function of the debris vertical trajectory, and it depends on the height of the source building and the debris initial condition on the roof (e.g., initial support configuration, initial angle of attack, etc.). Thus, logically, the probability distribution of $d_{s,i}$ varies with both debris type and the characteristics of the source building, as is also suggested by the subscripts. However, establishing the specific distribution of $d_{s,i}$ for each building may be too expensive in practice, and currently unsupported by available data. In studies of residential areas containing mostly similar houses, we assume that the most likely flight time is 2 s, with a standard deviation of 0.4 s, for every building and debris type. This assumption is based on observations from damage surveys and full-scale experiments (see Lin et al. [4]) that debris items generally fly in the air for 1–3 s. The flight time distribution may be assumed to be truncated Gaussian or lognormal. The flight time statistics could also be made to depend on debris types and house types (e.g., characterized by roof shapes, number of stories) in future studies of windborne debris risk.

The most likely horizontal impact speed $\bar{u}_{s,i|j}$ can be estimated for each type of debris by Eq. (A.7), for each pair of source (i) and target (j) houses in the study region, by substituting the flight distance variable with the physical distance between the two houses.

References

- [1] Twisdale LA, Vickery PJ, Steckley AC. Analysis of hurricane windborne debris risk for residential structures. Raleigh (NC): Applied Research Associates Inc.; 1996.
- [2] ASTM Standard E1886-05. Standard test method for performance of exterior windows, curtain walls, doors, and storm shutters impacted by missile(s) and exposed to cyclic pressure differentials. West Conshohocken (PA): American Society for Testing and Materials Inc.; 2005.
- [3] Cinlar E. Lecture notes in probability and stochastics. Princeton (NJ): Princeton University; 2008.

- [4] Lin N, Letchford CW, Holmes JD. Investigations of plate-type windborne debris. Part I. Experiments in wind tunnel and full scale. *J Wind Eng Ind Aerodyn* 2006;94(2):51–76.
- [5] Lin N, Holmes JD, Letchford CW. Trajectories of windborne debris and applications to impact testing. *ASCE J Struct Engrg* 2007;133(2):274–82.
- [6] Holmes JD, Letchford CW, Lin N. Investigations of plate-type windborne debris. II. Computed trajectories. *J Wind Eng Ind Aerodyn* 2006;94(1): 21–39.
- [7] Tachikawa M. A method for estimating the distribution range of trajectories of wind-borne missiles. *J Wind Eng Ind Aerodyn* 1988; 29(1–3):175–84.
- [8] Wills JAB, Lee BE, Wyatt TA. A model of wind-borne debris damage. *J Wind Eng Ind Aerodyn* 2002;90(4–5):555–65.
- [9] International Hurricane Research Center (IHRC) and Florida International University (FIU). Florida Public Hurricane Loss Projection Model. Engineering team final report. March 2005.
<http://www.cis.fiu.edu/hurricaneloss/html/research001.html>.
- [10] Vickery PJ, Skerlj PF, Lin J, Twisdale Jr LA, Young MA, Lavelle FM. HAZUS-MH Hurricane model methodology. ii: Damage and loss estimation. *Natural Hazards Rev* 2006;7(2):94–103.
- [11] Holmes JD, Baker CJ, Tamura Y. The Tachikawa number: A proposal. *J Wind Eng Ind Aerodyn* 2006;94(1):41–7.