# Data Analytics and Geostatistics:
# Spatial Estimation

**Lecture outline . . .**

- **Variogram Interpretation**
- **Variogram Modeling**


Introduction

Modeling Prerequisites

Spatial Estimation

- Stationarity and Trends
- Spatial Continuity Calculation
- Spatial Continuity Modeling
- Spatial Continuity Estimation

Spatial Uncertainty

Multivariate, Spatial

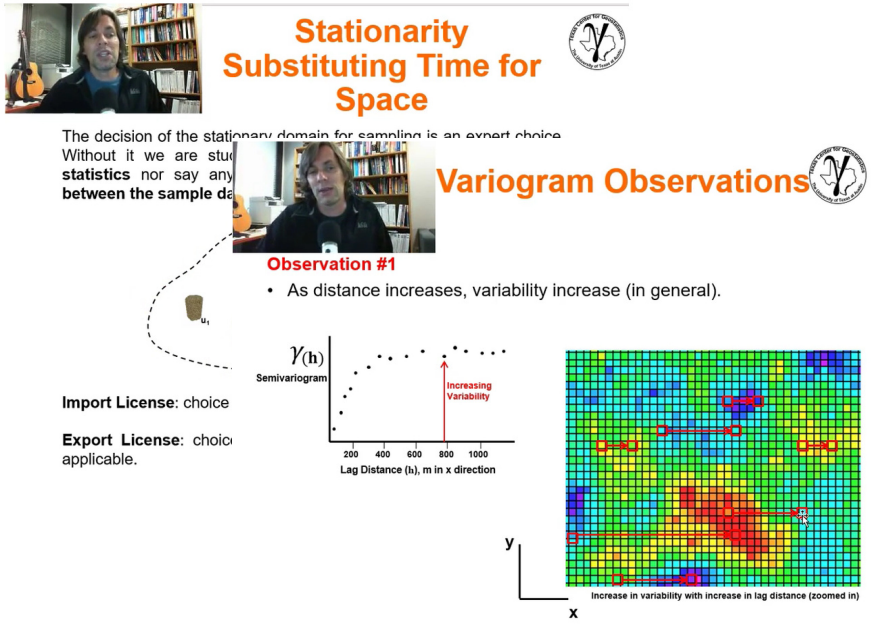
Novel Workflows

Conclusions

# Data Analytics and Geostatistics:
# Spatial Estimation

## Other Resources:

- **Lectures recorded on YouTube.**




Stationarity, spatial calculation and modeling on YouTube.

- **Worked out examples on GitHub in Excel and Python**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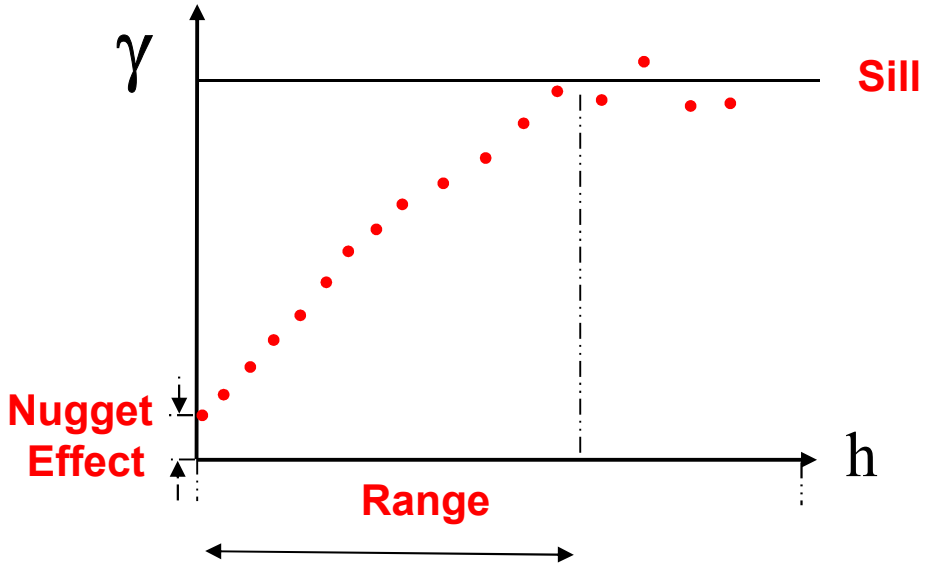
# Variogram Terminology

- The ***sill*** is the variance of the data used for variogram calculation (1.0 if the data are normal scores)
- The ***range*** is the distance at which the variogram reaches the sill
- The ***nugget effect*** is the behavior at distances less than the smallest experimental lag:

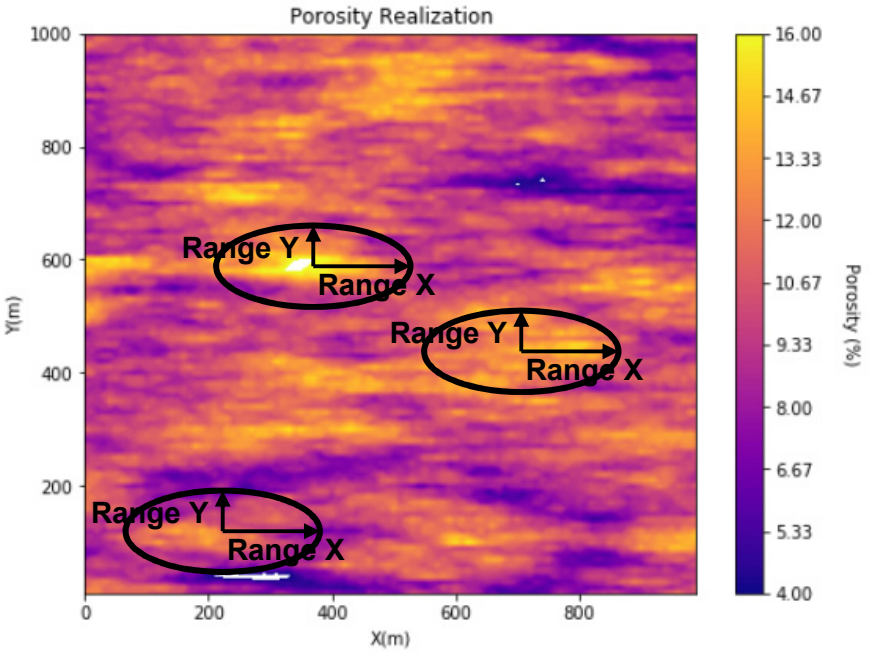
geological microstructure + measurement error

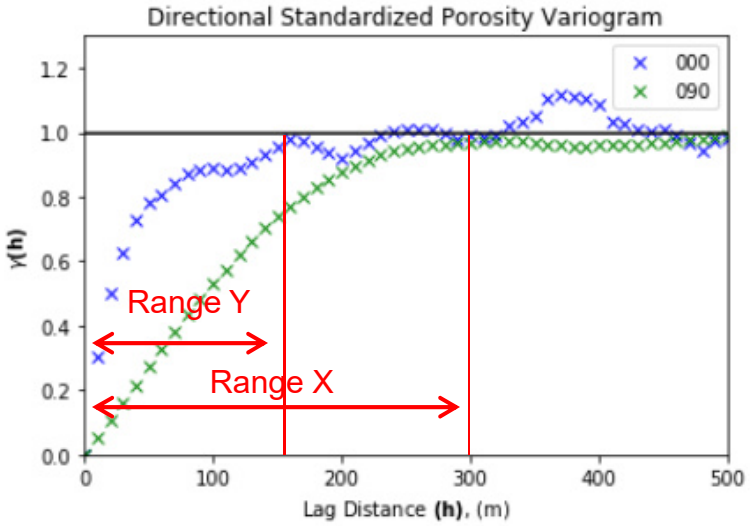
- Any error in the measurement value or the location assigned to the measurement translates to a higher nugget effect
- Sparse data may also lead to a higher than expected nugget effect



# Geometric Anisotropy

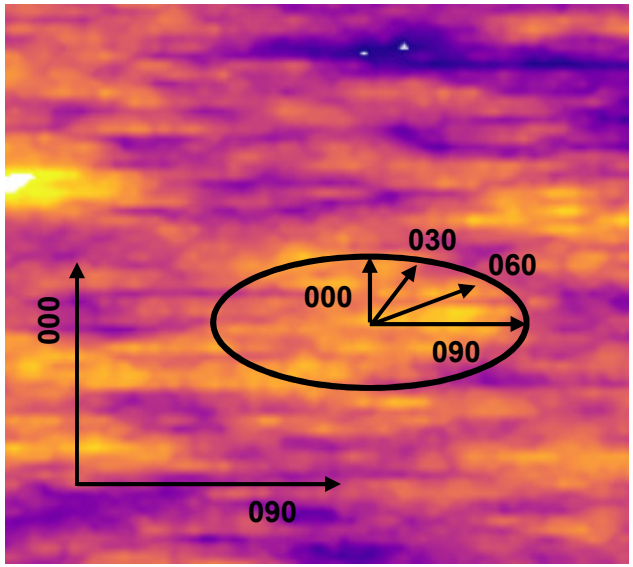
- The range of correlation / variogram range **depends on direction**
- Commonly, the vertical range of correlation is much less than horizontal range due to larger lateral distance during deposition.
- The ratio of the **horizontal: vertical range** is commonly known as the horizontal to vertical anisotropy ratio
- Geometric anisotropy is common in horizontal direction
- The ratio of horizontal **major direction: horizontal minor direction range** is commonly known as the horizontal major to minor anisotropy ratio

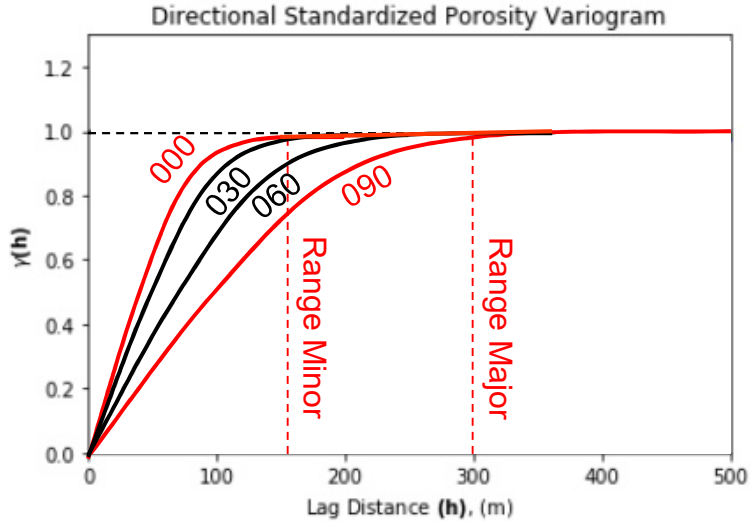


# Geometric Anisotropy

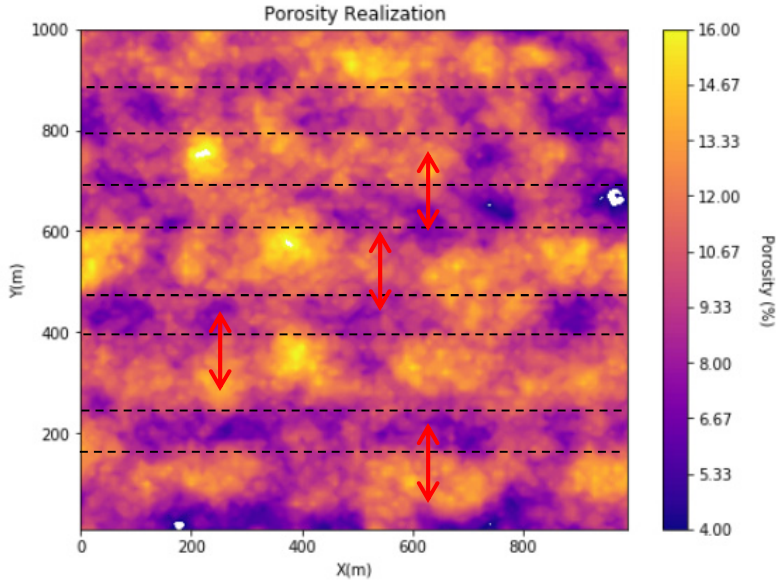
- We assume geometric anisotropy to model 2d and 3D variogram from experimental variograms calculated in primary directions.
- This model provides a valid interpolation of the variogram between the primary directions.
- When we start modeling this is critical to build nested variogram models with structures that:
  - Describe components of the variance
  - Act over all directions.

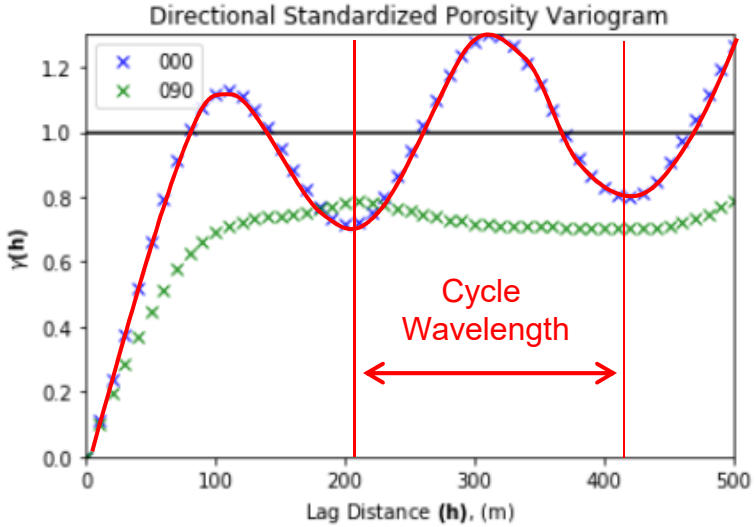


# Cyclicity

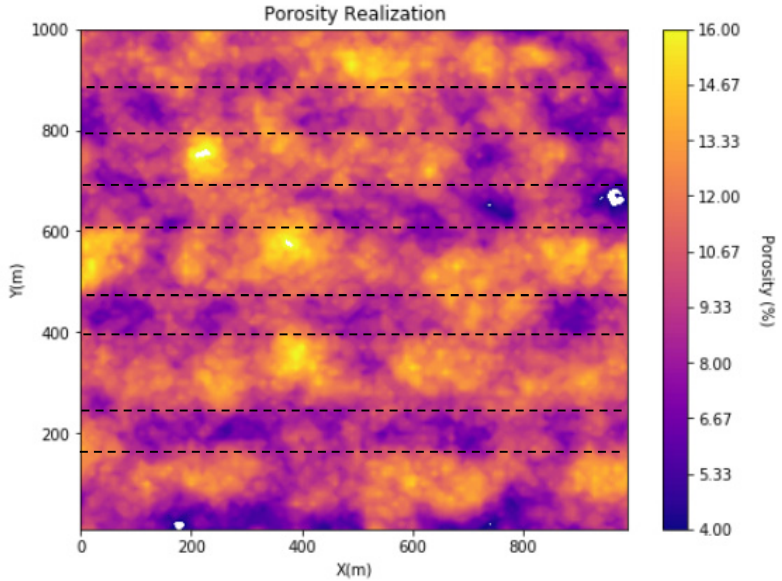
- Cyclicity may be linked to underlying geological periodicity, cycles in the deposition
- Sometimes noise in the experimental variogram due to too few data is mistaken as cyclicity
- The wavelength of the cycles in the experimental variogram is the wavelength of the spatial cycles.

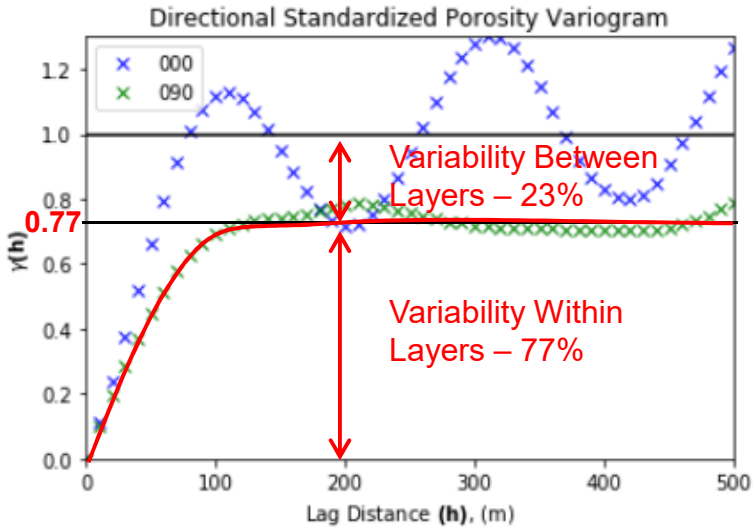


# Zonal Anisotropy

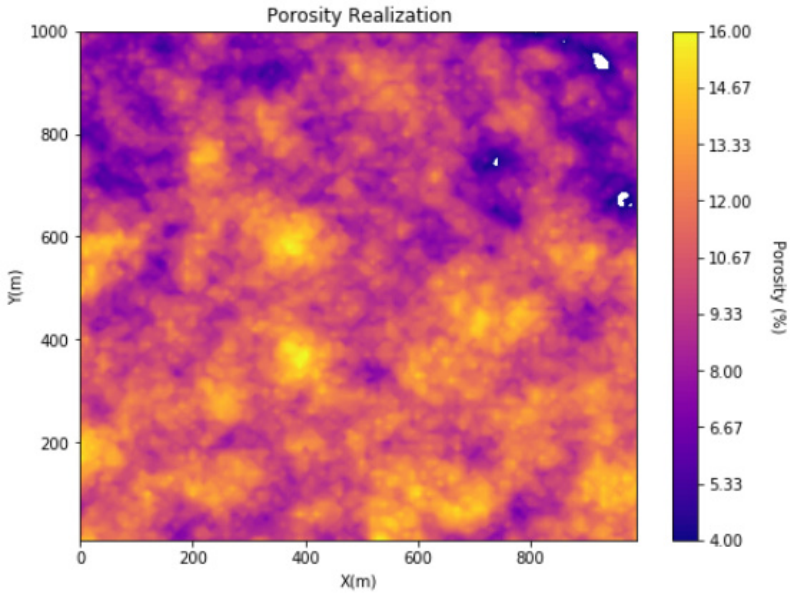
- When the experimental variogram does not reach the sill in a direction
- Often paired with cyclicity or trend in the other (orthogonal) direction.
- The variance at which the variogram levels off is called an apparent sill.

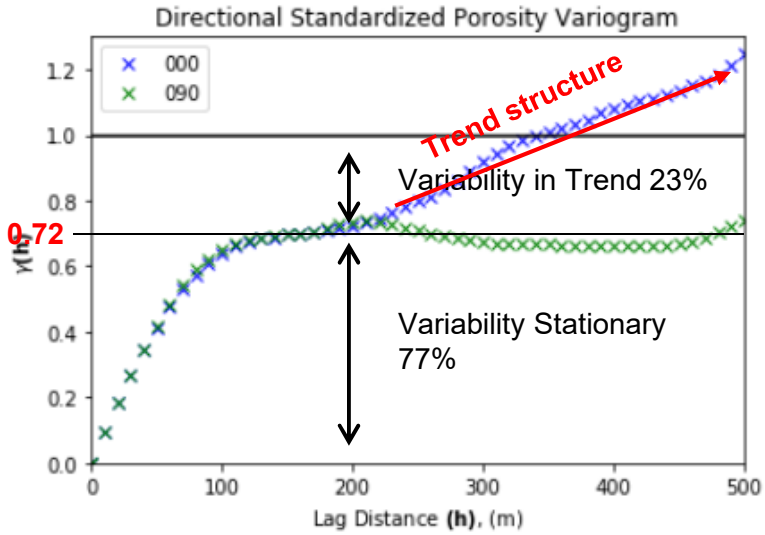


# Trend

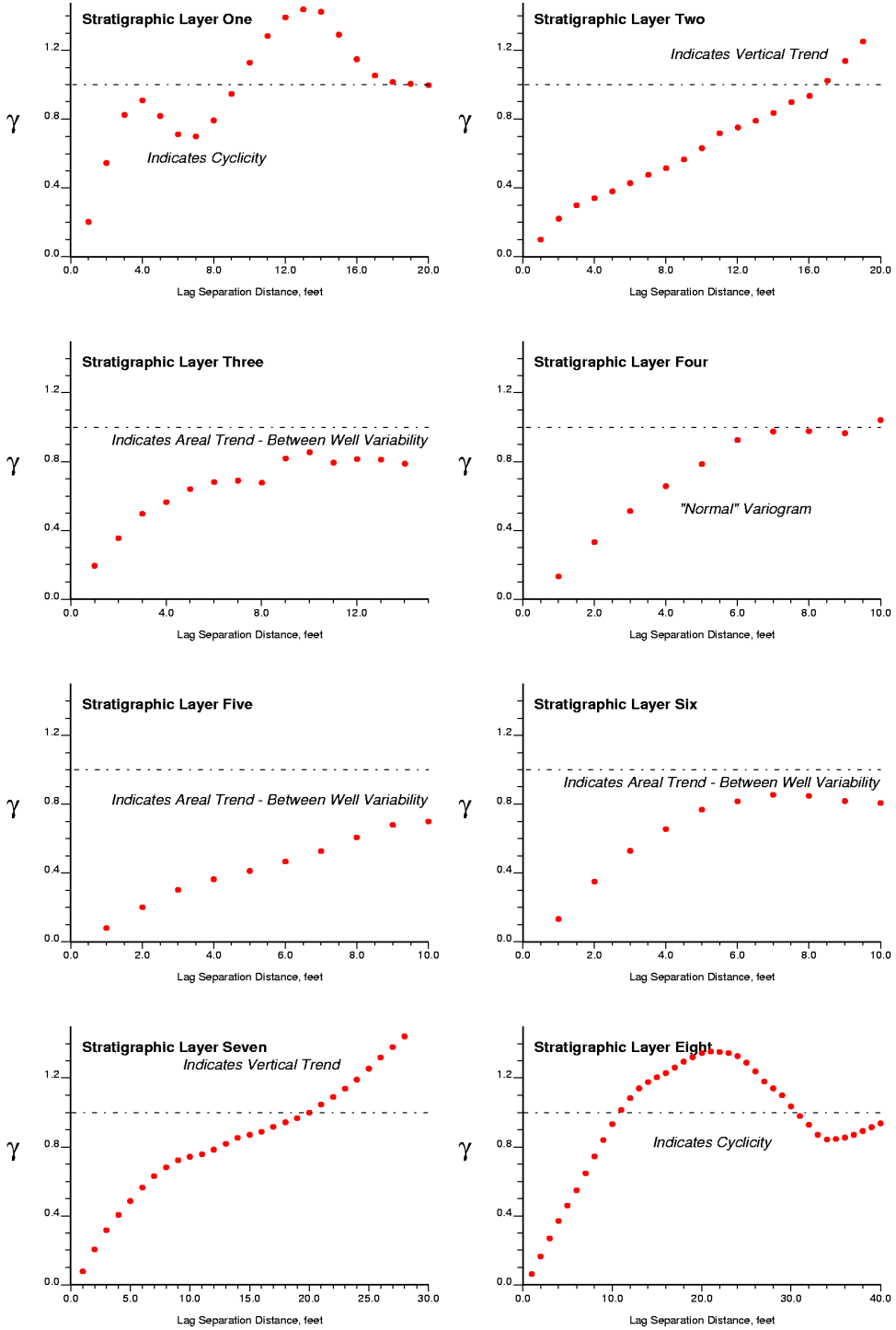
- Indicates a trend (fining upward, compacting with depth etc.)
- Could be interpreted as a fractal, fit a power law function
- May have to explicitly account for the trend in later simulation/modeling
  - Model, remove trend, work with the residual
  - If trend is removed the residual variogram will plateau at the sill



# Some Experimental Variograms

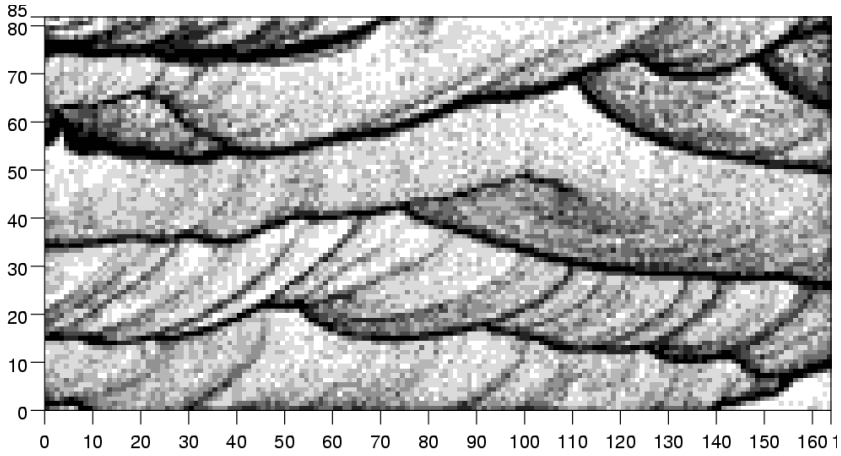
- Superposition of the four interpretation principles
  - Trend
  - Cyclicity
  - Zonal anisotropy
  - Geometric anisotropy
- Very few variograms are ‘textbook variograms’, natural settings a complicated!

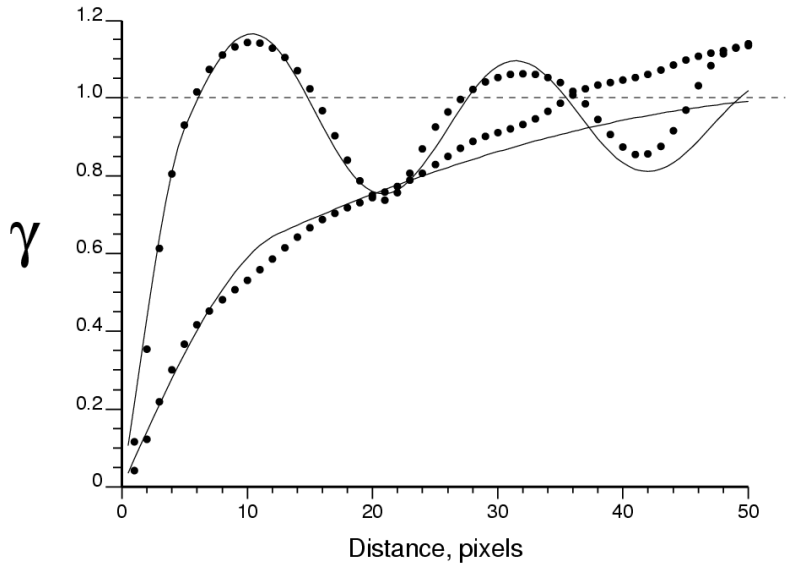



Variogram examples from Pyrcz and Deutsch, 2014.

# Variogram Interpretation

- Variogram interpretation consists of explaining the variability over different distance scales.
- The variogram is a chart of *variance versus distance* or *geological variability versus direction and Euclidean distance.*
- Establish the reasonableness of the variogram…




Slide from Clayton Deutsch's MinE 612 Class

# Review of Variogram Interpretation

- Variogram is very important in geostatistical study; measure of geological distance
- Initial coordinate and data transformation may be required.
- Interpretation Principles:
  - Trend
  - Cyclicity
  - Geometric Anisotropy
  - Zonal Anisotropy
- Short scale structure is most important
  - nugget due to measurement error should not be modeled
  - size of geological modeling cells
- Vertical direction is typically well informed
  - can have artifacts due to spacing of core data
  - handle vertical trends and areal variations
- Horizontal direction is not well informed
  - take from analog field or outcrop
  - typical horizontal vertical anisotropy ratios

Slide from Clayton Deutsch's MinE 612 Class

# Spoiler Alert

**We need to practically calculate and model spatial continuity. From the available and often sparse subsurface data.**

1. Calculate variogram with irregularly spaced data
   – Search templates with parameters

**Complete**

2. **Valid spatial model**
   – Fit with a couple different, nested (additive) spatial continuity models e.g. nugget, spherical, exponential and Gaussian

3. **Full 3D spatial continuity model**
   – Model primary directions, i.e. major horizontal, minor horizontal and vertical and combine together with assumption of geometric anisotropy

**Variogram Modeling**

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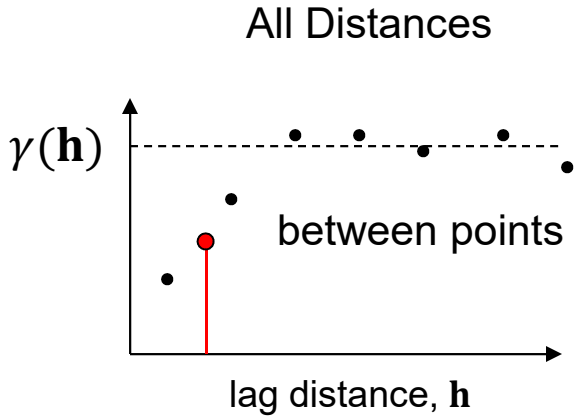
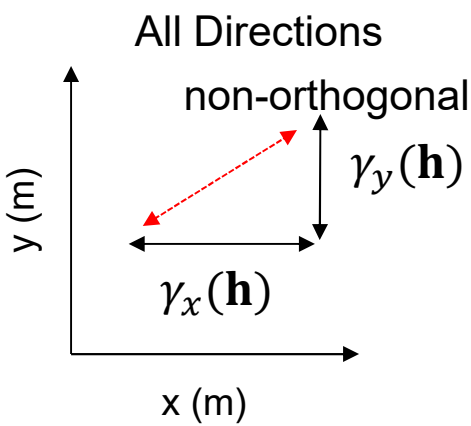
3. **Full 3D spatial continuity model**
   - Model primary directions, i.e. major horizontal, minor horizontal and vertical and combine together with assumption of geometric anisotropy

**Variogram Modeling**

# Reasons for Variogram Modeling

1. Need to know the variogram for *all* possible **h** lags, distances and directions – not just the ones calculated



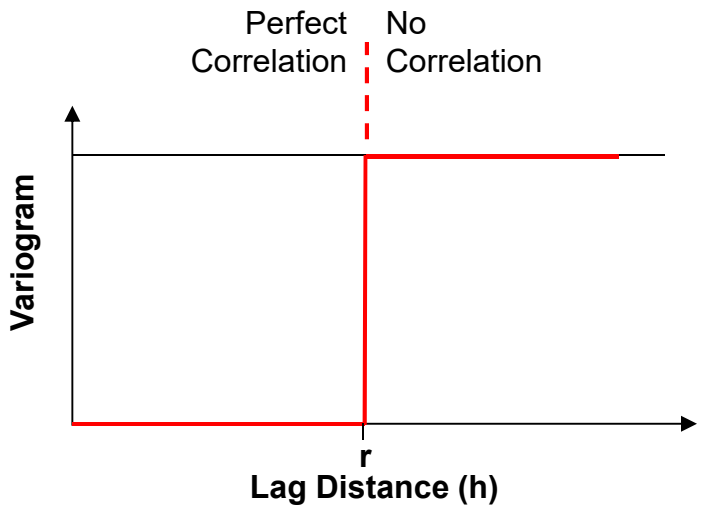

**a specific lag offset may not exist in our data!**

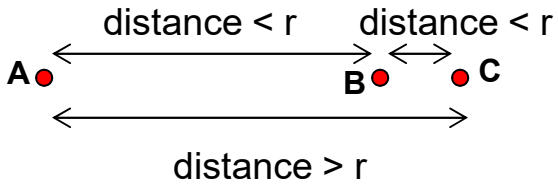
2. Incorporate additional geological knowledge (such as analogue information or information on directions of continuity …)
3. The variogram model must be **positive definite** (a legitimate measure of distance), that is, the variance of any linear combination must be positive

Slide modified from Clayton Deutsch's MinE 612 Class

# Reasons for Variogram Modeling

Extreme Example to Demonstrate the Need for Using Positive Definite Variogram




A and B and B and C are perfectly correlated,
but A and C are not correlated!

-Spatial paradox!

**Positive definite variogram models** ensure for all possible spatial configurations there are no paradoxes.

# Reasons for Variogram Modeling

- The variogram will be used in Kriging (next section)
- Kriging uses variogram (covariance) values in a linear system of equations
- Solves for estimate and estimation variance (uncertainty in the estimate)

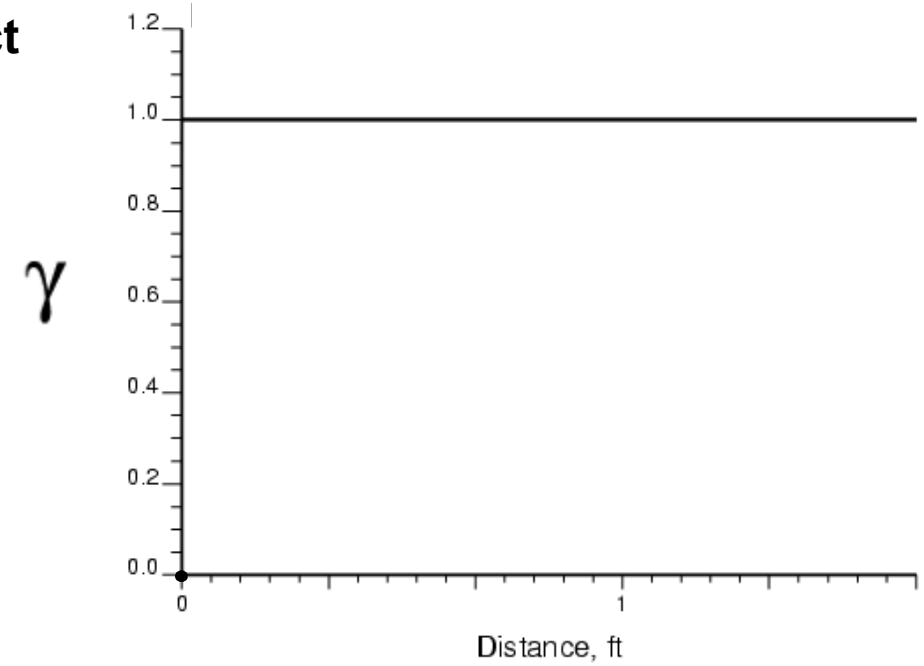
**estimation variance =** $\sigma^2_{x^*} = \sigma^2_x - \sum_{\alpha=1}^{n} \lambda_\alpha C_x(\mathbf{u}_\alpha - \mathbf{u}_0) \geq 0$

- Cannot just fit a smooth interpolation to the experimental variogram points.
  - It will result in unrealistic values for estimation variance

Slide modified from Clayton Deutsch's MinE 612 Class

# Common Variogram Models

**Nugget Effect**

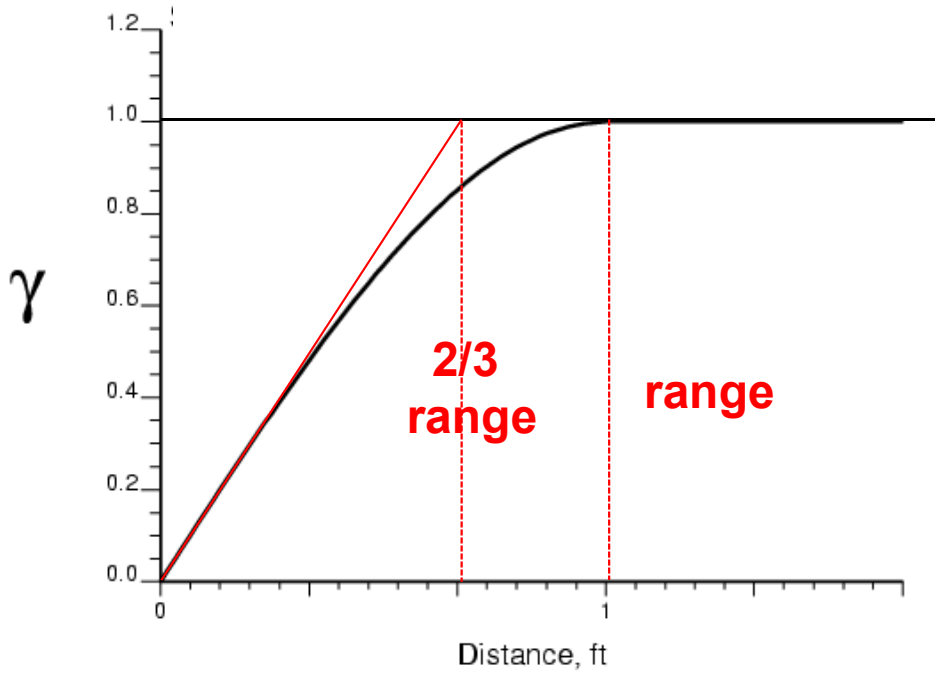



$$\gamma(\mathbf{h}) = C_1 \cdot Nugget = \begin{cases} 0 & ,if\ h = 0 \\ C_1 & ,if\ h > 0 \end{cases}$$

- No spatial correlation
- Should be a small component of the overall variance

# Common Variogram Models

**Spherical**



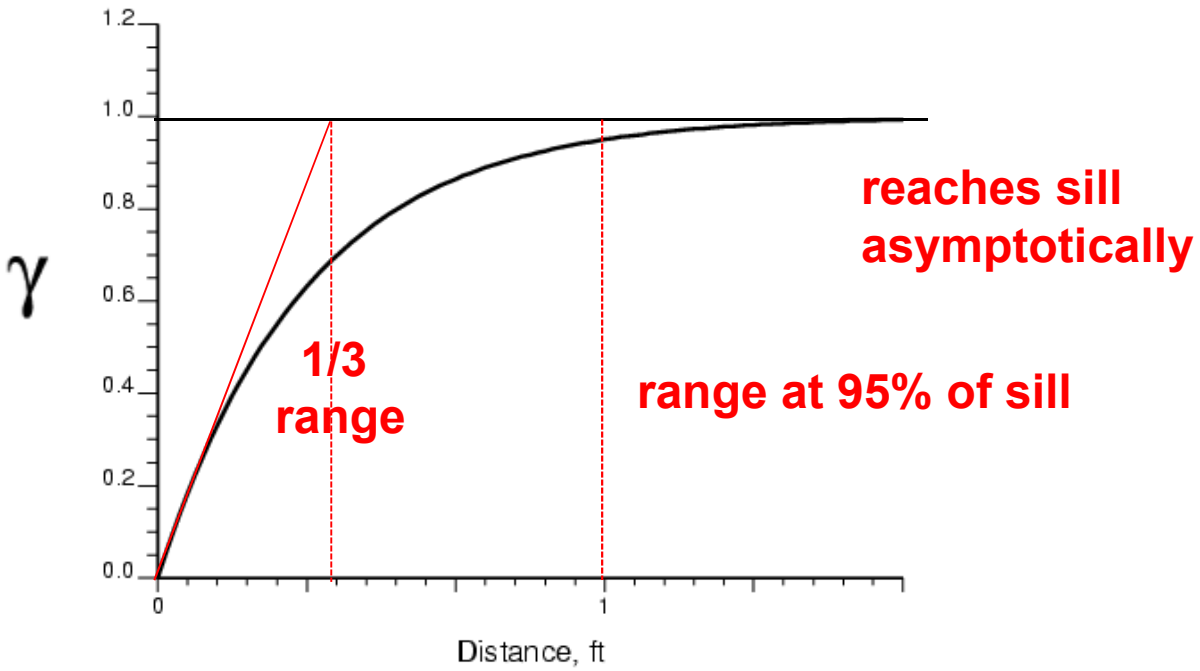

Commonly encountered variogram shape

$$\gamma(\mathbf{h}) = C_1 \cdot Sph\left(\frac{\mathbf{h}}{a}\right) = \begin{cases} C_1 \cdot \left[1.5\left(\frac{\mathbf{h}}{a}\right) - 0.5\left(\frac{\mathbf{h}}{a}\right)^3\right] & ,if\ h < a \\ C_1 & ,if\ h \geq a \end{cases},$$

where $a$ is the range.

# Common Variogram Models

**Exponential**



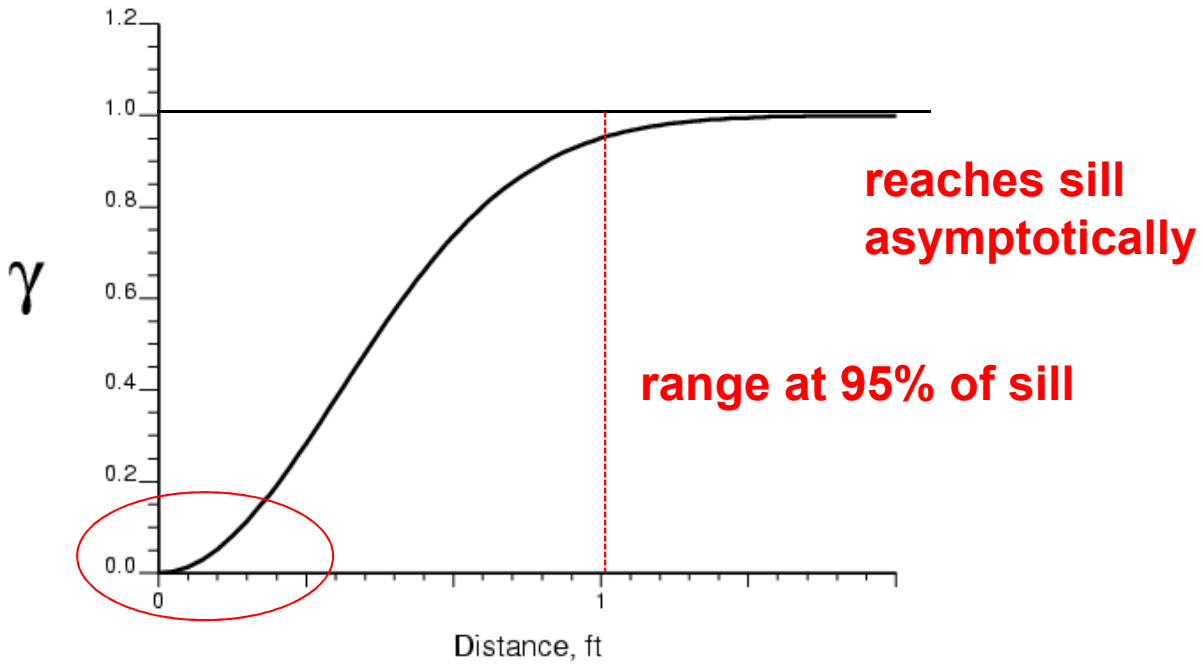

Similar to spherical but rises more steeply and reaches the sill asymptotically

$$\gamma(\mathbf{h}) = C_1 \cdot Exp\left(\frac{\mathbf{h}}{a}\right) = C_1 \cdot \left[1.0 - exp\left(-\frac{\mathbf{h}}{a}\right)\right]$$

where $a$ is the range.

# Common Variogram Models

**Gaussian**




Implies short scale continuity; parabolic behavior at the origin, instead of linear

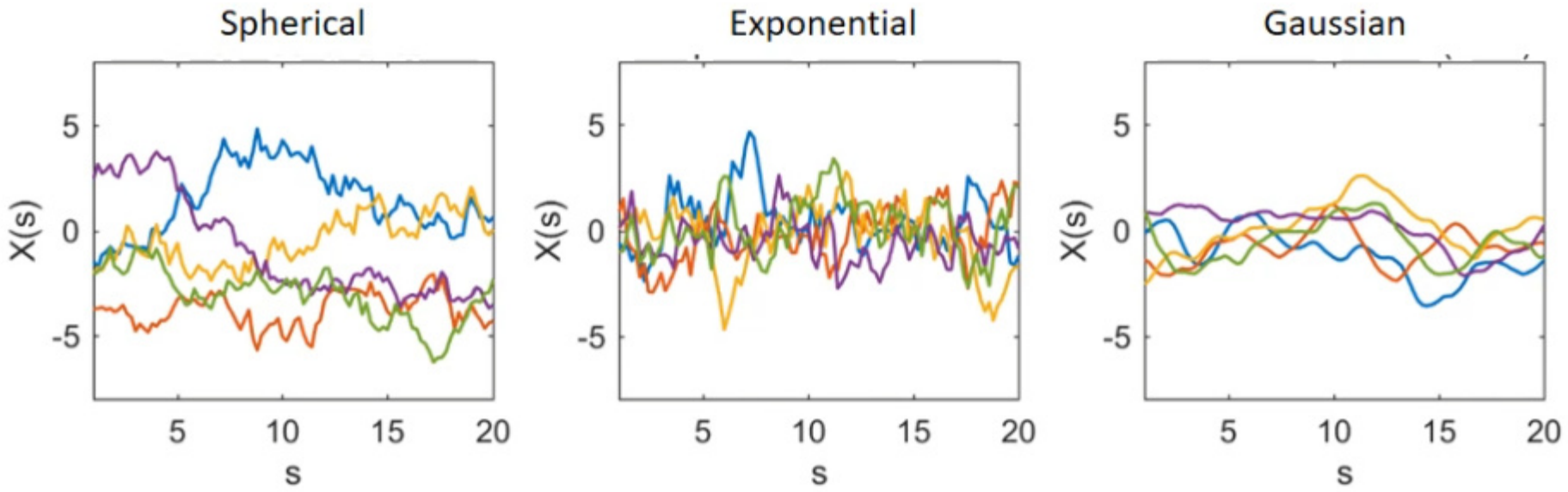
$$\gamma(\mathbf{h}) = C_1 \cdot Gaus\left(\frac{\mathbf{h}}{a}\right) = C_1 \cdot \left[1.0 - \exp\left(-3\frac{\mathbf{h}^2}{a^2}\right)\right]$$

where $a$ is the range.

# Common Variogram Models

**Behavior over short distances**

**Linear** —— **Exponential** —— **Gaussian**

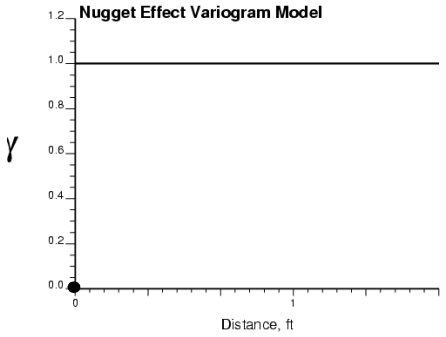


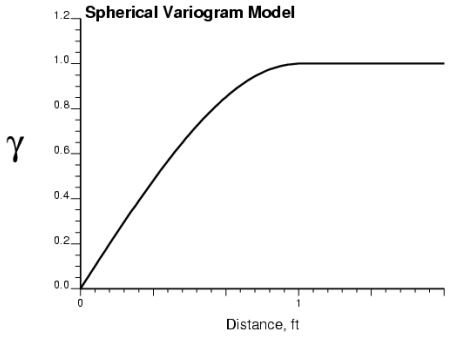

Slide from Prof. Jef Caers (https://www.youtube.com/watch?v=OxjrnVHO64c)

# Common Variogram Models

No spatial correlation

Should be a small component of the overall variance




Commonly encountered variogram shape

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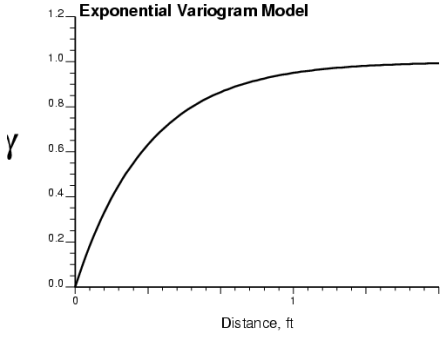
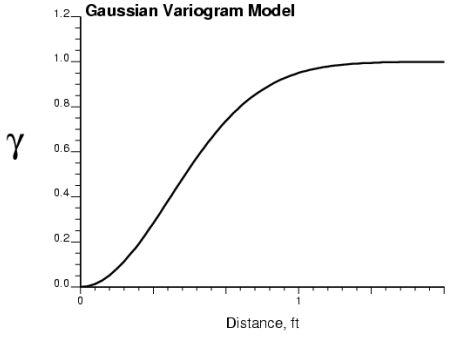
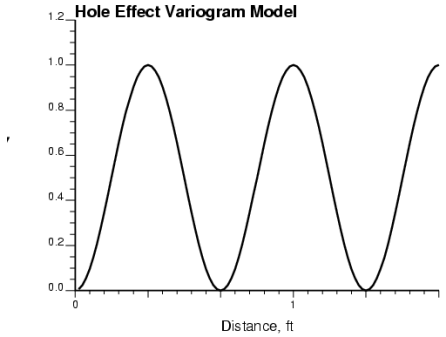
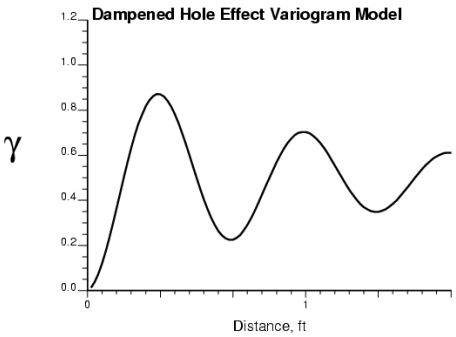



Implies short scale continuity; parabolic behavior at the origin, instead of linear

For periodic variables



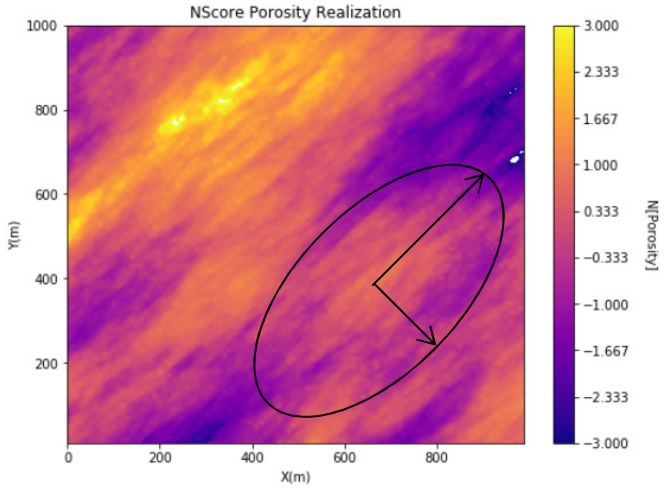



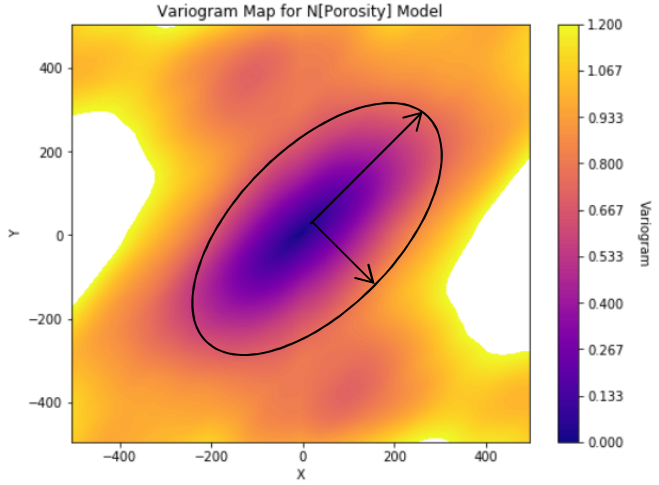

For periodic variables, when the period is not regular

Variogram examples from Pyrcz and Deutsch, 2014.

# 2D Variogram Models

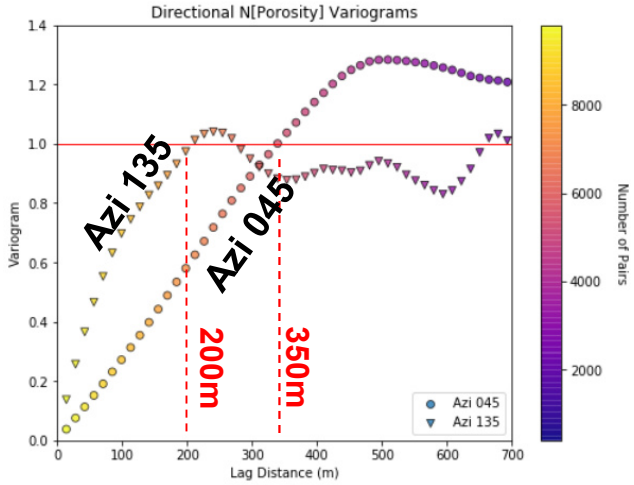
Calculate the variogram for all possible distances and directions (variogram map).



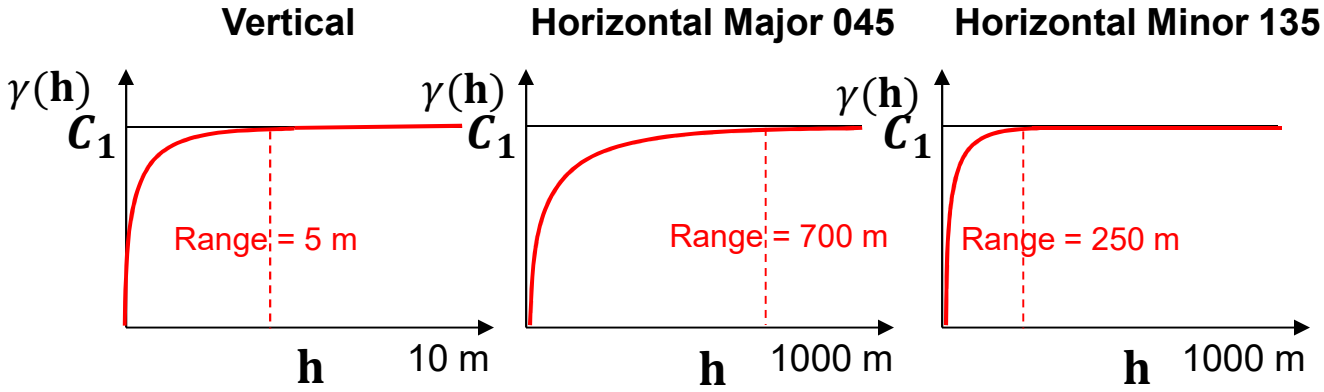




There is an ellipsoidal variation in continuity (geometric anisotropy):

- Parameters for a 2D variogram model:
  - direction, major and minor range, type of variogram



# Directional Variograms



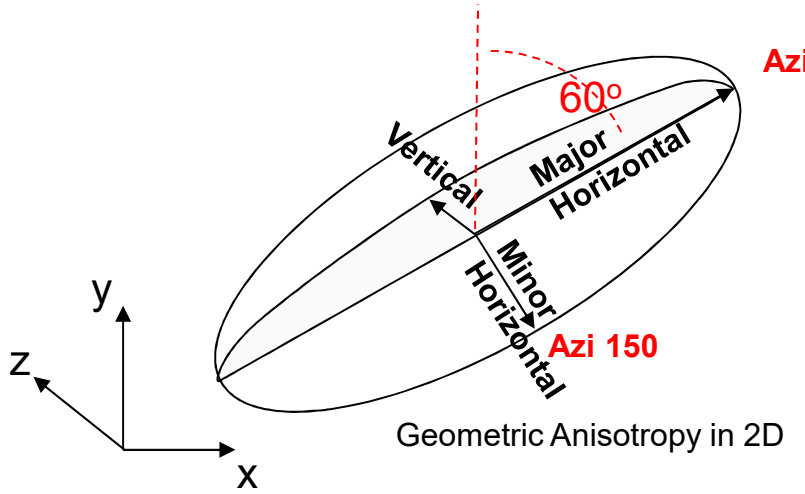

Variable range in each principal direction of a single variogram structure.

For each structure you determine the

- Contribution
- Orientation
- Type (nugget, spherical etc.)
- Range in the primary directions (geometric anisotropy).

# 2D / 3D Variogram Models

The variation of range along different directions is modeled using an ellipse in 2D and an ellipsoid in 3D




Geometric Anisotropy in 2D

**Geometric Distance**

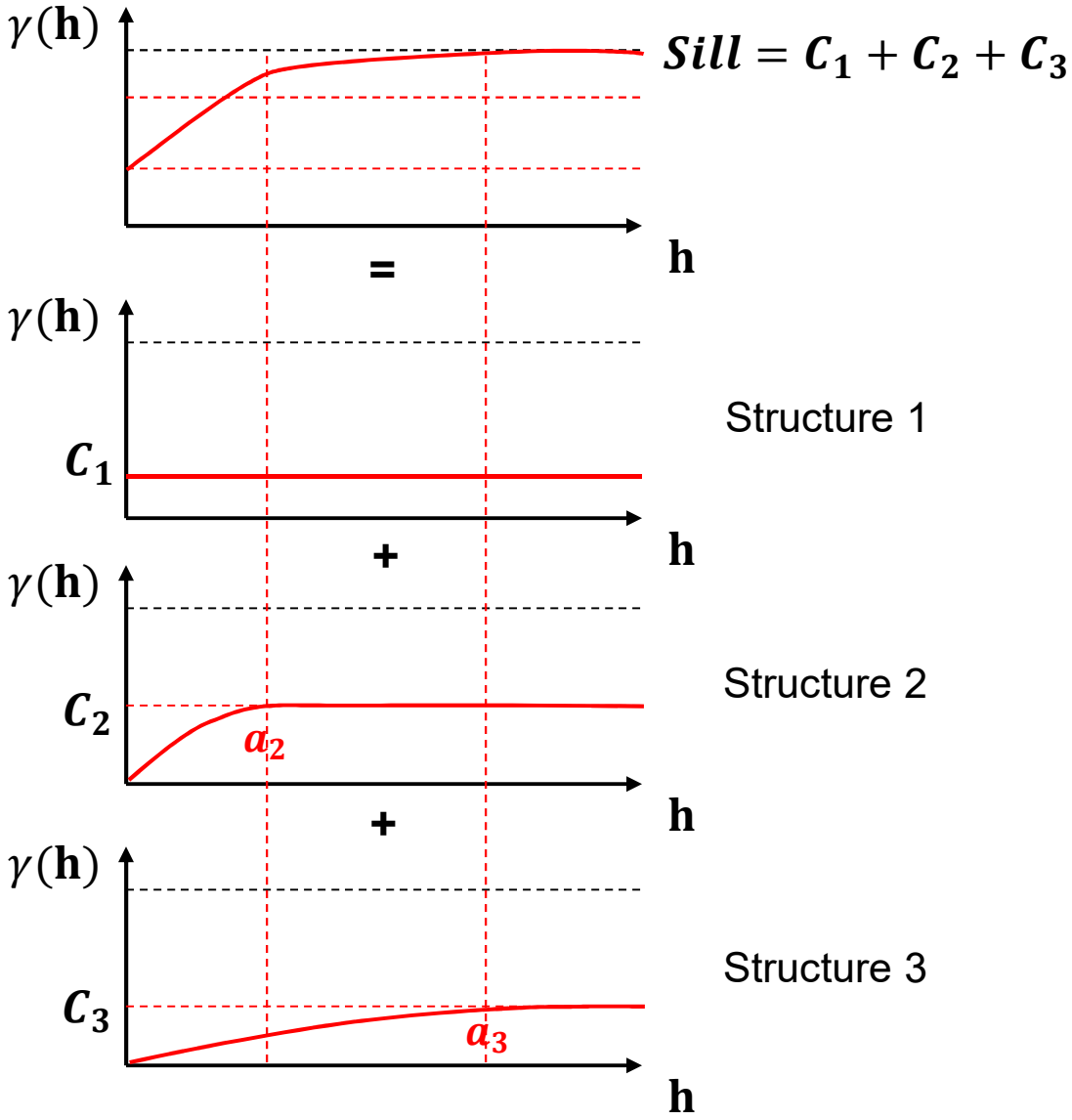
$$h = \sqrt{\left(\frac{h_{maj}}{a_{maj}}\right)^2 + \left(\frac{h_{min}}{a_{min}}\right)^2 + \left(\frac{h_{vert}}{a_{vert}}\right)^2}$$

**where $a_{maj}$, $a_{min}$ and $a_{vert}$ are range parameters**

- There is an ellipsoidal variation in continuity (geometric anisotropy):
  - Parameters for a 2D variogram model:
    - direction, major, minor, contribution and type of variogram
  - Parameters for a 3D variogram model:
    - direction, dip, major, minor and vertical range, contribution and type of variogram

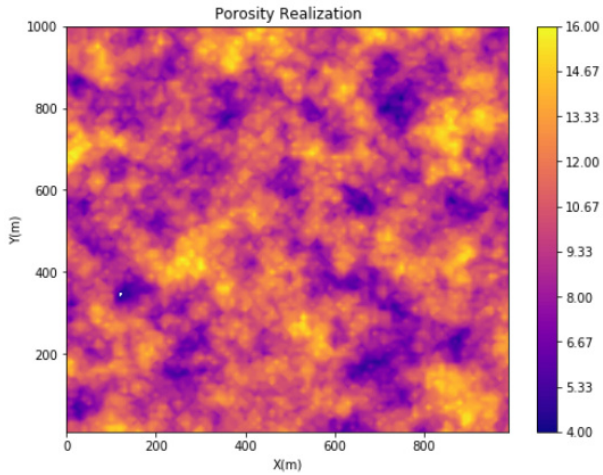
# Nested Structures

- The addition of positive definite variogram structures is positive definite.
- Each structure covers a proportion of the sill.
- For each structure we can change the:
  - orientation
  - range in major and minor
- We are spatially explaining parts of the variance!



# Nested Spatial Frequencies

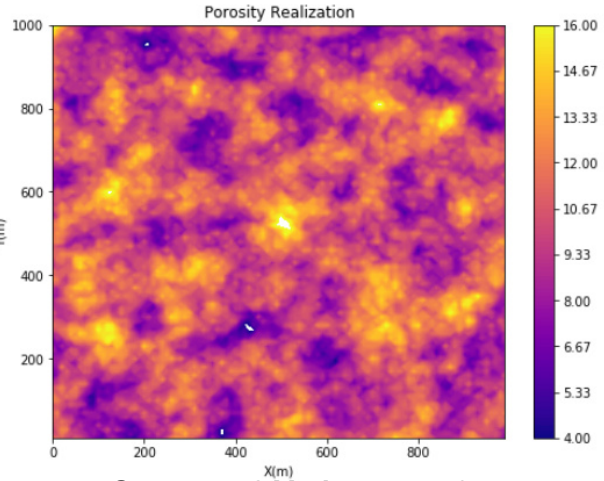
Isotropic 100 m Range, Sill = 4




**Structure 1 Variance = 4.0**

+

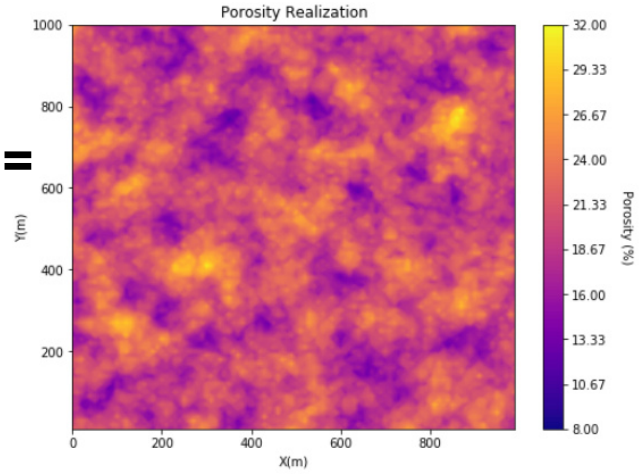
Isotropic 100 m Range, Sill = 4




**Structure 2 Variance = 4.0**

=

Isotropic 100m Range, Sill = 8



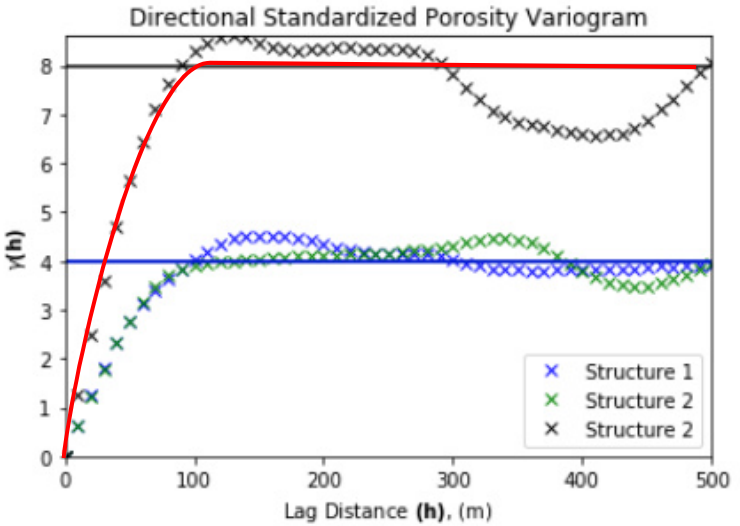

**Structure 1+2 Variance = 8.0**

**Superposition of the same structure only changes the sill.**

- The range stays the same:

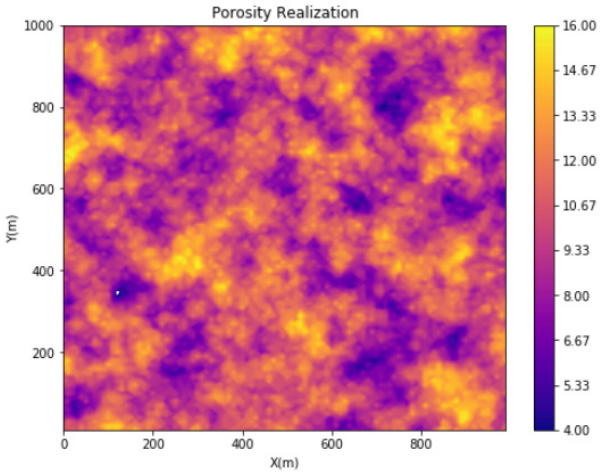
$$(C_0+C_1)\,\gamma_0(h) = C_1\gamma_0(h) + C_2\gamma_0(h)$$

- If you have a single structure in a direction just use the same range for all contributions.



# Nested Spatial Frequencies

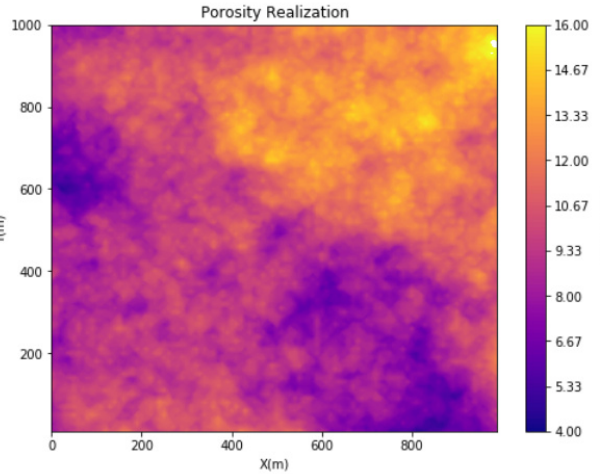
Isotropic 100 m Range, Sill = 4




Structure 1 Variance = 4.0

+

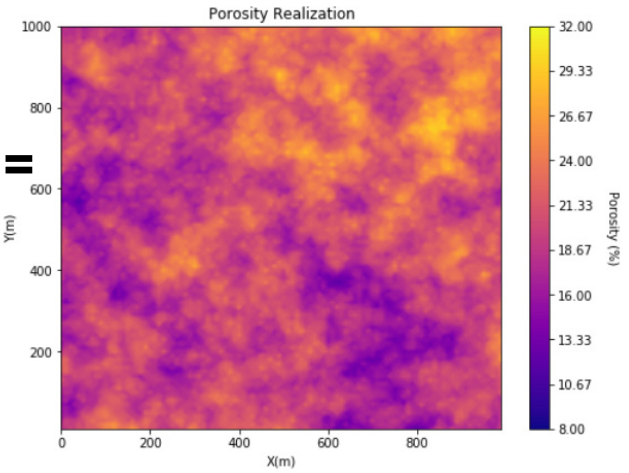
Isotropic 400 m Range, Sill = 4




Structure 2 Variance = 4.0

=

Isotropic Nested 400m Range, Sill = 8



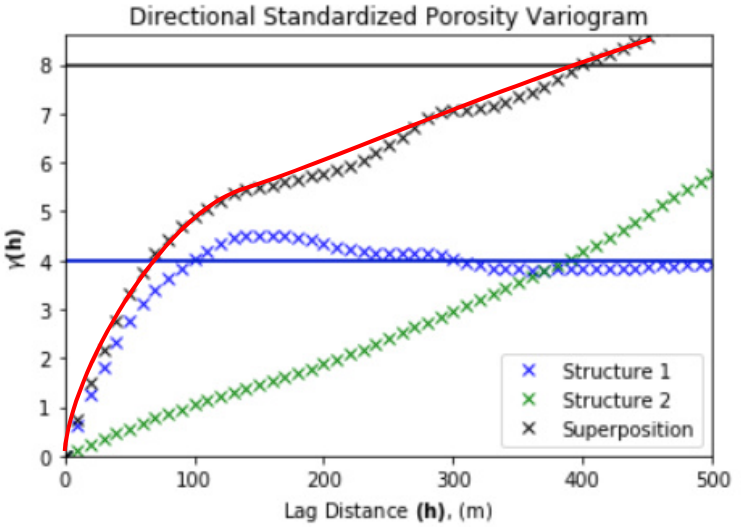

Structure 1+2 Variance = 8.0

**Superposition results if different ranges are applied.**

- New nested model:

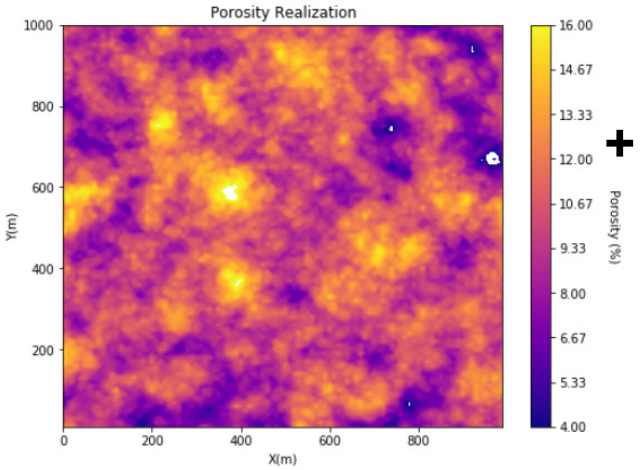
$$\gamma_{tot}(h) = C_1\gamma_1(h) + C_2\gamma_2(h)$$

- Forms an inflection point due to the combination of dissimilar ranges.



# Nested Spatial Frequencies

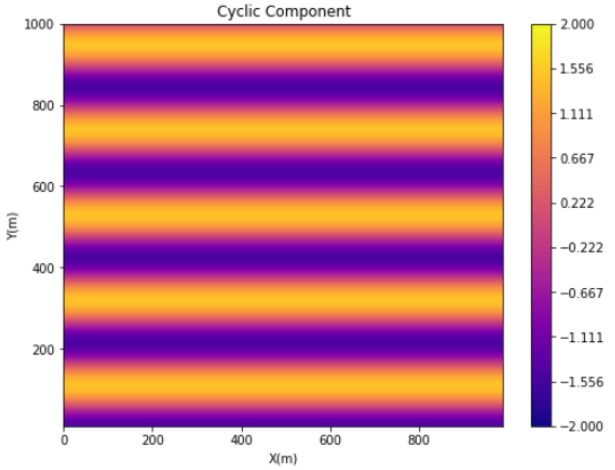
Isotropic 100 m Range




77% Variance

+

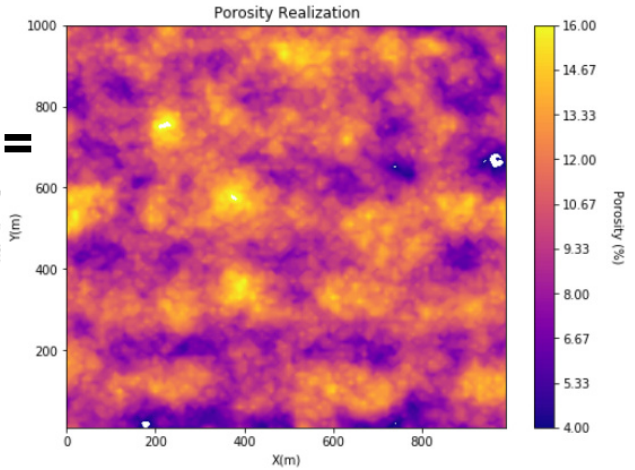
Cyclic Trend in Y 200 m Wavelength




23% Variance

=

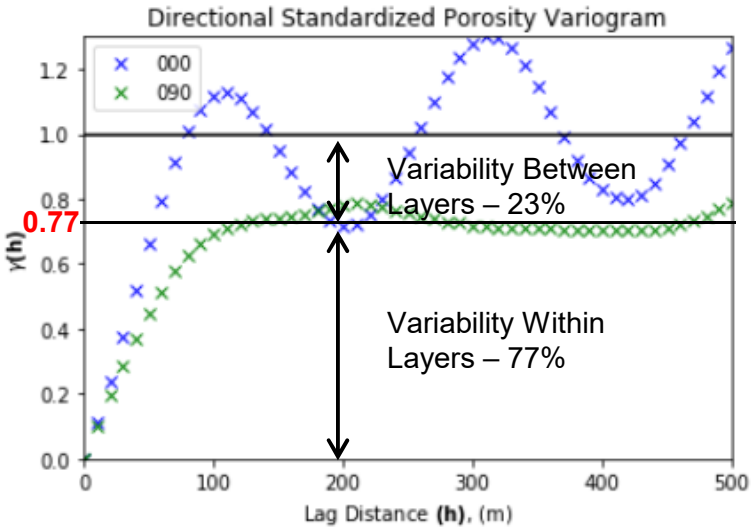
Isotropic 100m Range + Cyclic 200m




100% Variance

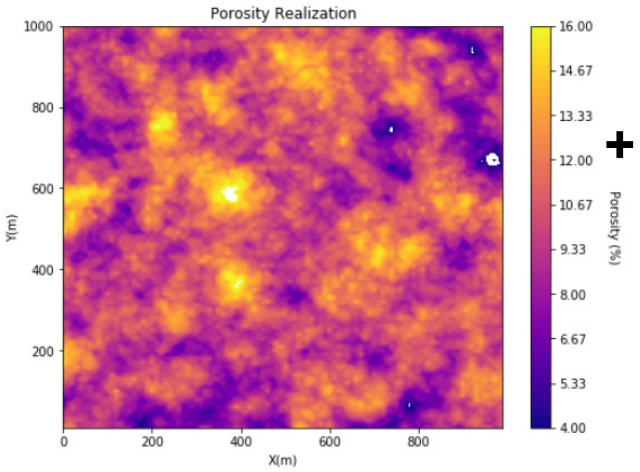
**Variance within each spatial component is the contribution of each variogram structure.**

- Superposition of multiple spatial structures each describing a proportion of the total variability



# Nested Spatial Frequencies

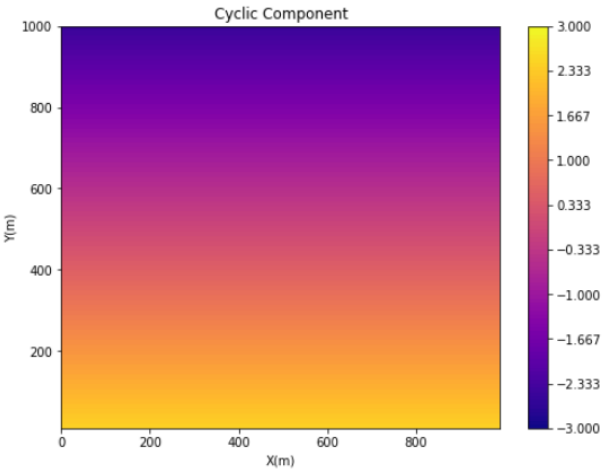
Isotropic 100 m Range




72% Variance

+

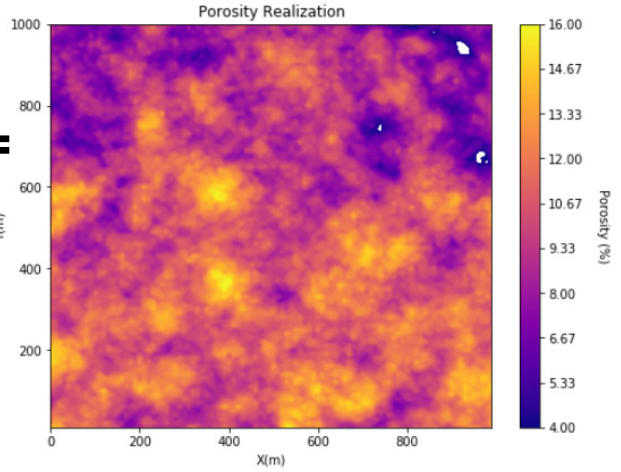
Trend in Y




28% Variance

=

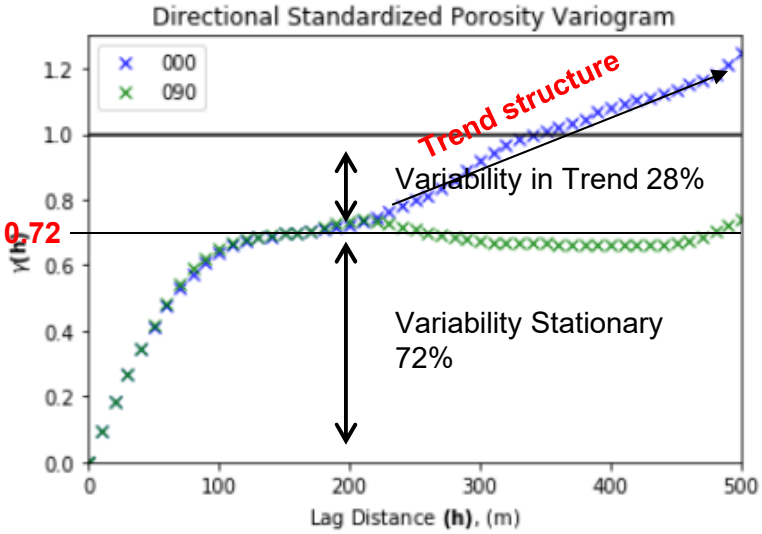
Isotropic 100m Range + Trend in Y




100% Variance

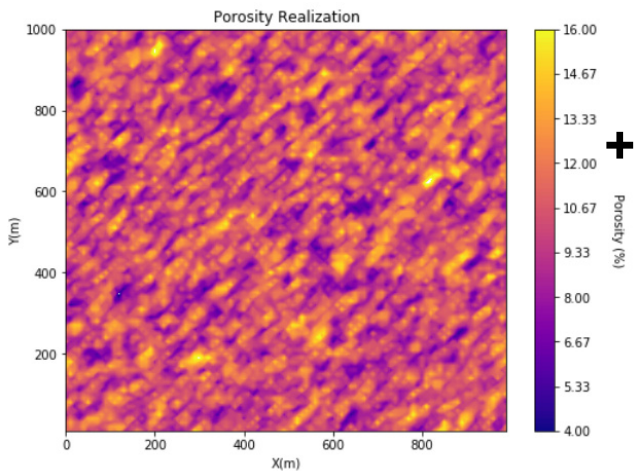
**Variance within each spatial component is the contribution of each variogram structure.**

- This illustrates the partitioning of spatial variance between trend and stationary, stochastic residual.



# Nested Spatial Frequencies

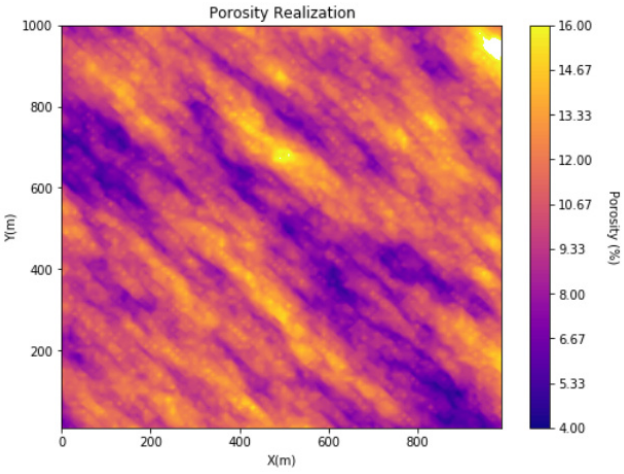
Anisotropic 045 50/20 m Range




50% Variance

+

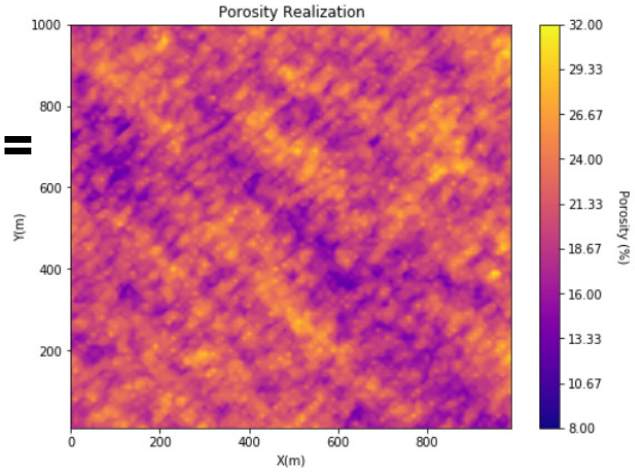
Anisotropic 135 300/100 m Range




50% Variance

=

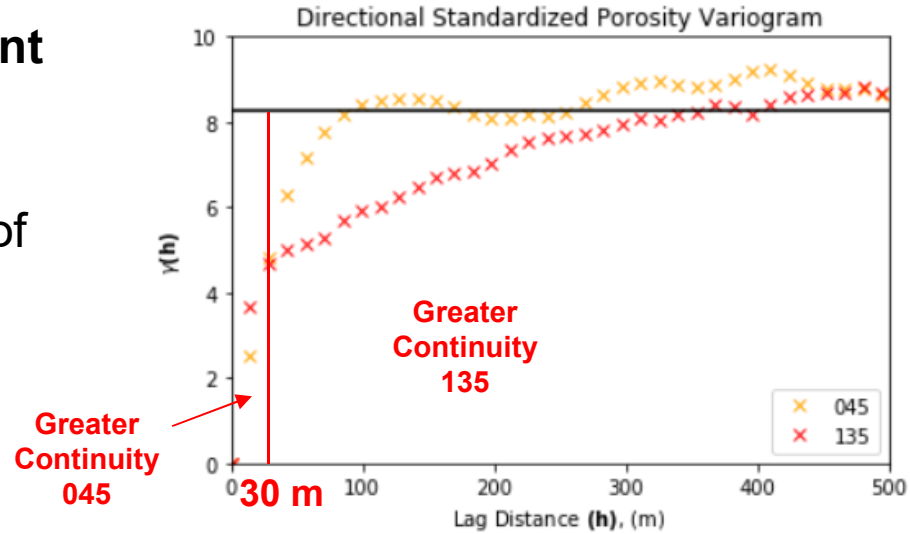
Multiscale, Orientation Variant




100% Variance

**Variance within each spatial component is the contribution of each variogram structure.**

- In this example the primary direction of continuity shifts over distance!
- < 30 m 045 has greater continuity
- > 30 m 135 has greater continuity



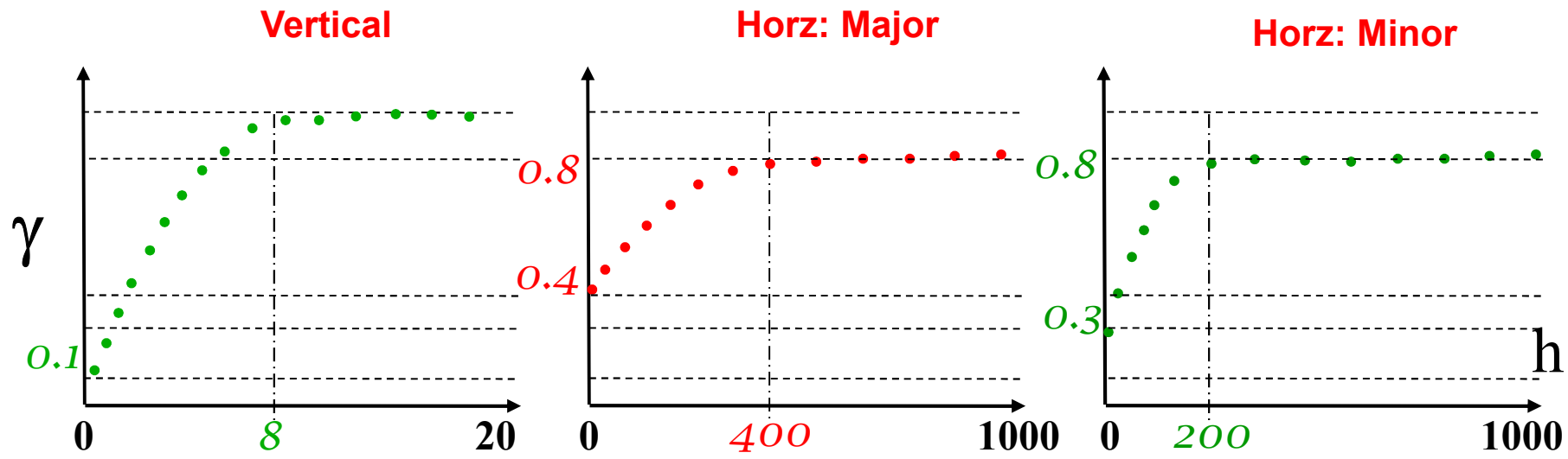
# Variogram Modeling

- The following procedure is used to ensure a legitimate model:
  - Pick a single (lowest) isotropic nugget effect
  - Choose the same number of variogram structures for all directions based on most complex direction
  - Ensure that the same contribution parameter is used for all variogram structures in all directions
  - Allow a **different range parameter** in each direction
  - Model a zonal anisotropy by setting a very large range parameter in one or more of the principal directions

- The responsibility is yours, but most software helps a little
  - force same structure and contributions in all directions, let you modify the range and observe the result in the 3 primary directions.

Slide modified from Clayton Deutsch's MinE 612 Class

# A Complicated Case




same ranges, behaves as one structure

nugget is isotropic over all distances

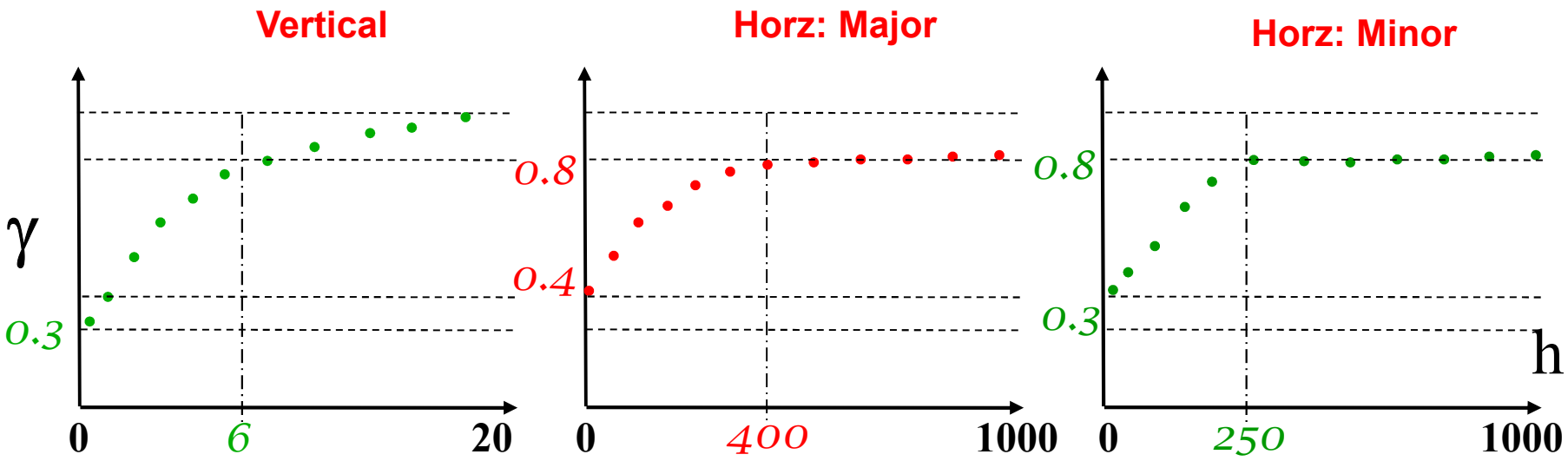
| | Type | $\sigma^2$ | $\nu$ | Hmaj | Hmin |
|---|---|---|---|---|---|
| 1.- | nugget | 0.1 | | | |
| 2.- | Sph | 0.2 | 8 | 0 | 0 |
| 3.- | Sph | 0.1 | 8 | 0 | 200 |
| 4.- | Sph | 0.4 | 8 | 400 | 200 |
| 5.- | Sph | 0.2 | 8 | 9999 | 9999 |
| | | 1.0 | | | |

no spatial correlation in those directions over those structures

$$\gamma(h) = 0.1 + 0.2 \cdot sph_{\substack{av=8 \\ ah1=0 \\ ah2=0}} + 0.1 \cdot sph_{\substack{av=8 \\ ah1=0 \\ ah2=200}} + 0.4 \cdot sph_{\substack{av=8 \\ ah1=400 \\ ah2=200}} + 0.2 \cdot sph_{\substack{av=8 \\ ah1=\infty \\ ah2=\infty}}$$

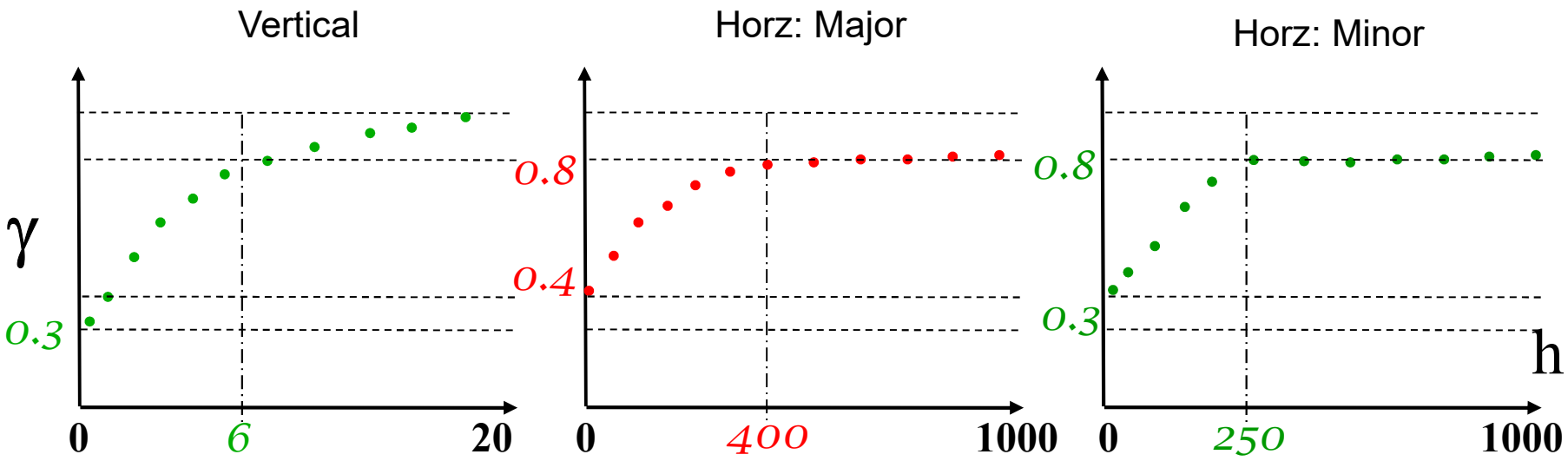
all contributions sum to sill
we are explaining each portion
of the variance

# Variogram Modeling Exercise




| Structure | Type | $\sigma^2$ | $a_{maj}$ | $a_{min}$ | $a_z$ |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |

# Variogram Modeling Exercise




| Structure | Type | $s^2$ | $a_{major}$ | $a_{minor}$ | $a_{vert}$ |
|---|---|---|---|---|---|
| 1 | Nugget | 0.3 | - | - | - |
| 2 | Sph | 0.1 | 0 | 0 | 6 |
| 3 | Sph | 0.4 | 400 | 250 | 6 |
| 4 | Sph | 0.2 | 9999 | 9999 | 20 |
| 5 | | | | | |

# Inference in Presence of Sparse Data

- Most often there are inadequate data to infer a reliable horizontal variogram.
- Horizontal wells have not significantly helped with horizontal variogram inference (yet!):
  - *hard* core data and sophisticated well log measurements are rarely collected from horizontal wells
  - horizontal wells rarely track the stratigraphic “time lines”; they typically intersect the formation at some angle that undulates along the length of the wellbore

  Horizontal well data will become increasingly important
- At present, we depend on analogue data deemed relevant to the site being considered such as:
  - other, more extensively sampled, reservoirs,
  - geological process simulation, or
  - outcrop measurements
- In all cases, care must be taken to integrate global information from analogues with sparse local data

# Variogram Modeling
# Hands-on Directional

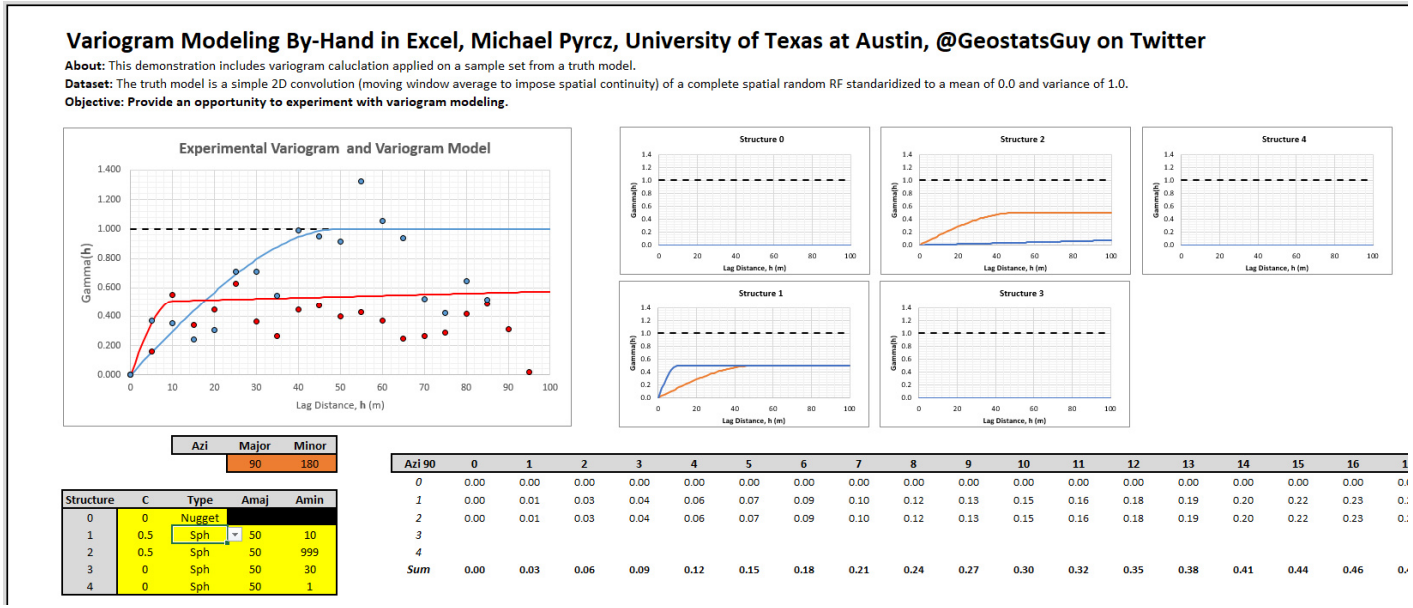
## Variogram Modeling:

### Things to try:

1. Calculate a 2D variogram model with 2 structures both spherical.

2. Vary structure type.

Formulate a 2D variogram model.



**Variogram Modeling By-Hand in Excel, Michael Pyrcz, University of Texas at Austin, @GeostatsGuy on Twitter**

**About:** This demonstration includes variogram caluclation applied on a sample set from a truth model.
**Dataset:** The truth model is a simple 2D convolution (moving window average to impose spatial continuity) of a complete spatial random RF standaridized to a mean of 0.0 and variance of 1.0.
**Objective: Provide an opportunity to experiment with variogram modeling.**



| Azi | Major | Minor |
|---|---|---|
| | 90 | 180 |

| Structure | C | Type | Amaj | Amin |
|---|---|---|---|---|
| 0 | 0 | Nugget | | |
| 1 | 0.5 | Sph | 50 | 10 |
| 2 | 0.5 | Sph | 50 | 999 |
| 3 | 0 | Sph | 50 | 30 |
| 4 | 0 | Sph | 50 | 1 |

| Azi 90 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *0* | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| *1* | 0.00 | 0.01 | 0.03 | 0.04 | 0.06 | 0.07 | 0.09 | 0.10 | 0.12 | 0.13 | 0.15 | 0.16 | 0.18 | 0.19 | 0.20 | 0.22 | 0.23 |
| *2* | 0.00 | 0.01 | 0.03 | 0.04 | 0.06 | 0.07 | 0.09 | 0.10 | 0.12 | 0.13 | 0.15 | 0.16 | 0.18 | 0.19 | 0.20 | 0.22 | 0.23 |
| *3* | | | | | | | | | | | | | | | | | |
| *4* | | | | | | | | | | | | | | | | | |
| ***Sum*** | **0.00** | **0.03** | **0.06** | **0.09** | **0.12** | **0.15** | **0.18** | **0.21** | **0.24** | **0.27** | **0.30** | **0.32** | **0.35** | **0.38** | **0.41** | **0.44** | **0.46** |

The file is at: https://git.io/fxhxr. The file is Variogram_Calc_Model_Demo_v2.0.xlsx

# Variogram Modeling Hands-on

**Here's an opportunity for experiential learning with Variogram Modeling.**

- **Things to try:**

1. Modify and improve the variogram model by changing the ranges.

2. Modify and improve the variogram model by changing the types of structures.

**GeostatsPy: Vairogram Modeling for Subsurface Data Analytics in Python**

**Michael Pyrcz, Associate Professor, University of Texas at Austin**

*Twitter* | *GitHub* | *Website* | *GoogleScholar* | *Book* | *YouTube* | *LinkedIn*

**PGE 383 Exercise: Variogram Modeling with GeostatsPy**

Here's a simple workflow on detecting the major spatial continuity directions in a spatial dataset with variogram analysis. This information is essential to optimum well placement and prectiction away from wells. First let's explain the concept of spatial continuity and the variogram.

**Spatial Continuity**

**Spatial Continuity** is the correlation between values over distance.

- No spatial continuity – no correlation between values over distance, random values at each location in space regardless of separation distance.
- Homogenous phenomenon have perfect spatial continuity, since all values as the same (or very similar) they are correlated.

We need a statistic to quantify spatial continuity! A convenient method is the Semivariogram.

**The Semivariogram**

Function of difference over distance.

- The expected (average) squared difference between values separated by a lag distance vector (distance and direction), $\boldsymbol{h}$:

$$\gamma(\mathbf{h}) = \frac{1}{2\mathrm{N}(\mathbf{h})} \sum_{\alpha=1}^{\mathrm{N}(\mathbf{h})} (\mathrm{z}(\mathbf{u}_\alpha) - \mathrm{z}(\mathbf{u}_\alpha + \mathbf{h}))^2$$

where $z(\mathbf{u}_\alpha)$ and $z(\mathbf{u}_\alpha + \mathbf{h})$ are the spatial sample values at tail and head locations of the lag vector respectively.

- Calculated over a suite of lag distances to obtain a continuous function.
- the $\frac{1}{2}$ term converts a variogram into a semivariogram, but in practice the term variogram is used instead of semivariogram.
- We prefer the semivariogram because it relates directly to the covariance function, $C_x(\mathbf{h})$ and univariate variance, $\sigma_x^2$:

$$C_x(\mathbf{h}) = \sigma_x^2 - \gamma(\mathbf{h})$$

Note the correlogram is related to the covariance function as:

$$\rho_x(\mathbf{h}) = \frac{\mathrm{C_x}(\mathbf{h})}{\sigma_x^2}$$

The correlogram provides of function of the $\mathbf{h} - \mathbf{h}$ scatter plot correlation vs. lag offset $\mathbf{h}$.

$$-1.0 \leq \rho_x(\mathbf{h}) \leq 1.0$$

File Name: GeostatsPy_variogram_modeling.ipynb File is at: https://git.io/fhxYc

# Data Analytics and Geostatistics:
# Spatial Estimation

**Lecture outline . . .**

- **Variogram Interpretation**
- **Variogram Modeling**


Introduction

Modeling Prerequisites

Spatial Estimation

- Stationarity and Trends
- Spatial Continuity Calculation
- Spatial Continuity Modeling
- Spatial Continuity Estimation

Spatial Uncertainty

Multivariate, Spatial

Novel Workflows

Conclusions