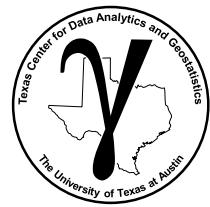


Data Analytics, Geostatistics and Machine Learning

Spatial Continuity / Prediction



Lecture outline . . .

- Stationarity
- Spatial Continuity
- Variogram Calculation
- Spatial Estimation

Introduction

Fundamental Concepts

Probability

Data Prep / Analytics

Spatial Continuity / Prediction

Multivariate Modeling

Uncertainty Modeling

Machine Learning

Instructor: Michael Pyrcz, the University of Texas at Austin

Other Resources

YouTube Lectures on:

Stationarity:

<https://youtu.be/QwxQ9xuUHIU>



Stationarity Definition 1: Geologic



Geological Definition: e.g. 'The rock over the stationary domain is sourced, deposited, preserved, and postdepositionally altered in a similar manner, the domain is map-able and may be used for local prediction or as information for analogous locations within the subsurface; therefore, it is useful to pool information over this expert mapped volume of the subsurface.'

Variogram Calculation:

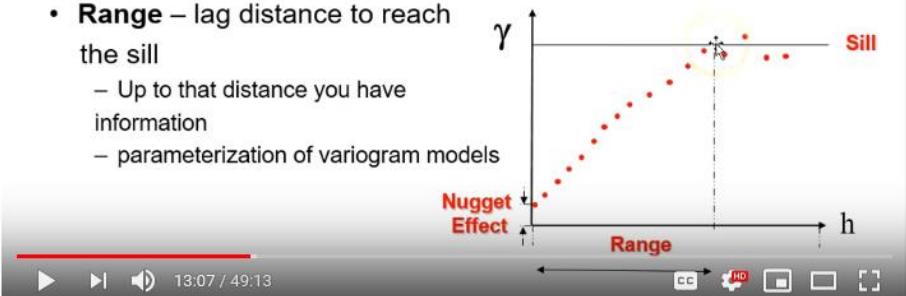
<https://youtu.be/mzPLicovE7Q>



Variogram Components Definition



- **Nugget Effect** – discontinuity in the variogram at distances less than the minimum data spacing
 - As a ratio of nugget / sill, is known as relative nugget effect (%)
 - Measurement error, mixing populations cause apparent nugget effect
- **Sill** – the sample variance
 - Interpret spatial correlation relative to the sill, level of no correlation
- **Range** – lag distance to reach the sill
 - Up to that distance you have information
 - parameterization of variogram models

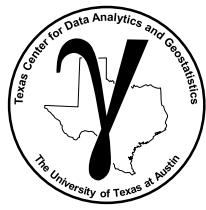


Variogram Interpretation:

<https://youtu.be/Li-Xzlu7hvs>

Variogram Modeling:

<https://youtu.be/-Bi63Y3u6TU>



Data Analytics, Geostatistics and Machine Learning

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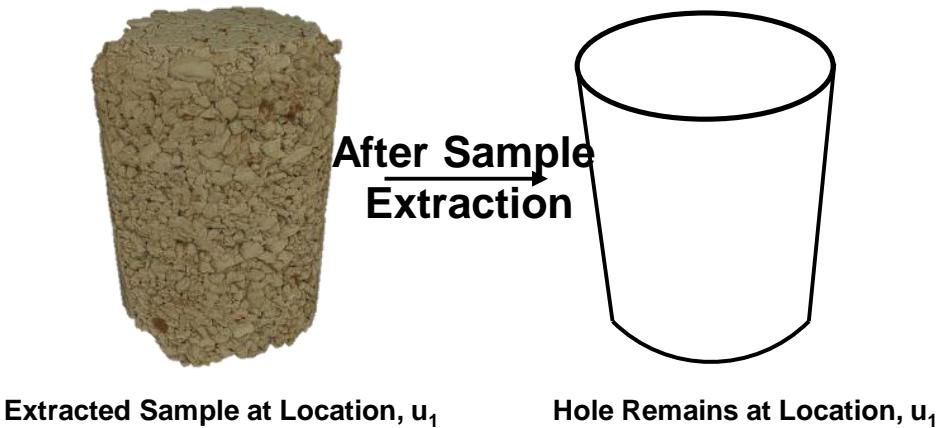
Uncertainty Modeling

Machine Learning

Instructor: Michael Pyrcz, the University of Texas at Austin

Stationarity Substituting Time for Space

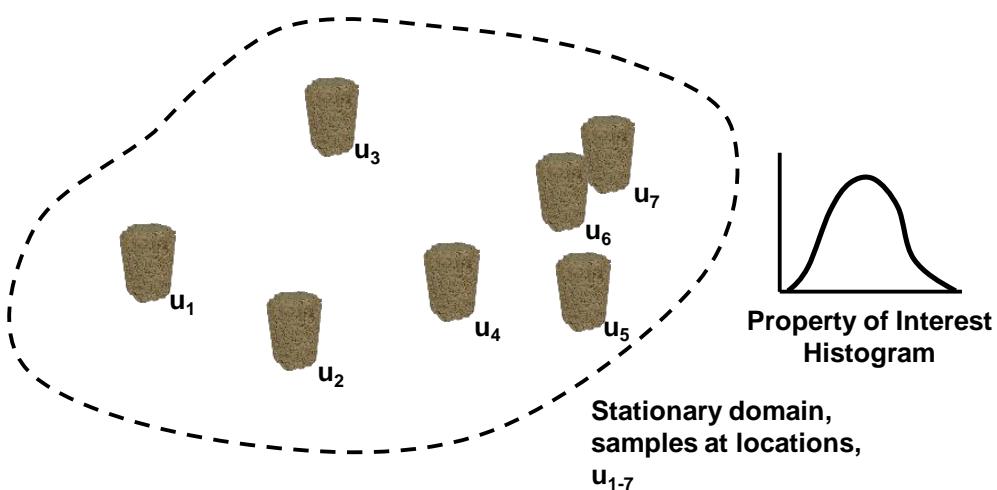
Any statistic requires replicates, repeated sampling (e.g. air or water samples from a monitoring station). In our geospatial problems repeated samples are not available at a location in the subsurface.



Instead of time, **we must pool samples over space** to calculate our statistics. This decision to pool is the decision of stationarity. It is the decision that the subset of the subsurface is all the “same stuff”.

Stationarity Substituting Time for Space

The decision of the stationary domain for sampling is an expert choice. Without it we are stuck in the “hole” and **cannot calculate any statistics** nor say anything about the behavior of the subsurface **between the sample data**.



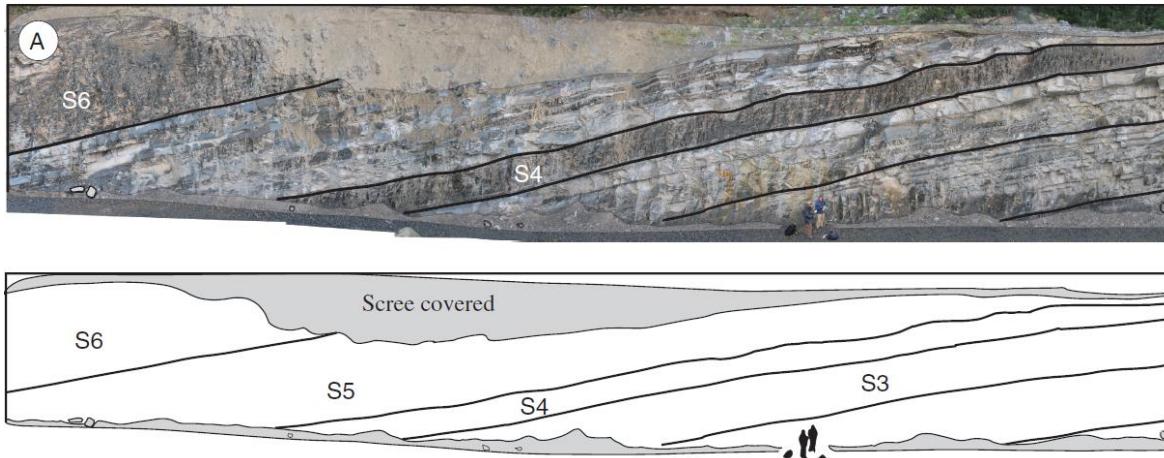
Import License: choice to pool specific samples to evaluate a statistic.

Export License: choice of where in the subsurface this statistic is applicable.

Stationarity

Definition 1: Geologic

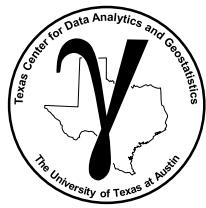
Geological Definition: e.g. ‘The rock over the stationary domain is sourced, deposited, preserved, and postdepositionally altered in a similar manner, the domain is map-able and may be used for local prediction or as information for analogous locations within the subsurface; therefore, it is useful to pool information over this expert mapped volume of the subsurface.’



Photomosaic, line drawing Punta Barrosa Formation sheet complex (Fildani et al. 2009).

Stationarity

Definition 2: Statistical



Statistical Definition: The metrics of interest are invariant under translation over the domain. For example, one point stationarity indicates that histogram and associated statistics do not rely on location, \mathbf{u} . Statistical stationarity for some common statistics:

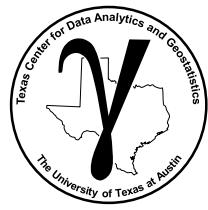
Stationary Mean: $E\{Z(\mathbf{u})\} = m, \forall \mathbf{u}$

Stationary Distribution: $F(\mathbf{u}; z) = F(z), \forall \mathbf{u}$

Stationary Semivariogram: $\gamma_z(\mathbf{u}; \mathbf{h}) = \gamma_z(\mathbf{h}), \forall \mathbf{u}$

Stationarity: *What metric / statistic? Over what volume?*

May be extended to any statistic of interest including, facies proportions, bivariate distributions and multiple point statistics.



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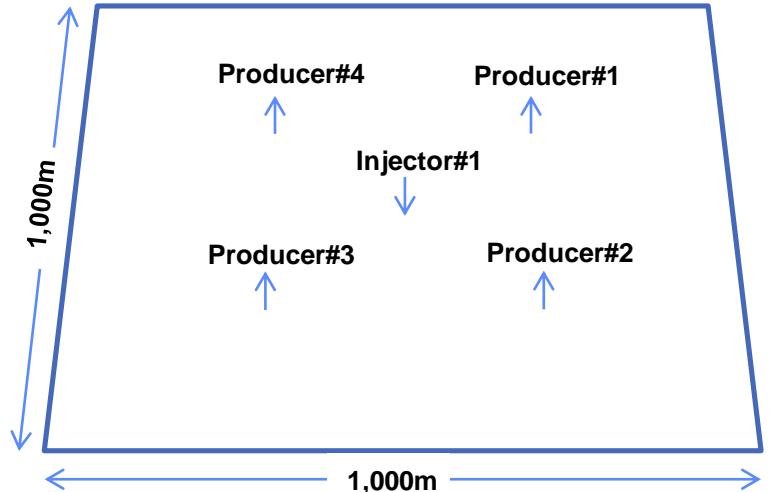
Machine Learning

Instructor: Michael Pyrcz, the University of Texas at Austin

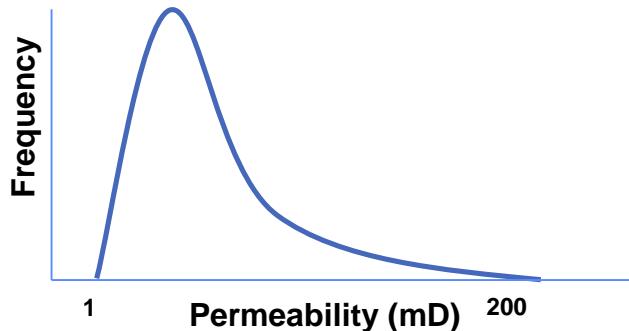
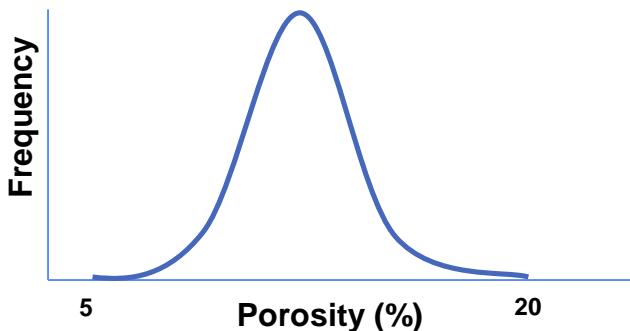
Motivation for Measuring Spatial Continuity

Simple Example

- Area of interest
- 1 Injector and 4 producers



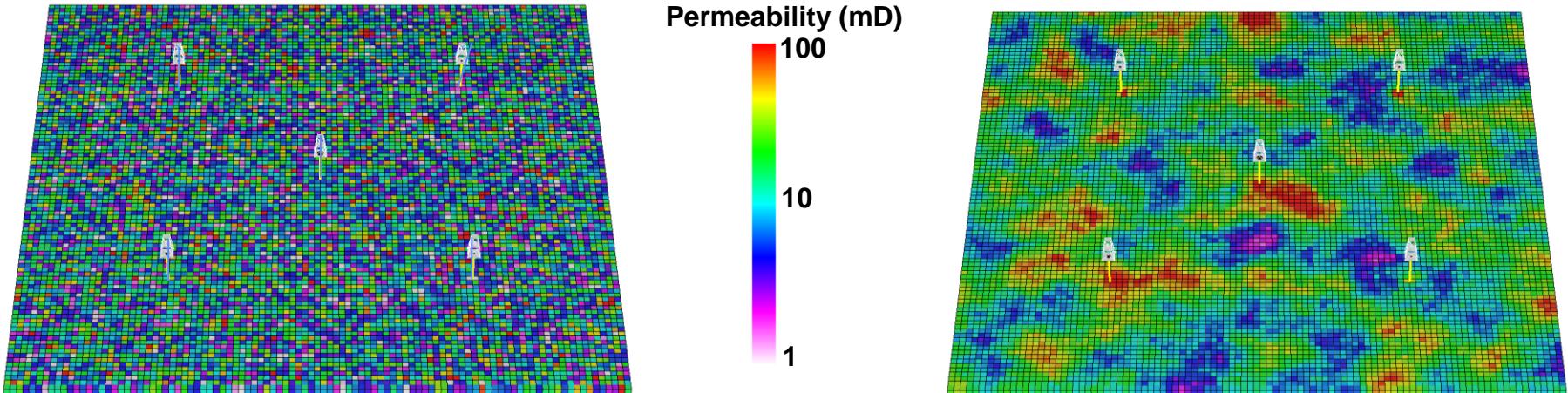
- Porosity and permeability distributions (held constant for all cases)



Motivation for Measuring Spatial Continuity

Does spatial continuity of reservoir properties matter?

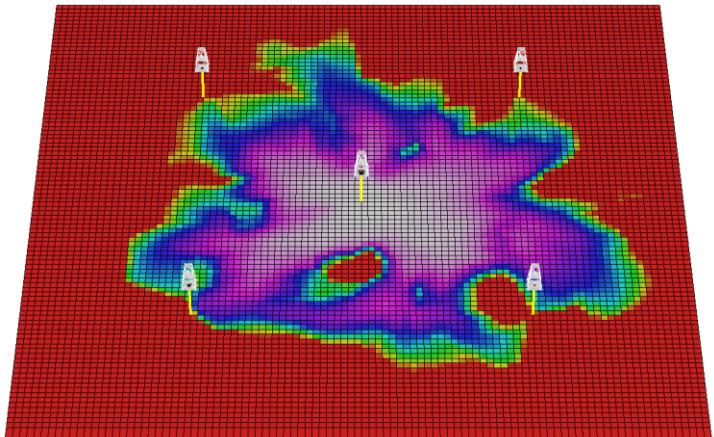
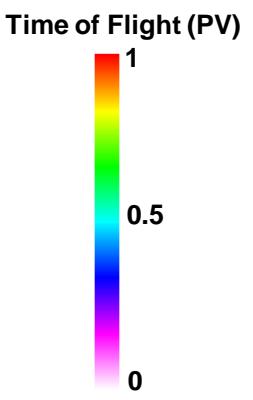
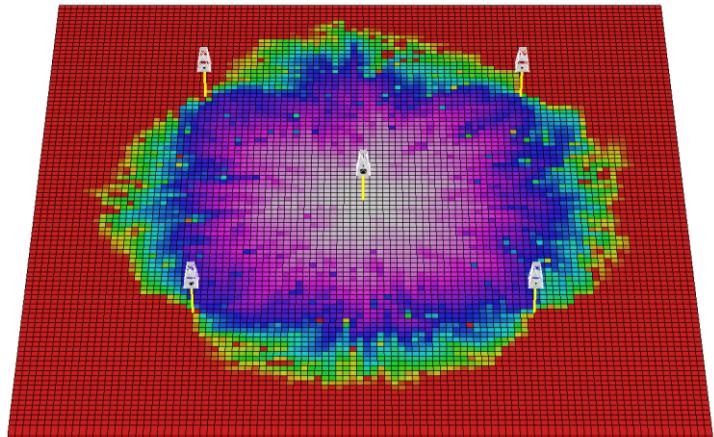
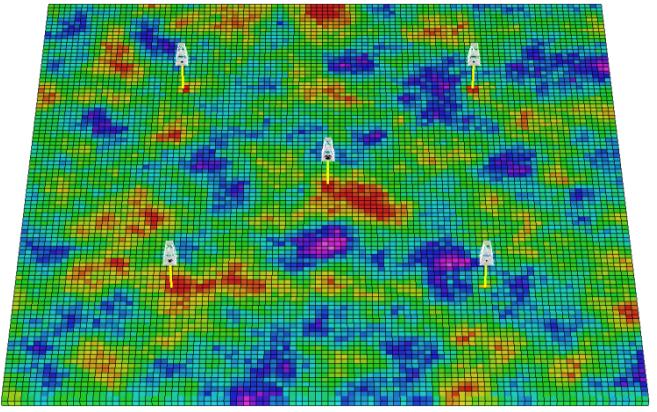
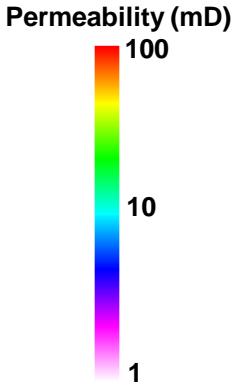
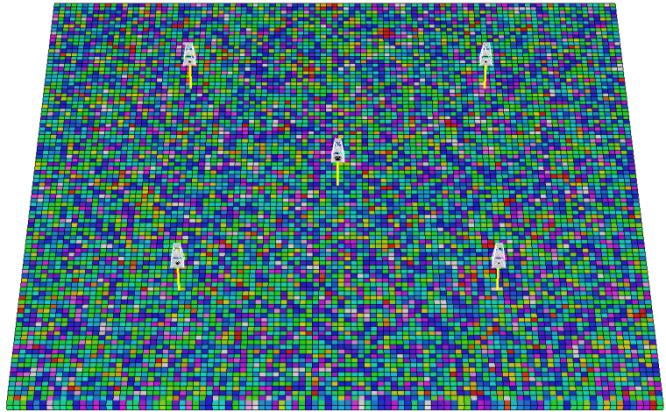
Consider these models of permeability



Recall – all models have the same porosity and permeability distributions

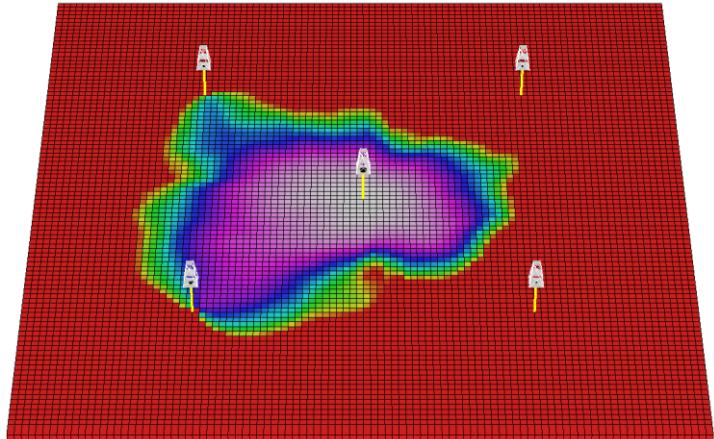
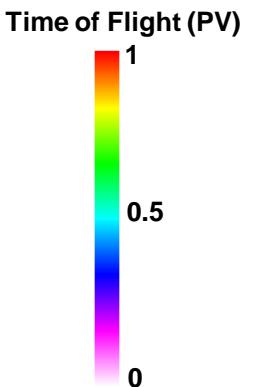
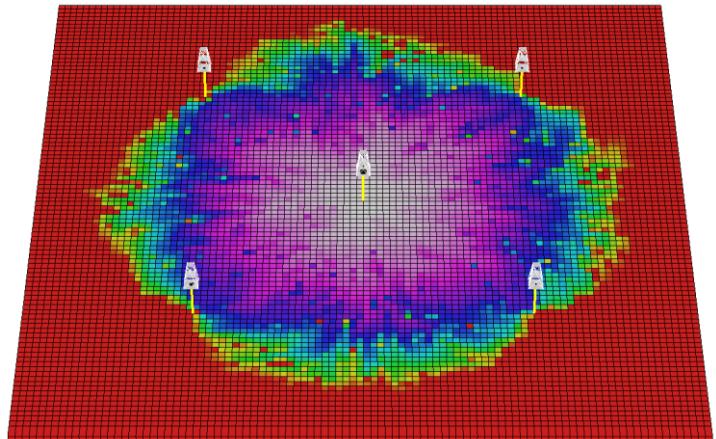
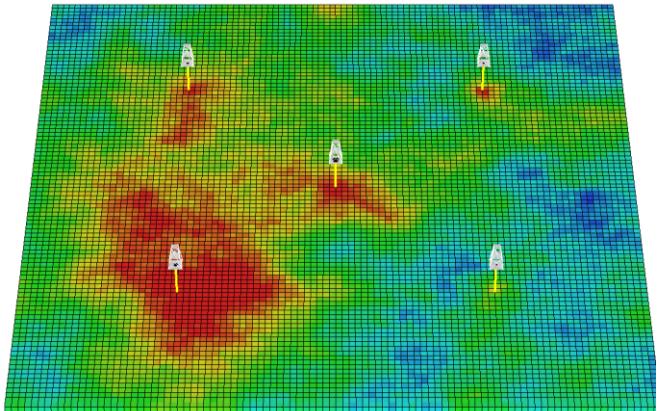
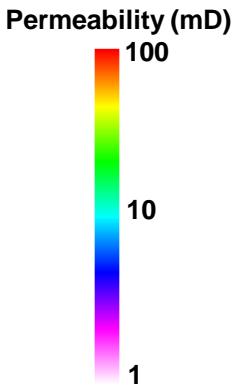
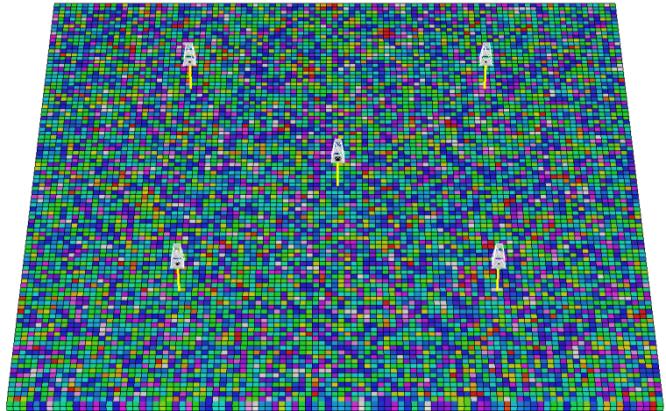
- Mean, variance, P10, P90 ...
- Same static oil in place!

Motivation for Measuring Spatial Continuity



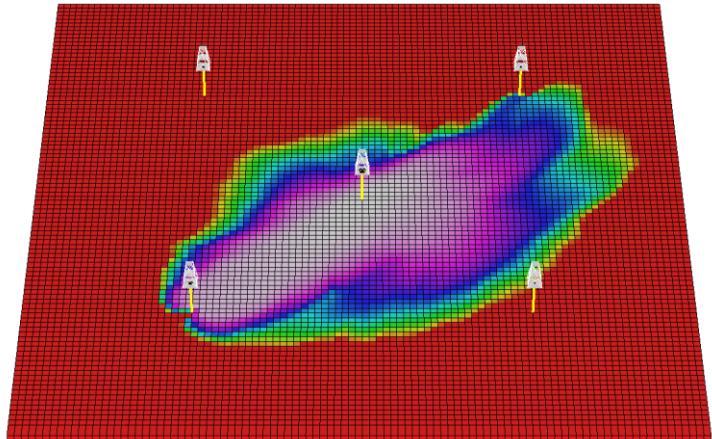
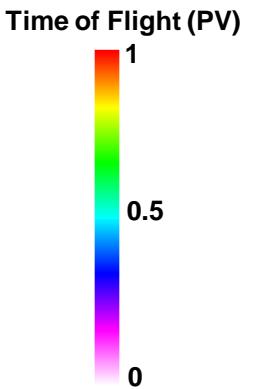
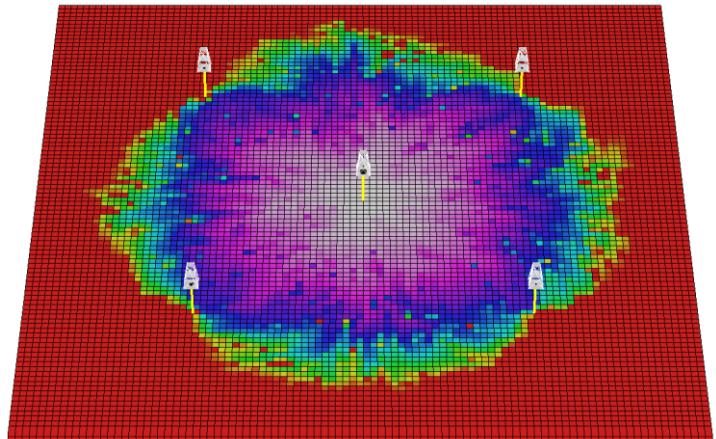
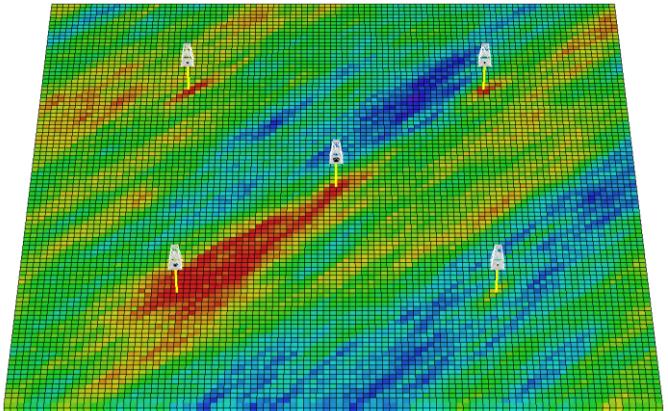
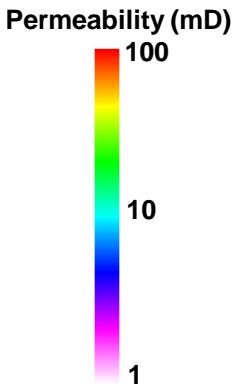
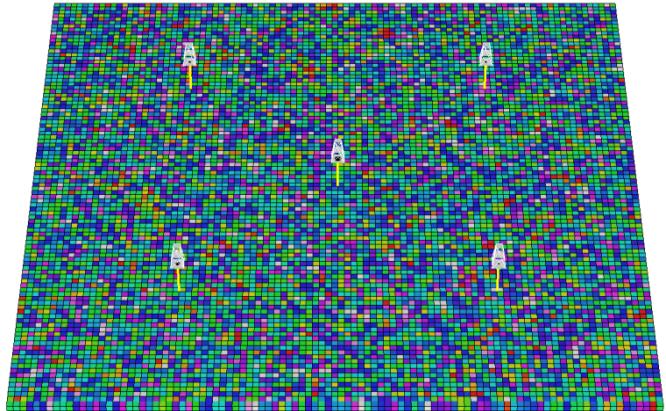
How does heterogeneity impact recovery factor?

Motivation for Measuring Spatial Continuity



How does heterogeneity impact recovery factor?

Motivation for Measuring Spatial Continuity



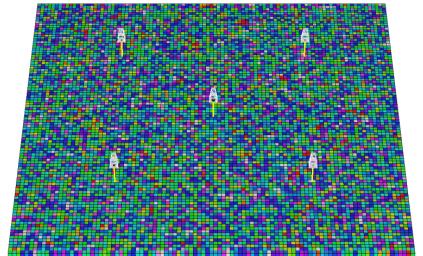
How does heterogeneity impact recovery factor?

Motivation for Measuring Spatial Continuity

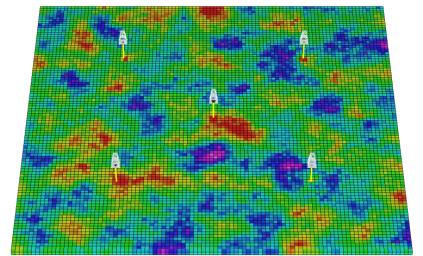
- For the same reservoir property distributions a wide range of spatial continuities are possible.
- Spatial continuity often impacts reservoir forecasts.
- Need to be able to:

Spatial Continuity

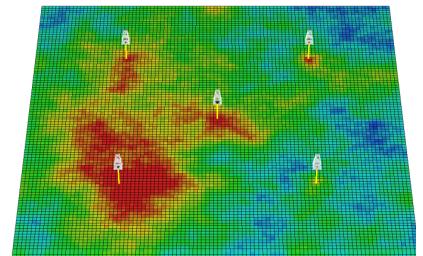
“Very Short”



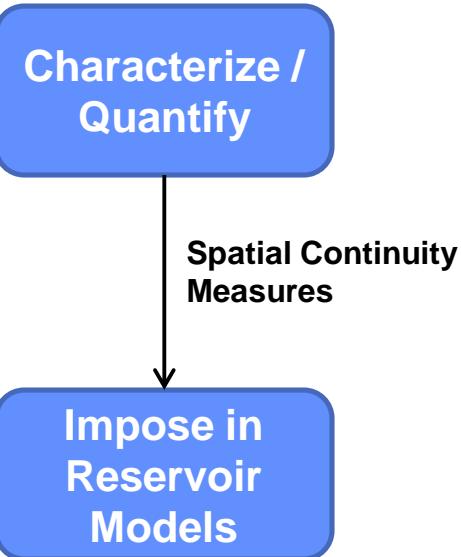
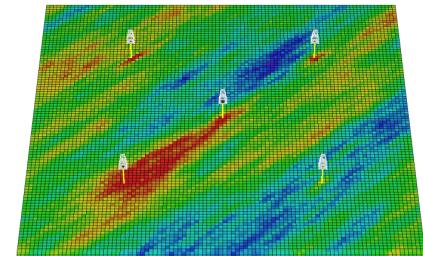
“Medium”

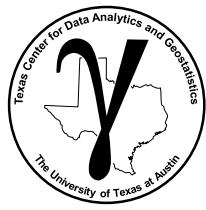


“Long”



“Anisotropic”





Spatial Continuity Definition

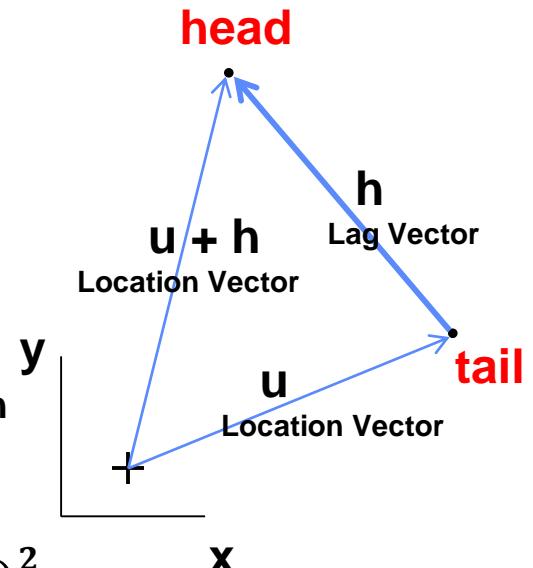
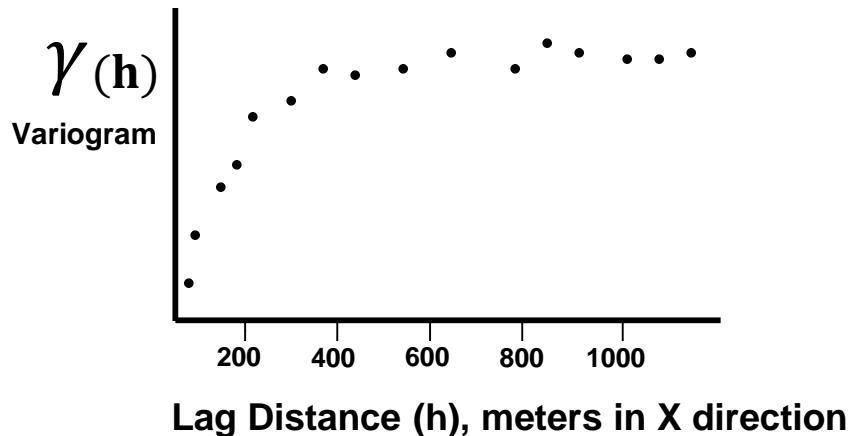
- **Spatial Continuity** – correlation between values over distance.
 - No spatial continuity – no correlation between values over distance, random values at each location in space regardless of separation distance.
 - Homogenous phenomenon have perfect spatial continuity, since all values as the same (or very similar) they are correlated.

Measuring Spatial Continuity

We need a statistic to quantify spatial continuity!

The Semivariogram:

- Function of difference over distance.

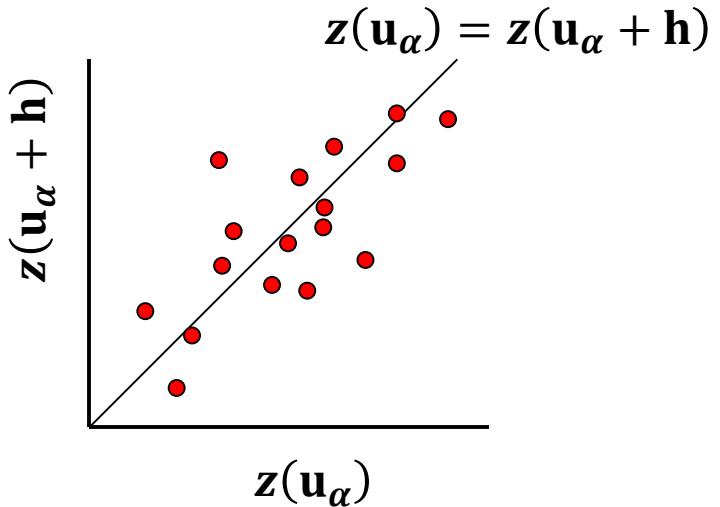


- The equation:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{\alpha=1}^{N(h)} (z(u_\alpha) - z(u_\alpha + h))^2$$

One half the average squared difference over lag distance, h , over all possible pairs of data, $N(h)$.

“h” Scatterplot



- The variogram calculated for lag distance, \mathbf{h} , corresponds to the expected value of squared difference:

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{\alpha=1}^{N(\mathbf{h})} (z(\mathbf{u}_\alpha) - z(\mathbf{u}_\alpha + \mathbf{h}))^2$$

- Calculate for a suite of lag distances to obtain a continuous function.

Variogram Definition

- **Variogram** – a measure of dissimilarity vs. distance. Calculated as $\frac{1}{2}$ the average squared difference of values separated by a lag vector.

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{\alpha=1}^{N(\mathbf{h})} (\mathbf{z}(\mathbf{u}_\alpha) - \mathbf{z}(\mathbf{u}_\alpha + \mathbf{h}))^2$$

- The precise term is semivariogram (variogram if you remove the $1/2$), but in practice the term variogram is used.
- The $\frac{1}{2}$ is used so that the covariance function and variogram may be related directly:

$$C_x(\mathbf{h}) = \sigma_x^2 - \gamma_x(\mathbf{h})$$

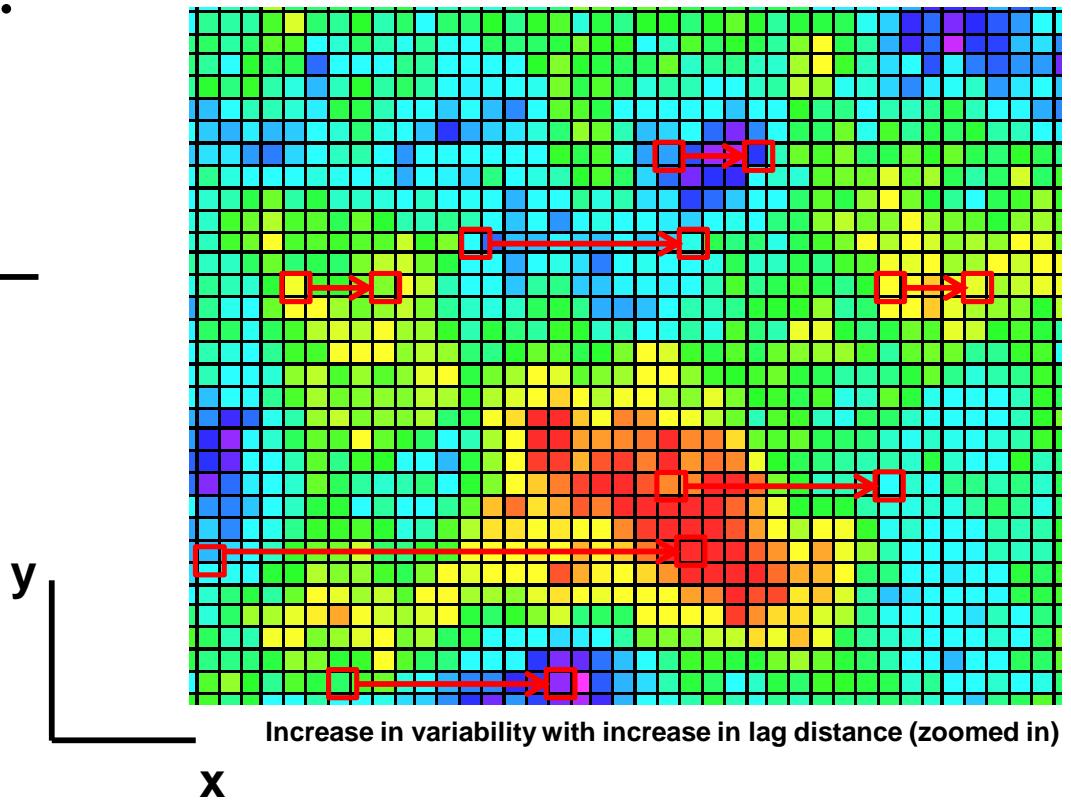
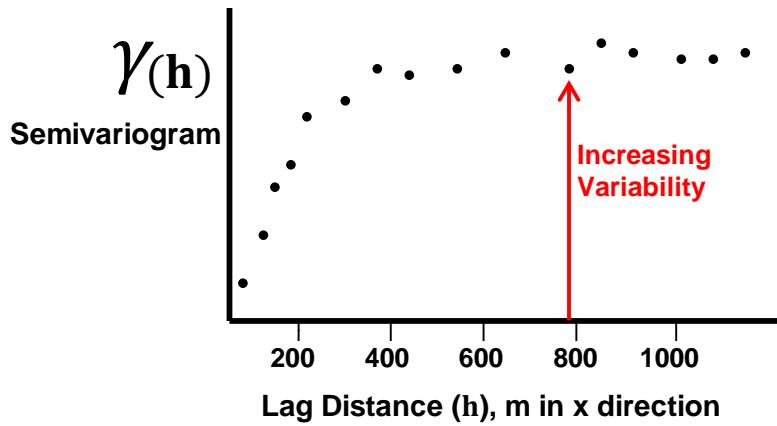
- Note the correlogram is related to the covariance function as:

$$\rho_x(\mathbf{h}) = \frac{C_x(\mathbf{h})}{\sigma_x^2} , \text{ h-scatter plot correlation vs. lag distance}$$

Variogram Observations

Observation #1

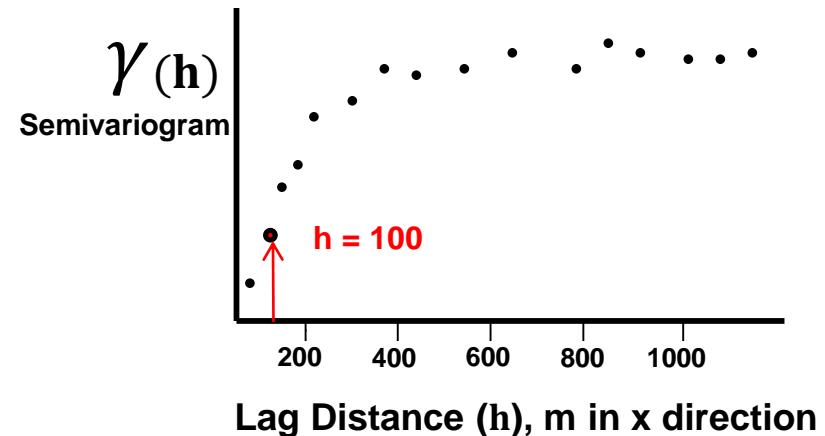
- As distance increases, variability increase (in general).



Variogram Observations

Observation #2

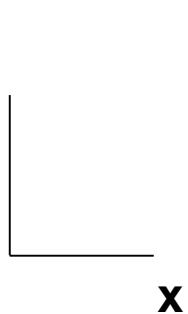
- Calculated with over all possible pairs separated by lag vector, \mathbf{h} .



- The variogram:

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{\alpha=1}^{N(\mathbf{h})} (z(\mathbf{u}_\alpha) - z(\mathbf{u}_\alpha + \mathbf{h}))^2$$

Given the number of pairs available $N(\mathbf{h})$.



Scan of all possible sets of pairs (zoomed in)

Variogram Observations

Observation #3

- Need to plot the sill to know the degree of correlation.

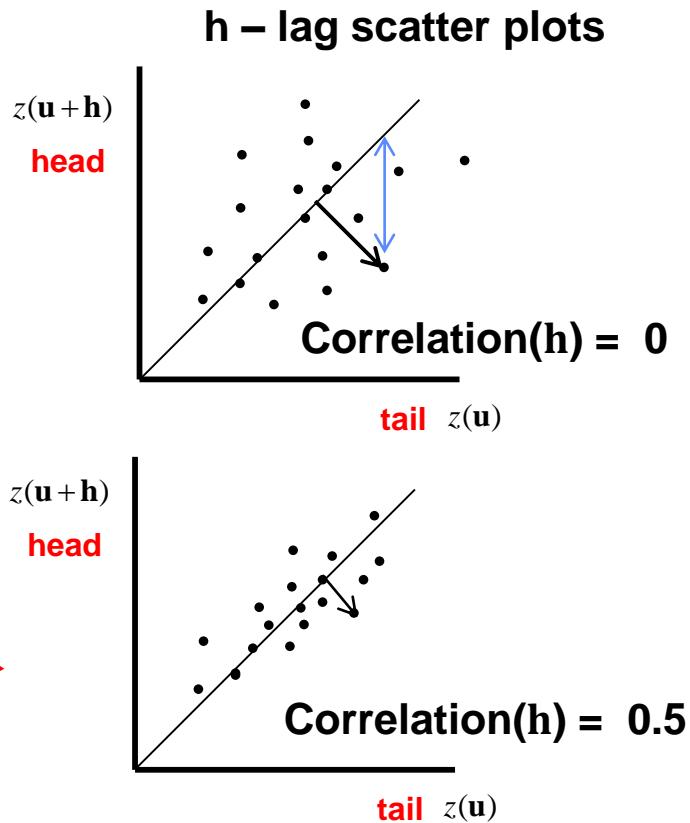
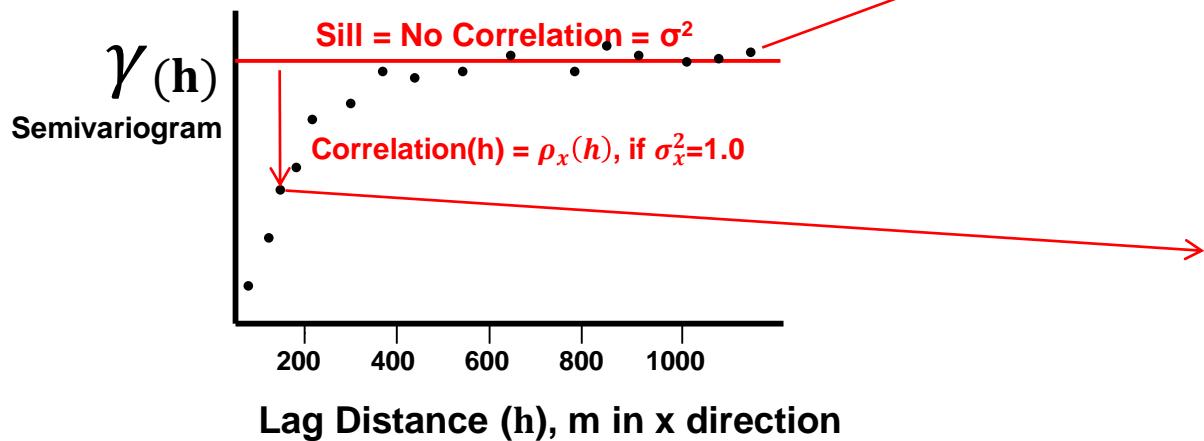
Sill is the Variance, σ^2

- Given stationarity of the variance and $\gamma_x(\mathbf{h})$:

Covariance Function: $C_x(\mathbf{h}) = \sigma_x^2 - \gamma_x(\mathbf{h})$

- Given a standardized distribution $\sigma_x^2 = 1.0$:

Correlogram: $\rho_x(\mathbf{h}) = \sigma_x^2 - \gamma_x(\mathbf{h})$

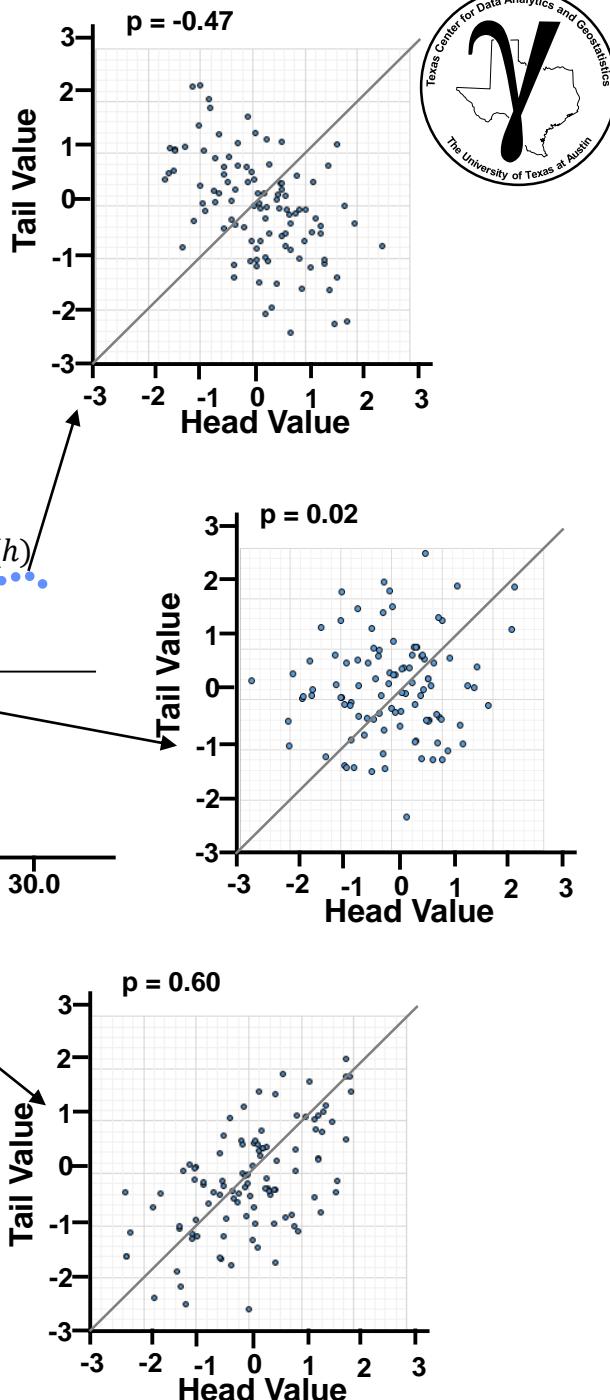
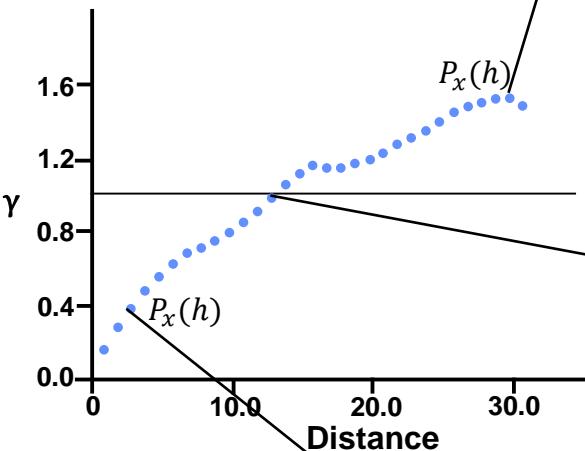


Variogram Interpretation

Observation #3

Need to plot the sill to know the degree of correlation.

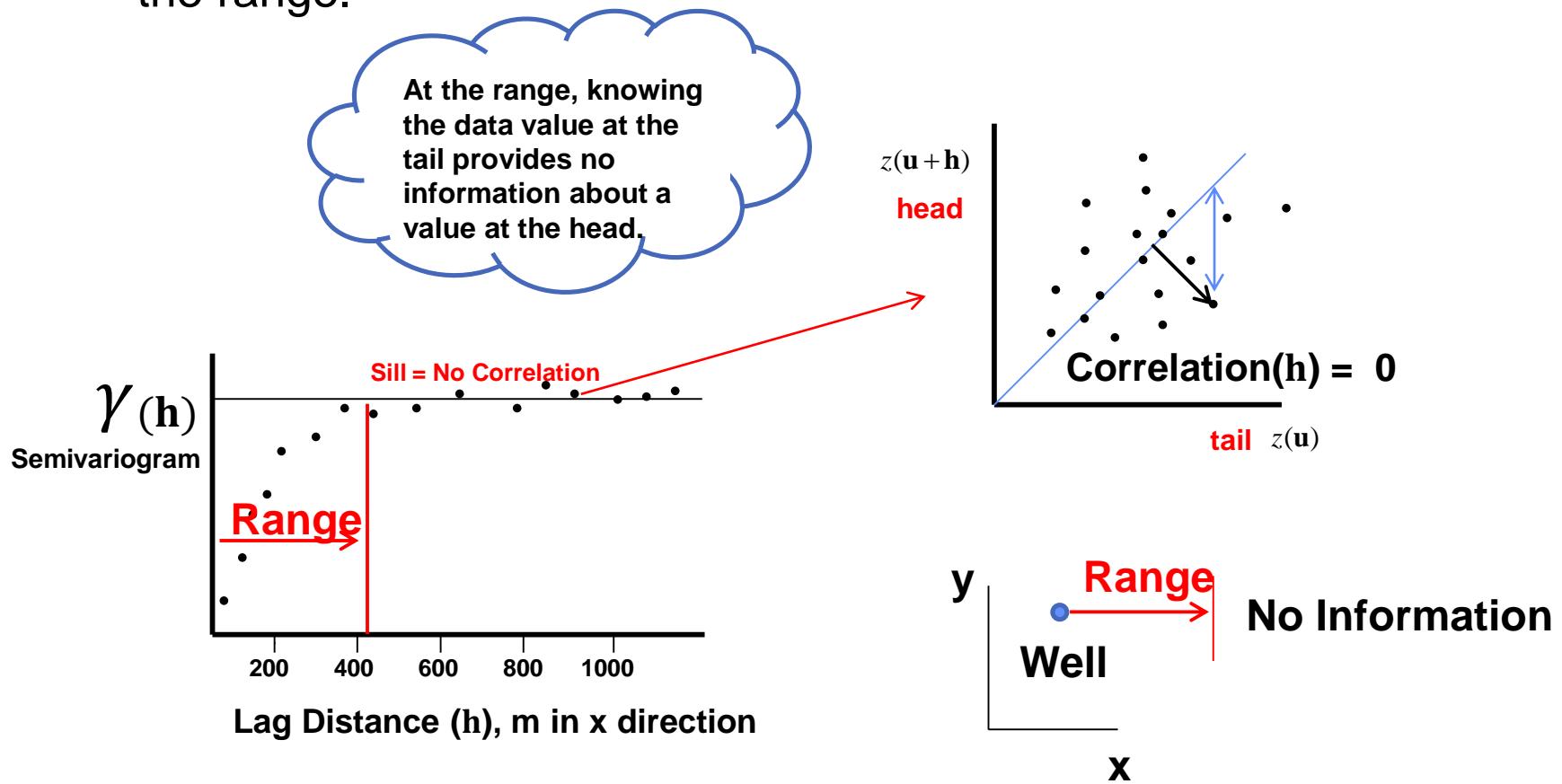
Another illustration of h-scatter plot correlation vs. lag distance.



Variogram Observations

Observation #4

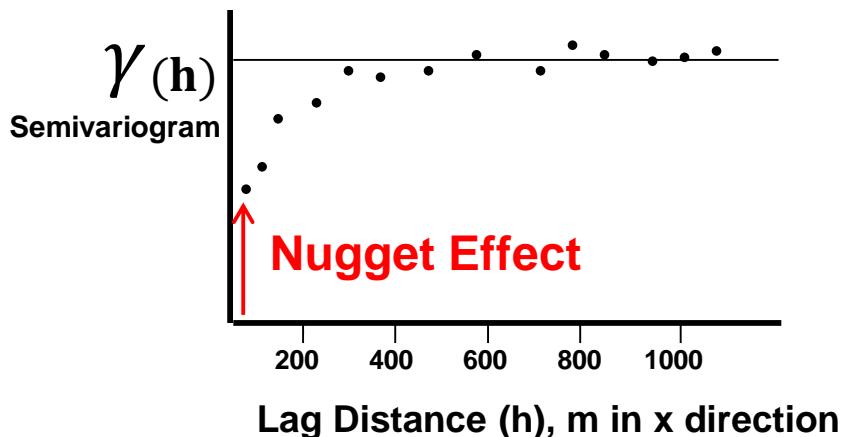
- The lag distance at which the variogram reaches the sill is known as the range.



Variogram Observations

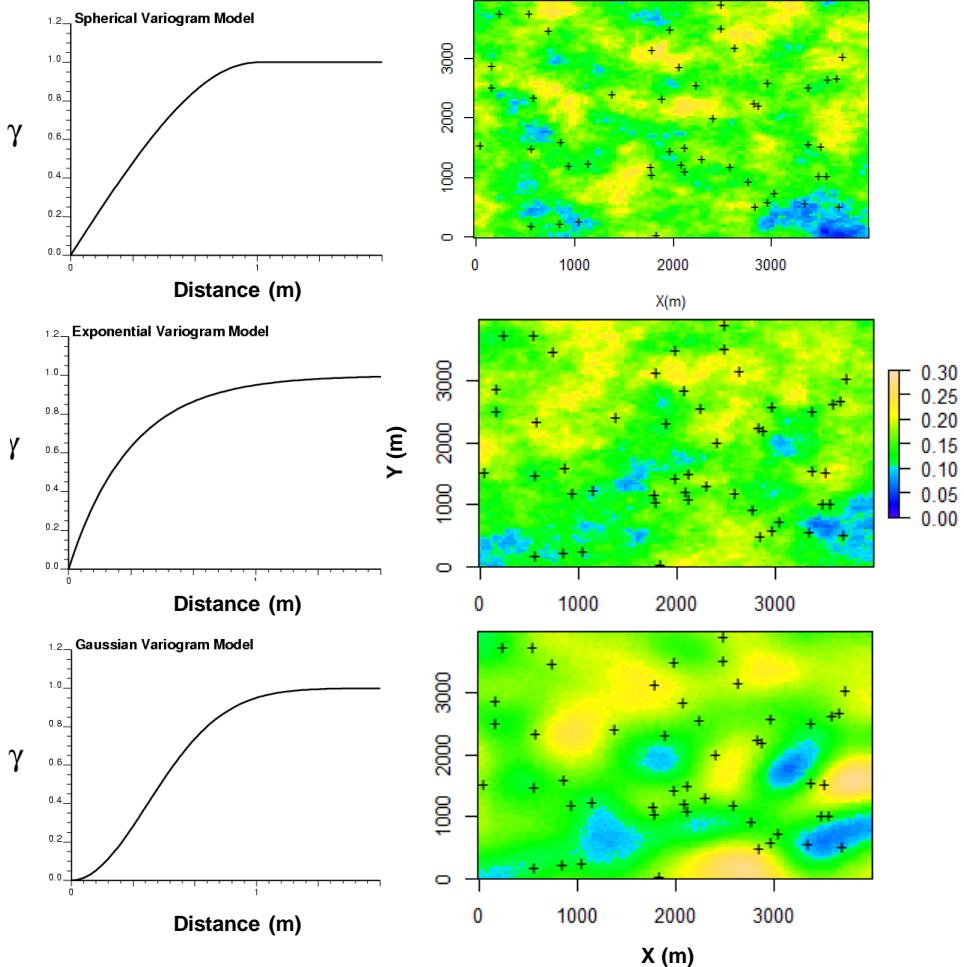
Observation #5

- Sometimes there is a discontinuity in the variogram at distances less than the minimum data spacing. This is known as nugget effect.
 - As a ratio of nugget / sill, is known as relative nugget effect (%)
 - Modeled as a no correlation structure that at lags, $h > \varepsilon$, an infinitesimal distance
 - Measurement error, mixing populations cause apparent nugget effect

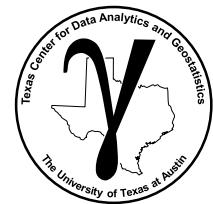


Spatial Variability

- The three maps are remarkably similar: all three have the same 50 data, same histograms and same range of correlation, and yet their **spatial variability/continuity** is quite different
- The spatial variability/continuity depends on the detailed distribution of the petrophysical attribute (ϕ, K)
- The charts on the left are “variograms”
- Our map-making efforts should consider the spatial variability/continuity of the variable we are mapping:
 - Variability
 - Uncertainty



Porosity Realizations with 3 isotropic variograms.



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Variogram Definition

- **Variogram** – a measure of dissimilarity vs. distance. Calculated as $\frac{1}{2}$ the average squared difference of values separated by a lag vector.

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{\alpha=1}^{N(\mathbf{h})} (\mathbf{z}(\mathbf{u}_\alpha) - \mathbf{z}(\mathbf{u}_\alpha + \mathbf{h}))^2$$

- The precise term is semivariogram (variogram if you remove the $1/2$), but in practice the term variogram is used.
- The $\frac{1}{2}$ is used so that the covariance function and variogram may be related directly:

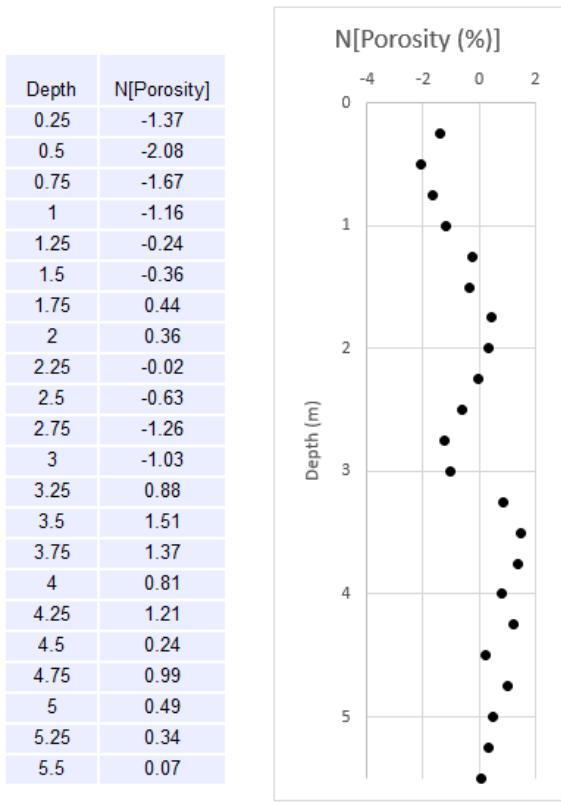
$$C_x(\mathbf{h}) = \sigma_x^2 - \gamma_x(\mathbf{h})$$

- Note the correlogram is related to the covariance function as:

$$\rho_x(\mathbf{h}) = \frac{C_x(\mathbf{h})}{\sigma_x^2} , \text{ h-scatter plot correlation vs. lag distance}$$

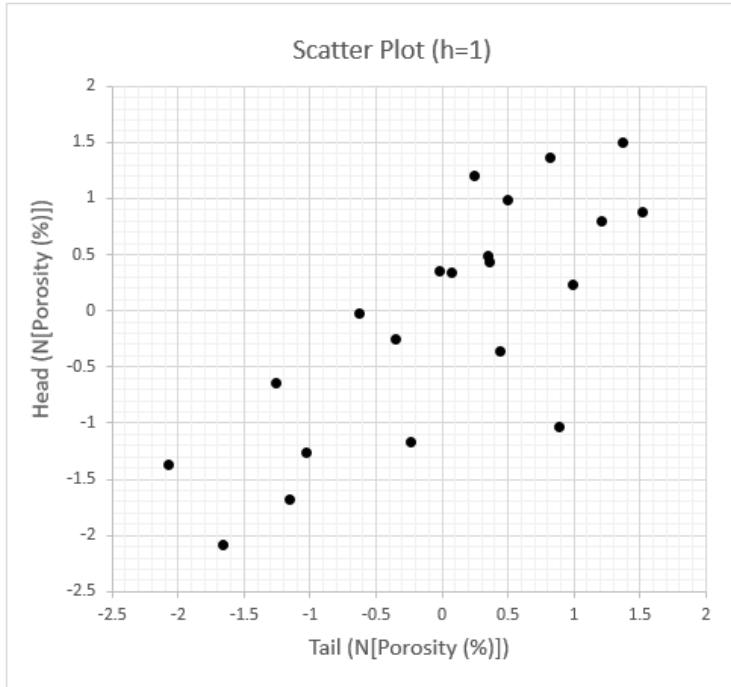
Variogram Calculation

- Consider data values separated by *lag* vectors (the h values)
- Here are two examples of a lag vector equal to the data spacing and then twice the data spacing:



Squared Difference

	-1.37	
0.50	-2.08	-1.37
0.17	-1.67	-2.08
0.26	-1.16	-1.67
0.85	-0.24	-1.16
0.01	-0.36	-0.24
0.64	0.44	-0.36
0.01	0.36	0.44
0.14	-0.02	0.36
0.37	-0.63	-0.02
0.40	0.40	-0.63
0.05	-1.26	0.40
3.65	0.05	-1.26
0.88	3.65	-1.03
0.40	0.88	0.88
0.02	1.51	0.88
0.31	1.37	1.51
0.81	0.81	1.37
0.16	1.21	0.81
0.94	0.24	1.21
0.56	0.99	0.24
0.25	0.49	0.99
0.02	0.34	0.49
0.07	0.07	0.34
Average / 2		0.07
		0.23

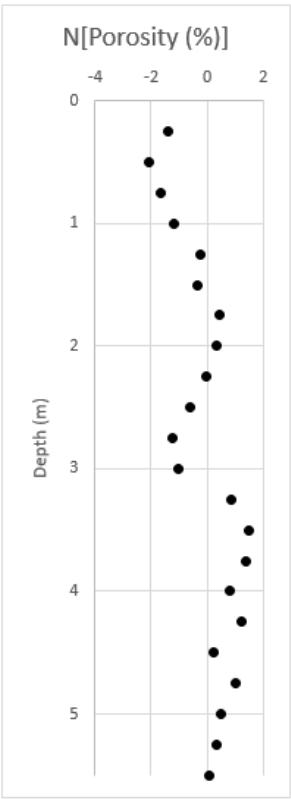


Correlation Coefficient (h=1)	0.77
Variogram (h=1)	0.23

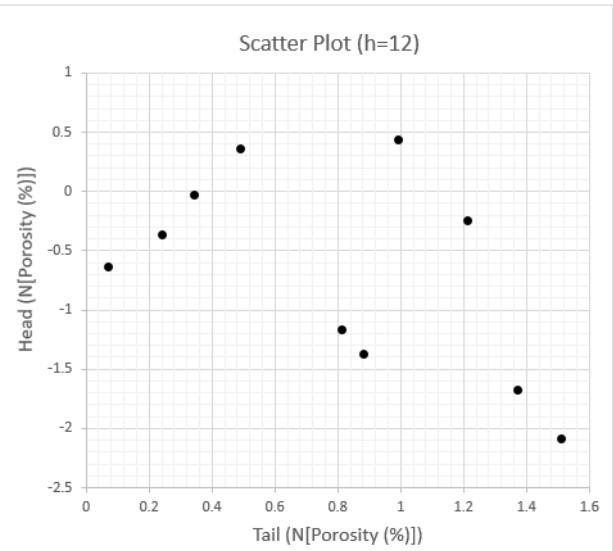
Variogram Calculation

- Consider data values separated by *lag* vectors (the h values)
- Here are two examples of a lag vector equal to the data spacing and then twice the data spacing:

Depth	N[Porosity (%)]
0.25	-1.37
0.5	-2.08
0.75	-1.67
1	-1.16
1.25	-0.24
1.5	-0.36
1.75	0.44
2	0.36
2.25	-0.02
2.5	-0.63
2.75	-1.26
3	-1.03
3.25	0.88
3.5	1.51
3.75	1.37
4	0.81
4.25	1.21
4.5	0.24
4.75	0.99
5	0.49
5.25	0.34
5.5	0.07



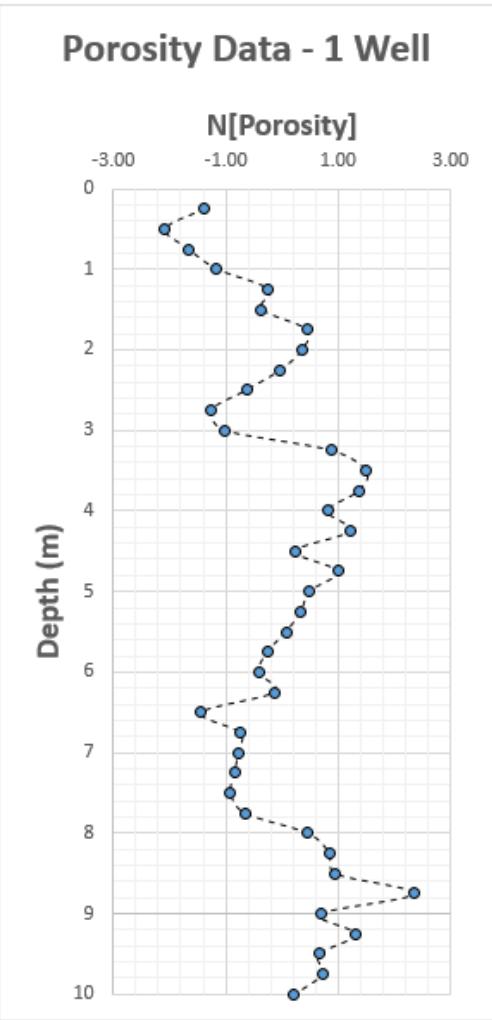
Squared Difference	-1.37	-2.08	-1.67	-1.16	-0.24	-0.36	0.44	0.36	-0.02	-0.63	-1.26	-1.03
5.06	0.88	-1.37										
12.89	1.51	-2.08										
9.24	1.37	-1.67										
3.88	0.81	-1.16										
2.10	1.21	-0.24										
0.36	0.24	-0.36										
0.30	0.99	0.44										
0.02	0.49	0.36										
0.13	0.34	-0.02										
0.49	0.07	-0.63										
Average / 2	1.72											
-1.26												
-1.03												
0.88												
1.51												
1.37												
0.81												
1.21												
0.24												
0.99												
0.49												
0.34												
0.07												



Correlation Coefficient (h=12)	-0.54
Variogram (h=12)	1.72

Variogram Calculation

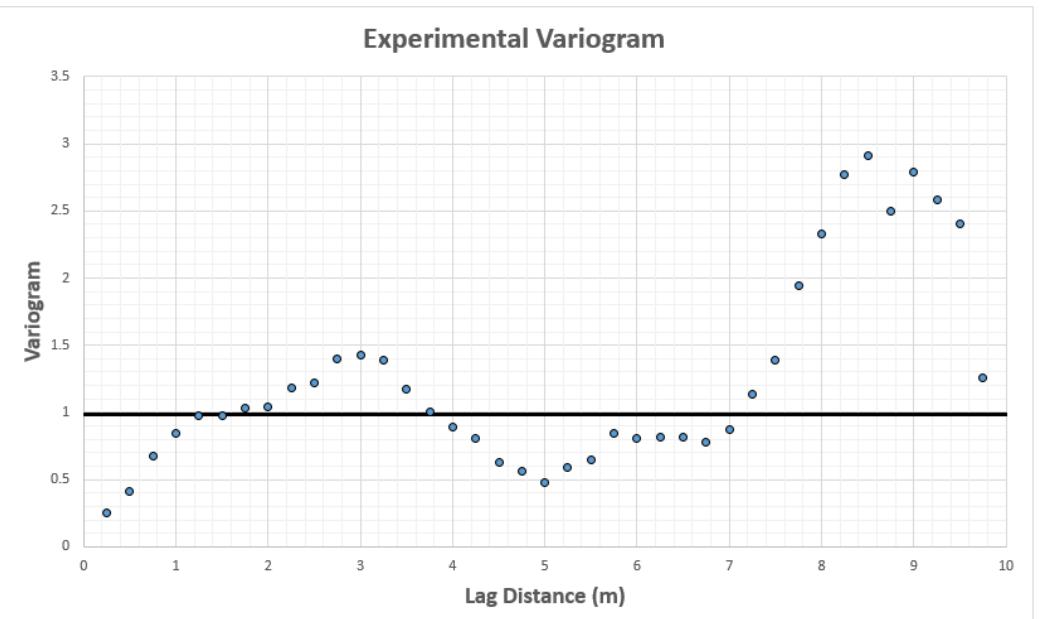
- Pick a lag distance and calculate the variogram for that one lag distance.



The file is Variogram_Simple_Calculation.xlsx

Variogram Calculation Example

- Pick a lag distance and calculate the variogram for that one lag distance.
- Here's all of them:



The Variogram and Covariance Function



- The variogram, covariance function and correlation coefficient are equivalent tools for characterizing spatial two-point correlation (assuming stationarity):

$$\begin{aligned}\gamma_x(\mathbf{h}) &= \sigma_x^2 - C_x(\mathbf{h}) \\ &= \sigma_x^2(1 - \rho_x(\mathbf{h}))\end{aligned}\quad \rho_x(\mathbf{h}) = \frac{C_x(\mathbf{h})}{\sigma_x^2}$$

where:

$$C_x(\mathbf{h}) = E\{X(\mathbf{u}) \cdot X(\mathbf{u} + \mathbf{h})\} - [E\{X(\mathbf{u})\} \cdot E\{X(\mathbf{u} + \mathbf{h})\}], \forall \mathbf{u}, \mathbf{u} + \mathbf{h} \in A$$

$$C_x(0) = \sigma_x^2$$

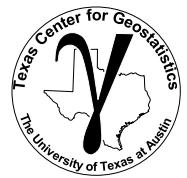
$$C_x(\mathbf{h}) = \frac{\sum_{\alpha=1}^n x(\mathbf{u}_\alpha) \cdot x(\mathbf{u}_\alpha + \mathbf{h})}{n} - (\bar{x})^2, \text{ if stationary mean}$$

Stationarity entails that:

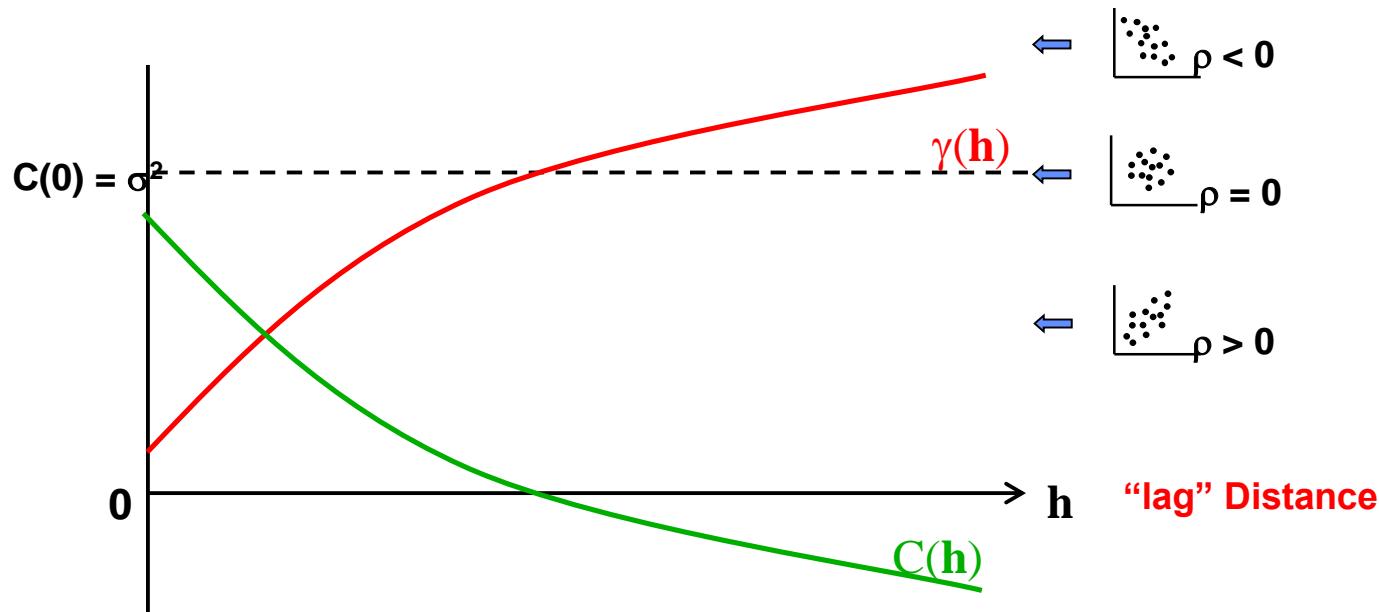
$$m(\mathbf{u}) = m(\mathbf{u} + \mathbf{h}) = m = E\{Z\}, \forall \mathbf{u} \in A$$

$$Var(\mathbf{u}) = Var(\mathbf{u} + \mathbf{h}) = \sigma^2 = Var\{Z\}, \forall \mathbf{u} \in A$$

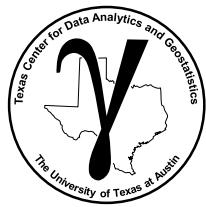
The Variogram and Covariance Function



- Must plot variance to interpret variogram:
 - Positive correlation when semivariogram less than variance
 - No correlation when the semivariogram is equal to the variance
 - Negative correlation when the semivariogram points above variance



Covariance Function Definition

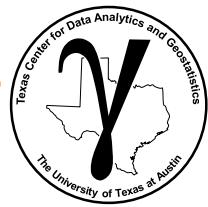


- **Covariance Function** – a measure of similarity vs. distance. Calculated as the average product of values separated by a lag vector centered by the square of the mean.

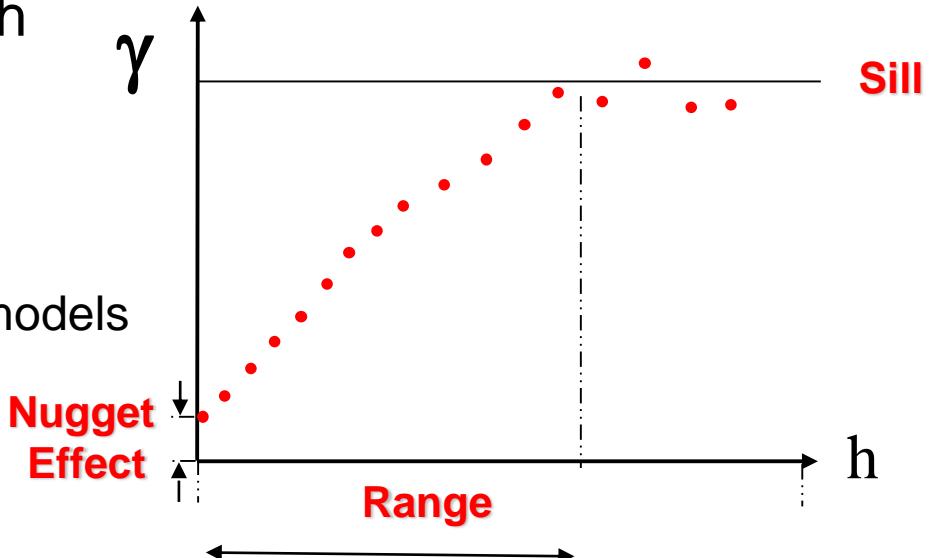
$$C_x(\mathbf{h}) = \frac{\sum_{\alpha=1}^n x(\mathbf{u}_\alpha) \cdot x(\mathbf{u}_\alpha + \mathbf{h})}{n} - (\bar{x})^2, \text{ if stationary mean}$$

- The covariance function is the variogram upside down. $\gamma_x(\mathbf{h}) = \sigma_x^2 - C_x(\mathbf{h})$
- We model variograms, but inside the kriging and simulation methods they are converted to covariance values for numerical convenience.

Variogram Components Definition



- **Nugget Effect** – discontinuity in the variogram at distances less than the minimum data spacing
 - As a ratio of nugget / sill, is known as relative nugget effect (%)
 - Measurement error, mixing populations cause apparent nugget effect
- **Sill** – the sample variance
 - Interpret spatial correlation relative to the sill, level of no correlation
- **Range** – lag distance to reach the sill
 - Up to that distance you have information
 - parameterization of variogram models





Spoiler Alert

We need to practically calculate and model spatial continuity. From the available and often sparse subsurface data.

1. Calculate variogram with irregularly spaced data
 - Search templates with parameters
2. Valid spatial model
 - Fit with a couple different, nest (additive) spatial continuity models
e.g. nugget, spherical, exponential and Gaussian
3. Full 3D spatial continuity model
 - Model primary directions, i.e. major horizontal, minor horizontal and vertical and combine together with assumption of geometric anisotropy

We will not cover variogram modeling.

Calculating Experimental Variograms



How do we get pairs separated by lag vector?

- Regular spaced data:
 - Specify as offsets of grid units
 - Fast calculation
 - Diagonal directions are awkward
- Irregular spaced data:
 - Nominal distance for each lag
 - Distance tolerance
 - Azimuth direction
 - Azimuth tolerance
 - Dip direction
 - Dip tolerance
 - Bandwidth (maximum deviation) in originally horizontal plane
 - Bandwidth in originally vertical plane

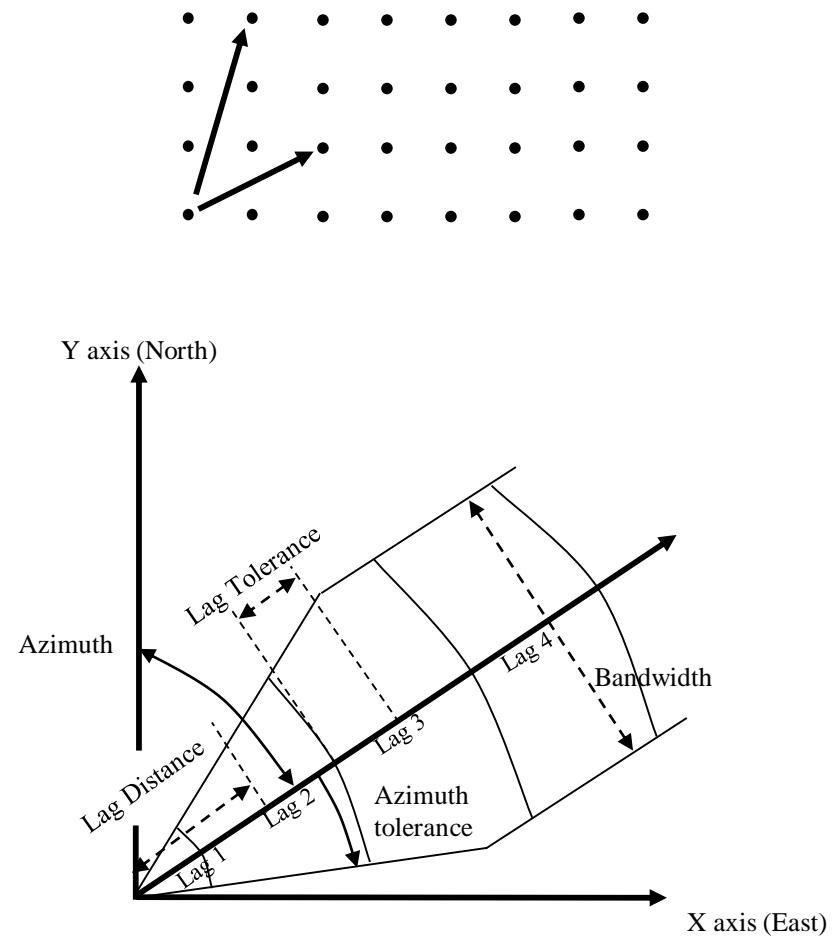
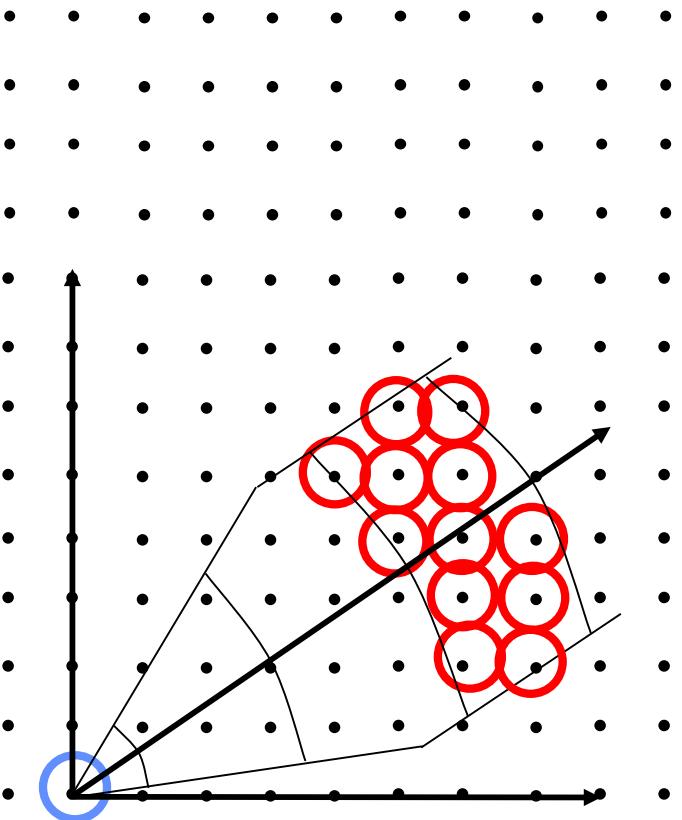


Image from Pyrcz and Deutsch, 2014

Calculating Experimental Variograms



Example: Starting With One Lag (i.e. #4)

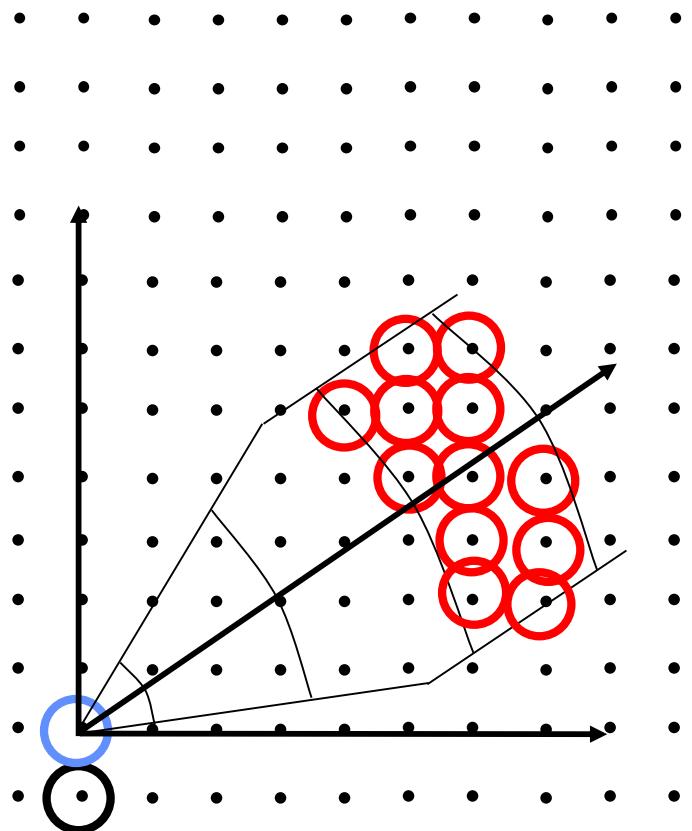


$$2\gamma(h) = \frac{1}{N(h)} \sum [z(u) - z(u + h)]^2$$

Start at a node, and compare value to all nodes which fall in the lag and angle tolerance.

...

Calculating Experimental Variograms



$$2\gamma(h) = \frac{1}{N(h)} \sum [z(u) - z(u+h)]^2$$

Move to next node.

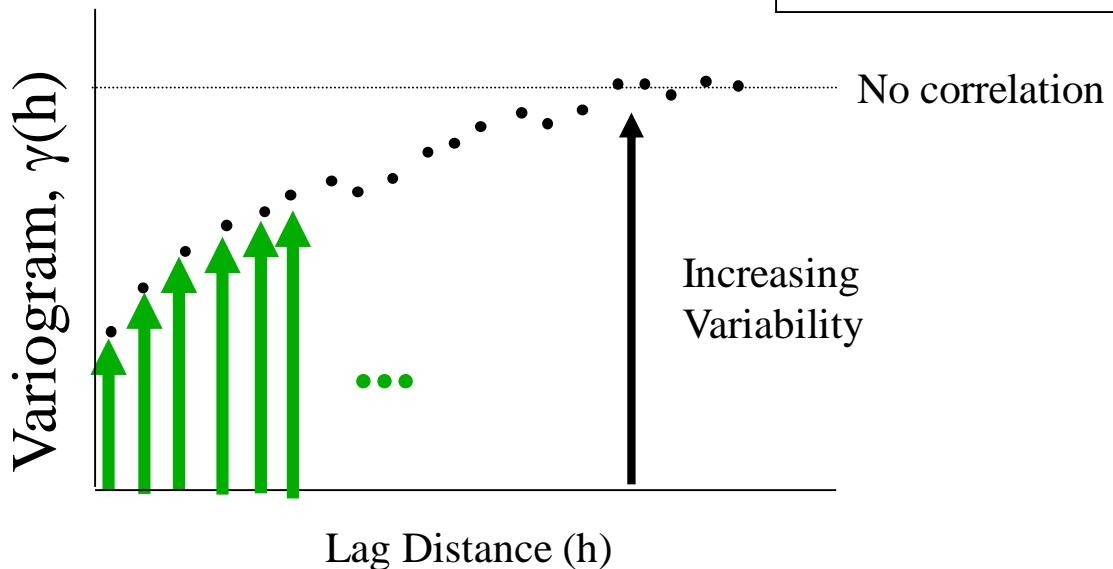
...

Calculating Experimental Variograms



Now Repeat for All Nodes

And Repeat for All Lags

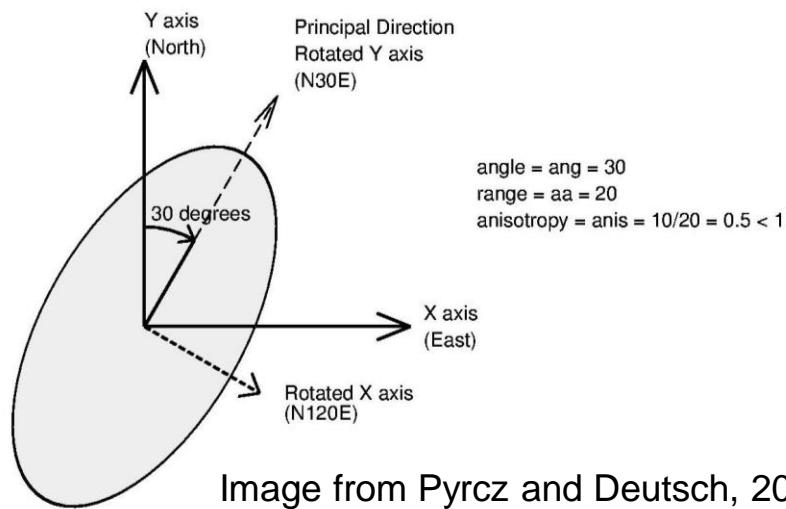


Some Options

- Data transformation:
 - Transform a continuous variable to a Gaussian or normal distribution, for use with / consistency with Gaussian simulation methods
 - Transform a categorical variable to a series of indicator variables for indicator methods and categorical to continuous for truncated Gaussian methods
- Coordinate transformation:
 - Variograms are calculated aligned with the stratigraphic framework
 - Otherwise the spatial continuity will be underestimated
- Should calculate the variogram on the variable being modeled with transforms (data and coordinates)
- Calculate the variogram, as this is what we model and apply in estimation and simulation (more later).

Choosing the Directions

- Inspect the data and interpretations, sections, plan views, ...
- Azimuth angles in degrees clockwise from north
- Review multiple directions before choosing a set of 3 perpendicular directions
 - Omnidirectional: all directions taken together → often yields the most well-behaved variograms.
 - Major horizontal direction & two perpendicular to major direction
 - All anisotropy in geostatistics is geometric – three mutually orthogonal directions with ellipsoidal change in the other directions:



Choosing the Lag Distances and Tolerances



Guidance for Variogram Calculation Parameters:

- Lag separation distance should coincide with data spacing
- Lag tolerance typically chosen to be $\frac{1}{2}$ lag separation distance
 - in cases of erratic variograms, may choose to overlap calculations so lag tolerance $> \frac{1}{2}$ lag separation, results in more pairs.
- The variogram is only valid for a distance one half of the field size: start leaving data out of calculations with larger distances

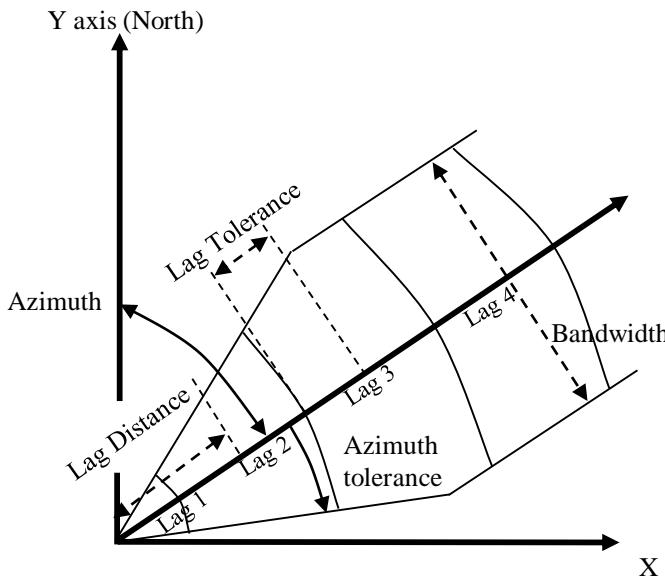


Image from Pyrcz and Deutsch, 2014

Spatial Calculation in Hands-on in Excel



Experiment with Variogram Calculation:

Variogram Calculation By-Hand in Excel, Michael Pyrcz, University of Texas at Austin, @GeostatsGuy on Twitter

About: This demonstration includes variogram calculation applied on a sample set from a truth model.

Dataset: The truth model is a simple 2D convolution (moving window average to impose spatial continuity) of a complete spatial random RIF standardized to a mean of 0.0 and variance of 1.0.

Objective: Provide an opportunity to experiment with variogram calculation.

Workflow:

- Set the sample locations $x \in [0, 100]$ and $y \in [0, 100]$, and samples are extracted from the truth model at those locations.
- Set the variogram major azimuth (azimuth is set to +90 degrees) and the azimuth tolerance, the lag distance and lag tolerance.
- Observe the changes in the experimental variogram.
- Change the major azimuth from 45 to 90, note that the major and minor variograms switch.
- Change the lag distance with the same distance tolerance, note the only change is adding and removing experimental points.
- Decrease and increase the lag tolerance and observe the change in the signal / noise.
- Increase the azimuth tolerance and the observe the directional variograms merge to the same.

Data Samples

Truth Model

Things to Attempt:

- 1. Change the major azimuth from 45 to 90, note that the major and minor variograms switch.
- 2. Change the lag distance with the same distance tolerance, note the only change is adding and removing experimental points.
- 3. Decrease and increase the lag tolerance and observe the change in the signal / noise.
- 4. Increase the azimuth tolerance and the observe the directional variograms merge to the same.

Variogram Calculation Parameters

Dir1	Dir2
30	180
Distance	22.22
Lag Distance	10
Lag Tolerance	10

This performs the analysis of 1000 points.

Experimental Variogram

pairwise square difference

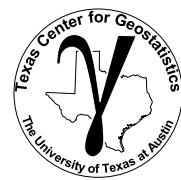
Pr1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Pr2	50.8	51.8	4.5	34.2	34.2	94.3	58.8	44.8	35.1	35.8	61.8	88.7	77.4	91.3	94.4	91.6	97.1	8.2	48.3	87.7	51.4	51.5	42.6	56.4	34.3	8.1	8.1	8.1	8.1	
Pr1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Pr2	50.8	51.8	4.5	34.2	34.2	94.3	58.8	44.8	35.1	35.8	61.8	88.7	77.4	91.3	94.4	91.6	97.1	8.2	48.3	87.7	51.4	51.5	42.6	56.4	34.3	8.1	8.1	8.1	8.1	

Things to try:

- Set azimuth / lag tolerance small and large
- Change directions.

The file is Variogram_Calc_Model_Demo_v2.0.xlsx.

Spatial Calculation in Demo in Python



Experiment with Variogram Calculation:

Things to Try:

Variogram maps

- Relate to the data location maps

Directional variograms

- Change lag tolerance
- Change lag distance

GeostatsPy: Spatial Continuity Directions for Subsurface Data Analytics in Python

Michael Pyrcz, Associate Professor, University of Texas at Austin

[Twitter](#) | [GitHub](#) | [Website](#) | [GoogleScholar](#) | [Book](#) | [YouTube](#) | [LinkedIn](#)

PGE 383 Exercise: Methods to Detect Directions of Continuity with GeostatsPy

Here's a simple workflow on detecting the major spatial continuity directions in a spatial dataset with variogram analysis. This information is essential to optimum well placement and prediction away from wells. First let's explain the concept of spatial continuity and the variogram.

Spatial Continuity

Spatial Continuity is the correlation between values over distance.

- No spatial continuity – no correlation between values over distance, random values at each location in space regardless of separation distance.
- Homogenous phenomenon have perfect spatial continuity, since all values are the same (or very similar) they are correlated.

We need a statistic to quantify spatial continuity! A convenient method is the Semivariogram.

The Semivariogram

Function of difference over distance.

- The expected (average) squared difference between values separated by a lag distance vector (distance and direction), \mathbf{h} :

$$\gamma(\mathbf{h}) = \frac{1}{2N(\mathbf{h})} \sum_{a=1}^{N(\mathbf{h})} (z(\mathbf{u}_a) - z(\mathbf{u}_a + \mathbf{h}))^2$$

where $z(\mathbf{u}_a)$ and $z(\mathbf{u}_a + \mathbf{h})$ are the spatial sample values at tail and head locations of the lag vector respectively.

- Calculated over a suite of lag distances to obtain a continuous function.
- the $\frac{1}{2}$ term converts a variogram into a semivariogram, but in practice the term variogram is used instead of semivariogram.
- We prefer the semivariogram because it relates directly to the covariance function, $C_x(\mathbf{h})$ and univariate variance, σ_x^2 :

$$C_x(\mathbf{h}) = \sigma_x^2 - \gamma(\mathbf{h})$$

Note the correlogram is related to the covariance function as:

$$\rho_x(\mathbf{h}) = \frac{C_x(\mathbf{h})}{\sigma_x^2}$$

The correlogram provides a function of the $\mathbf{h} - \mathbf{h}$ scatter plot correlation vs. lag offset \mathbf{h} .

$$-1.0 \leq \rho_x(\mathbf{h}) \leq 1.0$$

Variogram Observations

The following are common observations for variograms that should assist with their practical use.

Observation #1 - As distance increases, variability increase (in general).

Python notebook file: GeostatsPy_spatial_continuity_directions.ipynb

Spatial Continuity New Tools



Topic	Application to Subsurface Modeling
Stationarity	<p>In the presence of multivariate relationships, must jointly model variables.</p> <p><i>Summarize with bivariate statistics, and visualize and use conditional statistics to go beyond linear measures.</i></p>
Random Variables and Functions	<p>Random variables and random functions are used to represent spatial uncertainty.</p> <p><i>Porosity at a pre-drill location has the uncertainty model based on a random variable with Gaussian mean of 15% and standard deviation of 3%.</i></p>
Variogram Calculation	<p>Calculate spatial continuity from spatial data to use for spatial prediction.</p> <p><i>From the available wells the porosity spatial continuity range is 300 m in the 030 azimuth, we have no information beyond this spacing from existing wells.</i></p>

Data Analytics, Geostatistics and Machine Learning

Spatial Continuity / Prediction



Lecture outline . . .

- Spatial Estimation

Introduction

Fundamental Concepts

Probability

Data Prep / Analytics

Spatial Continuity / Prediction

Multivariate Modeling

Uncertainty Modeling

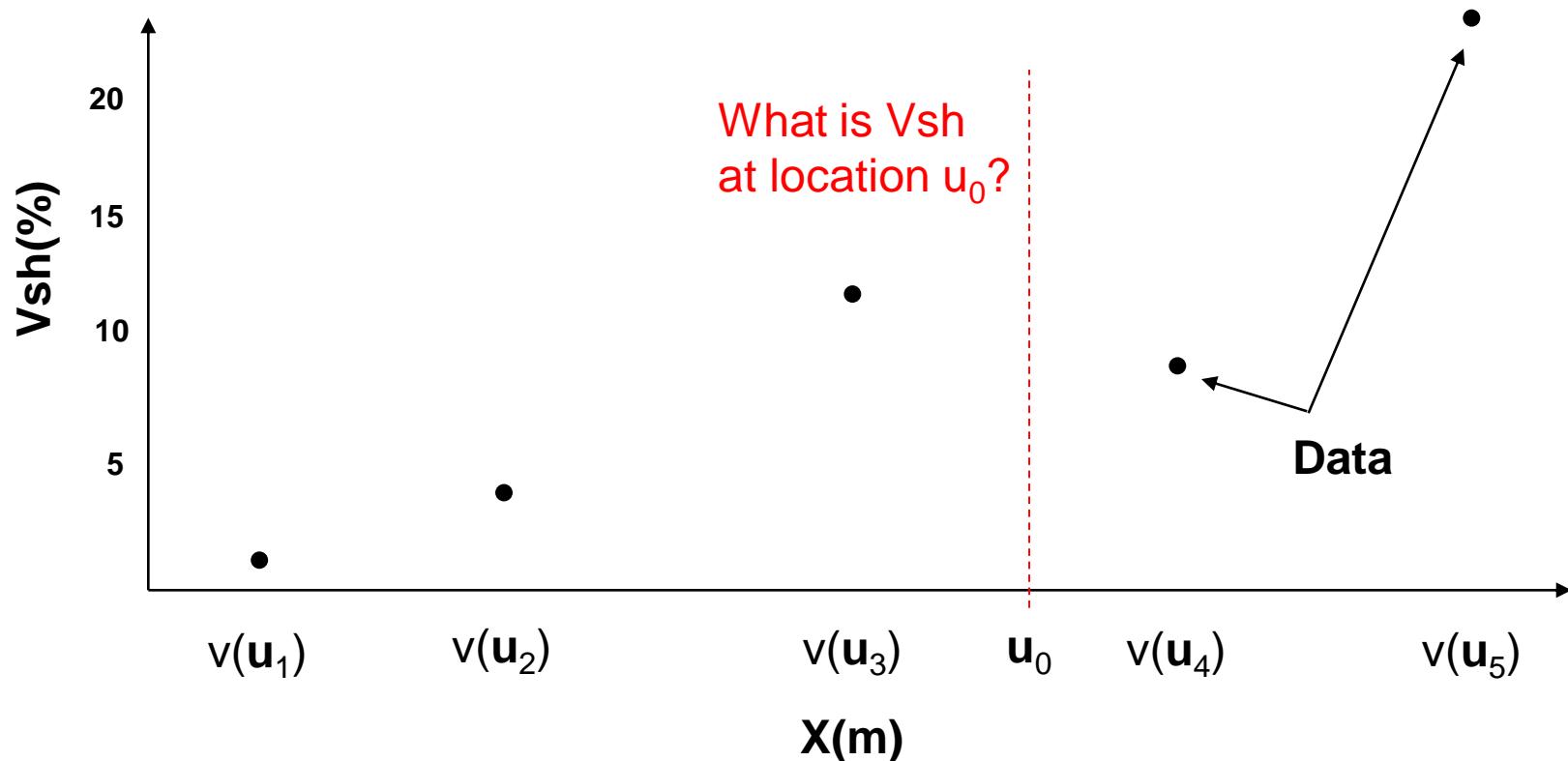
Machine Learning

Instructor: Michael Pyrcz, the University of Texas at Austin

Trend Modeling

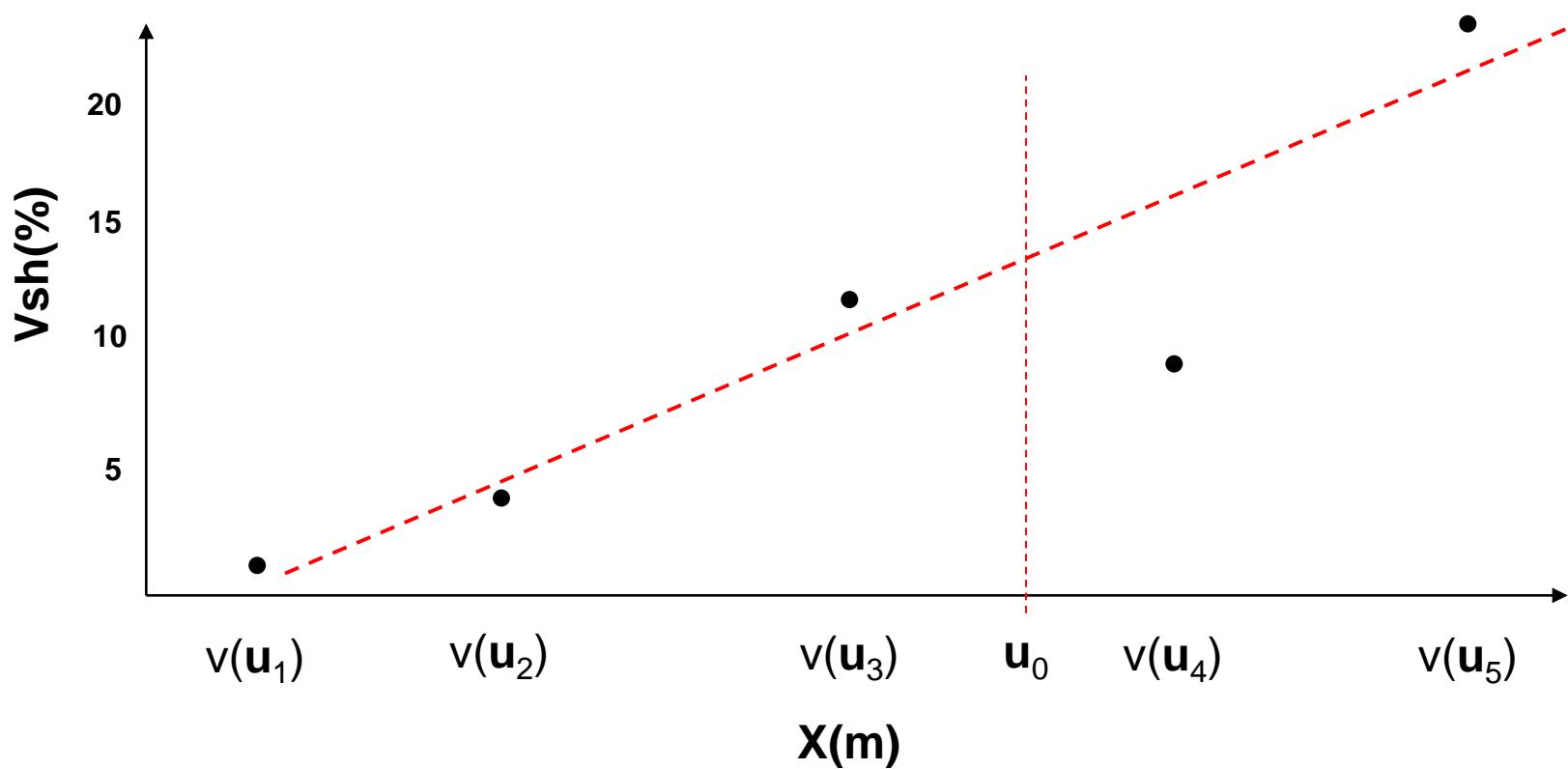
We Must Start with Trend Modeling

- Geostatistical spatial estimation methods make an assumption concerning stationarity
 - In the presence of significant nonstationarity we would not rely 100% for spatial estimation on data + spatial continuity model



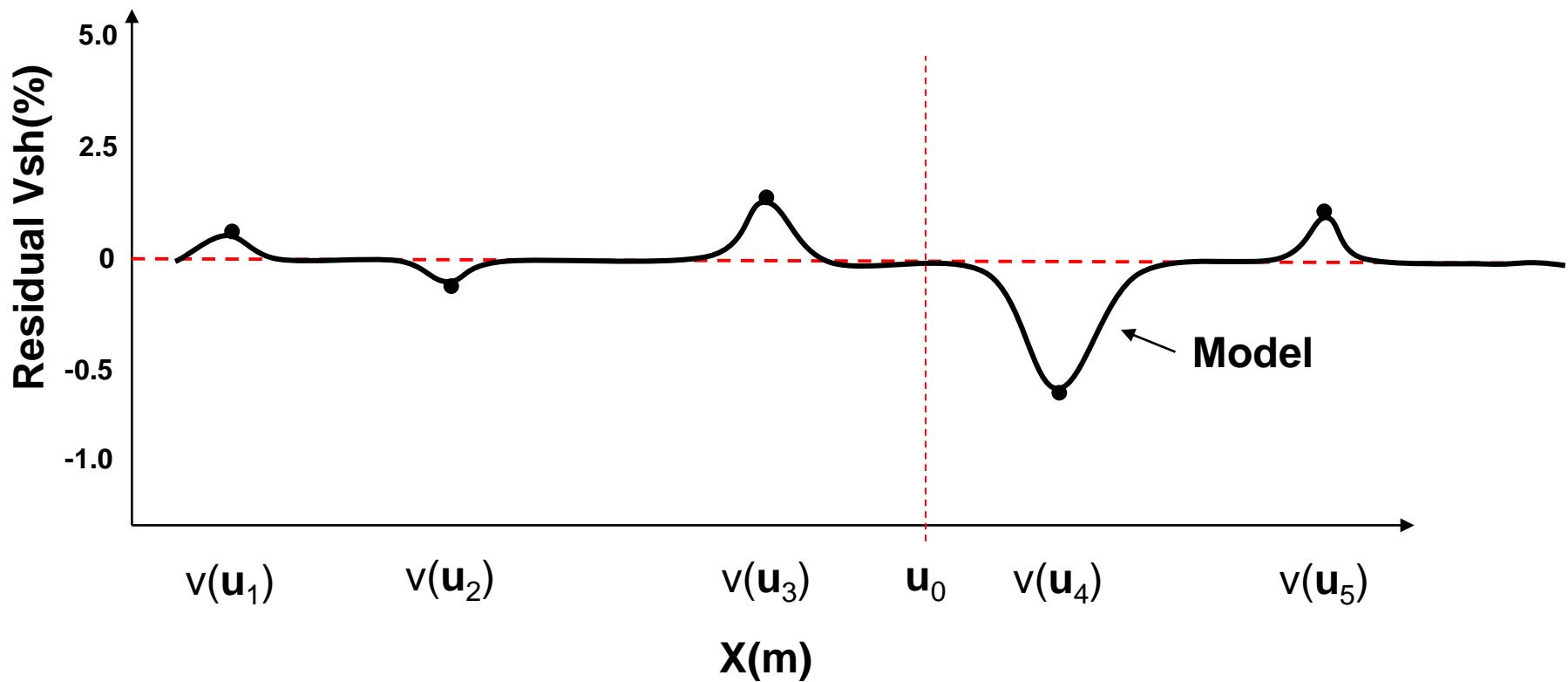
Trend Modeling

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
 - If we observe a trend, we should model the trend.



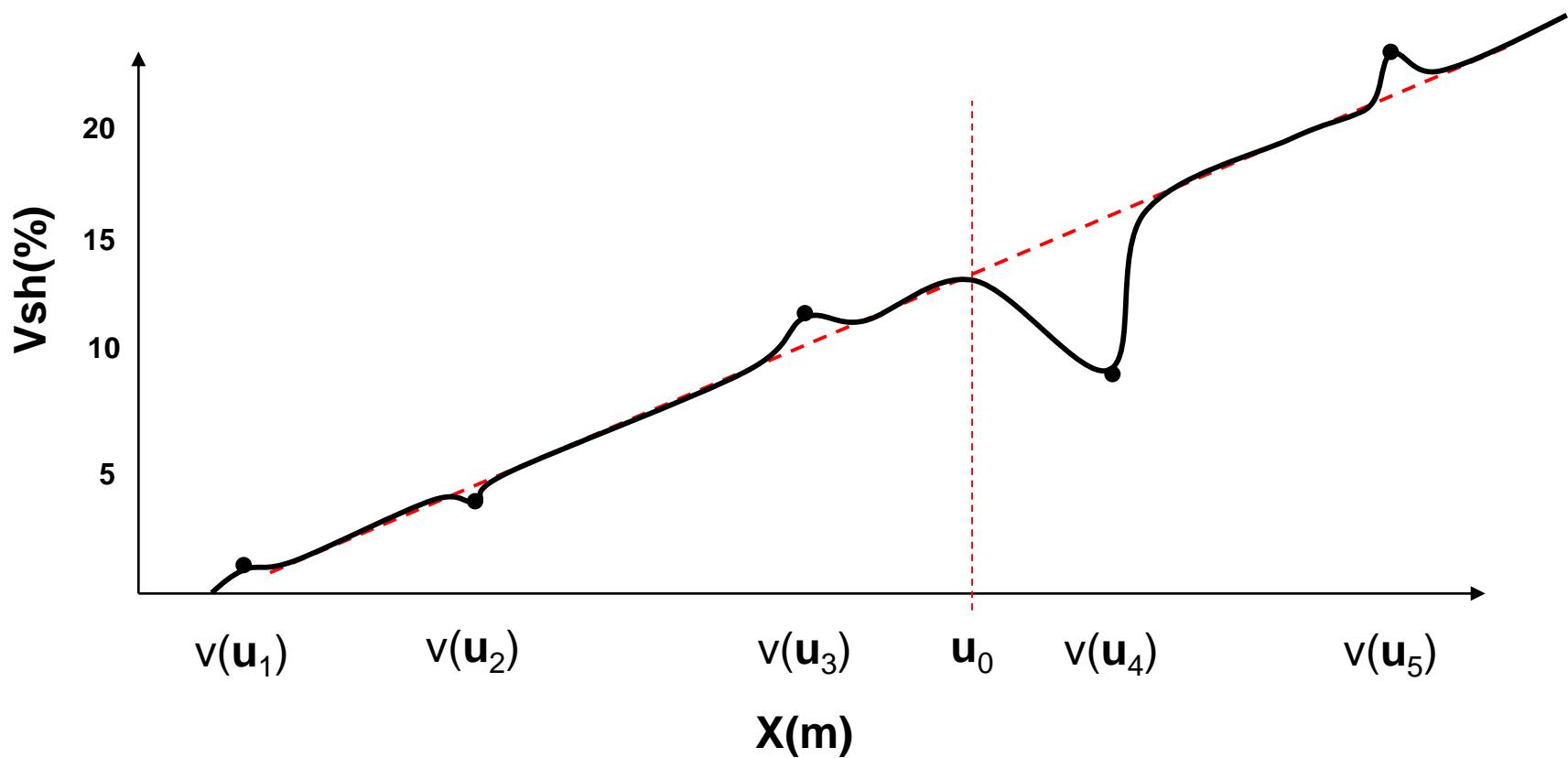
Trend Modeling

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
 - Then model the residuals.



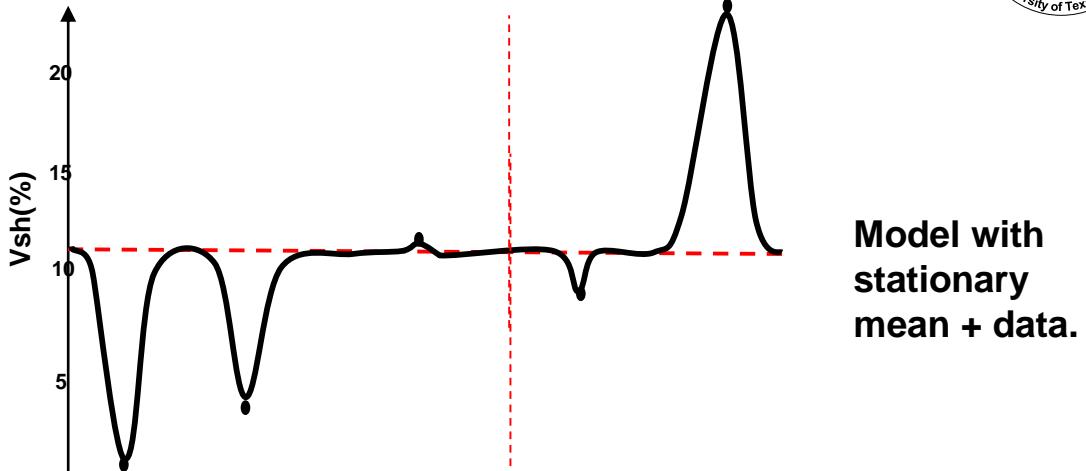
Trend Modeling

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
 - After modeling, add the trend back to the modelled residuals

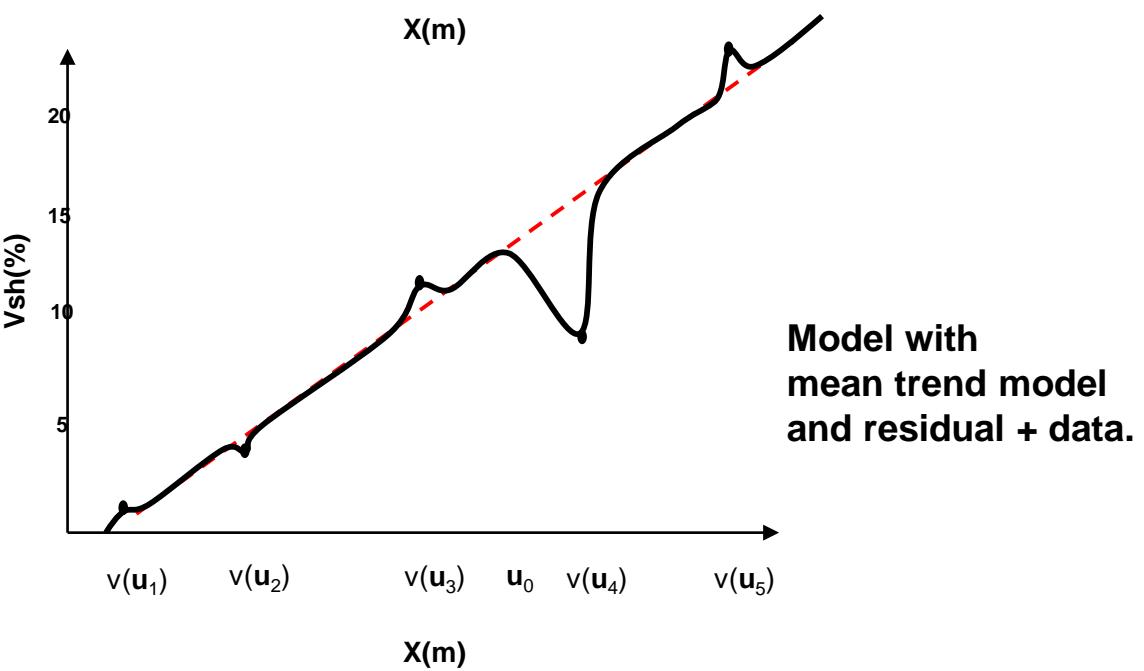


Trend Modeling

- How bad could it be if we did not model a trend?
- Geostatistical estimation would assume stationarity and away from data we would estimate with the global mean (simple kriging)!



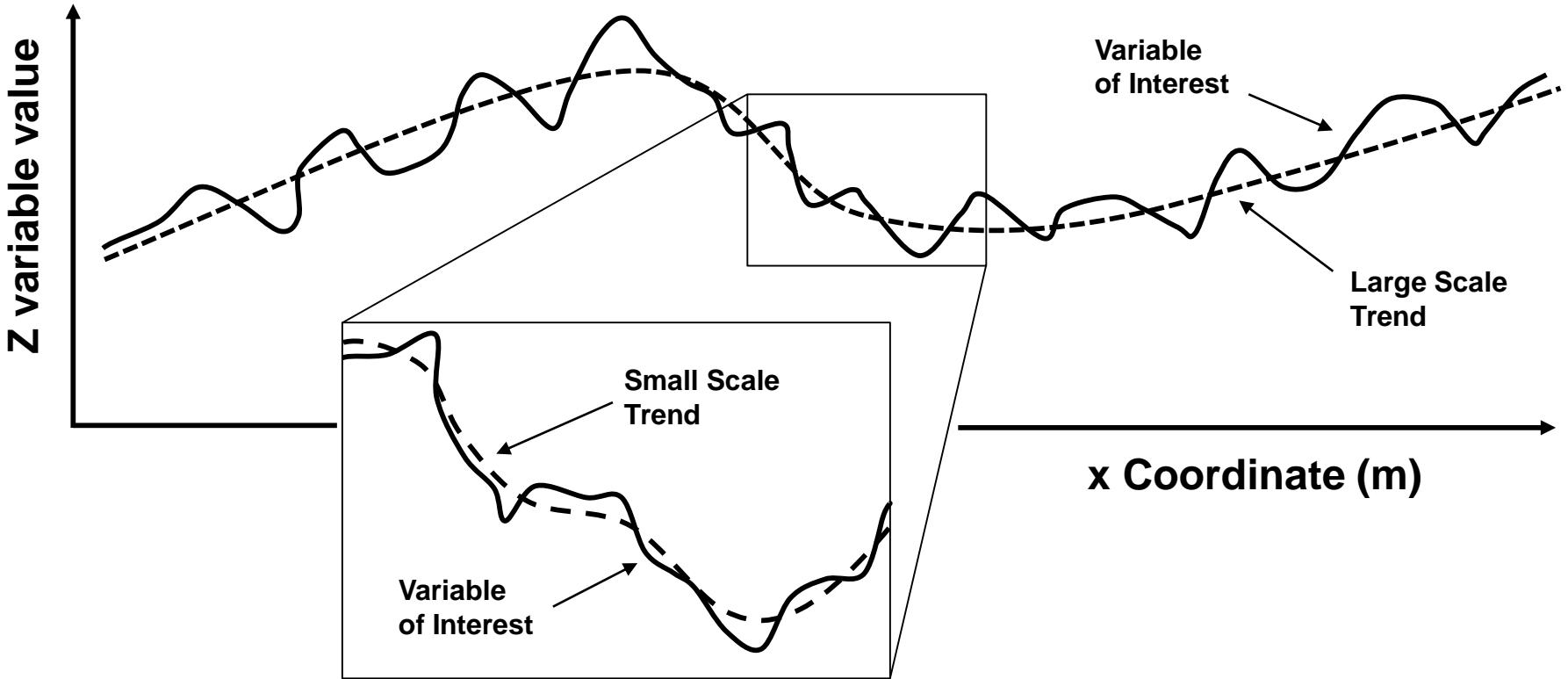
Model with stationary mean + data.



Model with mean trend model and residual + data.

Trend Modeling

- Trend Modeling
 - We must identify and model trends / nonstationarities



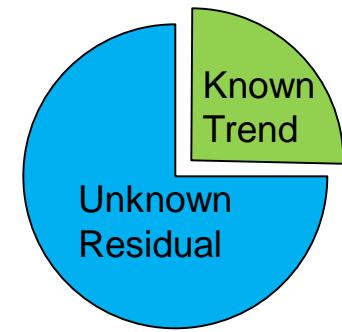
Trend Modeling

- Any variance in the trend is removed from the residual:

$$\sigma_X^2 = \sigma_{X_t}^2 + \sigma_{X_r}^2 + 2C_{X_t, X_r}$$

- if the $X_t \perp\!\!\!\perp X_r$, $C_{X_t, X_r} = 0$

$$\sigma_{X_r}^2 = \sigma_X^2 - \sigma_{X_t}^2$$



- So if σ_X^2 is the total variance (variability), and $\sigma_{X_t}^2$ is the variability that is deterministically modelled, treated as known, and $\sigma_{X_r}^2$ is the component of the variability that is treated as unknown.
- Result: the more variability explained by the trend the less variability that remains as uncertain.

Definition Deterministic Model



- Model that assumes perfect knowledge, without uncertainty
- Based on knowledge of the phenomenon or trend fitting to data
- Most subsurface models have a deterministic component (trend) to capture expert knowledge and to provide a stationary residual for geostatistical modeling.

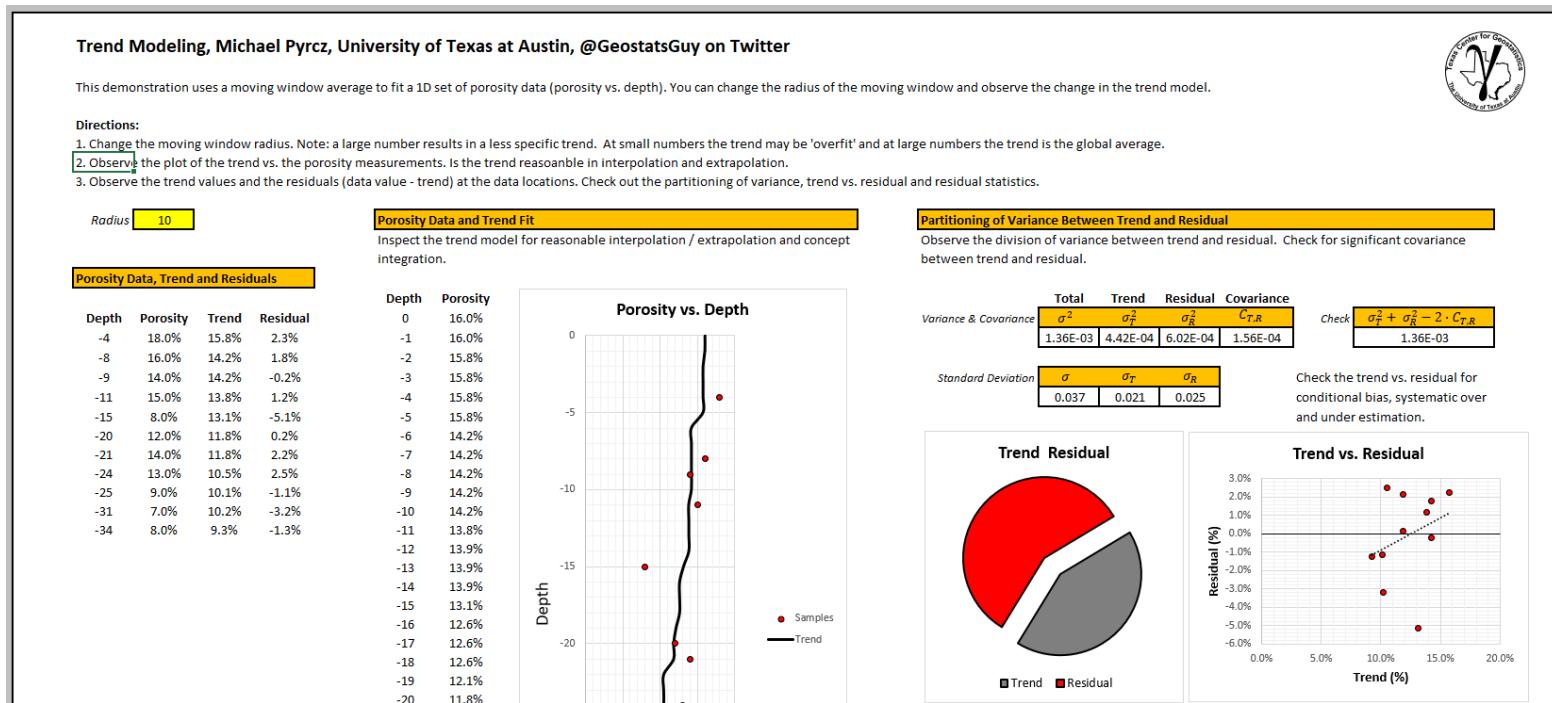
Trend Modeling Hands-on



Here's an opportunity for experiential learning with Trend Modeling.

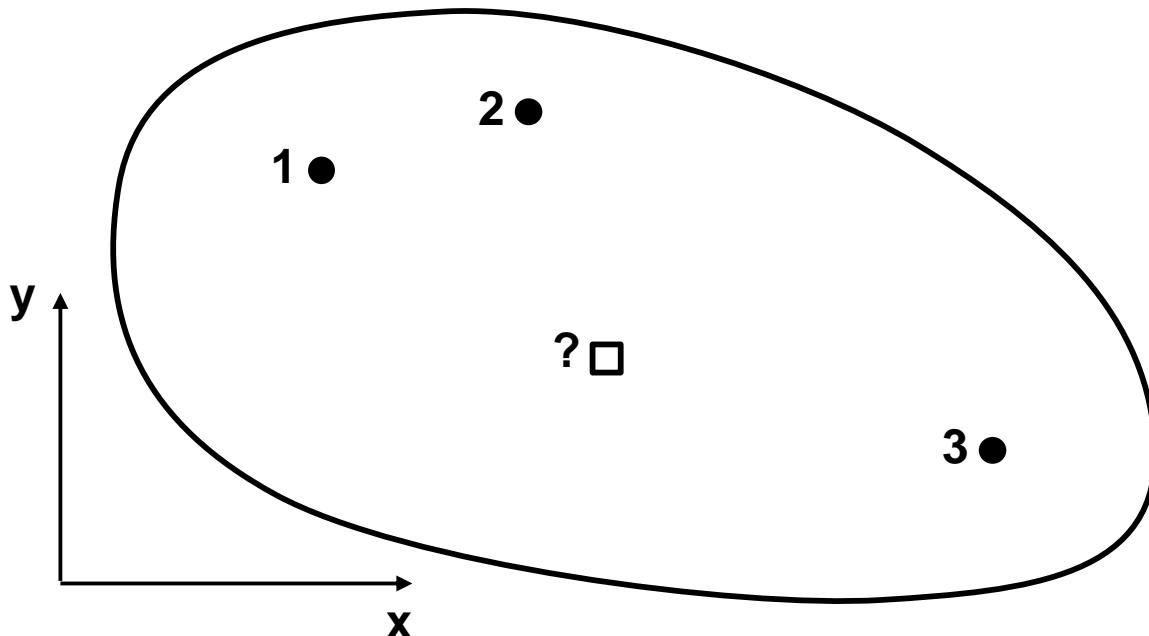
- **Things to try:**

1. Set the radius very large (50). How's the trend model performing? Try radius very small (1).
2. What do you think is the best radius to fit a trend to this spatial data?



Spatial Estimation

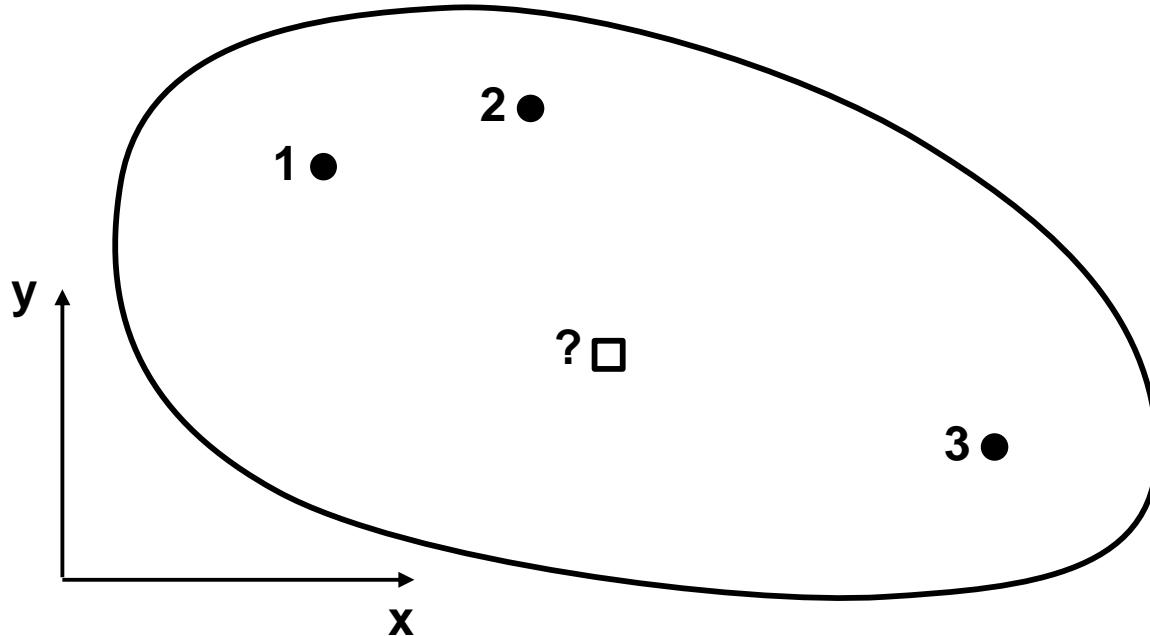
- Consider the case of estimating at some unsampled location:



- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?
- Note: z is the variable of interest (e.g. porosity etc.) and \mathbf{u}_i is the data locations.

Spatial Estimation

- Consider the case of estimating at some unsampled location:



$z(u_\alpha)$ is the data values

$z^*(u_0)$ is an estimate

λ_α is the data weights

m_z is the global mean

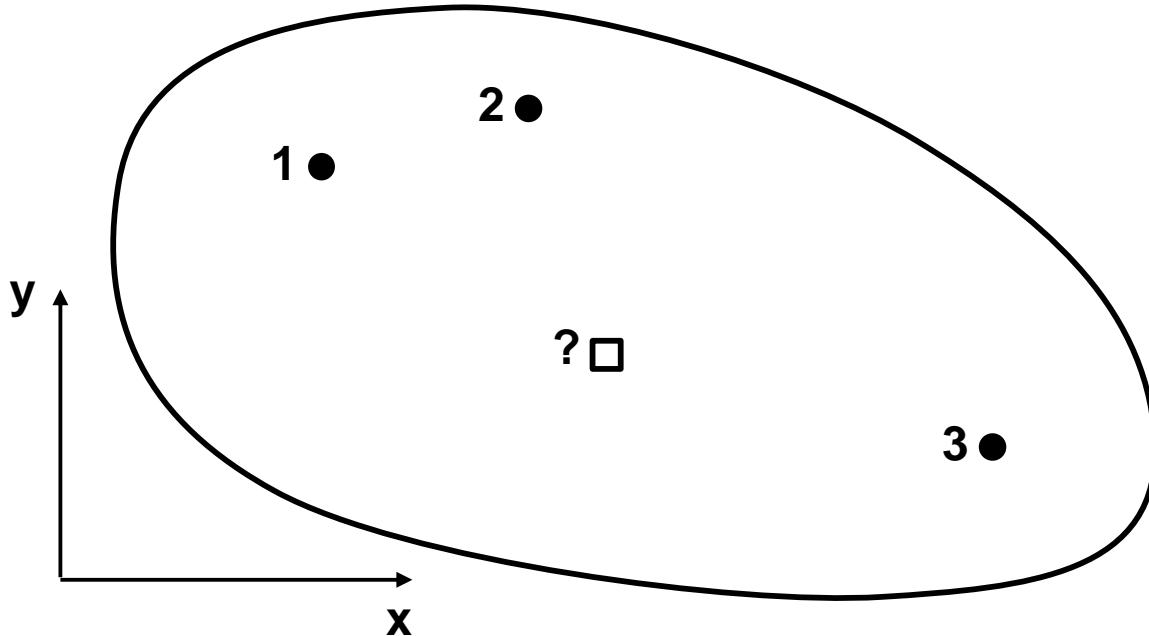
- How would you do this given data, $z(u_1)$, $z(u_2)$, and $z(u_3)$?

$$z^*(u_0) = \sum_{\alpha=1}^n \lambda_\alpha z(u_\alpha) + \left(1 - \sum_{\alpha=1}^n \lambda_\alpha\right) m_z$$

Unbiasedness
Constraint
Weights sum to 1.0.

Spatial Estimation

- Consider the case of estimating at some unsampled location:



- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?

$$z^*(\mathbf{u}_0) - m_z(\mathbf{u}_0) = \sum_{\alpha=1}^n \lambda_\alpha (z(\mathbf{u}_\alpha) - m_z(\mathbf{u}_\alpha))$$

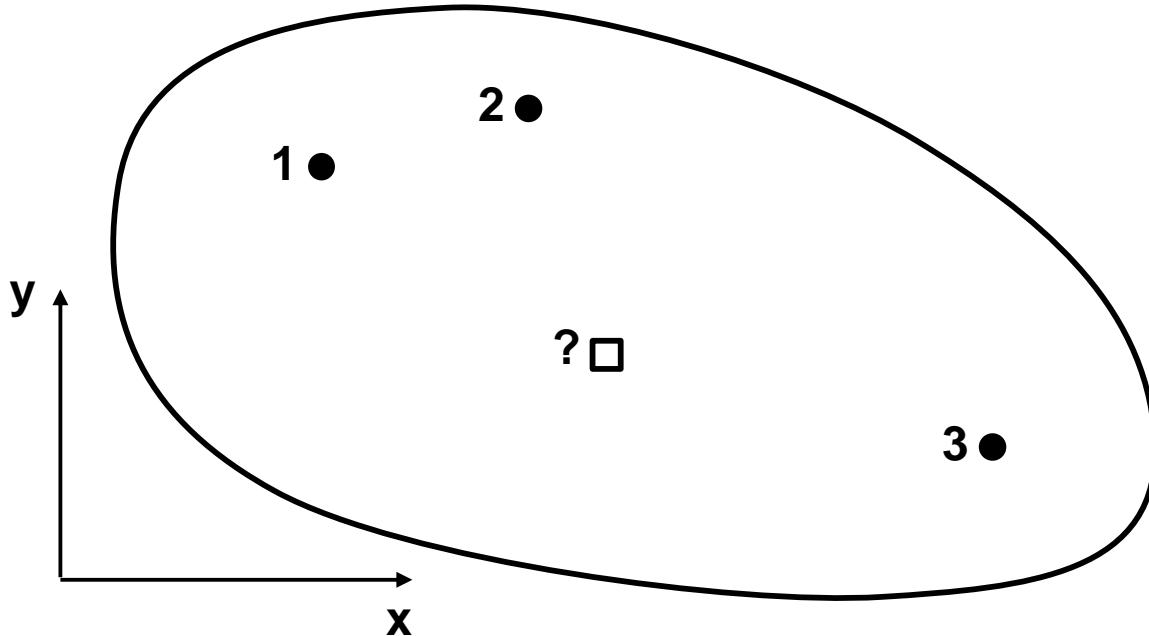
In the case where
the mean is non-stationary.

Given $y = z - m$, $y^*(\mathbf{u}_0) = \sum_{\alpha=1}^n \lambda_\alpha y(\mathbf{u}_\alpha)$

Simplified with residual, y .

Spatial Estimation

- Consider the case of estimating at some unsampled location:

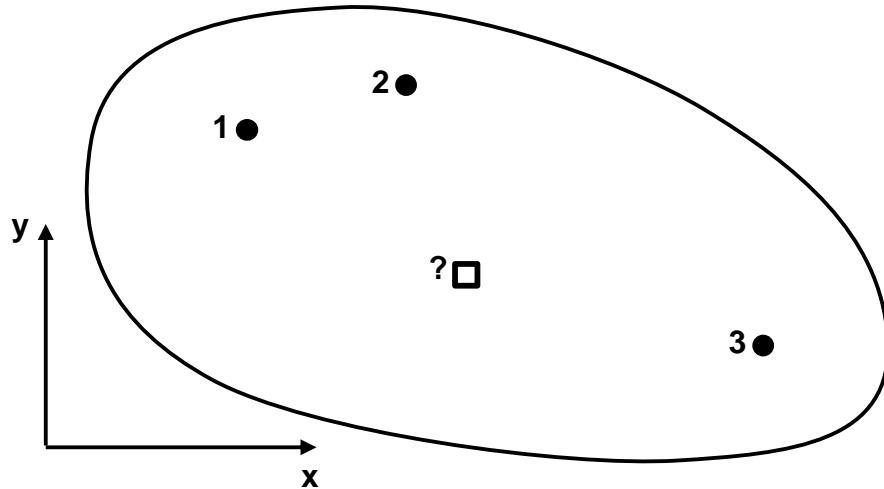


- Linear weighted, sound good. How do we get the weights? $\lambda_\alpha, \alpha = 1, \dots, n$

$$y^*(\mathbf{u}_0) = \sum_{\alpha=1}^n \lambda_\alpha y(\mathbf{u}_\alpha) \quad \text{Simplified with residual, } y.$$

Spatial Estimation

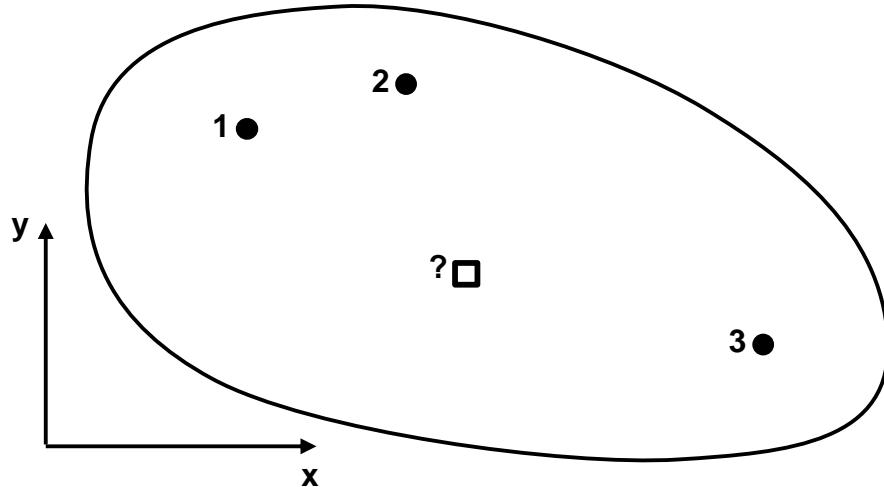
- Consider the case of estimating at some unsampled location:



- Linear weighted, sound good. How do we get the weights? $\lambda_\alpha, \alpha = 1, \dots, n$
- Equal weighted / average? $\lambda_\alpha = 1/n$ **Equal weight average of data**
- What's wrong with that?

Spatial Estimation

- Consider the case of estimating at some unsampled location:



- How do we get the weights? $\lambda_\alpha, \alpha = 1, \dots, n$

Inverse distance to power
standardized so weights
sum to 1.0.

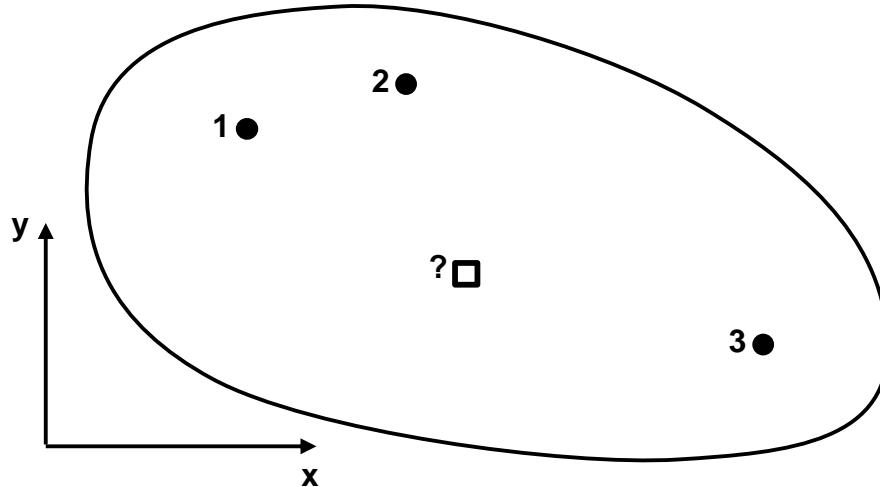
- Inverse distance?

$$\lambda_\alpha = \frac{1}{dist(\mathbf{u}_0, \mathbf{u}_\alpha)^p} / \sum_{\alpha=1}^n \lambda_\alpha$$

- What's wrong with that?

Spatial Estimation

- Consider the case of estimating at some unsampled location:



- How do we get the weights? $\lambda_\alpha, \alpha = 1, \dots, n$
- It would be great to use weight that account for closeness (spatial correlation > distance alone), redundancy (once again with spatial correlation).
- How can we do that?



Spatial Estimation

Kriging Derivations Removed

The results is a very useful and interpretable linear system of equations.

Simple Kriging: Some Details



There are three equations to determine the three weights:

$$\lambda_1 \cdot C(\mathbf{u}_1, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_1, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_1, \mathbf{u}_3) = C(\mathbf{u}_0, \mathbf{u}_1)$$

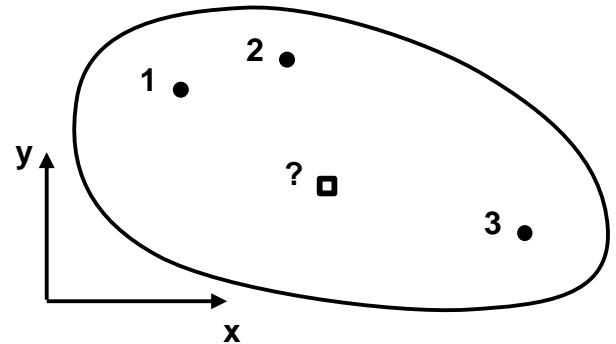
$$\lambda_1 \cdot C(\mathbf{u}_2, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_2, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_2, \mathbf{u}_3) = C(\mathbf{u}_0, \mathbf{u}_2)$$

$$\lambda_1 \cdot C(\mathbf{u}_3, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_3, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_3, \mathbf{u}_3) = C(\mathbf{u}_0, \mathbf{u}_3)$$

In matrix notation: Recall that $C(\mathbf{h}) = C(0) - \gamma(\mathbf{h})$

$$\begin{bmatrix} C(\mathbf{u}_1, \mathbf{u}_1) & C(\mathbf{u}_1, \mathbf{u}_2) & C(\mathbf{u}_1, \mathbf{u}_3) \\ C(\mathbf{u}_2, \mathbf{u}_1) & C(\mathbf{u}_2, \mathbf{u}_2) & C(\mathbf{u}_2, \mathbf{u}_3) \\ C(\mathbf{u}_3, \mathbf{u}_1) & C(\mathbf{u}_3, \mathbf{u}_2) & C(\mathbf{u}_3, \mathbf{u}_3) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} C(\mathbf{u}_0, \mathbf{u}_1) \\ C(\mathbf{u}_0, \mathbf{u}_2) \\ C(\mathbf{u}_0, \mathbf{u}_3) \end{bmatrix}$$

↑
redundancy
↑
weights
↑
closeness



Properties of Simple Kriging



What Does Kriging Provide?:

- **Best Estimate:** Minimum error estimator (just try to pick weights, you won't beat it)
- **Estimation Variance:** Provides a measure of the estimation (or kriging) variance (uncertainty in the estimate):

$$\sigma_E^2(\mathbf{u}) = C(0) - \sum_{\alpha=1}^n \lambda_\alpha C(\mathbf{u} - \mathbf{u}_\alpha)$$

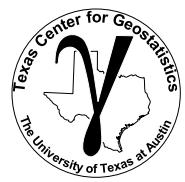
$\sigma_E^2 \rightarrow [0, \sigma_x^2]$

Diagram illustrating the components of the Estimation Variance formula:

- Estimation Variance** (red text) points to $\sigma_E^2(\mathbf{u})$.
- Variance** (red text) points to $C(0)$.
- Kriging Weights** (red text) points to λ_α .
- Covariance Between Data and Estimate Location** (red text) points to $C(\mathbf{u} - \mathbf{u}_\alpha)$.

More Properties

- Exact interpolator: at data location
- Kriging variance can be calculated before getting the sample information, homoscedastic!
- Kriging takes into account:
 - distance of the information: $C(\mathbf{u}, \mathbf{u}_i)$
 - configuration of the data: $C(\mathbf{u}_i, \mathbf{u}_j)$
 - structural continuity of the variable being considered: $C(\mathbf{h})$
- The smoothing effect of kriging can be forecast – we will return to this with simulation.
- Kriging theory is part of the probabilistic theory of *projectors*: orthogonal projection onto space of linear combinations of the n data (Hilbert space)



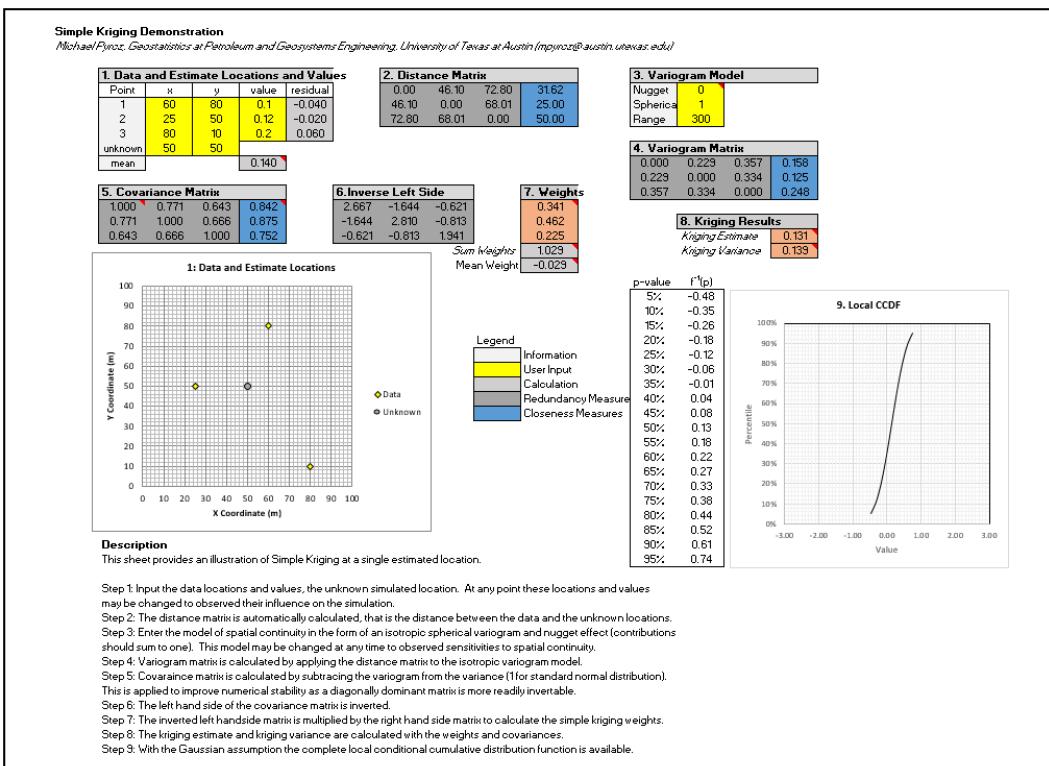
Simple Kriging Hands-on

Here's an opportunity for experiential learning with Simple Kriging.

- **Things to try:**

Pay attention to the kriging weights, kriging estimate and kriging variance while you:

1. Set points 1 and 2 closer together.
2. Put point 1 behind point 2 to create screening.
3. Put all points outside the range.
4. Set the range very large.



Spatial Uncertainty Hands-on

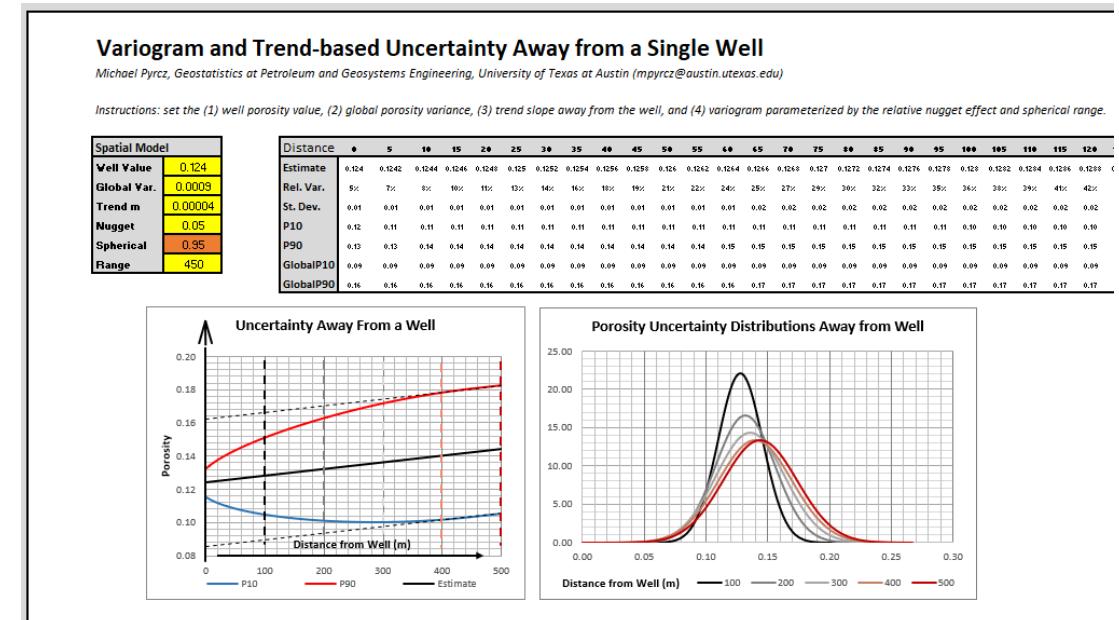


Here's an opportunity for experiential learning with Simple Kriging for spatial uncertainty. The kriging estimation variance is very useful.

- **Things to try:**

Pay attention to the kriging uncertainty P10, mean and P90 away from the well as you:

1. Change the spatial continuity range.
2. Add and adjust the nugget effect.
3. Modify the trend slope.



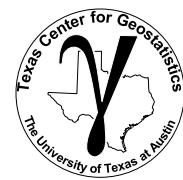
Spatial Estimation New Tools



Topic	Application to Subsurface Modeling
Trend Modeling	<p>Decompose variance into deterministic trend and stochastic residual.</p> <p><i>30% of porosity variance is described by a linear depth trend and 70% is described by a 3D variogram model.</i></p>
Kriging Estimates and Kriging Variances	<p>Kriging provides the best estimate and a measure of estimation variance.</p> <p><i>Given a kriging estimate of 13% and kriging variance of 9% and the assumption of a Gaussian distribution we have a complete local distribution of uncertainty for pre-drill porosity.</i></p>

Data Analytics, Geostatistics and Machine Learning

Spatial Continuity / Prediction



Lecture outline . . .

- Spatial Estimation

Introduction

Fundamental Concepts

Probability

Data Prep / Analytics

Spatial Continuity / Prediction

Multivariate Modeling

Uncertainty Modeling

Machine Learning

Instructor: Michael Pyrcz, the University of Texas at Austin