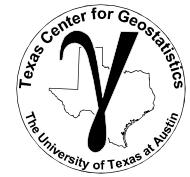


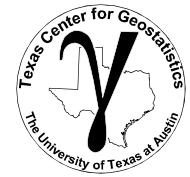
Data Analytics and Geostatistics: Sparse Data



Lecture outline . . .

- **Random Variable**
- **Random Function**
- **Stationarity**
- **Trends**

Data Analytics and Geostatistics: Sparse Data



Lecture outline . . .

- **Random Variable**
- **Random Function**

Facies

What are the Criteria for Facies?



These are the **criteria for facies** (or any categories in reservoir models).

Criteria	Considerations	Example
Separation of Rock Properties	Facies must divide the properties of interest that impact subsurface environmental and economic performance (e.g. grade, porosity and permeability).	A scatter plot with 'Porosity' on the x-axis and 'Permeability' on the y-axis. Three overlapping circles represent different facies: one red circle containing mostly low-permeability points, one green circle containing mostly high-permeability points, and one blue circle containing intermediate points.
Identifiable in Data	Facies must be identifiable with the most common data available. e.g. facies identifiable only in cores are not useful if most wells have only logs.	Two well logs side-by-side. Well 1 has four distinct horizontal layers, each outlined in red and labeled with a letter (top to bottom: A, B, C, D). Well 2 has five distinct horizontal layers, each outlined in red and labeled with a letter (top to bottom: E, F, G, H, I). The layers represent different rock facies identified from the logs.
Map-able Away from Data	Facies must be easier to predict away from data than the rock properties of interest directly, facies improves prediction.	A geological cross-section with two vertical wells labeled 'Well 1' and 'Well 2'. Between them is a dashed green rectangular area representing a region where no data is available. A red square with a question mark is placed in this area, indicating that facies information can be predicted here even though no direct data exists.
Sufficient Sampling	There must be enough data to allow for reliable inference of reliable statistics for rock properties for each facies.	Two plots side-by-side. The left plot shows a bell-shaped red curve labeled 'PDF' on the x-axis. The right plot shows a red line with a dashed continuation labeled 'γ(h)' on the y-axis, with the x-axis labeled 'h'.



Now We Begin Spatial / Geostatistics

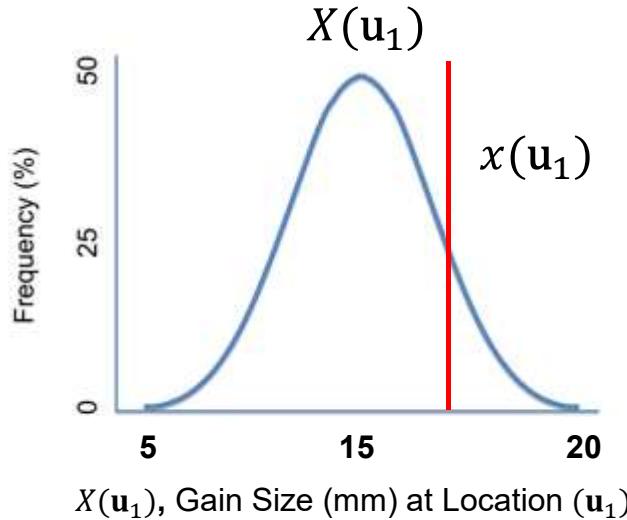
- **We Build on the past lectures:**
 - probability theory
 - univariate and bivariate statistics
 - sampling and bootstrap
- To begin let's provide a concise and practical definition for random variable.

Random Variable (RV) Definition



Random Variable

- we do not know the value at a location / time, it can take on a range of possible values, fully described with a PDF.
- represented as an upper case variable, e.g. X , while possible outcomes or data measures are represented with lower case, e.g. x .
- in spatial context common to use a location vector, \mathbf{u} , to describe a location, e.g. $x(\mathbf{u})$, $X(\mathbf{u})$

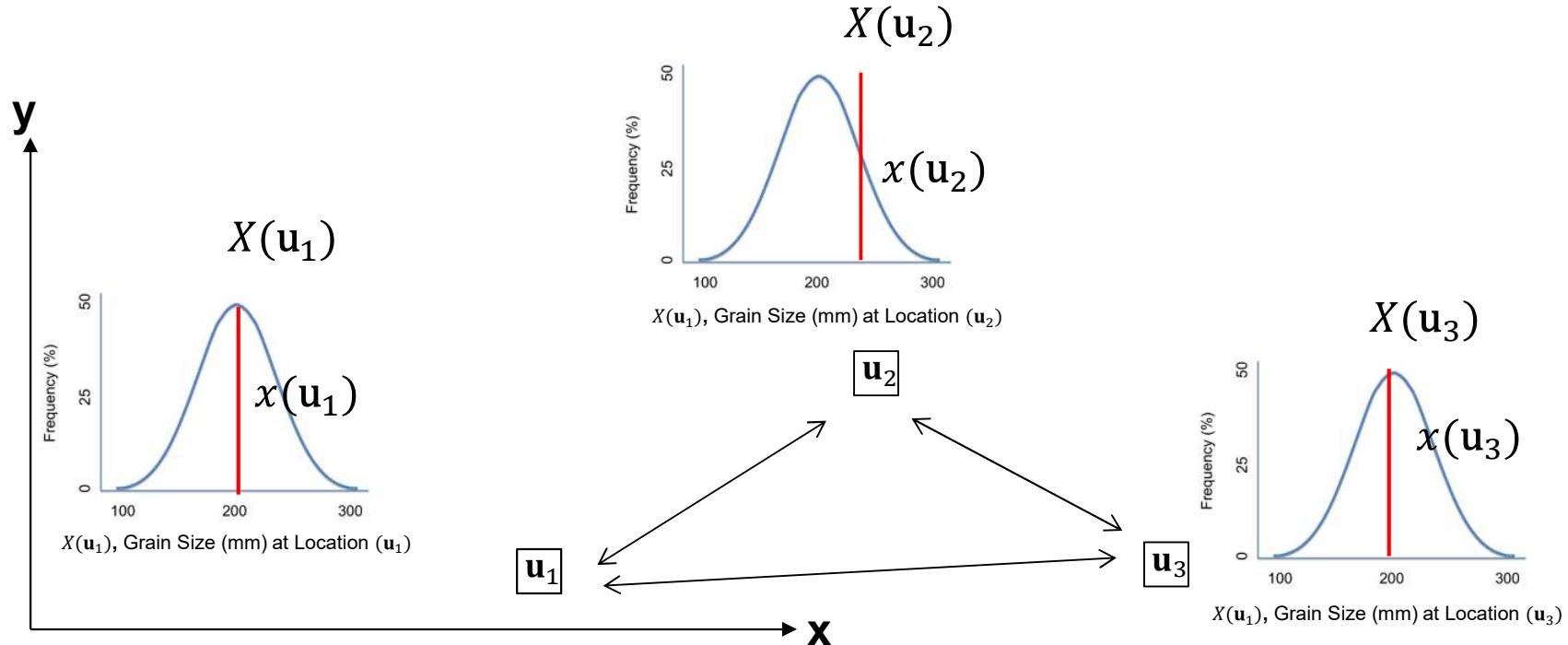


Random Function (RF) Definition



Random Function

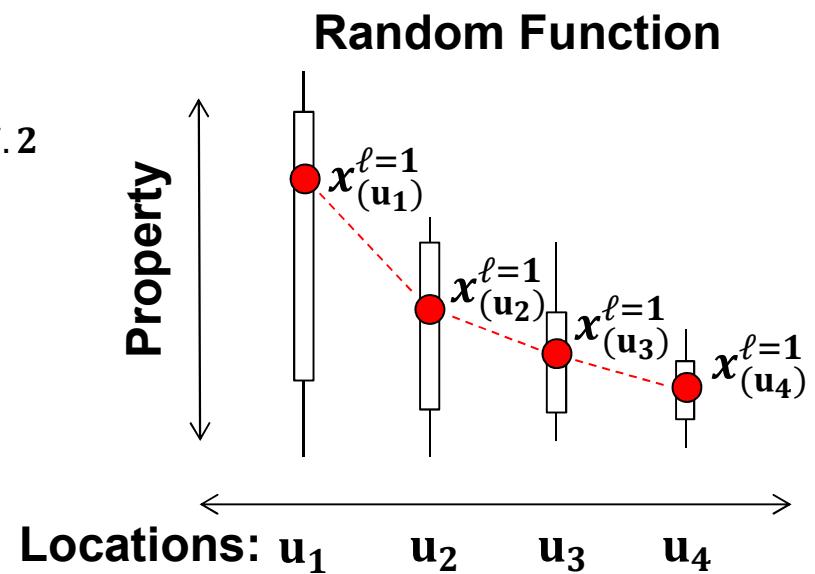
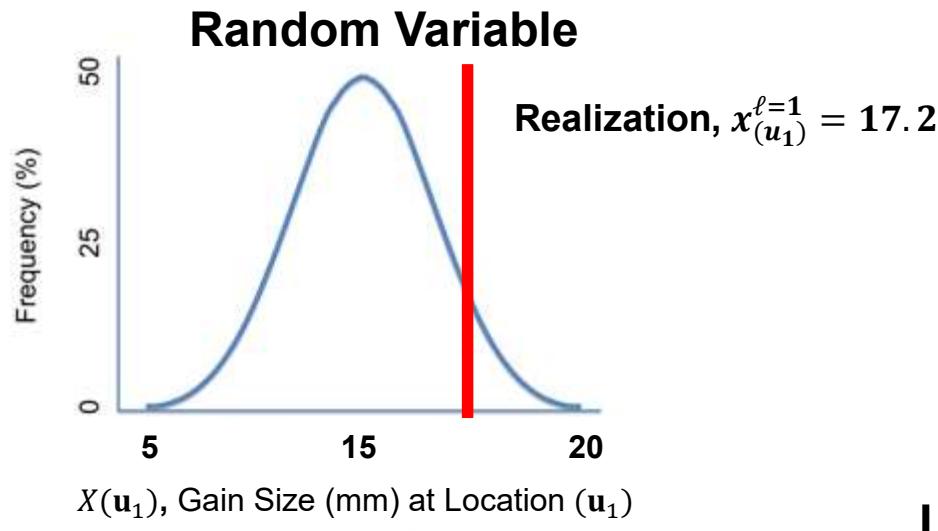
- set of random variables correlated over space and / or time
- represented as an upper case variable, e.g. X_1, X_2, \dots, X_n , while possible joint outcomes or data measures are represented with lower case, e.g. x_1, x_2, \dots, x_n
- in spatial context common to use a location vector, \mathbf{u}_α , to describe a location, e.g. $x(\mathbf{u}_1), x(\mathbf{u}_2), \dots, x(\mathbf{u}_n)$, and $X(\mathbf{u}_1), X(\mathbf{u}_2), \dots, X(\mathbf{u}_n)$



Realization Definition

Realization

- an outcome from a random variable or joint set of outcomes from a random function.
- represented with lower case, e.g. x .
- in spatial context common to use a location vector, \mathbf{u} , to describe a location, e.g. $x(\mathbf{u})$, $X(\mathbf{u})$
- resulting from simulation, e.g. Monte Carlo simulation, sequential Gaussian simulation ← a method to sample (jointly) from the RV (RF)
- each realization is considered equiprobable

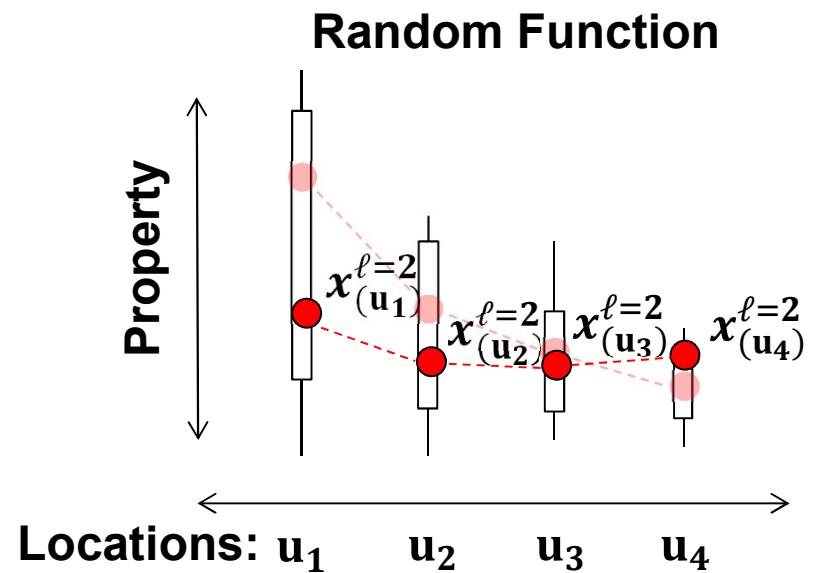
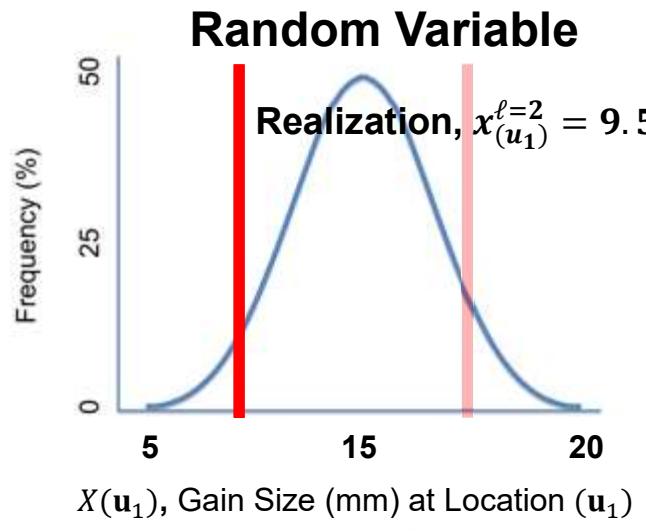


Realization Definition

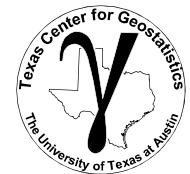


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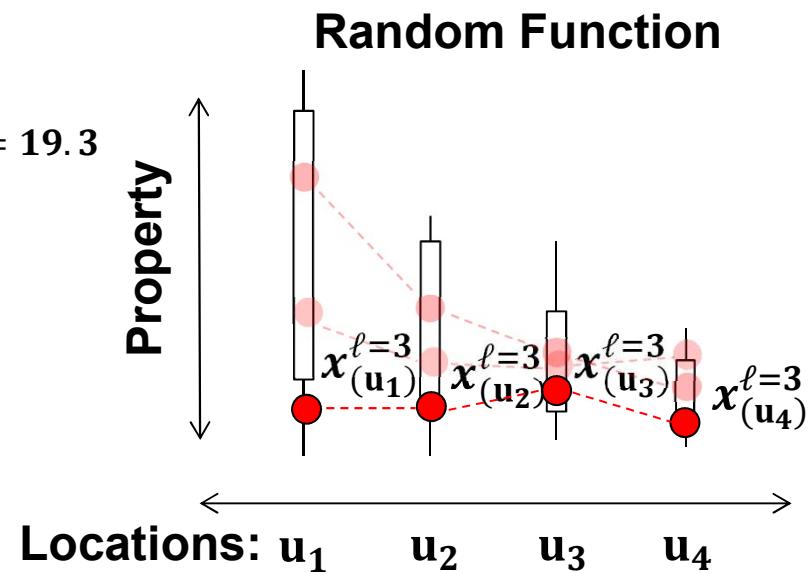
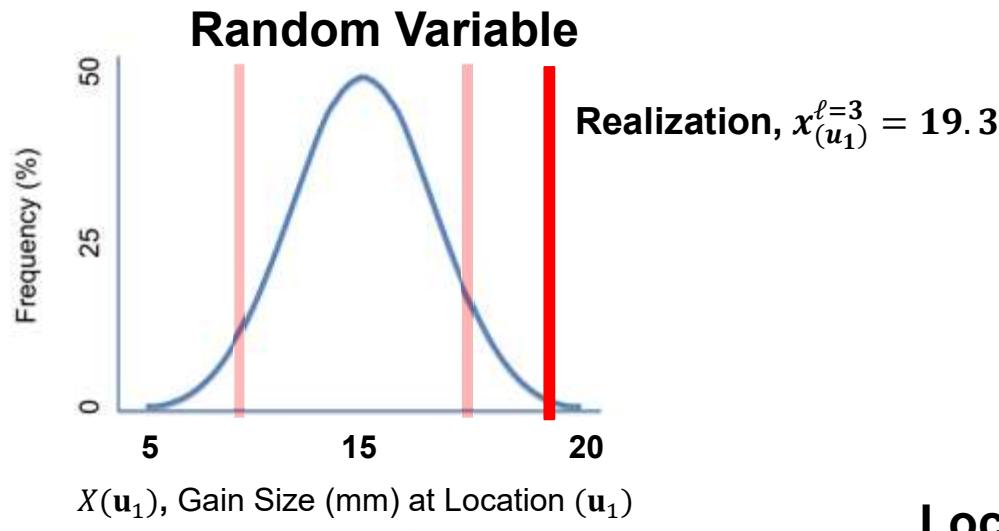


Realization Definition

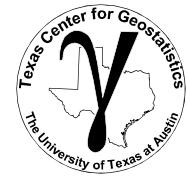


Realization

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Data Analytics and Geostatistics: Sparse Data



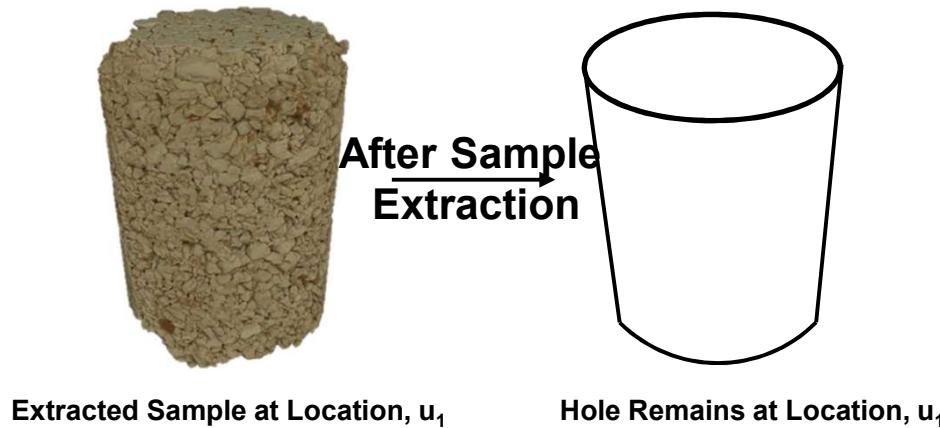
Lecture outline . . .

- Stationarity

Stationarity Substituting Time for Space

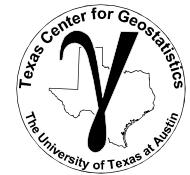


Any statistic requires replicates, repeated sampling (e.g. air or water samples from a monitoring station). In our geospatial problems repeated samples are not available at a location in the subsurface.

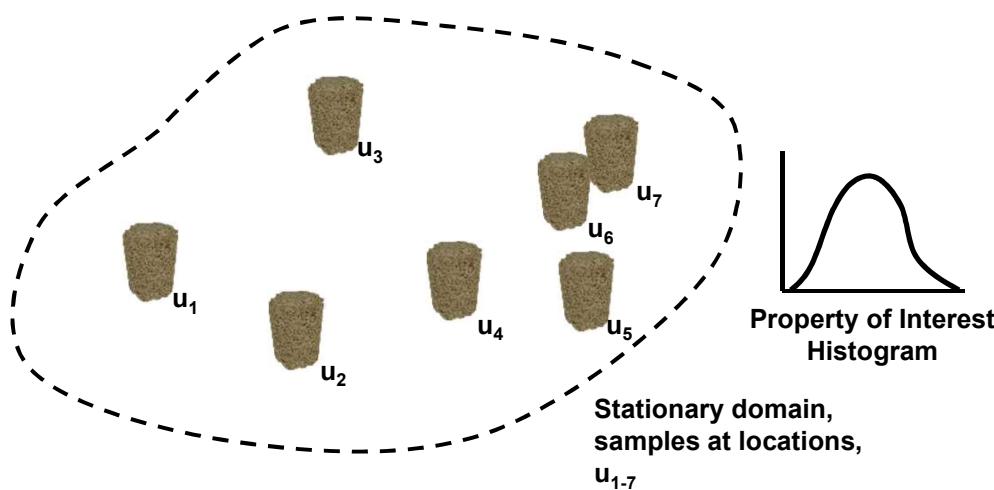


Instead of time, **we must pool samples over space** to calculate our statistics. This decision to pool is the decision of stationarity. It is the decision that the subset of the subsurface is all the “same stuff”.

Stationarity Substituting Time for Space



The decision of the stationary domain for sampling is an expert choice. Without it we are stuck in the “hole” and **cannot calculate any statistics** nor say anything about the behavior of the subsurface **between the sample data**.

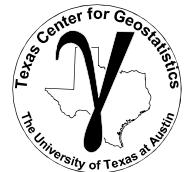


Import License: choice to pool specific samples to evaluate a statistic.

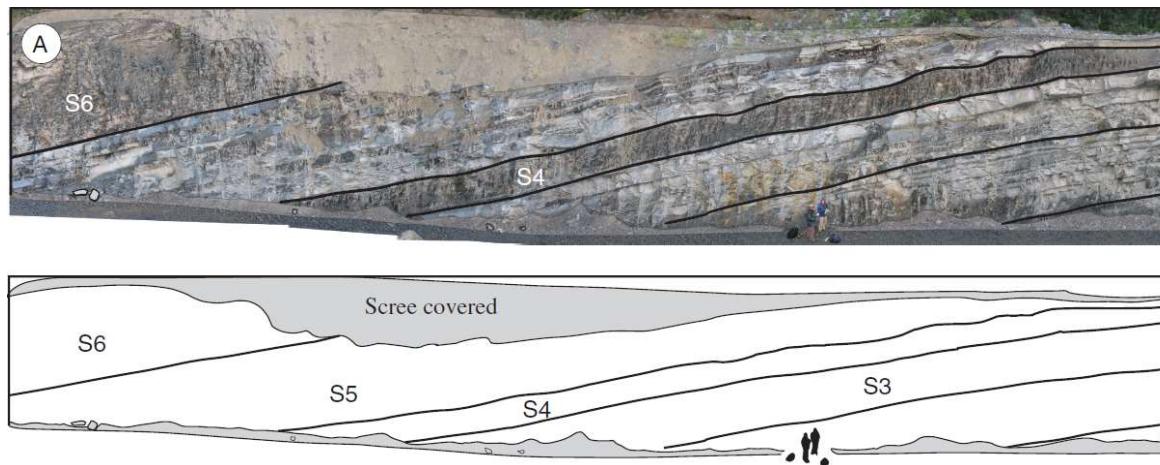
Export License: choice of where in the subsurface this statistic is applicable.

Stationarity

Definition 1: Geologic



Geological Definition: e.g. 'The rock over the stationary domain is sourced, deposited, preserved, and postdepositionally altered in a similar manner, the domain is map-able and may be used for local prediction or as information for analogous locations within the subsurface; therefore, it is useful to pool information over this expert mapped volume of the subsurface.'



Photomosaic, line drawing Punta Barrosa Formation sheet complex (Fildani et al. (2009)).

Stationarity

Definition 2: Statistical



Statistical Definition: The metrics of interest are invariant under translation over the domain. For example, one point stationarity indicates that histogram and associated statistics do not rely on location, \mathbf{u} . Statistical stationarity for some common statistics:

Stationary Mean: $E\{Z(\mathbf{u})\} = m, \forall \mathbf{u}$

Stationary Distribution: $F(\mathbf{u}; z) = F(z), \forall \mathbf{u}$

Stationary Semivariogram: $\gamma_z(\mathbf{u}; \mathbf{h}) = \gamma_z(\mathbf{h}), \forall \mathbf{u}$

Stationarity: *What metric / statistic? Over what volume?*

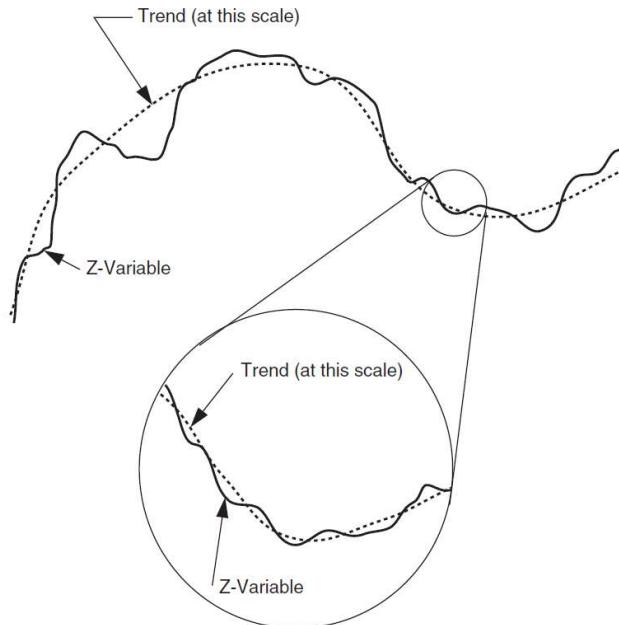
May be extended to any statistic of interest including, facies proportions, bivariate distributions and multiple point statistics.

Stationarity

Comments on Stationarity

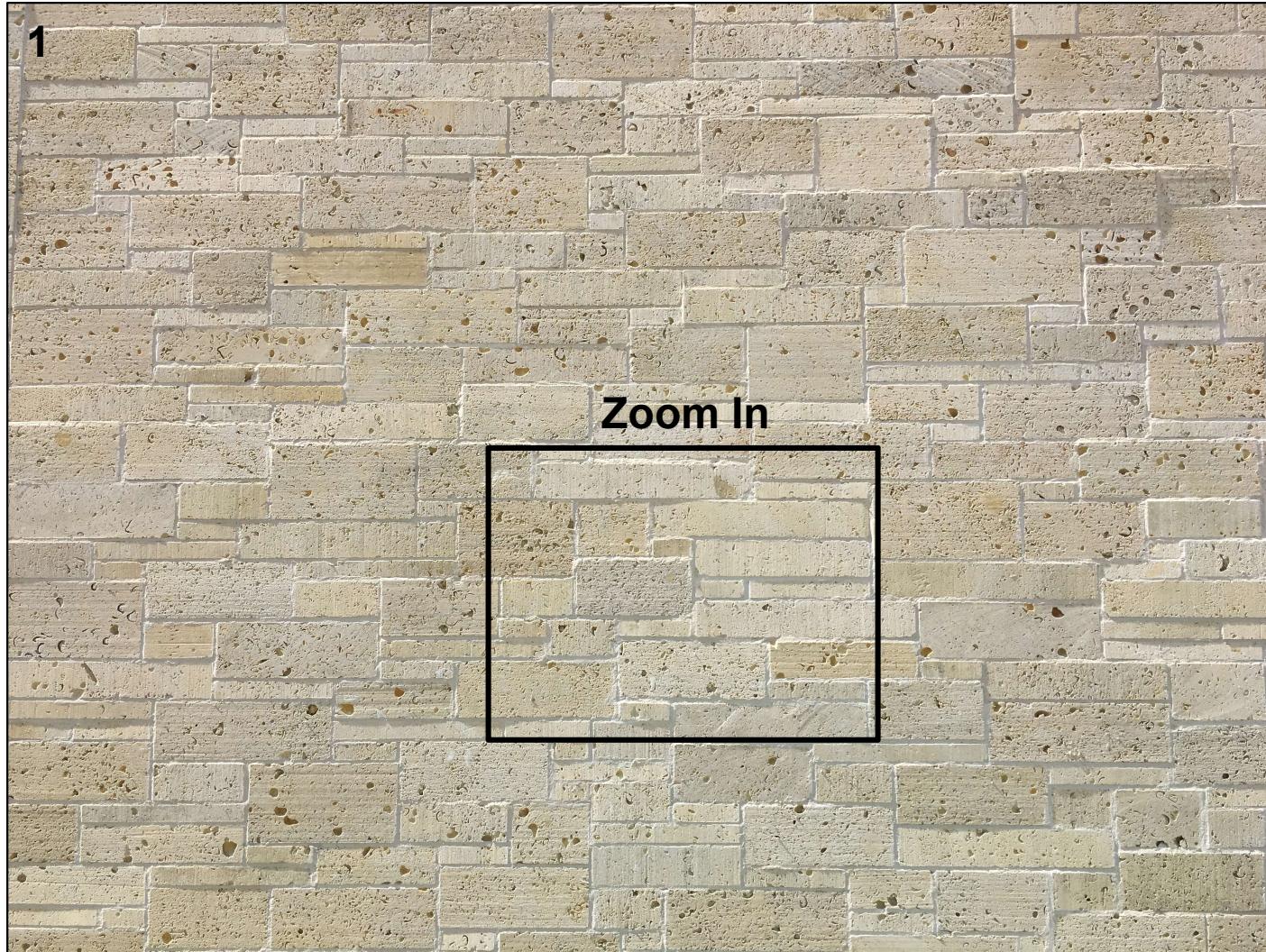
Stationarity is a decision, not an hypothesis; therefore it cannot be tested. Data may demonstrate that it is inappropriate.

The stationarity assessment depends on scale. This choice of modeling scale(s) should be based on the specific problem and project needs.



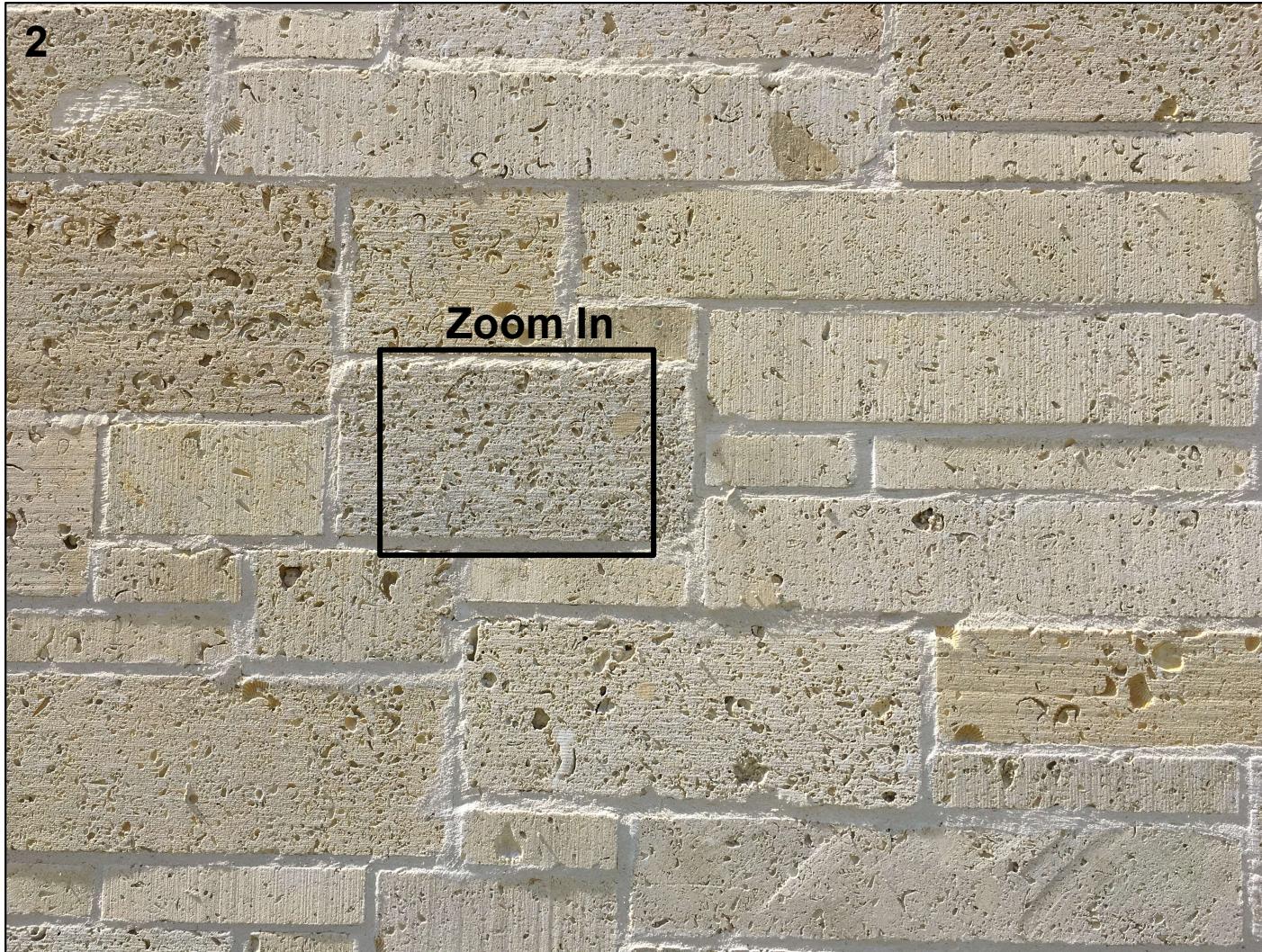
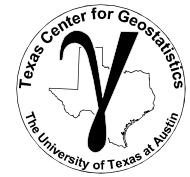
Scales of stationarity from Pyrcz and Deutsch (2014).

Stationarity Example



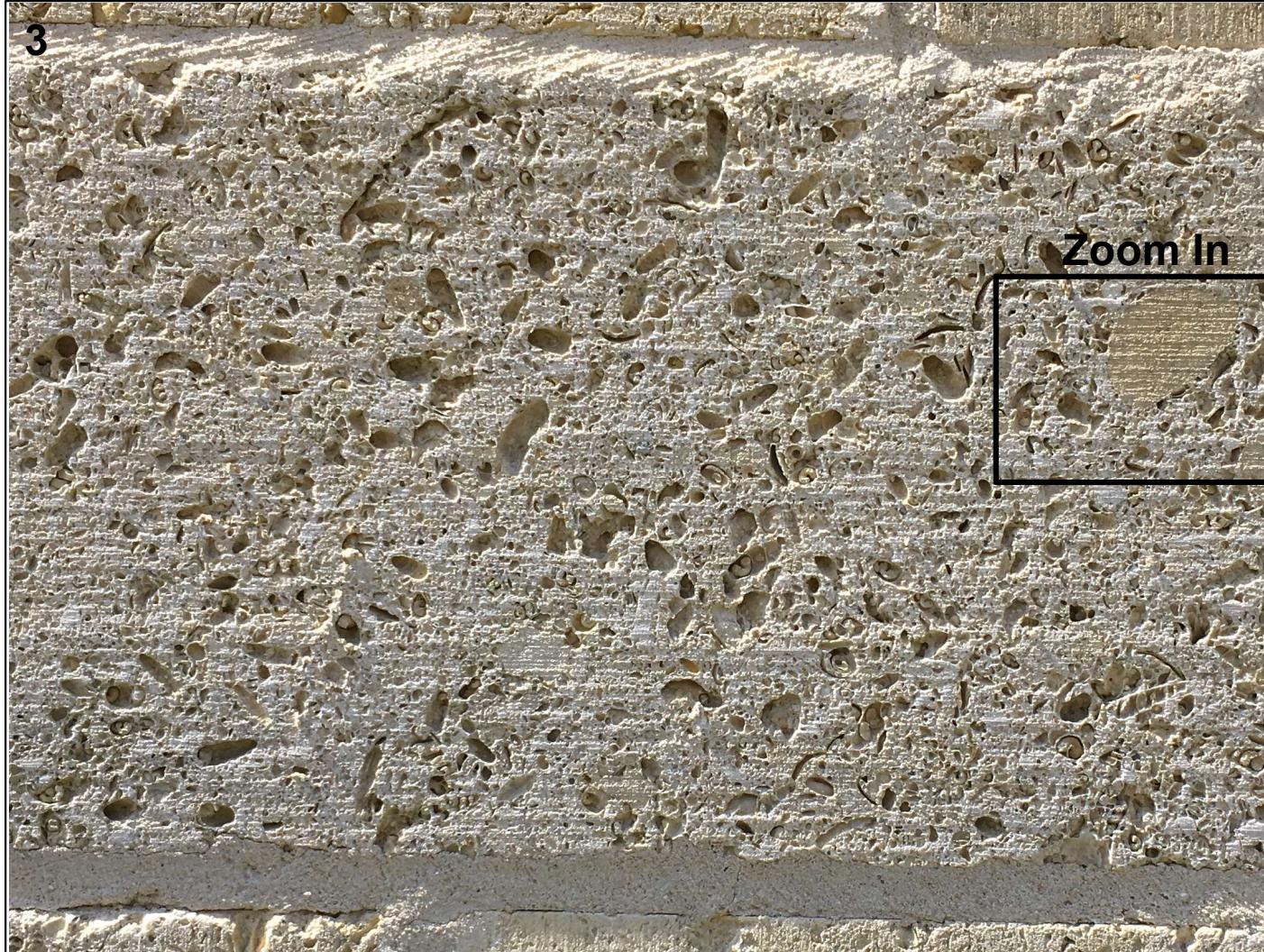
- Is this image stationary? What metric do you consider?

Stationarity Example



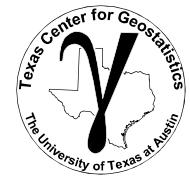
- A smaller group of bricks?

Stationarity Example



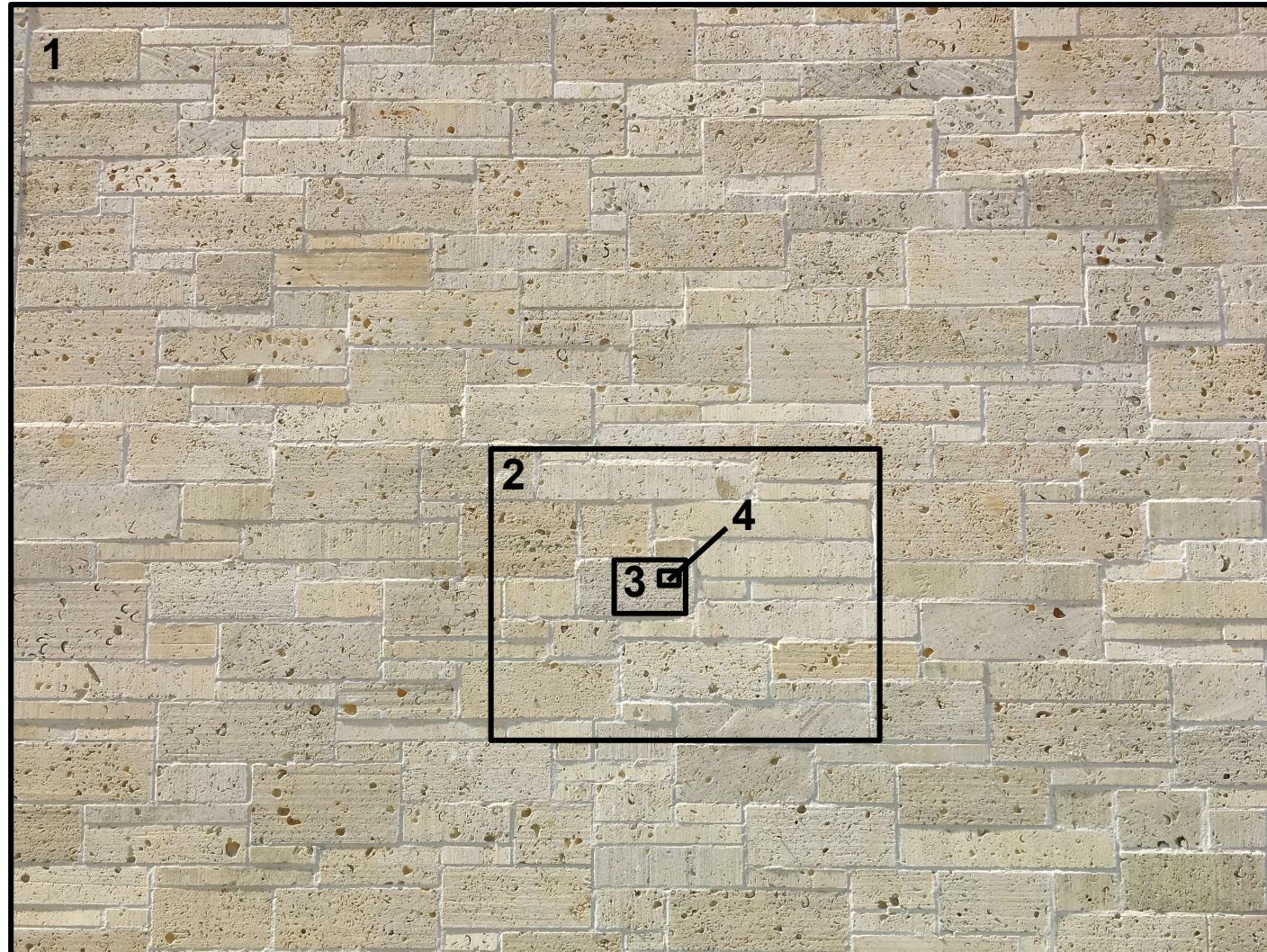
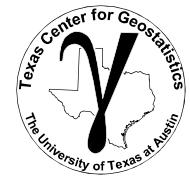
- A single brick?

Stationarity Example

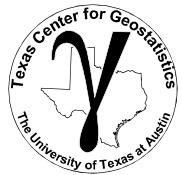


- Small part of a brick?

Stationarity Example



- Is this image stationary? What metric do you consider?



Comments on Stationarity

We cannot avoid a decision of stationarity. No stationarity decision and we cannot move beyond the data. Conversely, assuming broad stationarity over all the data and over large volumes of the earth is naïve.

Geomodeling stationarity is the decision: (1) over what region to pool data (import license) and (2) over what region to use the resulting statistics (export license).

Nonstationary trends may be mapped and the remaining stationary residual modelled statistically / stochastically, trends may be treated uncertain.

Good geological mapping and data integration is essential!

it is the framework of any subsurface model.

Stationarity Definition



- **Stationarity** – “statistic/metric” is invariant under translation over an *interval* e.g. region, time period etc.
 - **What is stationary?** Need a metric.
 - **Over what interval?** Need a time or volume of interest
 - Depends on the model purpose
 - Depends on the scale of observation
 - Decision not an hypothesis; therefore, if cannot be tested

Stationarity Summary



- Consider a random variable $X(\mathbf{u}_\alpha) \rightarrow F_x(x; \mathbf{u}_\alpha) = Prob(X \leq x)$
- What is the practical meaning of $F_x(x; \mathbf{u}_\alpha)$? There can only be one sample at any specific time/location.
- There is a need to pool samples coming from different times and/or locations to come up with $F_x(x; \mathbf{u}_\alpha)$ (or any statistic).

Choice of the Pool = Decision of Stationarity

Import License to pool samples over an area / volume.

Export License to use these statistics over an area / volume.

- Stationarity in the mean, variance and entire CDF.
 - stationary mean, $m_x(\mathbf{u}) = m_x$
 - stationary variance, $\sigma_x^2(\mathbf{u}) = \sigma_x^2$
 - stationary CDF, $F_x(x; \mathbf{u}) = F_x(x)$
 - etc.
- Depends on scale of observation

Stationarity Summary on GitHub / Twitter



An explanation of **STATIONARITY** for geoscientists and geo-engineers.

Michael Pyrcz, University of Texas at Austin, @GeostatsGuy

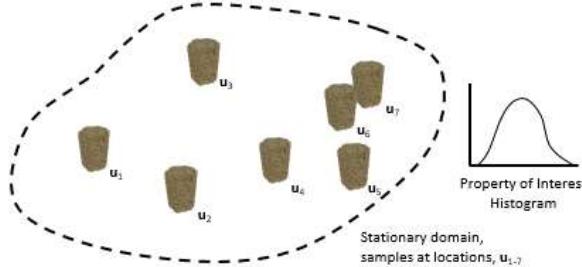
A description of the concepts of stationarity that are central to collecting geoscience information and applying it in subsurface modeling.

1. Substituting time for space.

Any statistic requires replicates, repeated sampling (e.g. air or water samples from a monitoring station). In our geospatial problems repeated samples are not available at a location in the subsurface.



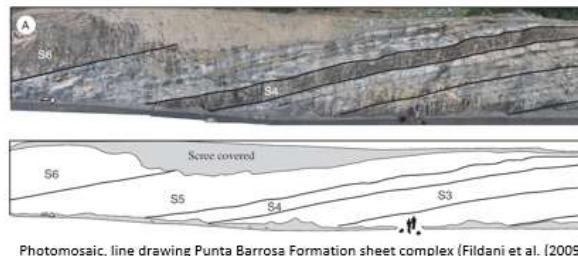
Instead of time, we must pool samples over space to calculate our statistics. This decision to pool is the decision of stationarity. It is the decision that the subset of the subsurface is all the "same stuff".



The decision of the stationary domain for sampling is an expert choice. Without it we are stuck in the "hole" and cannot calculate any statistics nor say anything about the behavior of the subsurface between the sample data. Core image from <https://www.fei.com/oil-gas/>

2. Definitions of Stationarity.

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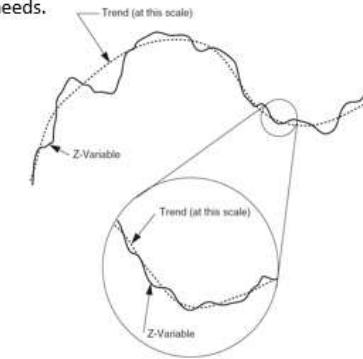
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May be extended to any statistic of interest including, facies proportions, bivariate distributions and multiple point statistics.

3. Comments on Stationarity.

Stationarity is a decision, not an hypothesis; therefore it cannot be tested. Data may demonstrate that it is inappropriate.

The stationarity assessment depends on scale. This choice of modeling scale(s) should be based on the specific problem and project needs.



We cannot avoid a decision of stationarity. No stationarity decision and we cannot move beyond the data. Conversely, assuming broad stationarity over all the data and over large volumes of the earth is naive. Good geological mapping is essential.

Geomodeling stationarity is the decision (1) over what region to pool data and (2) over what region to use the resulting statistics.

Nonstationary trends may be mapped and the remaining stationary residual modelled stochastically, trends may be treated uncertain.

For more information check out Pyrcz, M.J., and Deutsch, C.V., 2014, Geostatistical Reservoir Modeling, 2nd edition, Oxford University Press.

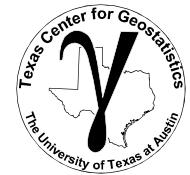
Data Analytics and Geostatistics: Sparse Data



Lecture outline . . .

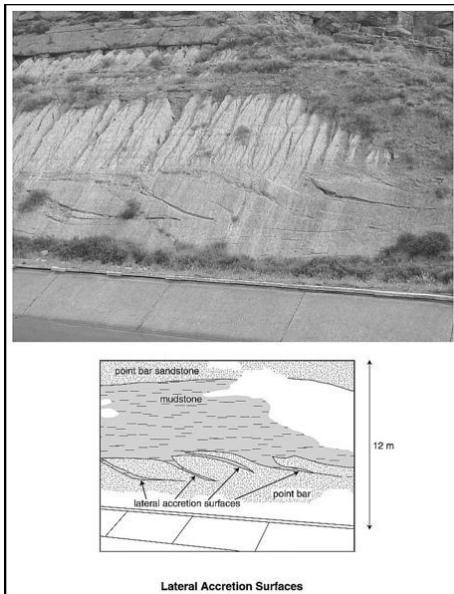
- Trends

Spatial Estimation

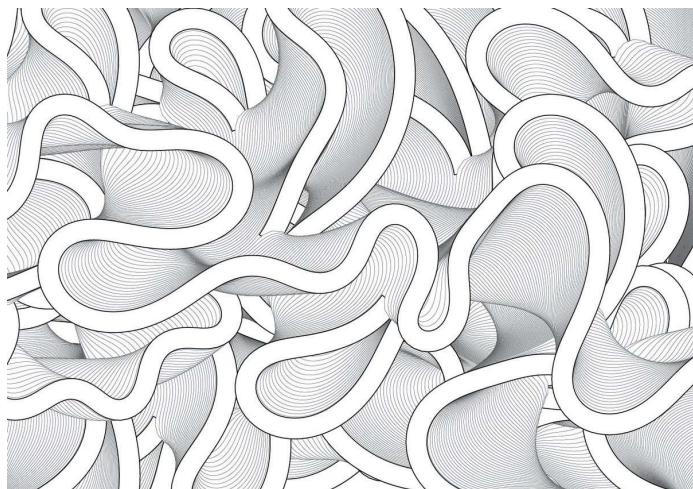


- First we will cover trend modeling and the concept of overfit
- Then we will cover spatial ccontinuity calculation and modeling and then spatial estimation.

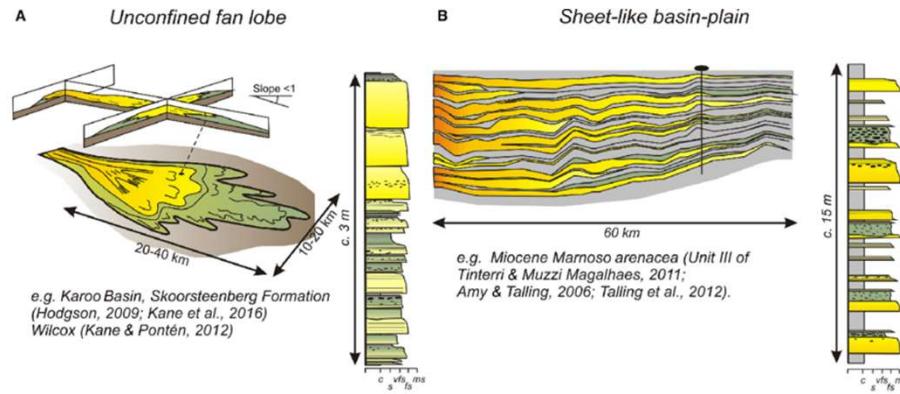
Trend Examples



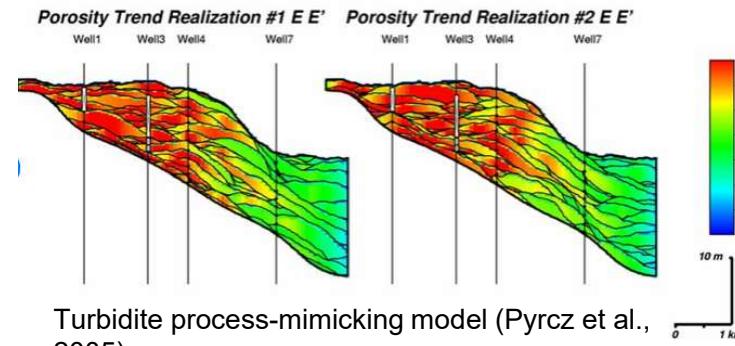
Fluvial trends observable in outcrop
Ebro, Spain (Shepherd, 2009)



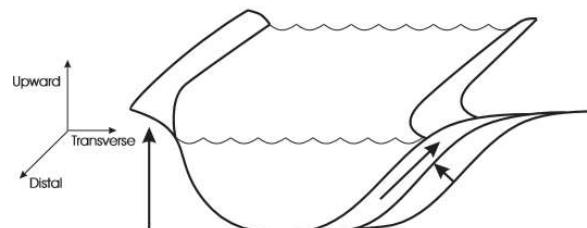
River meander model (Sylvester, 2009)



Deepwater trends (see authors above)

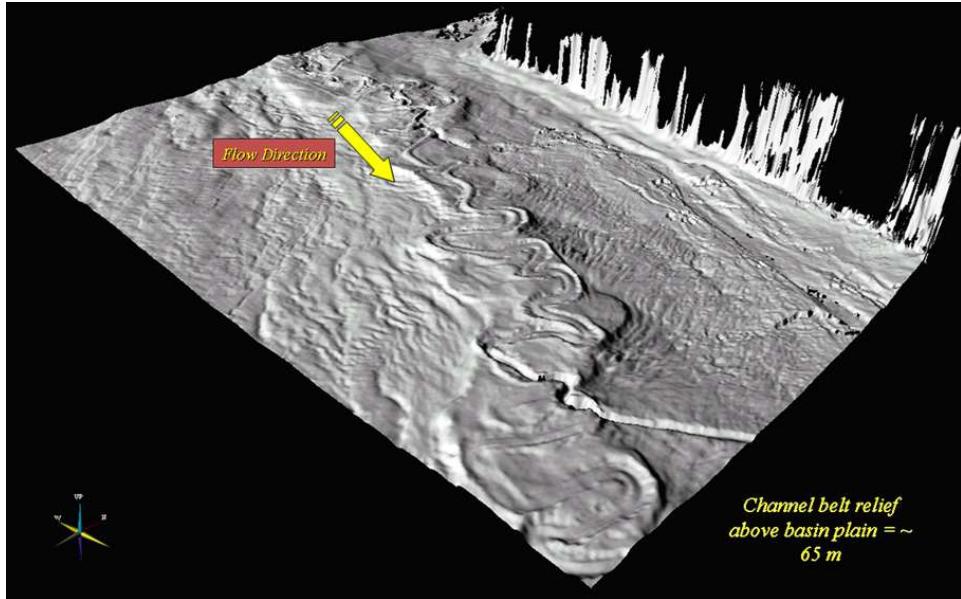


Turbidite process-mimicking model (Pyrcz et al., 2005)

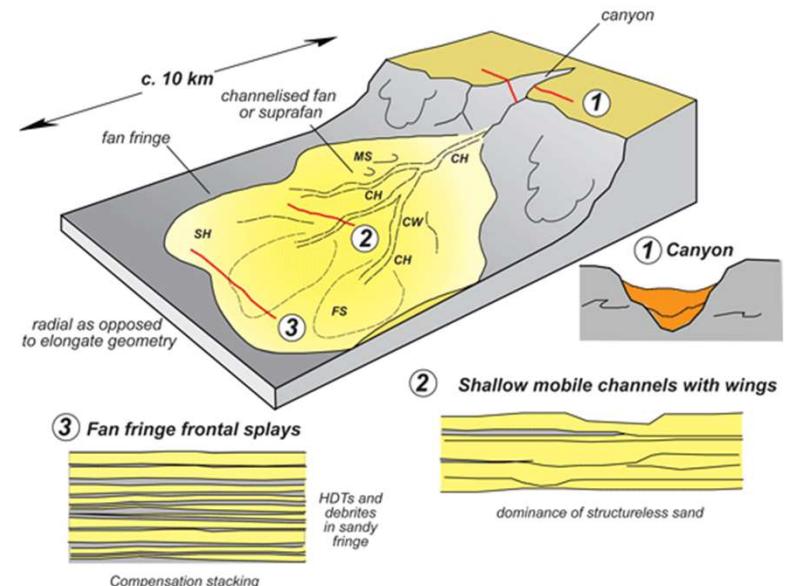


Three of the Thomas et al., 1998, lateral accretion trends (Pyrcz and Deutsch, 2004)

Trend Sources



Shallow seismic investigation (Posamentier, 2010)
<https://www.seismicatlas.org/entity?id=a10eb91f-5d3e-4f66-8fa3-9b2e63192823>



Information and process integration into conceptual models (SEPM Online Resources)



Outcrop, Ross Formation, Ireland
 (Sylvester, 2012)

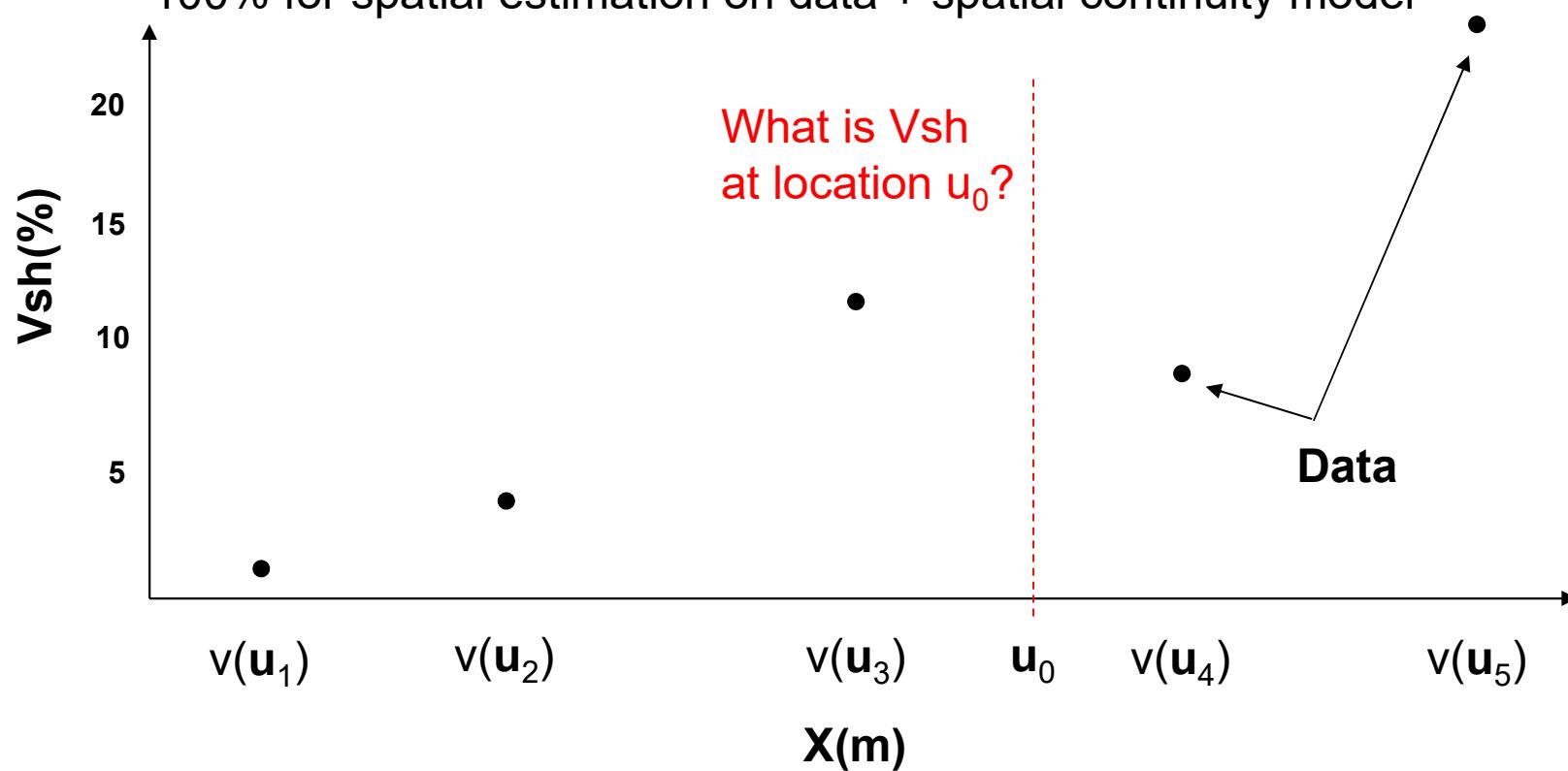


Jurassic Tank 'XES Run 02, Strong and Paola
https://www.sepml.org/CM_Files/da_strong_9_XES_02_stratigraphy_dip_LR.pdf

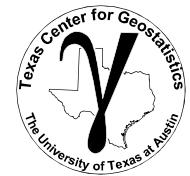
I will demonstrate data driven, but we often must move beyond the data.

Nonstationarity

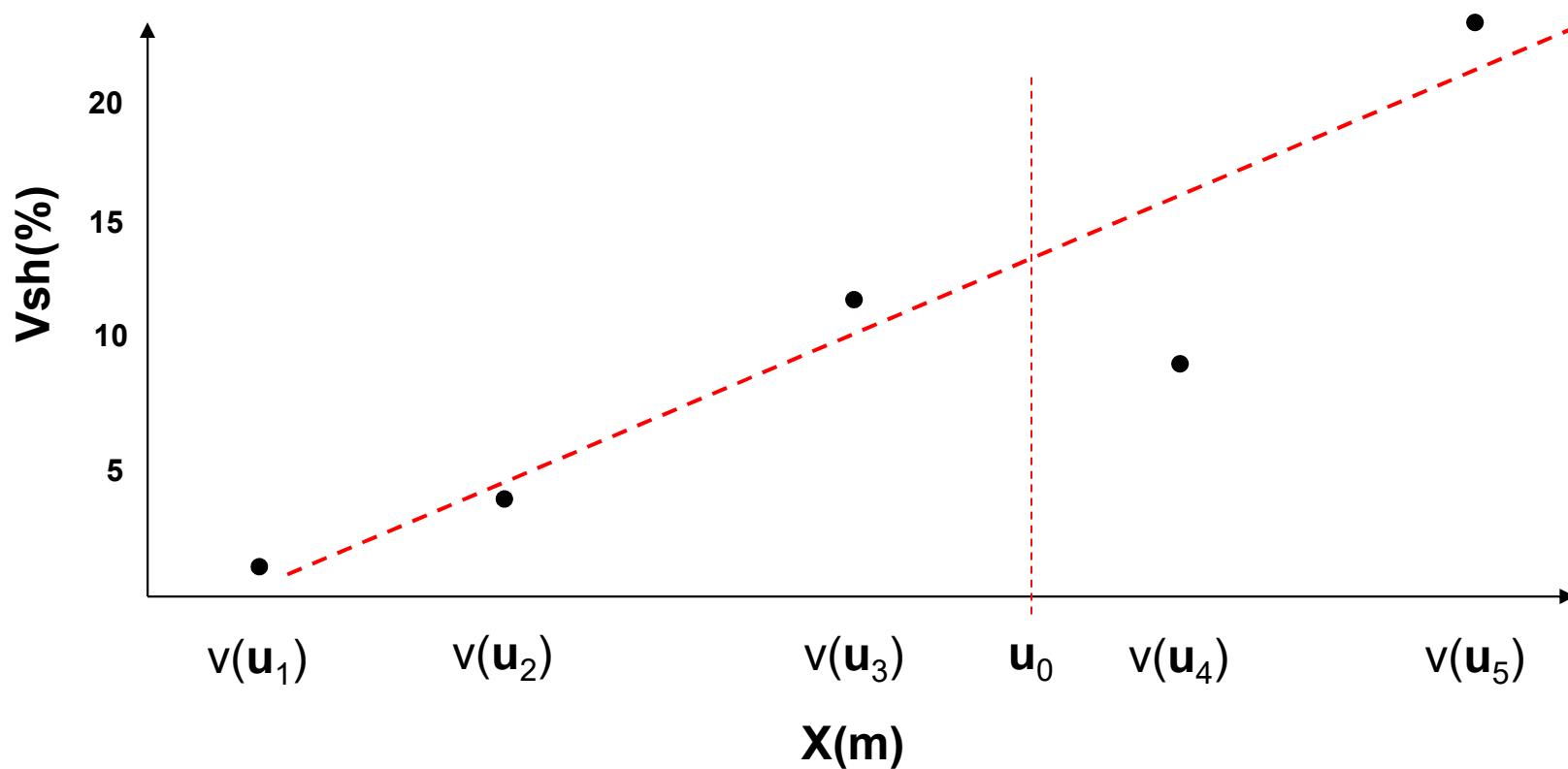
- Geostatistical spatial estimation methods will make an assumption concerning stationarity
 - In the presence of significant nonstationarity we would not rely 100% for spatial estimation on data + spatial continuity model



Nonstationarity



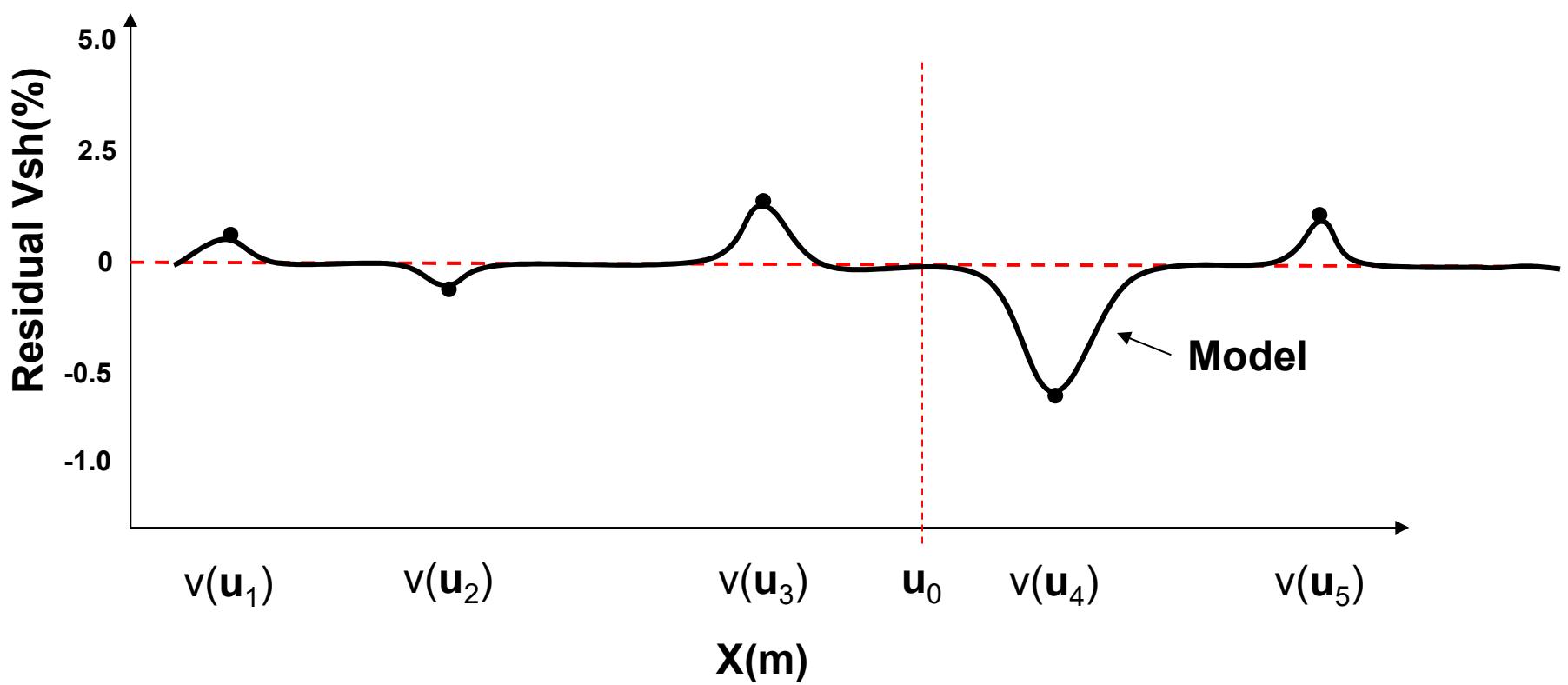
- Geostatistical spatial estimation methods will make an assumption concerning stationarity
 - If we observe a trend, we should model the trend.





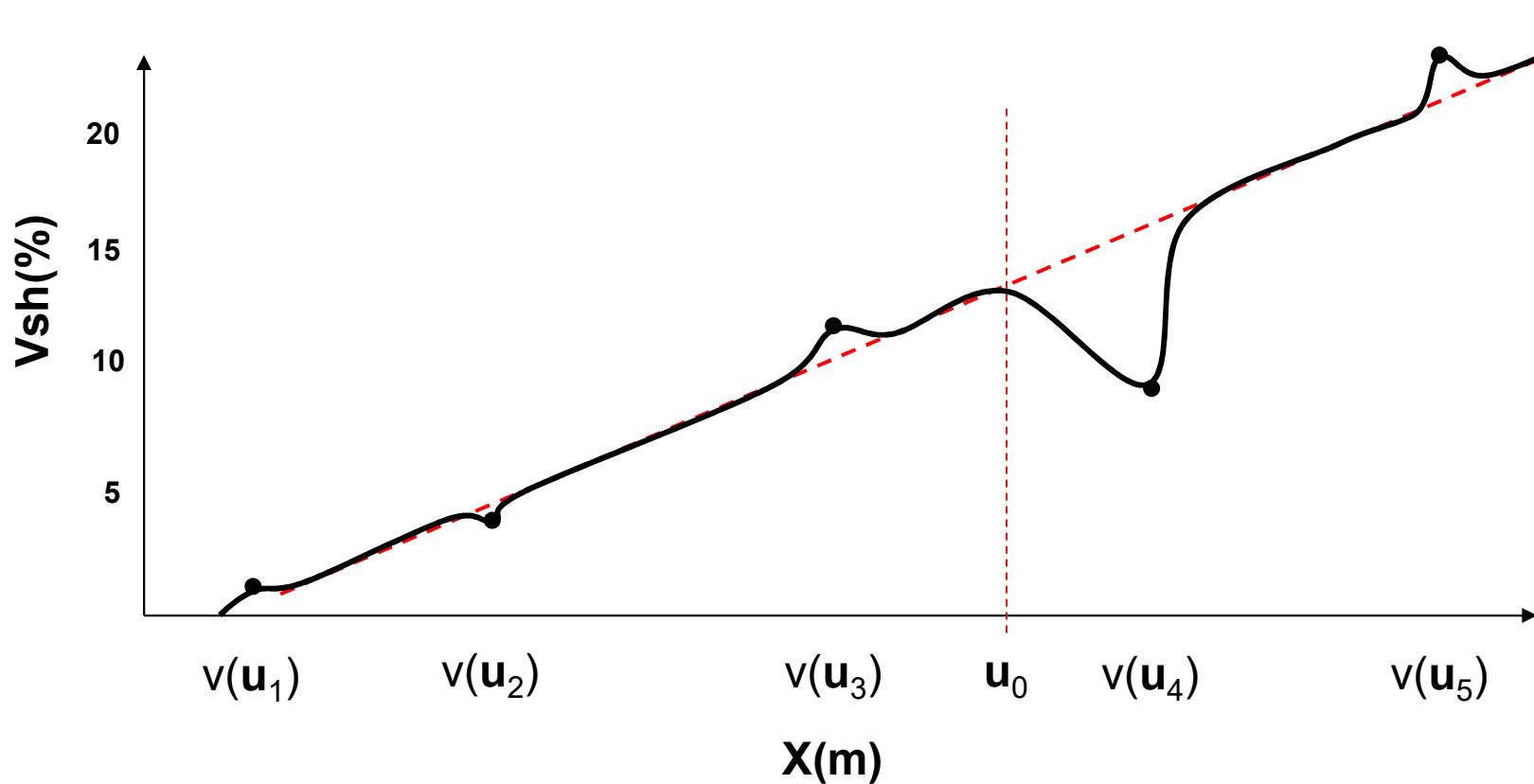
Nonstationarity

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
 - Then model the residuals.



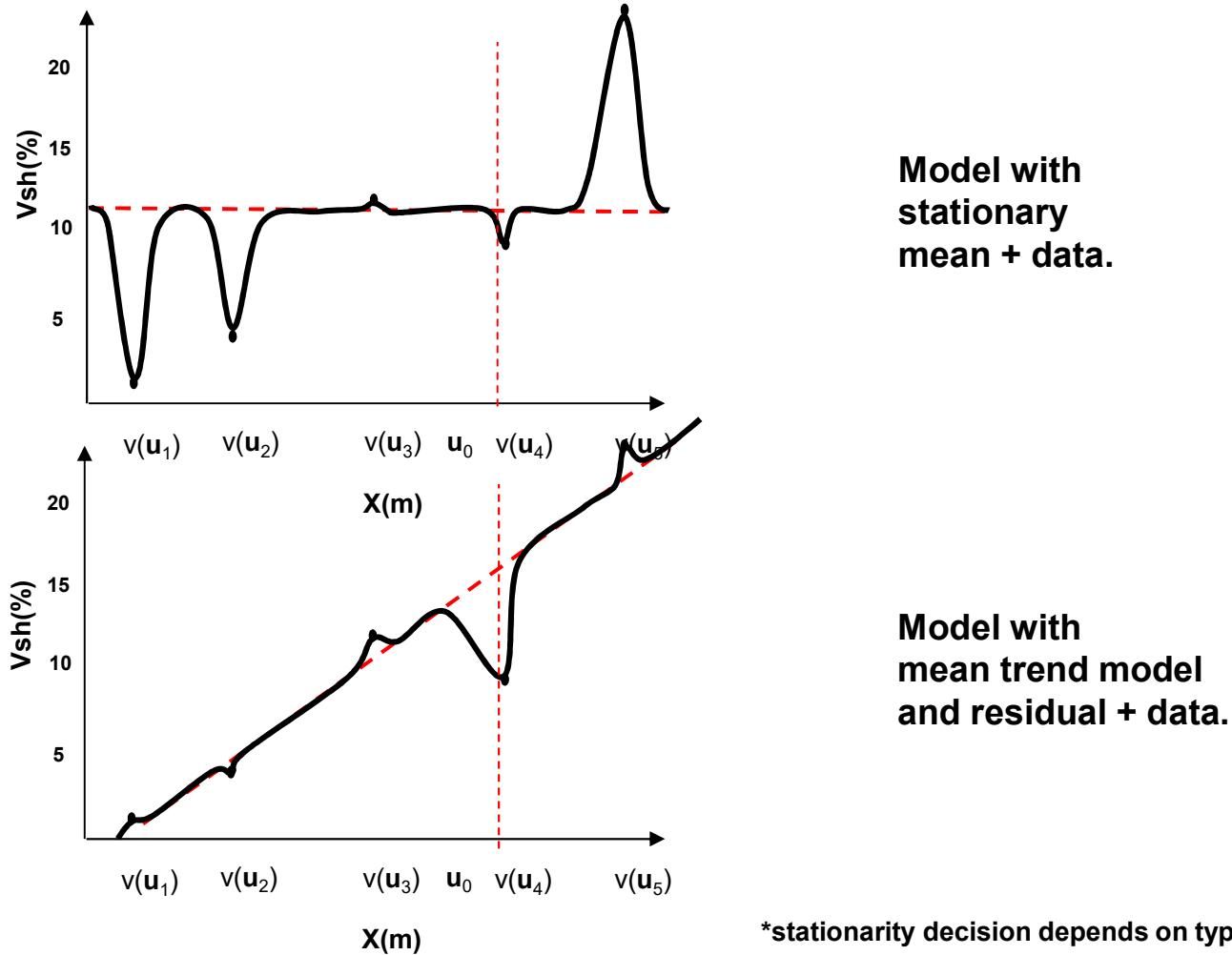
Nonstationarity

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
 - After modeling, add the trend back to the modelled residuals



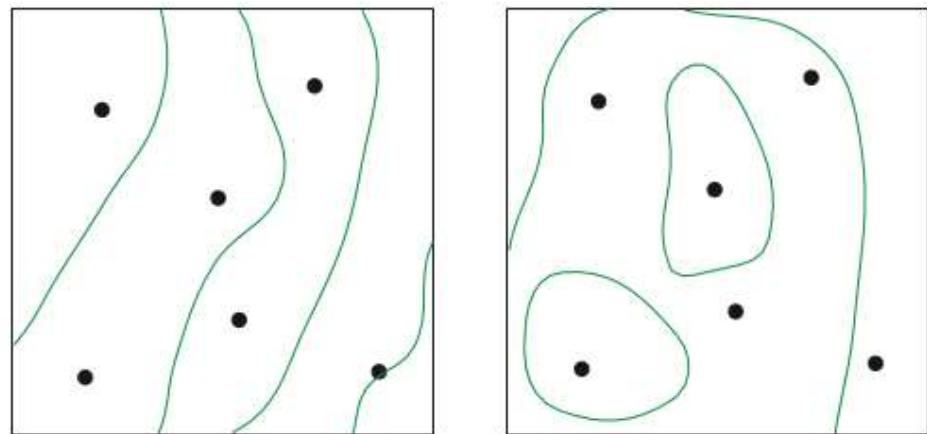
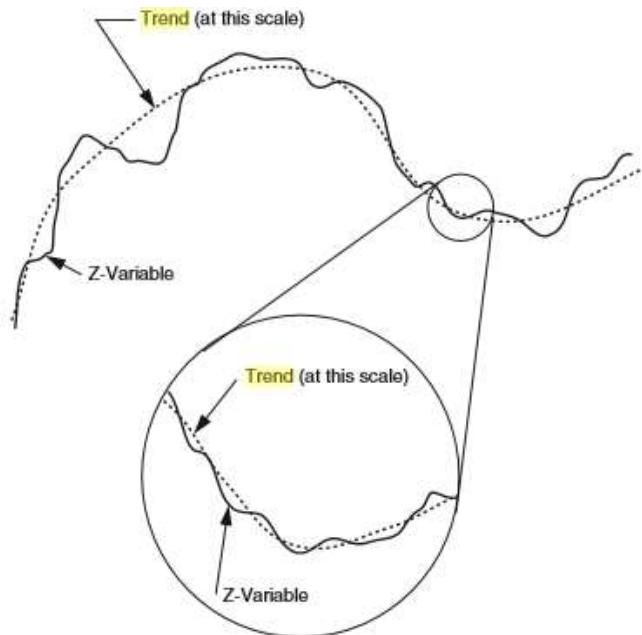
Nonstationarity

- How bad could it be if we did not model a trend?
- Geostatistical estimation would assume stationarity* and away from data we would estimate with the global mean (simple kriging)!



Nonstationarity

- Trend Modeling
 - We must identify and model trends / nonstationarities



- While we discuss data-driven trend modeling here any **trend modeling should include data integration** over the entire asset team
 - Geology
 - Geophysics
 - Petrophysics
 - Reservoir Engineering

Images from Pyrcz and Deutsch (2014)

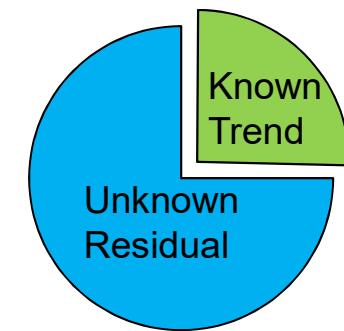
Nonstationarity

- Any variance in the trend is removed from the residual:

$$\sigma_X^2 = \sigma_{X_t}^2 + \sigma_{X_r}^2 + 2C_{X_t, X_r}$$

- if the $X_t \perp\!\!\! \perp X_r$, $C_{X_t, X_r} = 0$

$$\sigma_{X_r}^2 = \sigma_X^2 - \sigma_{X_t}^2$$



- So if σ_X^2 is the total variance (variability), and $\sigma_{X_t}^2$ is the variability that is deterministically modelled, treated as known, and $\sigma_{X_r}^2$ is the component of the variability that is treated as unknown.
- Result: the more variability explained by the trend the less variability that remains as uncertain.

Trend and Residual Assumptions



- The residual is stationary
- The residual variance is homoscedastic (does not depend on the magnitude of the trend) or distribution is wrong.
- Trend is smoothly varying or spatial continuity is wrong
- Alternatives to trend model include cosimulations with secondary data to impose nonstationarity

Additivity of Variance for Decomposing Trend and Residual



Can we partition variance of random variable Z between trend (X) and residual (Y)?

$$\sigma_Z^2 = E(Z^2) - [E(Z)]^2$$

- Start with the variance of Z:

- Substitute: $Z = X + Y$

$$\sigma_{X+Y}^2 = E((X+Y)^2) - [E(X+Y)]^2$$

$$\sigma_{X+Y}^2 = E(X^2 + 2XY + Y^2) - [E(X) + E(Y)]^2$$

$$\sigma_{X+Y}^2 = E(X^2) + 2E(XY) + E(Y^2) - (E(X)^2 + 2E(X)E(Y) + E(Y)^2)$$

$$\sigma_{X+Y}^2 = \boxed{E(X^2) - E(X)^2} + \boxed{E(Y^2) - E(Y)^2} + 2\boxed{(E(XY) - E(X)E(Y))}$$

$$\sigma_X^2 \qquad \qquad \qquad \sigma_Y^2 \qquad \qquad \qquad C_{XY}(0)$$

- Note covariance: $C_{XY} = E(XY) - E(X)E(Y)$

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2C_{XY}(0) \quad \triangleleft \text{ Additivity of variance}$$

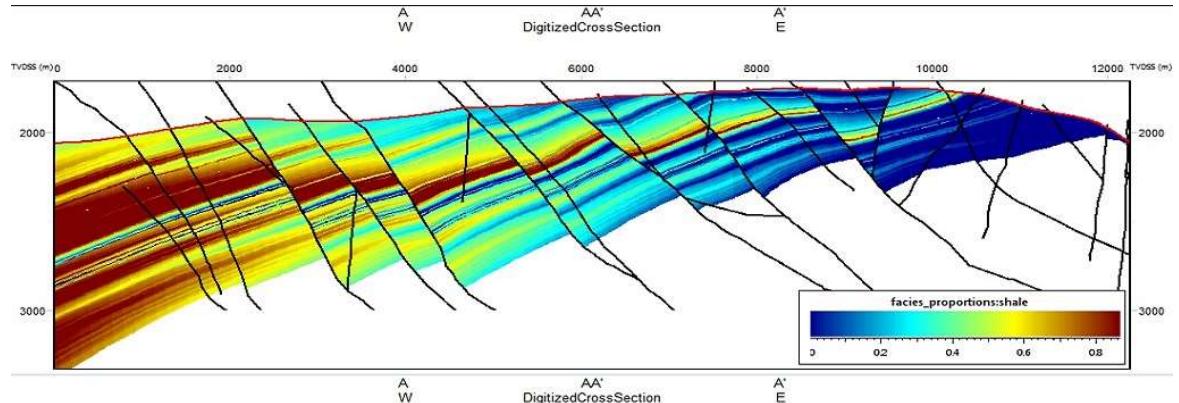
- If ~~the~~ $X \perp Y$, $C_{XY}(0) = 0$, then $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$ \triangleleft In practice

Definition Deterministic Model



- Model that assumes perfect knowledge, without uncertainty
- Based on knowledge of the phenomenon or trend fitting to data
- Most subsurface models have a deterministic component (trend) to capture expert knowledge and to provide a stationary residual for geostatistical modeling.

Trend Modeling



- Trend models:
 - Tend to be smooth, based on data and interpretation
 - May be complicated (see above)
 - Parameterized by vertical proportion curves (see below) and areal trend maps

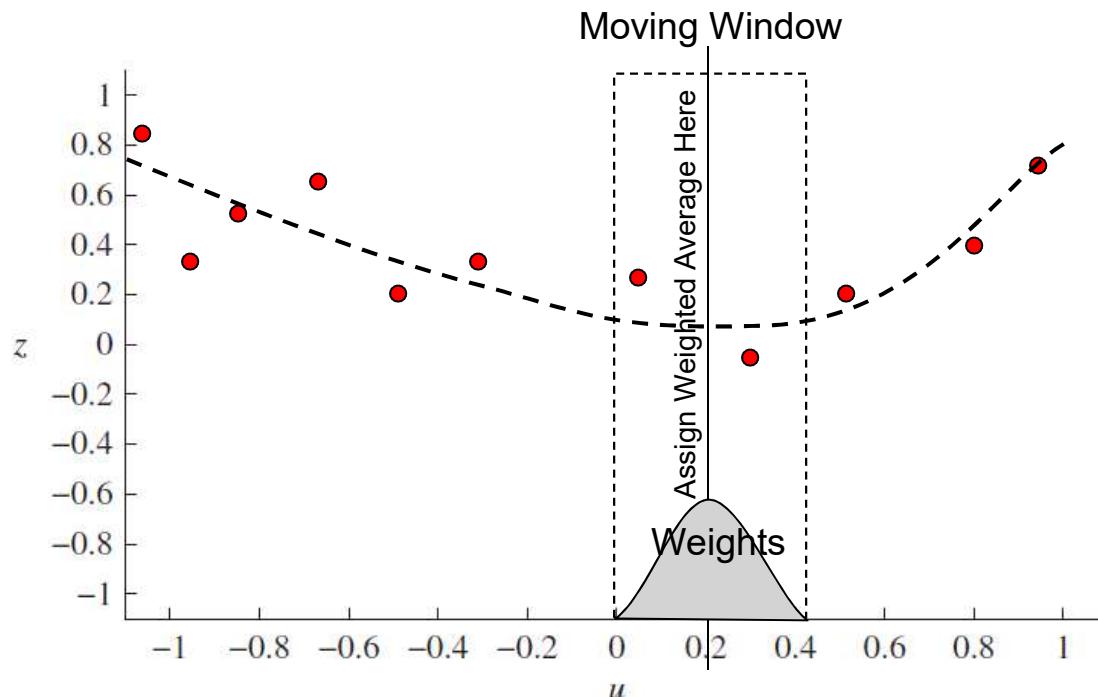
Example facies trend model Gocad SKUA. <http://www.pdgm.com/getdoc/b24891f9-7470-4728-8cb7-0ddd7df196df/skua-facies-modeling/>

Example facies trend model Gocad.
<http://www.pdgm.com/getdoc/bd9ab6b6-7dbb-4023-8f79-de34238d5136/skua-reservoir-data-analysis/>

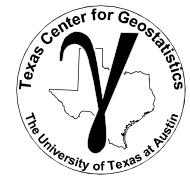
Trend Modeling Workflow



- How to calculate a trend model:
 - Moving window average of the available data
 - Weighting scheme within the window
 - » Uniform weights can cause discontinuities
 - » Reduce weight at edges of moving window to reduce discontinuities (e.g. Gaussian weights).



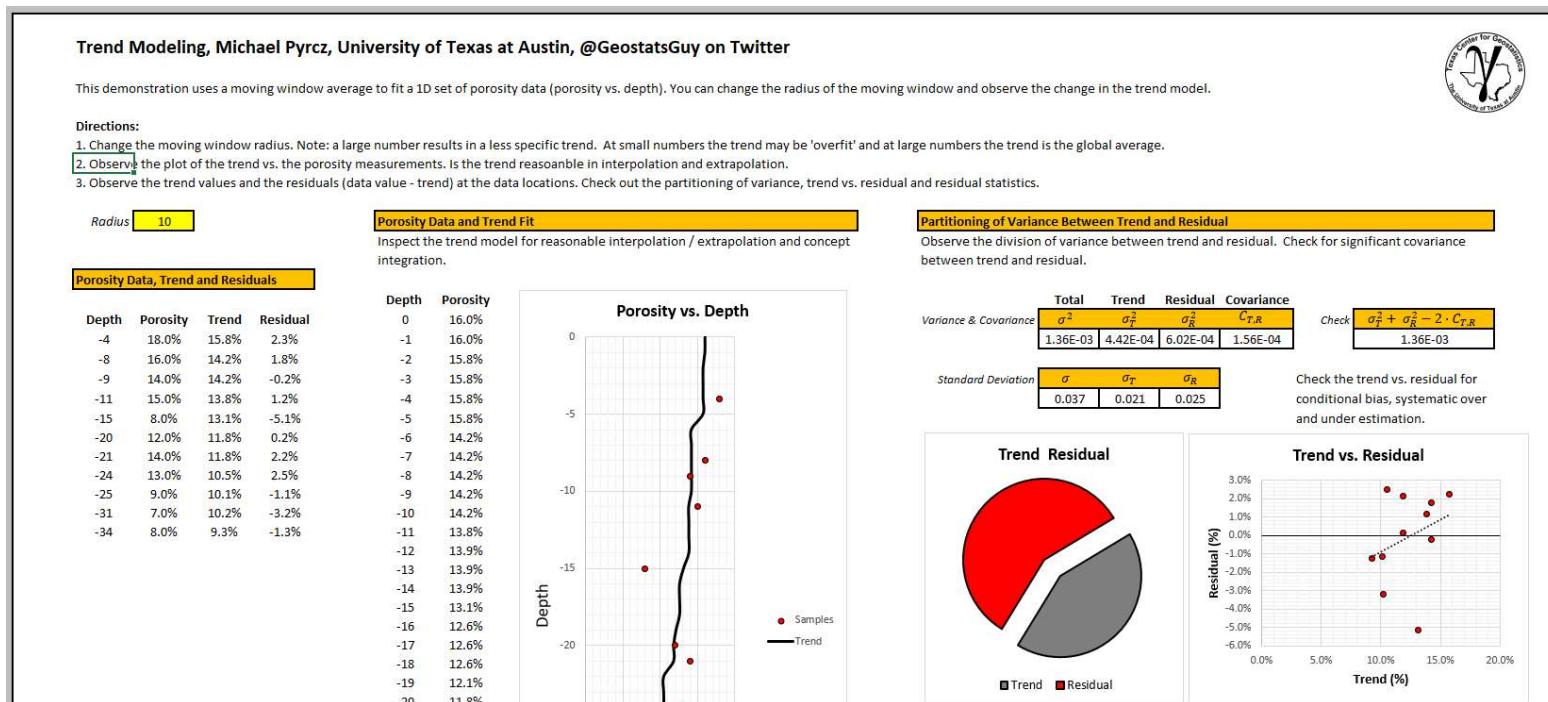
Trend Modeling Hands-on



Here's an opportunity for experiential learning with Trend Modeling.

- **Things to try:**

1. Set the radius very large (50). How's the trend model performing? Try radius very small (1).
2. What do you think is the best radius to fit a trend to this spatial data?



File Name: Trend_Modeling.xlsx

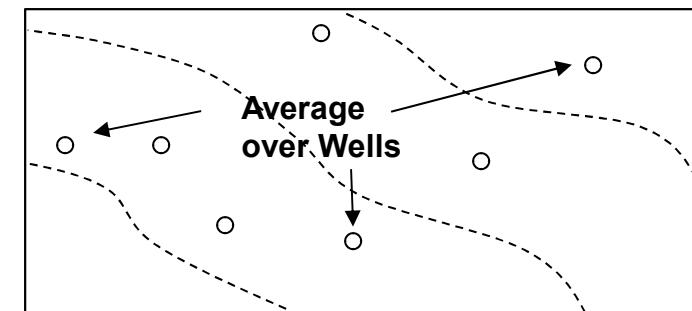
File is at: <https://git.io/fhALP>

Trend 2D + 1D Workflow

- Calculate 2D Area and 1D Vertical trends:



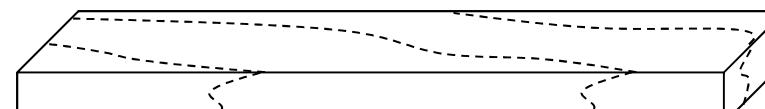
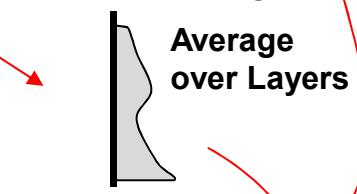
2D Areal Trend From Well Averages



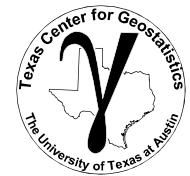
- Combine 1D and 2D \rightarrow 3D.

$$\bar{X}(x, y, z) = \bar{X}(z) \cdot \frac{\bar{X}(x, y)}{\bar{X}}$$

1D Vertical Trend Layer Averages



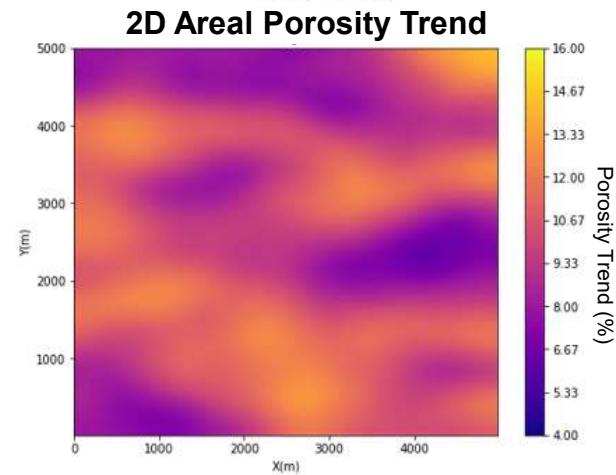
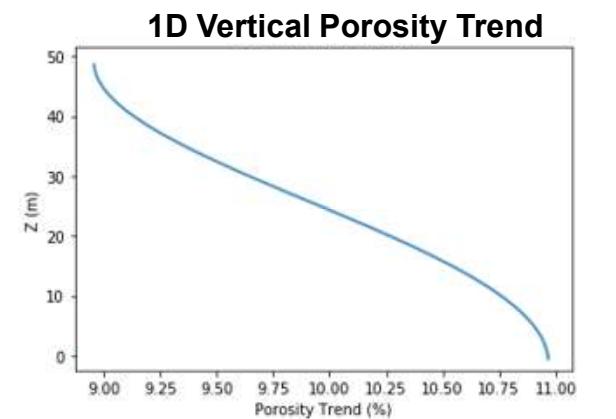
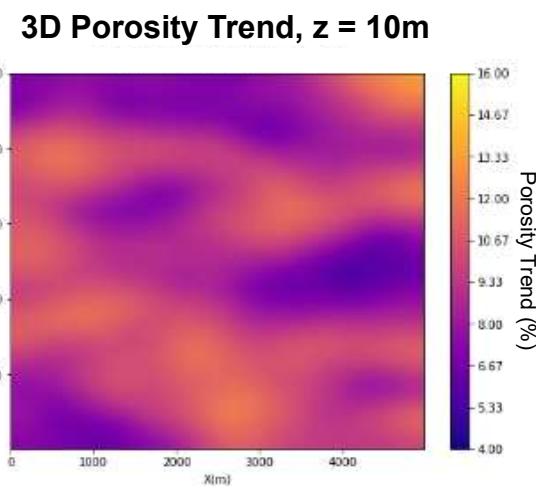
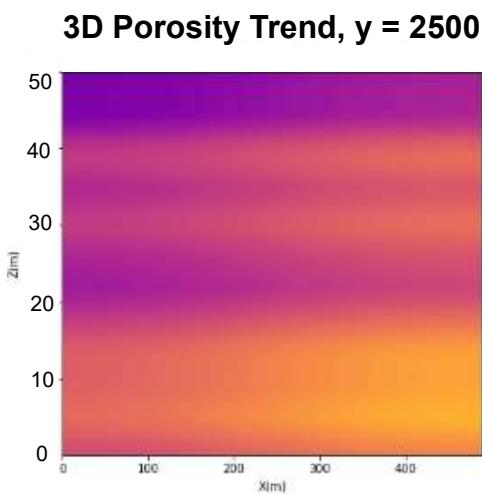
Trend Modeling Workflow



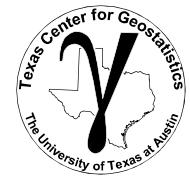
- How to calculate a trend model:
 - Calculate the 2D areal trend by interpolating over vertically averaged wells.
 - Calculate the 1D vertical trend by averaging layers
 - Combine the 1D vertical and the 2D areal trends:

$$\bar{X}(x, y, z) = \bar{X}(z) \cdot \frac{\bar{X}(x, y)}{\bar{X}}$$

3D Trend 1D Vertical Trend 2D Areal Trend
→ → →
→ →



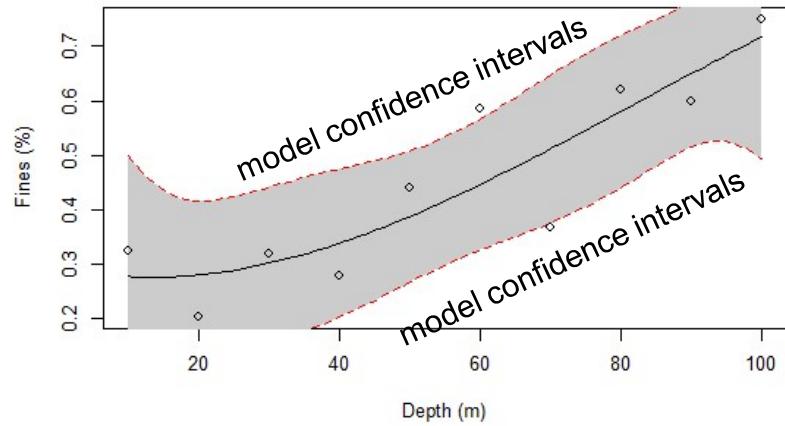
Trend Definition



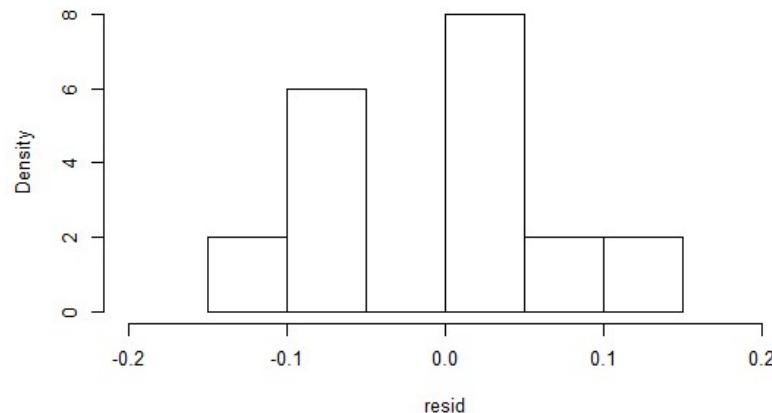
- Observation of nonstationarity in any statistic, metric of interest
- A model of the nonstationarity in any statistic, metric of interest
- Typically modeled with support of data and expert knowledge in a deterministic manner (without uncertainty).

Overfitting

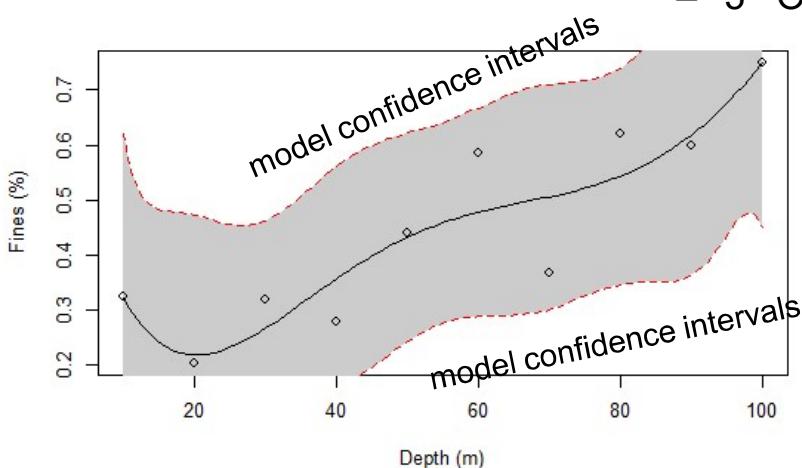
- Example of trend fits:
 - 3rd Ordered Polynomial



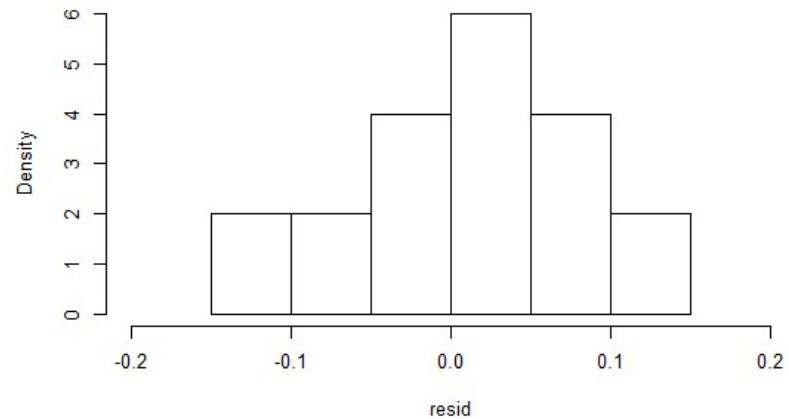
Distribution of Residuals



- 5th Order Polynomial



Distribution of Residuals

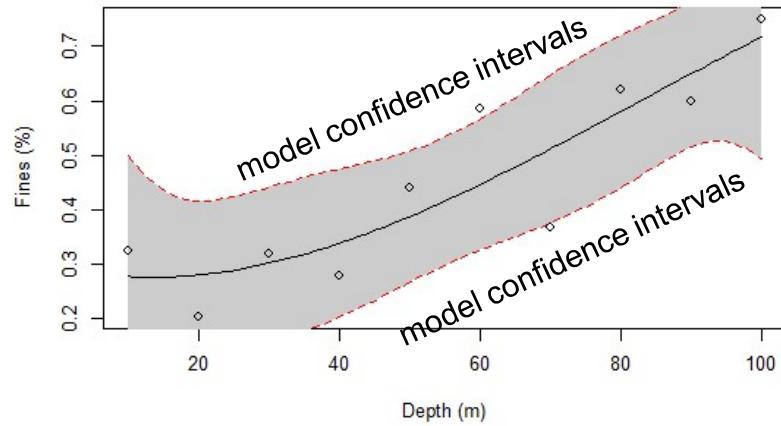


Overfit demonstration in R, code is here:
<https://github.com/GeostatsGuy/geostatsr/blob/master/overfit.R>

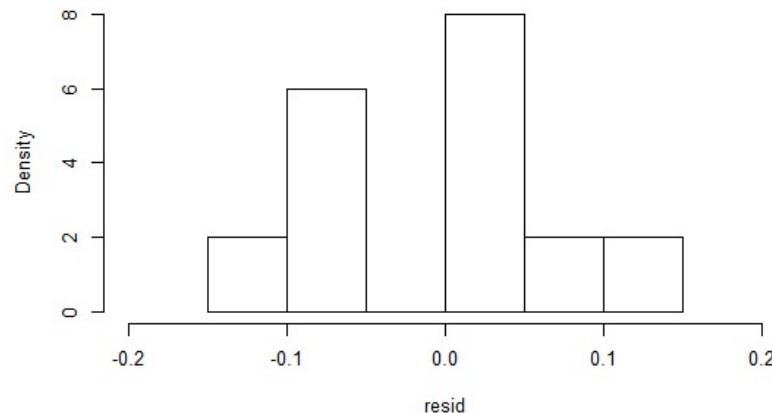
R code at [Code/Overfit.R](#)

Overfitting

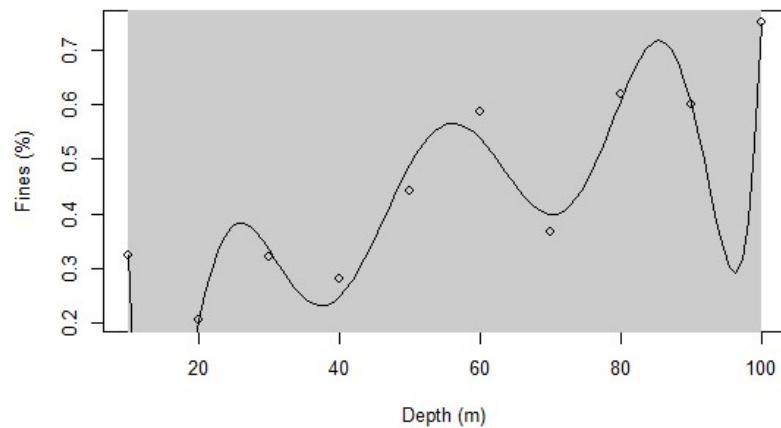
- Example of trend fits:
 - 3rd Ordered Polynomial



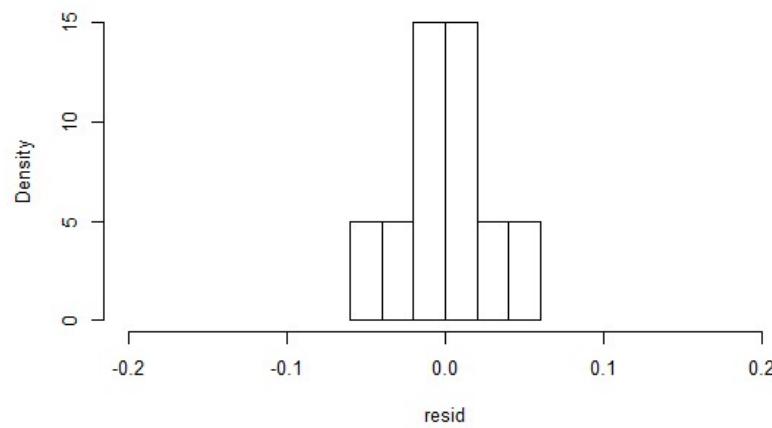
Distribution of Residuals



- 8th Order Polynomial



Distribution of Residuals



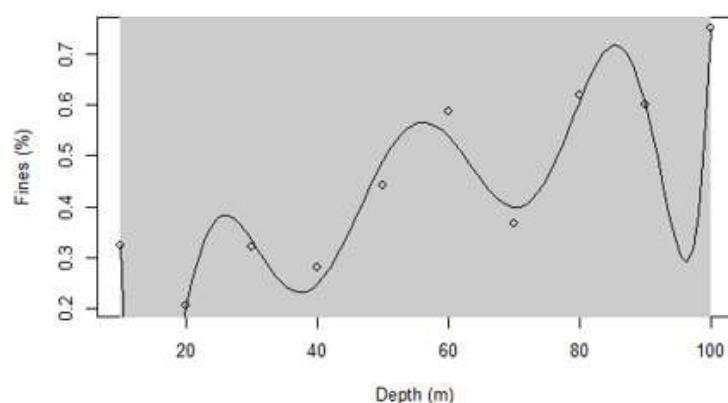
Overfit demonstration in R, code is here:

<https://github.com/GeostatsGuy/geostatsr/blob/master/overfit.R>

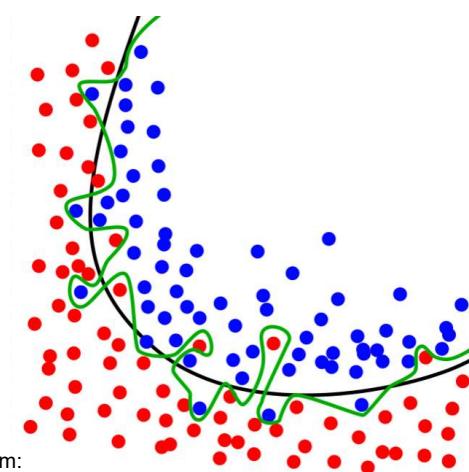
R code at [Code/Overfit.R](#)

Definition of Overfitting

- Overly complicated model to explain “idiosyncrasies” of the data, capturing data noise in the model
- More parameters than can be justified with the data
- Results in likely very high error away from the data
- But, results in low residual variance!
- High R^2
- Very accurate at the data! - Claim you know more than you actually do!



Overfit demonstration in R, code is here:
<https://github.com/GeostatsGuy/geostatsr/blob/master/overfit.R>



Overfit classification model example from:
<https://en.wikipedia.org/wiki/Overfitting#/media/File:Overfitting.svg>

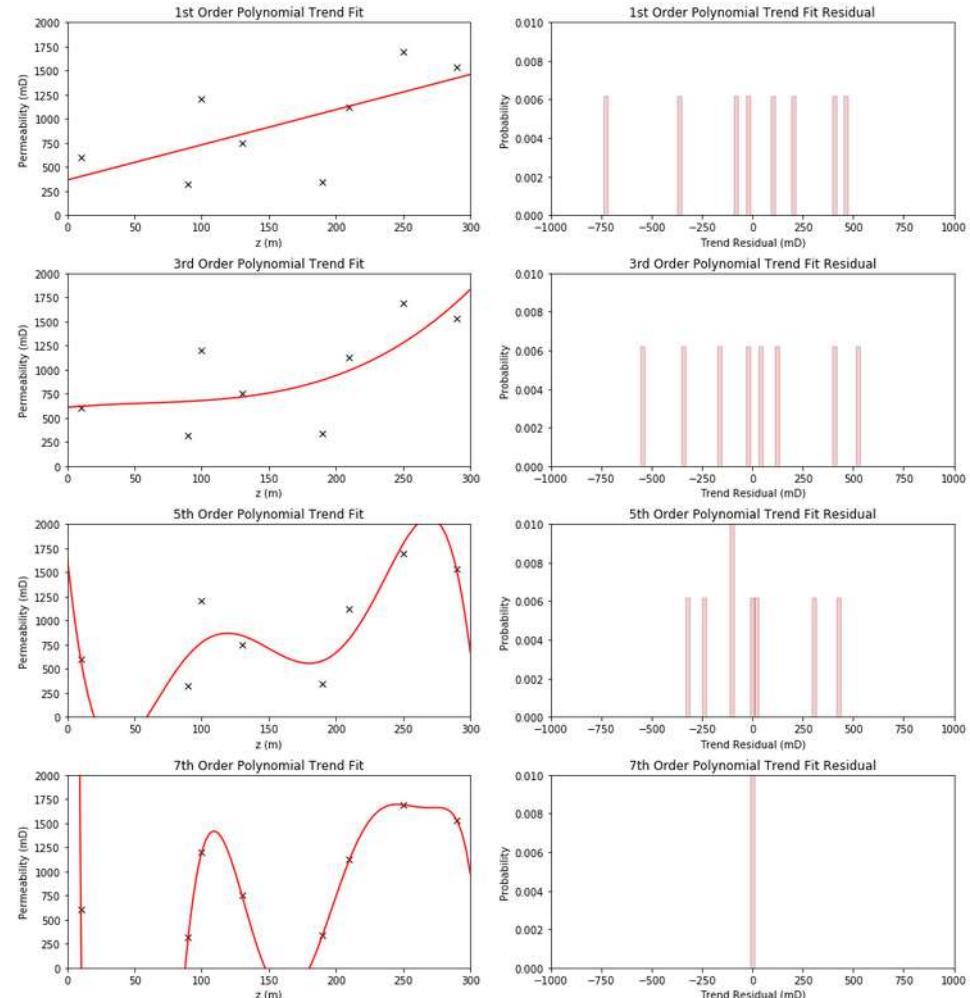
Overfit Hands-on

Here's an opportunity for experiential learning with the concept of overfit.

- Things to observe:**

While we move from simple to complicated models note:

1. The behavior away from the training data.
2. Residual at the training data locations.
3. Fit relative to testing data (withheld).
4. Confidence interval for the model given the training data.



Trend Modeling In Python Demo



Here's an opportunity for experiential learning with the concept of overfit.

• Things to try:

1. Evaluate the trend and residual statistics.
2. Build an overfit trend.
3. Build an underfit trend.
4. Observed the residual spatial distribution.
5. Consider how else you could build a trend.

GeostatsPy: Univariate Spatial Trend Modeling for Subsurface Data Analytics in Python

Michael Pyrcz, Associate Professor, University of Texas at Austin

[Twitter](#) | [GitHub](#) | [Website](#) | [Google Scholar](#) | [Book](#) | [YouTube](#) | [LinkedIn](#)

PGE 383 Exercise: Univariate Spatial Trends Modeling for Subsurface Data Analytics in Python

Here's a simple workflow with basic univariate spatial trend modeling for subsurface modeling workflows. This should help you get started with building subsurface models that include deterministic and stochastic components.

Trend Modeling

Trend modeling is the modeling of local features, based on data and interpretation, that are deemed certain (known). The trend is subtracted from the data, leaving a residual that is modeled stochastically with uncertainty (treated as unknown).

- geostatistical spatial estimation methods will make an assumption concerning stationarity
 - in the presence of significant nonstationarity we can not rely on spatial estimates based on data + spatial continuity model
 - if we observe a trend, we should model the trend.
 - then model the residuals stochastically

Steps:

1. model trend consistent with data and interpretation at all locations within the area of interest, integrate all available information and expertise.

$$m(\mathbf{u}_\beta), \forall \beta \in AOI$$

1. subtract trend from data at the n data locations to formulate a residual at the data locations.

$$y(\mathbf{u}_\alpha) = z(\mathbf{u}_\alpha) - m(\mathbf{u}_\alpha), \forall \alpha = 1, \dots, n$$

1. characterize the statistical behavior of the residual $y(\mathbf{u}_\alpha)$ integrating any information sources and interpretations. For example the global cumulative distribution function and a measure of spatial continuity shown here.

$$F_y(y) \quad r_y(\mathbf{h})$$

1. model the residual at all locations with L multiple realizations.

$$Y^\ell(\mathbf{u}_\beta), \forall \beta \in AOI; \ell = 1, \dots, L$$

1. add the trend back to the stochastic residual realizations to calculate the multiple realizations, L , of the property of interest based on the composite model of known deterministic trend, $m(\mathbf{u}_\alpha)$ and unknown stochastic residual, $y(\mathbf{u}_\alpha)$

$$Z^\ell(\mathbf{u}_\beta) = Y^\ell(\mathbf{u}_\beta) + m(\mathbf{u}_\beta), \forall \beta \in AOI; \ell = 1, \dots, L$$

1. check the model, including quantification of the proportion of variance treated as known (trend) and unknown (residual).

$$\sigma_Z^2 = \sigma_Y^2 + \sigma_m^2 + 2 \cdot C_{Y, m}$$

given $C_{Y, m} \rightarrow 0$:

$$\sigma_Z^2 = \sigma_Y^2 + \sigma_m^2$$

I can now describe the proportion of variance allocated to known and unknown components as follows:

$$Prop_{Known} = \frac{\sigma_m^2}{\sigma_Y^2 + \sigma_m^2} \quad Prop_{Unknown} = \frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_m^2}$$

I provide some practical, data-driven methods for trend model, but I should indicate that:

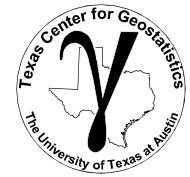
1. trend modeling is very important in reservoir modeling as it has large impact on local model accuracy and on the uncertainty model
2. trend modeling is used in almost every subsurface model, unless the data is dense enough to impose local trends
3. trend modeling includes a high degree of expert judgement combined with the integration of various information sources

Univariate Statistics New Tools



Topic	Application to Subsurface Modeling
Random Variable	<p>A value at a location (and time) that can take multiple possible outcomes.</p> <p><i>Represent uncertainty in measures, provide predrill assessments.</i></p>
Stationarity	<p>Pooling samples over space (and time) for inference.</p> <p><i>While aware of the limitations, use them method to calculate uncertainty in e.g. mean porosity and carry through workflow as scenarios</i></p>

Data Analytics and Geostatistics: Sparse Data



Lecture outline . . .

- **Random Variable**
- **Random Function**
- **Stationarity**
- **Trends**