Multivariate Modeling
Spatial Estimation

Lecture outline . . .

- **Trend Modeling**
- **Kriging**

Multivariate Modeling
Spatial Estimation

Lecture outline . . .

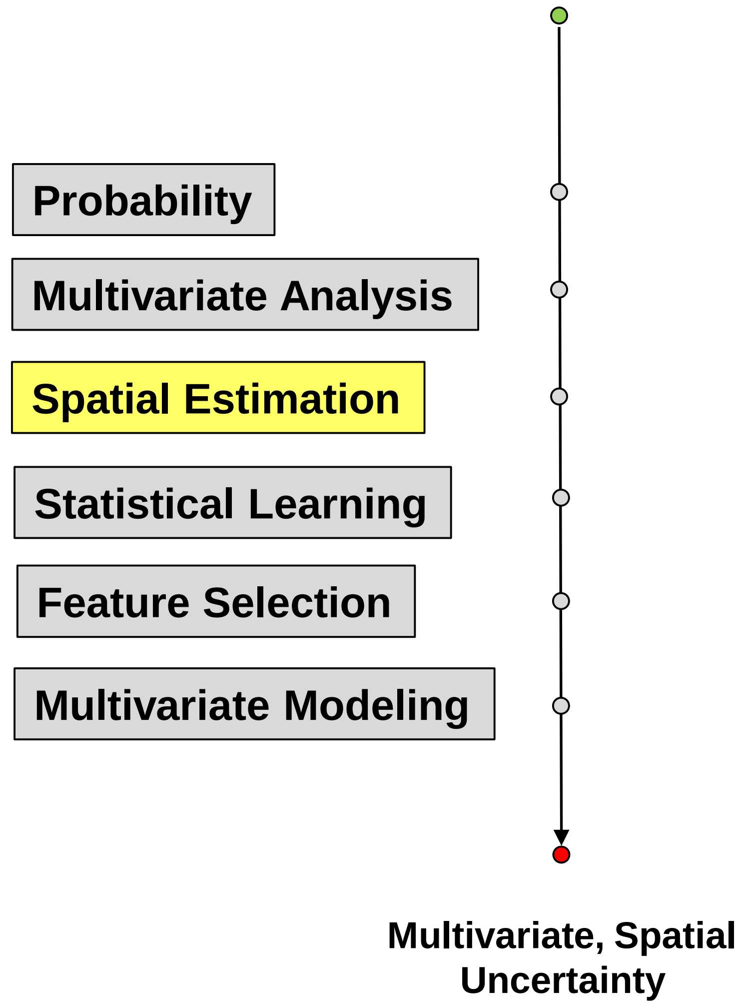
- **Trend Modeling**

What Will You Learn?

Why Cover Spatial Estimation?

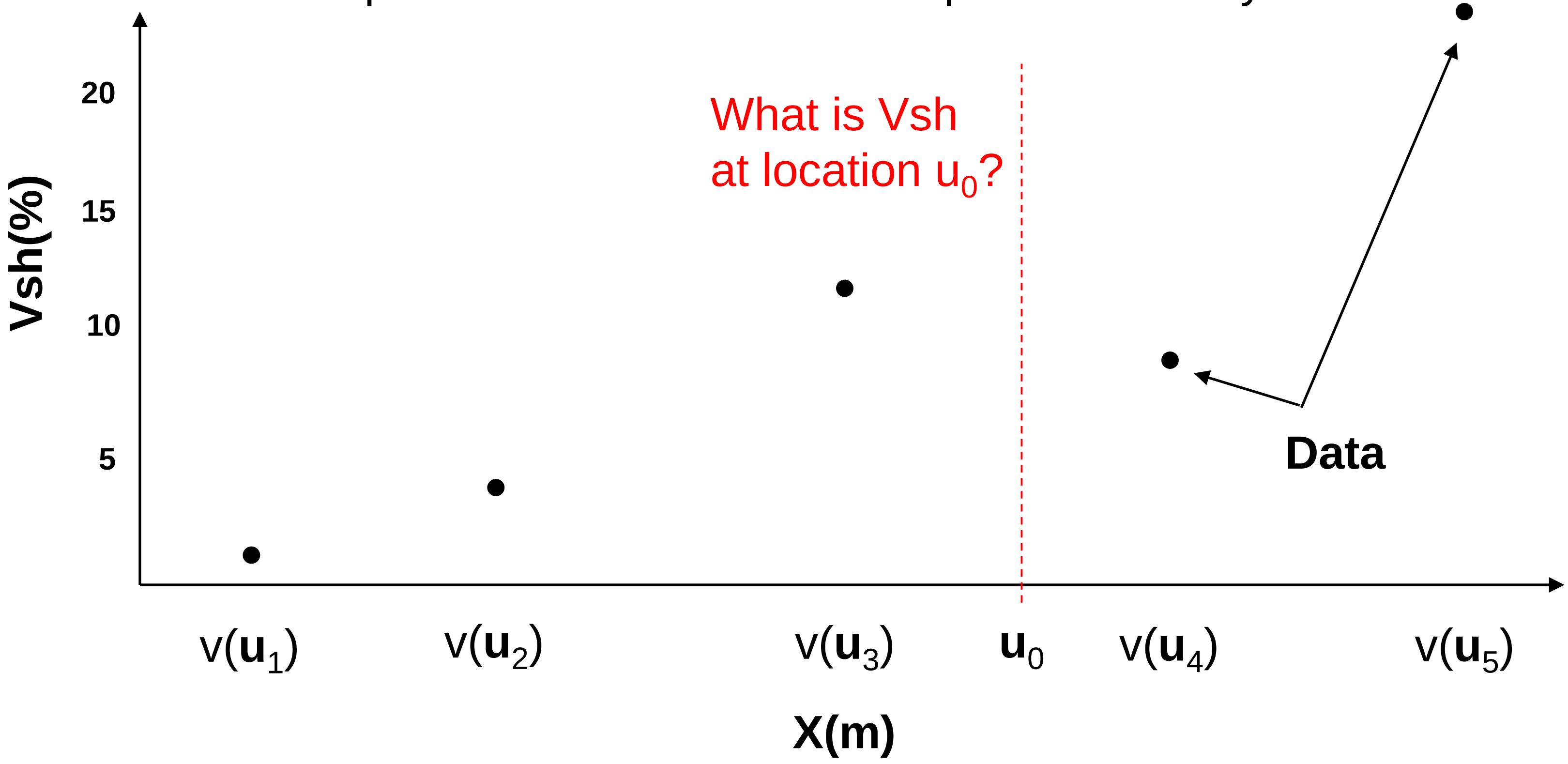
- We will use kriging estimate and variance combined with an assumption of Gaussian distribution to calculate the prior distribution (given well data) at each location.

- We will use multivariate kriging to combine the secondary variables to estimate the likelihood distribution.



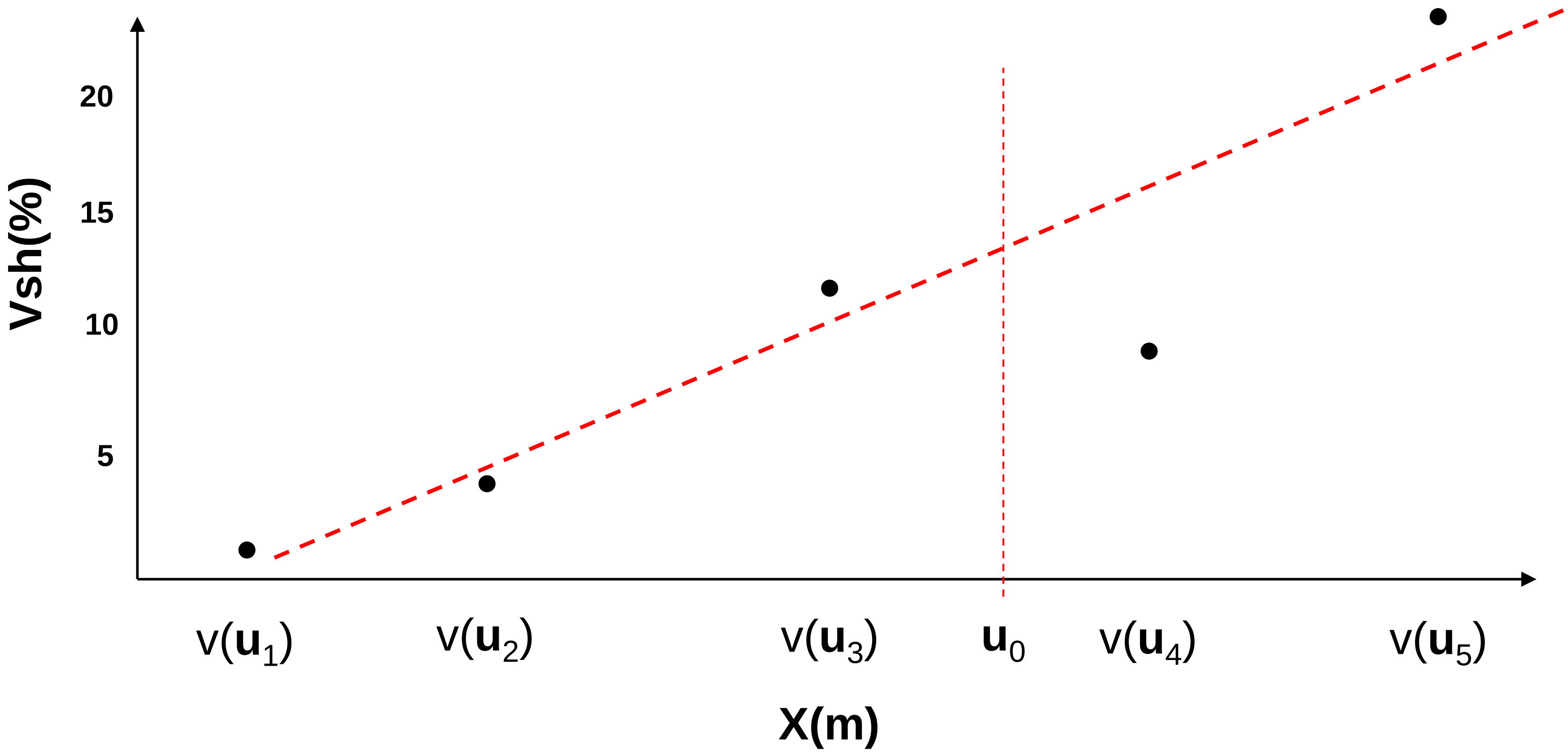
Nonstationarity

- Geostatistical spatial estimation methods make an assumption concerning stationarity
  - In the presence of significant nonstationarity we would not rely 100% for spatial estimation on data + spatial continuity model



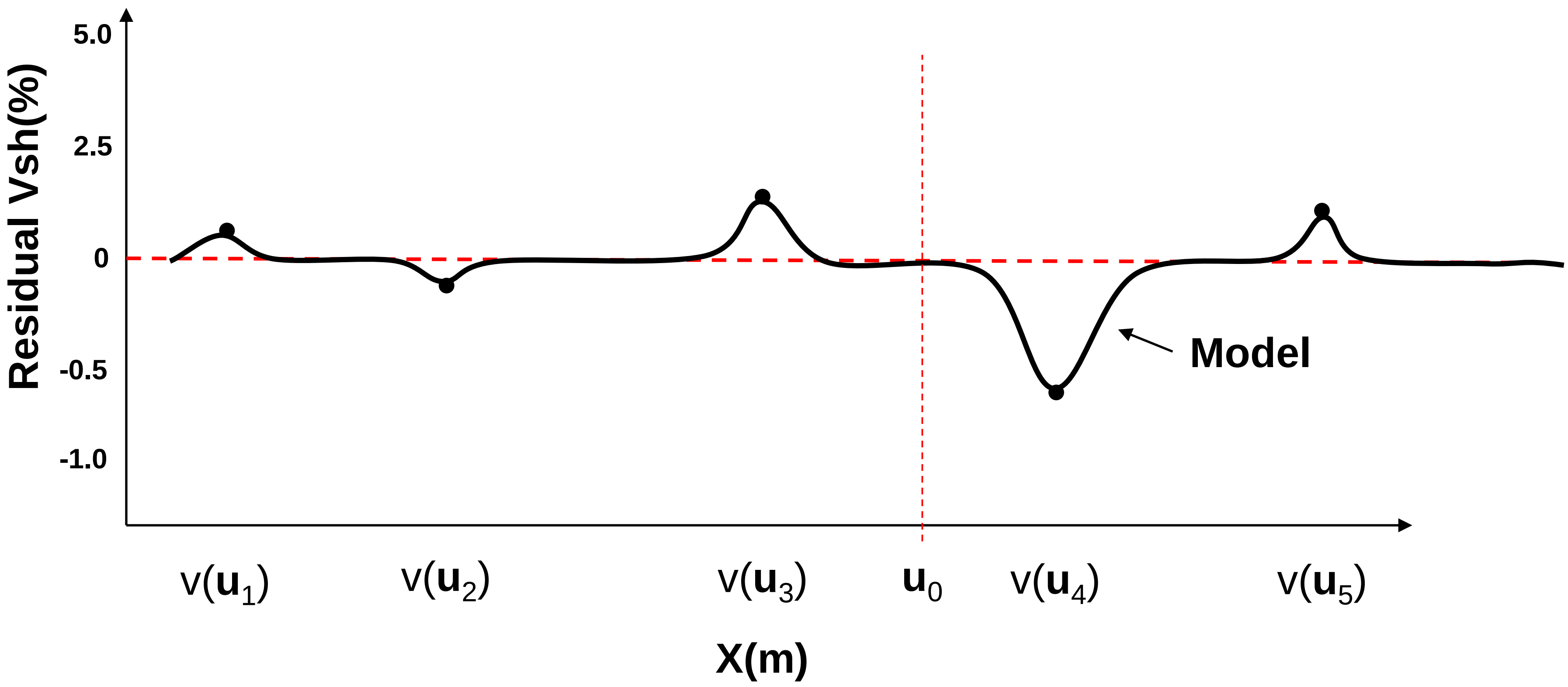
Nonstationarity

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
  - If we observe a trend, we should model the trend.



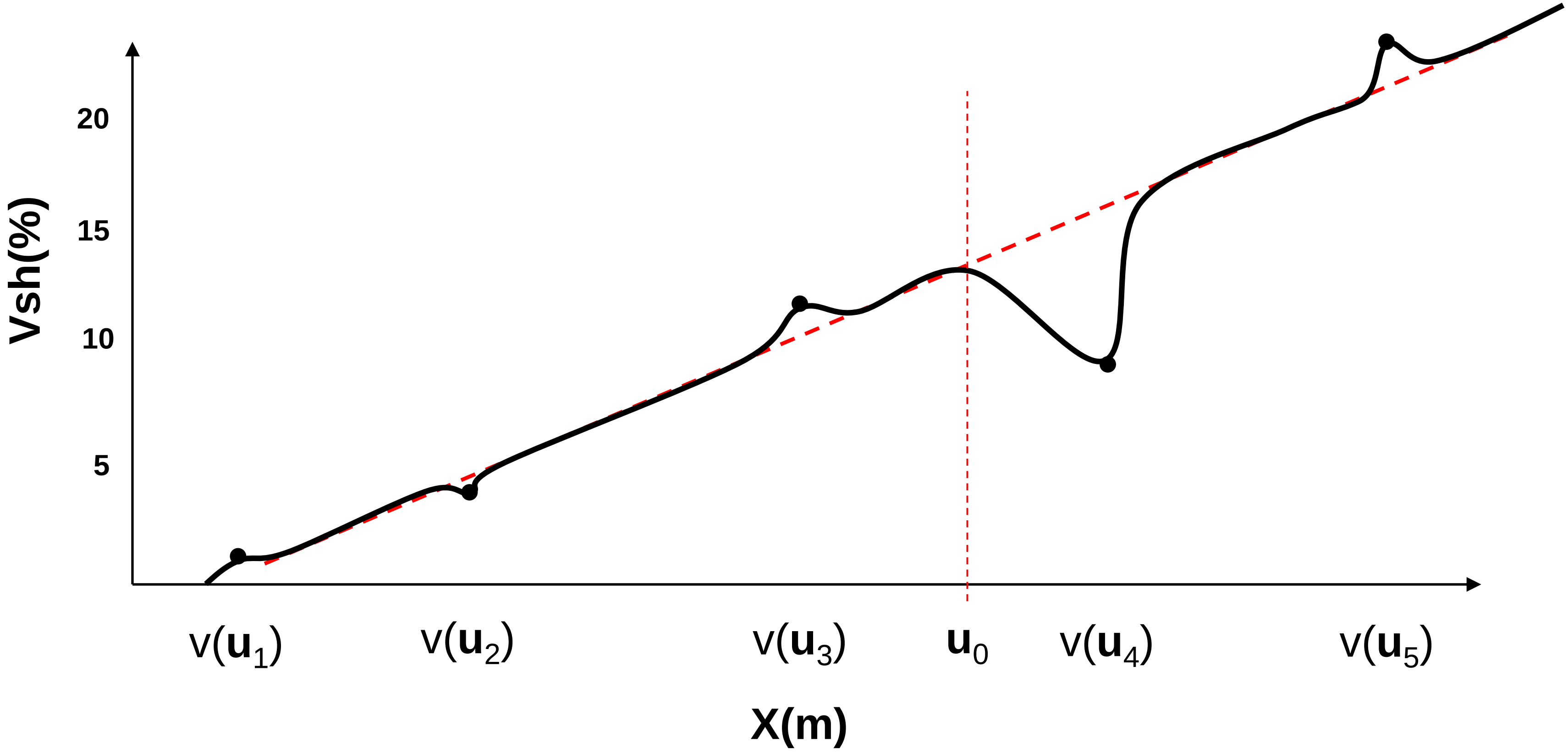
Nonstationarity

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
  - Then model the residuals.



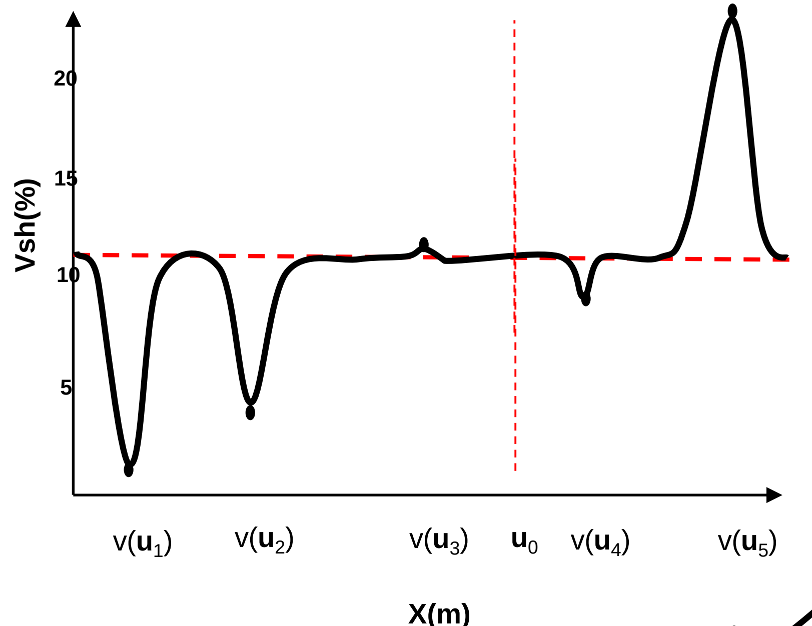
Nonstationarity

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
  - After modeling, add the trend back to the modelled residuals

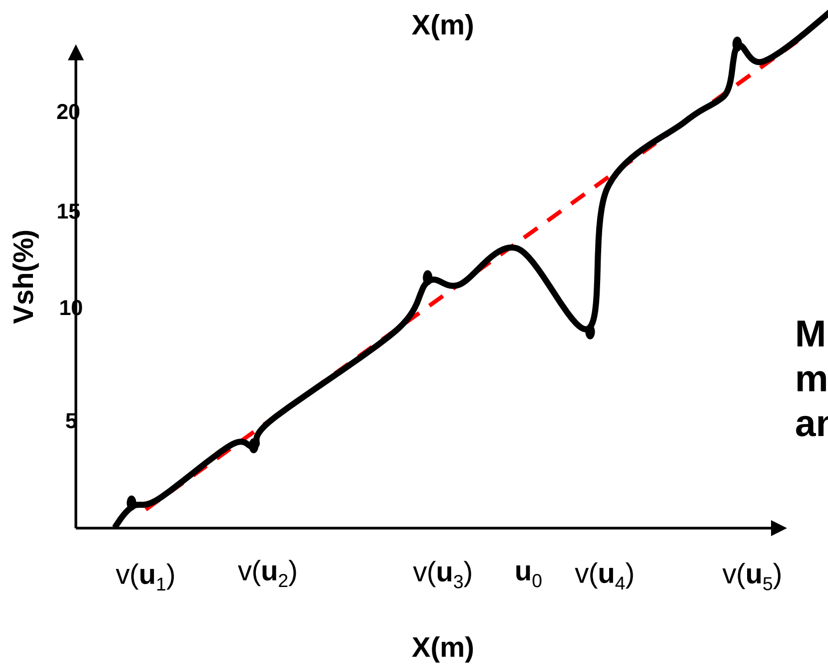


Nonstationarity

– How bad could it be if we did not model a trend?
– Geostatistical estimation would assume stationarity and away from data we would estimate with the global mean (simple kriging)!



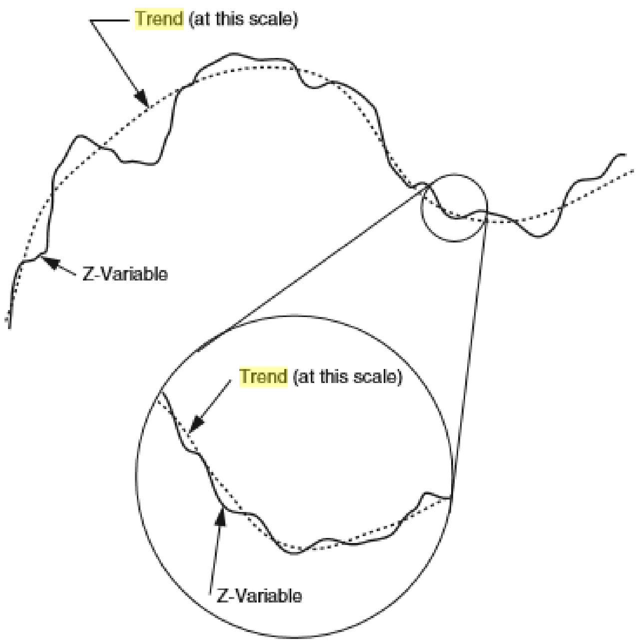
Model with stationary mean + data.

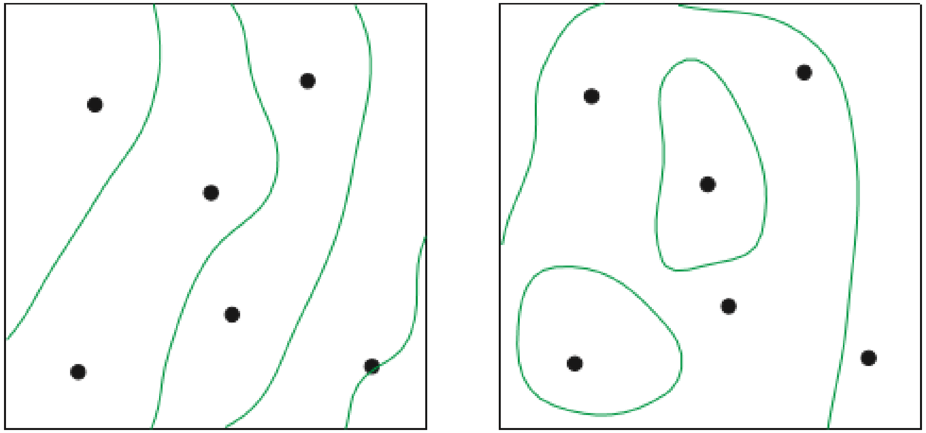


Model with mean trend model and residual + data.

Nonstationarity

- Trend Modeling
  - We must identify and model trends / nonstationarities





- While we discuss data-driven trend modeling here any **trend modeling should include data integration** over the entire asset team
  - Geology
  - Geophysics
  - Petrophysics
  - Reservoir Engineering

Images from Pyrcz and Deutsch (2014)

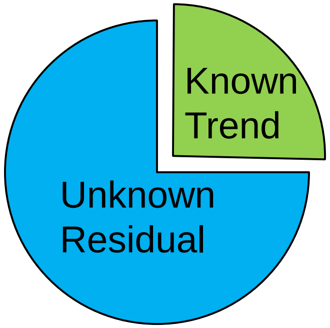
Nonstationarity

- Any variance in the trend is removed from the residual:

$$\sigma_X^2 = \sigma_{X_t}^2 + \sigma_{X_r}^2 + 2C_{X_t,X_r}$$

  - if the $X_t \perp\!\!\!\perp X_r$, $C_{X_t,X_r} = 0$

$$\sigma_{X_r}^2 = \sigma_X^2 - \sigma_{X_t}^2$$



  – So if σ_X^2 is the total variance (variability), and $\sigma_{X_t}^2$ is the variability that is deterministically modelled, treated as known, and $\sigma_{X_r}^2$ is the component of the variability that is treated as unknown.

  – Result: the more variability explained by the trend the less variability that remains as uncertain.

Additivity of Variance for Decomposing Trend and Residual

Can we partition variance of random variable Z between trend (X) and residual (Y)?

- Start with the variance of Z:

$$\sigma_Z^2 = E\left(Z^2\right) - [E(Z)]^{\,2}$$

- Substitute: $Z = \mathrm{X} + \mathrm{Y}$

$$\sigma_{X+Y}^2 = E\left((X+Y)^2\right) - [E(X+Y)]^{\,2}$$

$$\sigma_{X+Y}^2 = E\left(X^2 + 2XY + Y^2\right) - [E(X) + E(Y)]^{\,2}$$

$$\sigma_{X+Y}^2 = E\left(X^2\right) + 2E(XY) + E\left(Y^2\right) - \left(E(X)^2 + 2E(X)E(Y) + E(Y)^2\right)$$

$$\sigma_{X+Y}^2 = \underbrace{\boxed{E\left(X^2\right) - E(X)^2}}_{\sigma_X^2} + \underbrace{\boxed{E\left(Y^2\right) - E(Y)^2}}_{\sigma_Y^2} + 2\underbrace{\boxed{\left(E(XY) - E(X)E(Y)\right)}}_{C_{XY}(0)}$$

- Note covariance: $C_{XY} = E(XY) - E(X)E(Y)$

$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2C_{XY}(0) \quad \triangleleft$$ **Additivity of variance**

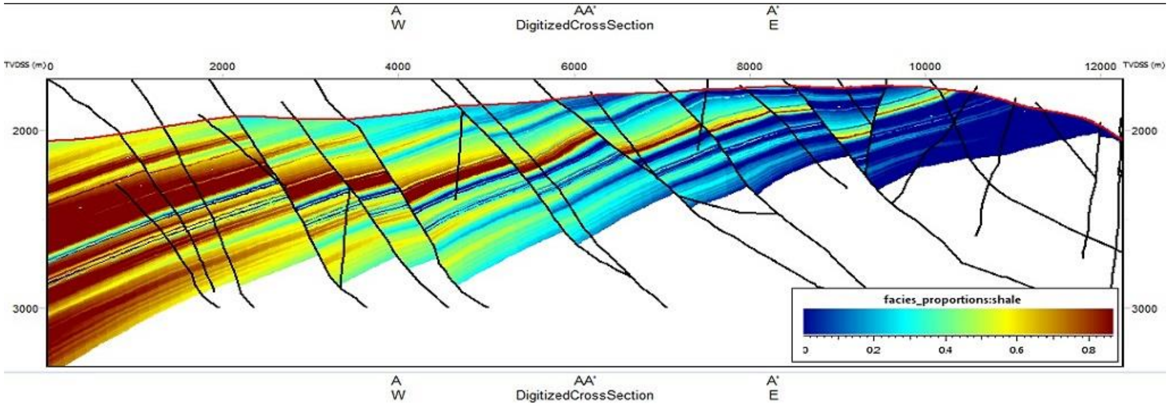
- If the X __ Y, $C_{XY}(0)$ = 0, then $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$ $\triangleleft$ **In practice**

Definition
Deterministic Model

- Model that assumes perfect knowledge, without uncertainty

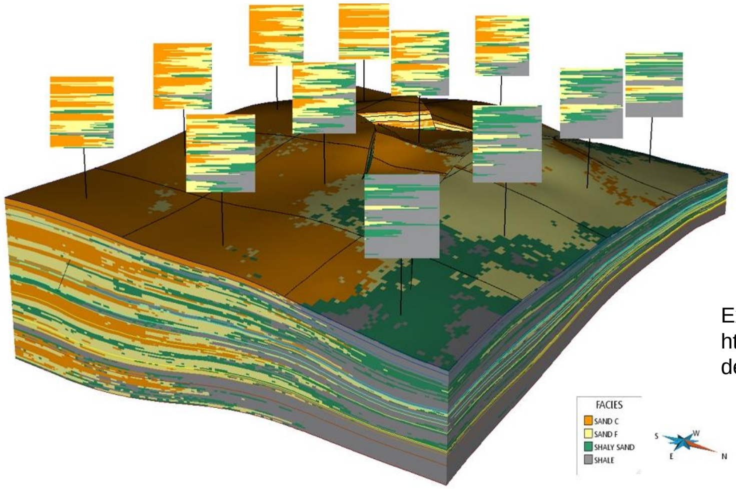
- Based on knowledge of the phenomenon or trend fitting to data
- Most subsurface models have a deterministic component (trend) to capture expert knowledge and to provide a stationary residual for geostatistical modeling.

Trend Modeling



Example facies trend model Gocad SKUA. http://www.pdgm.com/getdoc/b24891f9-7470-4728-8cb7-0ddd7df196df/skua-facies-modeling/

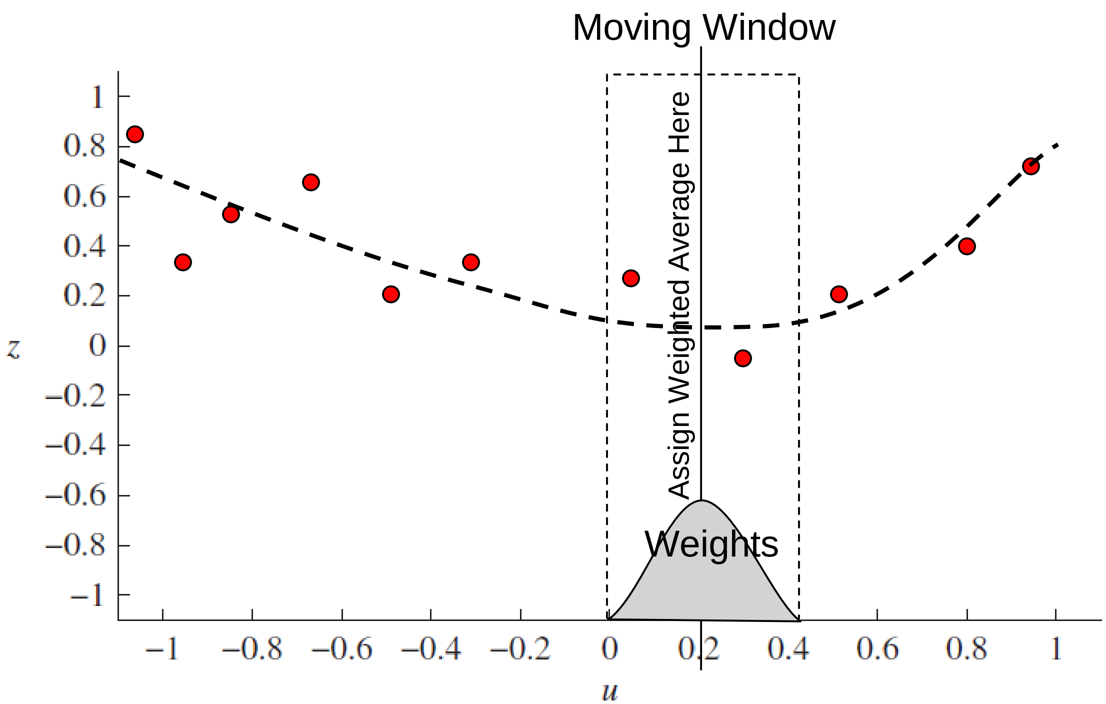
- Trend models:
  - Tend to be smooth, based on data and interpretation
  - May be complicated (see above)
  - Parameterized by vertical proportion curves (see below) and areal trend maps



Example facies trend model Gocad. http://www.pdgm.com/getdoc/bd9ab6b6-7dbb-4023-8f79-de34238d5136/skua-reservoir-data-analysis/

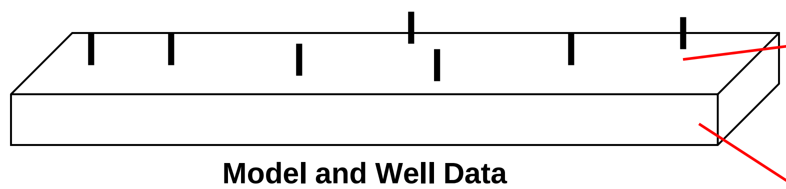
Trend Modeling Workflow

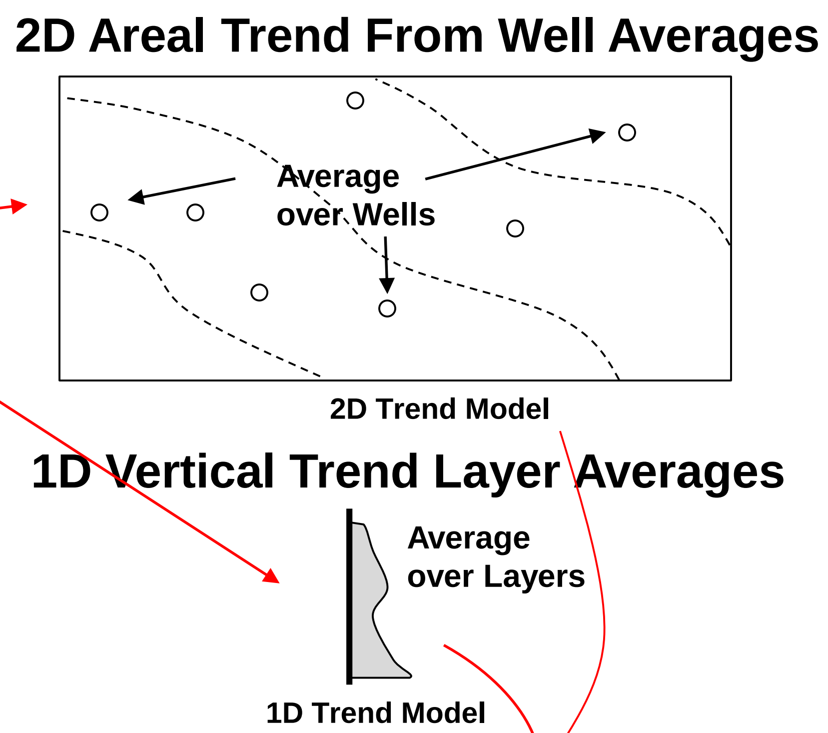
- How to calculate a trend model:
  - Moving window average of the available data
  - Weighting scheme within the window
    - » Uniform weights can cause discontinuities
    - » Reduce weight at edges of moving window to reduce discontinuities (e.g. Gaussian weights).



Trend 2D + 1D Workflow

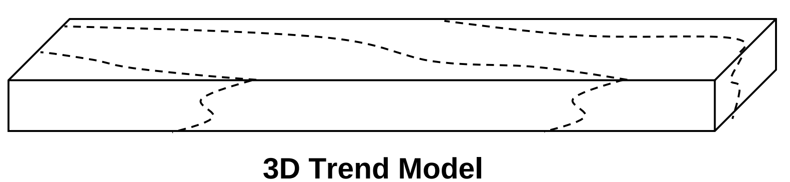
- Calculate 2D Area and 1D Vertical trends:





- Combine 1D and 2D ⇨ 3D.

$$\bar{X}(x,y,z) = \bar{X}(z) \cdot \frac{\bar{X}(x,y)}{\bar{X}}$$



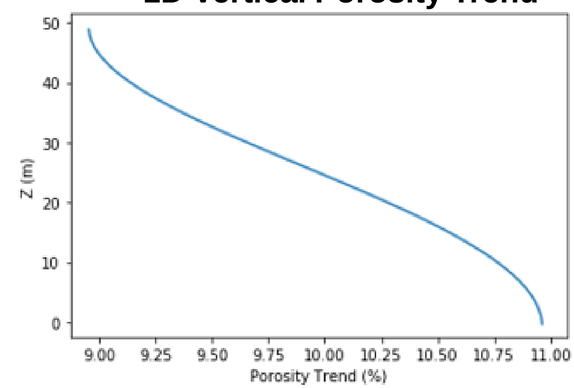
Trend Modeling Workflow

- How to calculate a trend model:
  - Calculate the 2D areal trend by interpolating over vertically averaged wells.
  - Calculate the 1D vertical trend by averaging layers
  - Combine the 1D vertical and the 2D areal trends:

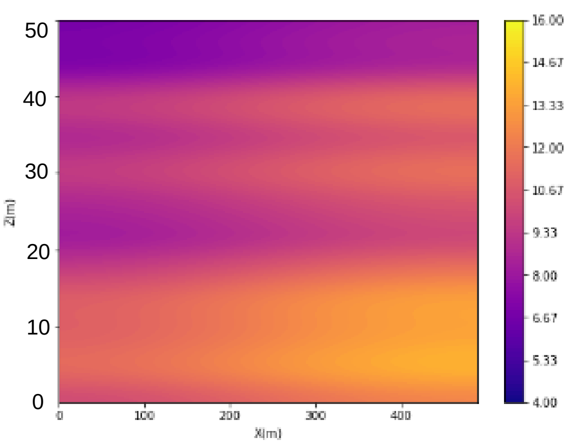
3D Trend — 1D Vertical Trend — 2D Areal Trend — Global Mean

$$\bar{X}(x,y,z) = \bar{X}(z) \cdot \frac{\bar{X}(x,y)}{\bar{X}}$$

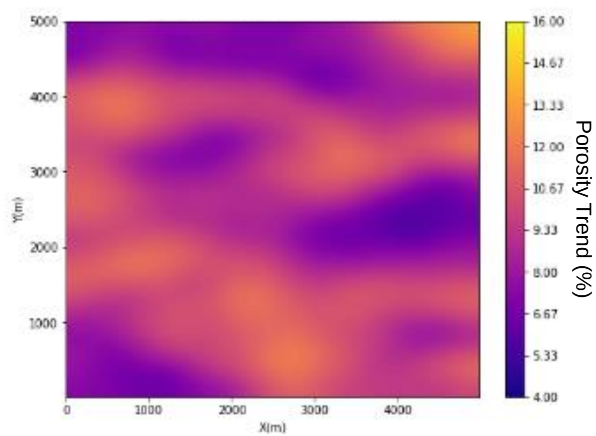
1D Vertical Porosity Trend



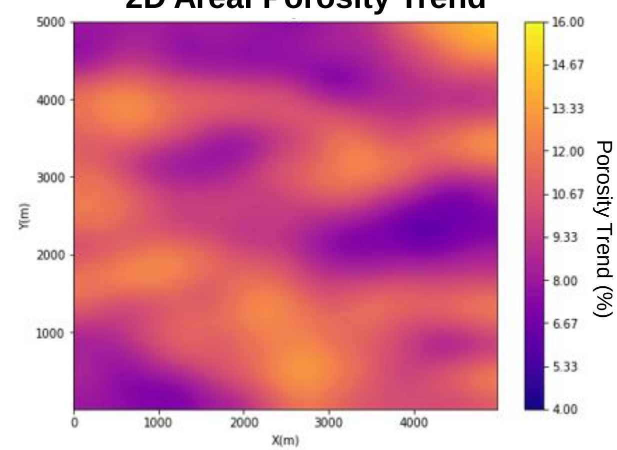
3D Porosity Trend, y = 2500m



3D Porosity Trend, z = 10m



2D Areal Porosity Trend



Trend Definition

- Observation of nonstationarity in any statistic, metric of interest
- A model of the nonstationarity in any statistic, metric of interest
- Typically modeled with support of data and expert knowledge in a deterministic manner (without uncertainty).

Multivariate Modeling
Spatial Estimation

Lecture outline . . .

- **Kriging**

Introduction

Prerequisites

Probability

Multivariate Analysis

Spatial Estimation

Statistical Learning

Feature Selection

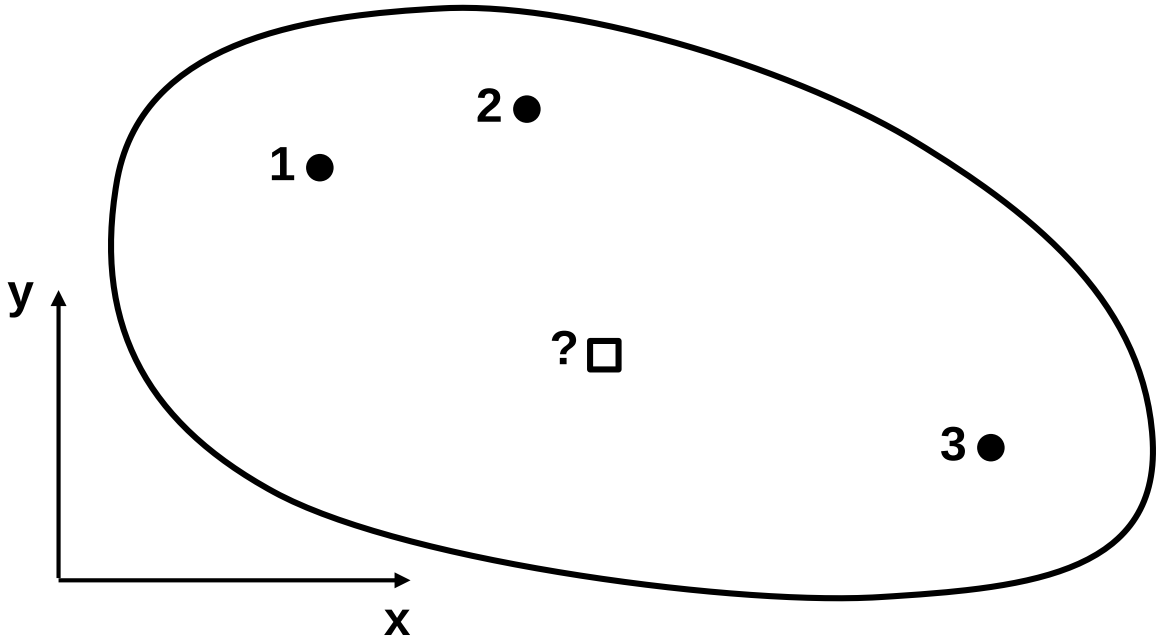
Multivariate Modeling

Conclusions

Lecture based on Pyrcz and Deutsch (2014)

Spatial Estimation

- Consider the case of estimating at some unsampled location:

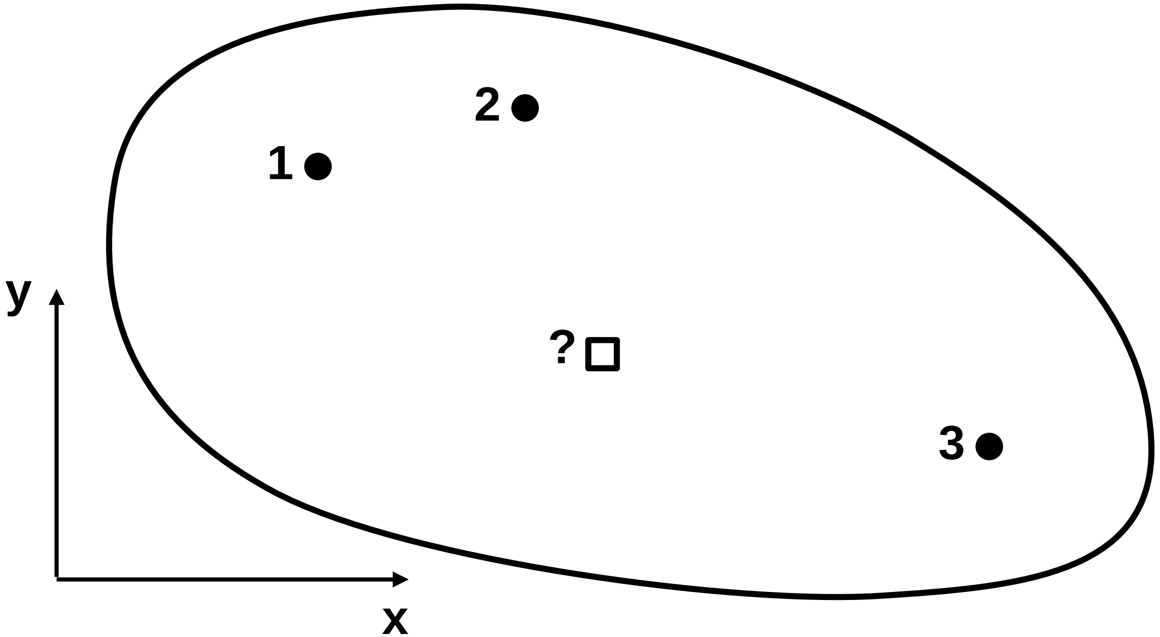


- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?

- Note: z is the variable of interest (e.g. porosity etc.) and $\mathbf{u}_i$is the data locations.

Spatial Estimation

- Consider the case of estimating at some unsampled location:



$\mathbf{z}(\mathbf{u}_\alpha)$ **is the data values**

$z^*(\mathbf{u}_0)$ **is an estimate**

λ_α **is the data weights**

m_z **is the global mean**

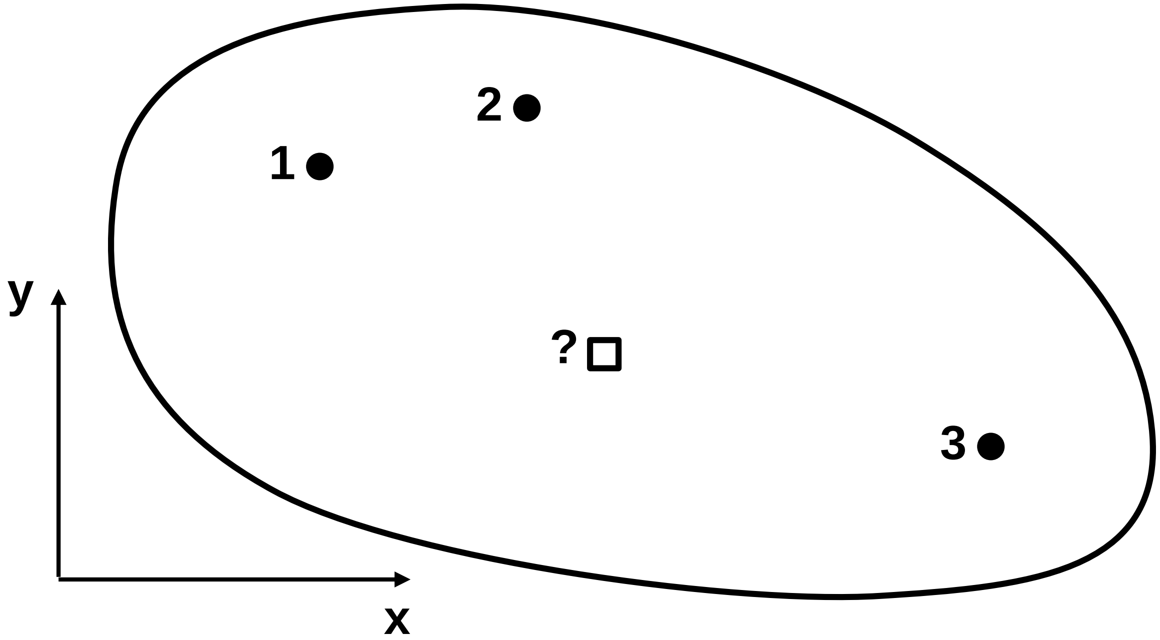
- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?

$$z^*(\mathbf{u}_0) = \sum_{\alpha=1}^{n} \lambda_\alpha \mathbf{z}(\mathbf{u}_\alpha) + \boxed{\left(1 - \sum_{\alpha=1}^{n} \lambda_\alpha\right) m_z}$$

Unbiasedness Constraint Weights sum to 1.0.

Spatial Estimation

- Consider the case of estimating at some unsampled location:



- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?

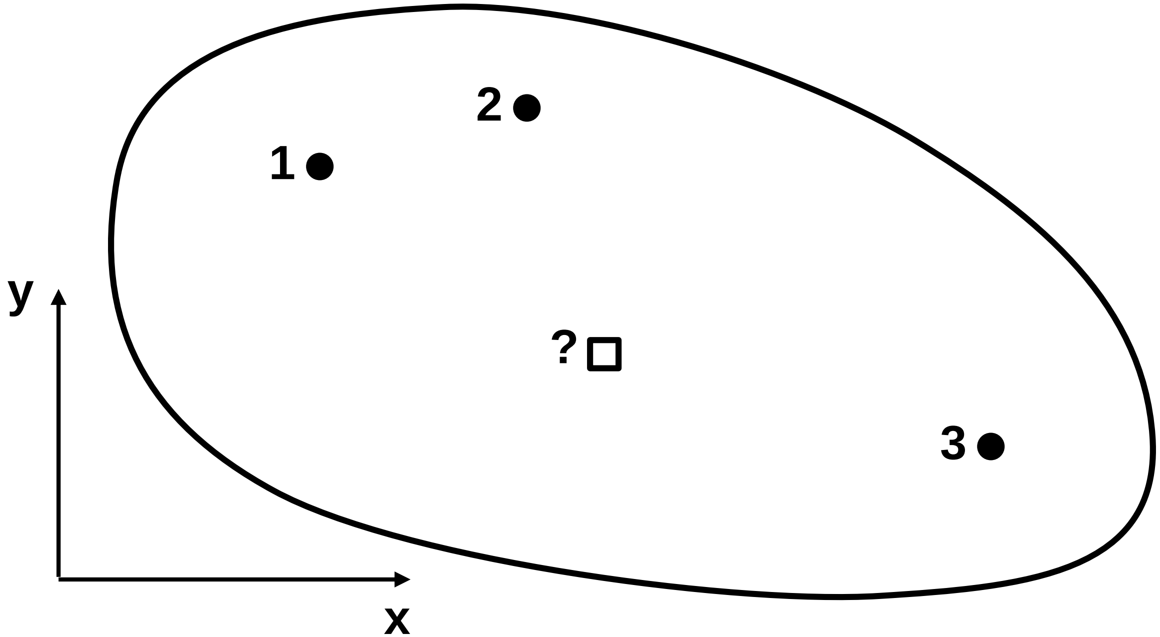
$$z^*(\mathbf{u_0}) - m_z(\mathbf{u_0}) = \sum_{\alpha=1}^{n} \lambda_\alpha \left(\mathbf{z}(\mathbf{u}_\alpha) - m_z(\mathbf{u}_\alpha)\right)$$

In the case where the mean is non-stationary.

Given y$= z - m,\quad y^*(\mathbf{u_0}) = \sum_{\alpha=1}^{n} \lambda_\alpha\, \mathbf{y}(\mathbf{u}_\alpha)$ **Simplified with residual, y.**

Spatial Estimation

- Consider the case of estimating at some unsampled location:



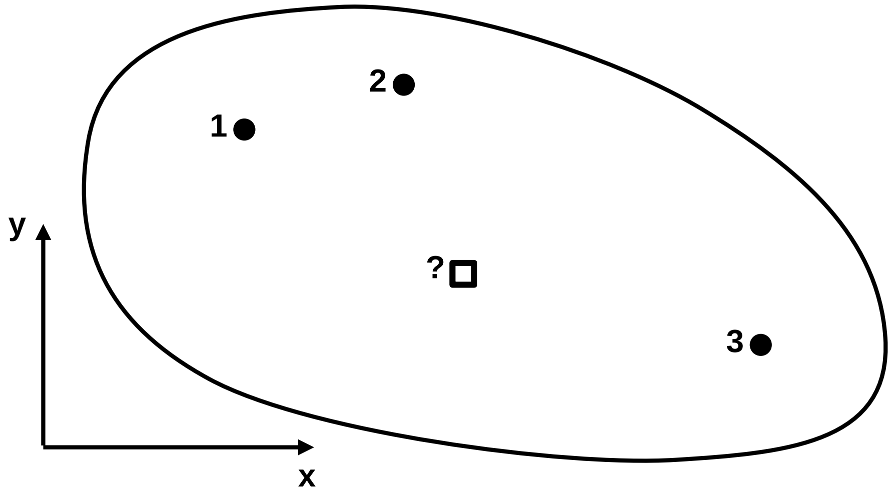
- Linear weighted, sound good. How do we get the weights? $\boldsymbol{\lambda_\alpha, \alpha = 1, \ldots, n}$

$$\boldsymbol{y^*(\mathbf{u_0}) = \sum_{\alpha=1}^{n} \lambda_\alpha \, \mathbf{y}(\mathbf{u}_\alpha)}$$

Simplified with residual, y.

Spatial Estimation

- Consider the case of estimating at some unsampled location:



- Linear weighted, sound good. How do we get the weights? $\lambda_\alpha, \alpha = 1, \dots, n$

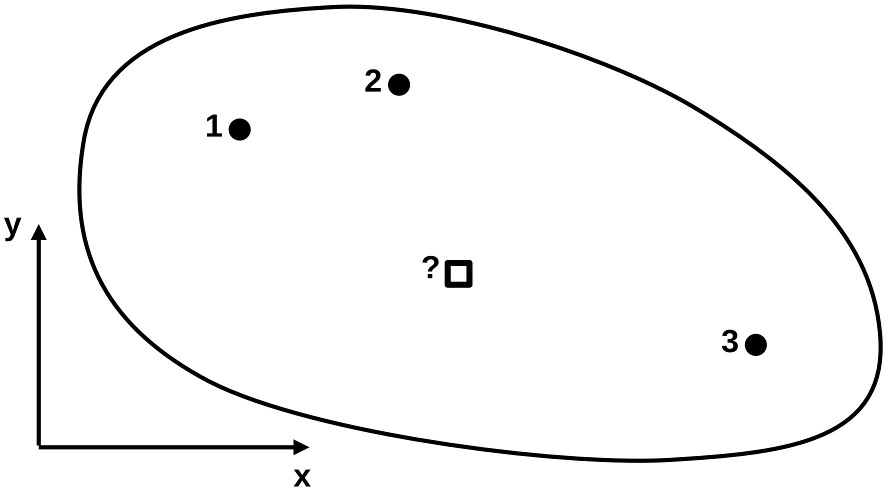
- Equal weighted / average? $\lambda_\alpha = {}^{1}\!/_{n}$

Equal weight average of data

- What's wrong with that?

Spatial Estimation

- Consider the case of estimating at some unsampled location:



- How do we get the weights? $\boldsymbol{\lambda_\alpha, \alpha = 1, \ldots, n}$

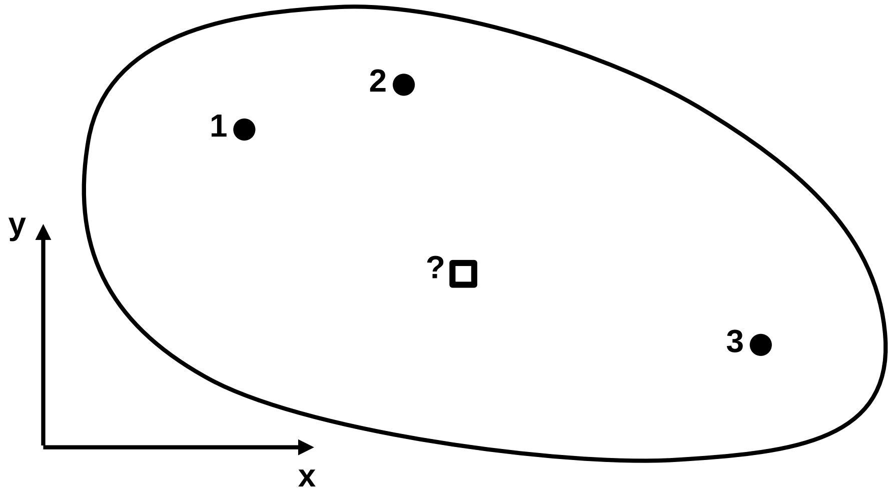
- Inverse distance? $$\lambda_\alpha = \frac{1}{dist(\mathbf{u_0}, \mathbf{u}_\alpha)^p} \Big/ \sum_{\alpha=1}^{n} \lambda_\alpha$$

Inverse distance to power standardized so weights sum to 1.0.

- What's wrong with that?

Spatial Estimation

- Consider the case of estimating at some unsampled location:



- How do we get the weights? $\lambda_\alpha, \alpha = 1, \ldots, n$

- It would be great to use weight that account for closeness (spatial correlation > distance alone), redundancy (once again with spatial correlation).

- How can we do that?

Derivation of Simple Kriging Equations

- Consider a linear estimator:

$$Y^*(\mathbf{u}) = \sum_{i=1}^{n} \lambda_i \cdot Y(\mathbf{u}_i)$$

where $Y(u_i)$ are the residual data (data values minus the mean) and $Y^*(u_i)$ is the estimate (add the mean back in when we are finished)

- The **estimation variance** is defined as:

Stationary Mean, Variogram

$$E\{Y\} = 0$$

$$2\gamma(\mathbf{h}) = E\left\{\left[Y(\mathbf{u}) - Y(\mathbf{u}+\mathbf{h})\right]^2\right\}$$

$$E\left\{\left[Y^*(u) - Y(u)\right]^2\right\} = \ldots$$

$$= E\left\{\left[Y^*(u)\right]^2\right\} - 2\ E\left\{Y^*(u)\ Y(u)\right\} + E\left\{\left[Y(u)\right]^2\right\}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda_j E\left\{Y(u_i)\ Y(u_j)\right\} - 2\sum_{i=1}^{n} \lambda_i E\left\{Y(u)\ Y(u_i)\right\} + C(0)$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda_i \lambda_j C(u_i, u_j) - 2\sum_{i=1}^{n} \lambda_i C(u, u_i) + C(0)$$

redundancy **closeness** **variance**

$C(\mathbf{u}_i, \mathbf{u}_j)$ – covariance between data i and j, $C(\mathbf{u}_i, \mathbf{u})$covariance between data and unknown location and $C(0)$ is the variance.

More Derivation

- Optimal weights $\lambda_i, i = 1, \ldots, n$ may be determined by taking partial derivatives of the error variance w.r.t. the weights

$$\frac{\partial[\quad\quad]}{\partial \lambda_i} = \sum_{j=1}^{n} \lambda_j C(\mathbf{u}_i, \mathbf{u}_i) - 2 \cdot C(\mathbf{u}, \mathbf{u}_i), i = 1, \ldots, n$$

and setting them to zero

$$\sum_{j=1}^{n} \lambda_j C(\mathbf{u}_i, \mathbf{u}_i) = \cdot\, C(\mathbf{u}, \mathbf{u}_i), i = 1, \ldots, n$$

- This system of *n* equations with *n* unknown weights is the simple kriging (SK) system

Kriging Definition

- Estimation approach that relies on linear weights that account for spatial continuity, data closeness and redundancy.
- Weights are unbiased and minimize the estimation variance.

Simple Kriging: Some Details

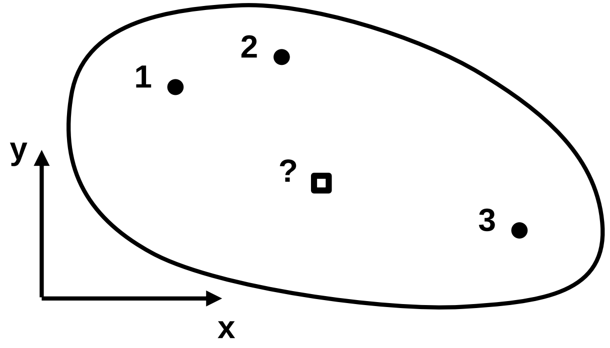
There are three equations to determine the three weights:

$$\lambda_1 \cdot C(\mathbf{u}_1, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_1, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_1, \mathbf{u}_3) = C(\mathbf{u}_0, \mathbf{u}_1)$$

$$\lambda_1 \cdot C(\mathbf{u}_2, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_2, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_2, \mathbf{u}_3) = C(\mathbf{u}_0, \mathbf{u}_2)$$

$$\lambda_1 \cdot C(\mathbf{u}_3, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_1, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_1, \mathbf{u}_3) = C(\mathbf{u}_0, \mathbf{u}_1)$$

In matrix notation: Recall that $C(\mathbf{h}) = C(0) - \gamma(\mathbf{h})$



$$\begin{bmatrix} C(\mathbf{u}_1, \mathbf{u}_1) & C(\mathbf{u}_1, \mathbf{u}_2) & C(\mathbf{u}_1, \mathbf{u}_3) \\ C(\mathbf{u}_2, \mathbf{u}_1) & C(\mathbf{u}_2, \mathbf{u}_2) & C(\mathbf{u}_2, \mathbf{u}_3) \\ C(\mathbf{u}_3, \mathbf{u}_1) & C(\mathbf{u}_3, \mathbf{u}_2) & C(\mathbf{u}_3, \mathbf{u}_3) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} C(\mathbf{u}_o, \mathbf{u}_1) \\ C(\mathbf{u}_o, \mathbf{u}_2) \\ C(\mathbf{u}_o, \mathbf{u}_3) \end{bmatrix}$$

redundancy **closeness**

Properties of Simple Kriging

- Solution exists and is unique of matrix $\left[C(v_i, v_j) \right]$ is positive definite

- Kriging estimator is unbiased: $$E\left\{\left[Z - Z^* \right]\right\} = 0$$

- Minimum error variance estimator (just try to pick weights, you won't bet it)

- Best Linear Unbiased Estimator

- Provides a measure of the estimation (or kriging) variance (uncertainty in the estimate):

$$\sigma_E^2(\mathbf{u}) = C(0) - \sum_{\alpha=1}^{n} \lambda_\alpha C(\mathbf{u} - \mathbf{u}_\alpha) \qquad \boldsymbol{\sigma_E^2} \rightarrow [\mathbf{0}, \boldsymbol{\sigma_x^2}]$$

More Properties

- Exact interpolator: at data location
- Kriging variance can be calculated before getting the sample information, homoscedastic!
- Kriging takes into account:
  - distance of the information: $C(\mathbf{u},\mathbf{u}_i)$
  - configuration of the data: $C(\mathbf{u}_i,\mathbf{u}_j)$
  - structural continuity of the variable being considered: $C(\mathbf{h})$
- The smoothing effect of kriging can be forecast – we will return to this with simulation.
- Kriging theory is part of the probabilistic theory of *projectors*: orthogonal projection onto space of linear combinations of the *n* data (Hilbert space)

Simple Kriging Hands-on

File Simple_Kriging_Demo.xls

Simple Kriging Demonstration

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1. Data and Estimate Locations and Values

Point	x	y	value	residual
1	60	80	0.1	-0.040
2	25	50	0.12	-0.020
3	80	10	0.2	0.060
unknown	50	50		
mean			0.140	

2. Distance Matrix

0.00	46.10	72.80	31.62
46.10	0.00	68.01	25.00
72.80	68.01	0.00	50.00

3. Variogram Model

Nugget	0
Spherica	1
Range	300

4. Variogram Matrix

0.000	0.229	0.357	0.158
0.229	0.000	0.334	0.125
0.357	0.334	0.000	0.248

5. Covariance Matrix

1.000	0.771	0.643	0.842
0.771	1.000	0.666	0.875
0.643	0.666	1.000	0.752

6. Inverse Left Side

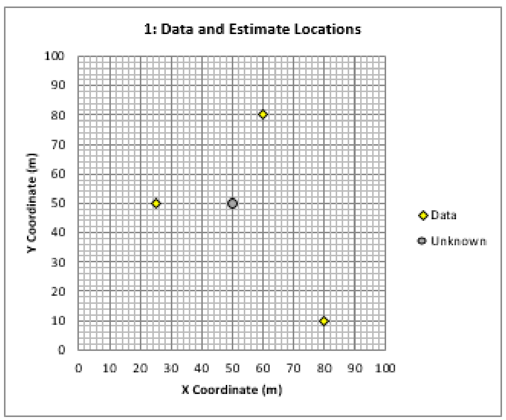
2.667	-1.644	-0.621
-1.644	2.810	-0.813
-0.621	-0.813	1.941

7. Weights

	0.341
	0.462
	0.225
Sum Weights	1.029
Mean Weight	-0.029

8. Kriging Results

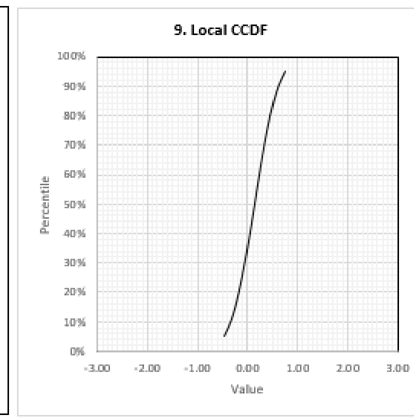
Kriging Estimate	0.131
Kriging Variance	0.139



Legend

	Information
	User Input
	Calculation
	Redundancy Measure
	Closeness Measures

p-value	$f^{-1}(p)$
5%	-0.48
10%	-0.35
15%	-0.26
20%	-0.18
25%	-0.12
30%	-0.06
35%	-0.01
40%	0.04
45%	0.08
50%	0.13
55%	0.18
60%	0.22
65%	0.27
70%	0.33
75%	0.38
80%	0.44
85%	0.52
90%	0.61
95%	0.74



Description

This sheet provides an illustration of Simple Kriging at a single estimated location.

Step 1: Input the data locations and values, the unknown simulated location. At any point these locations and values may be changed to observed their influence on the simulation.
Step 2: The distance matrix is automatically calculated, that is the distance between the data and the unknown locations.
Step 3: Enter the model of spatial continuity in the form of an isotropic spherical variogram and nugget effect (contributions should sum to one). This model may be changed at any time to observed sensitivities to spatial continuity.
Step 4: Variogram matrix is calculated by applying the distance matrix to the isotropic variogram model.
Step 5: Covaraince matrix is calculated by subtracing the variogram from the variance (1 for standard normal distribution). This is applied to improve numerical stability as a diagonally dominant matrix is more readily invertable.
Step 6: The left hand side of the covariance matrix is inverted.
Step 7: The inverted left handside matrix is multiplied by the right hand side matrix to calculate the simple kriging weights.
Step 8: The kriging estimate and kriging variance are calculated with the weights and covariances.
Step 9: With the Gaussian assumption the complete local conditional cumulative distribution function is available.

Excel file available at: https://github.com/GeostatsGuy/ExcelNumericalDemos/blob/master/Simple_Kriging_Demo.xlsx

Simple Kriging Hands-on

- Some ideas for experimenting with simple kriging. Do the following and pay attention to the weights, the estimate and the estimation variance.

1. Set points 1 and 2 closer together.

2. Put point 1 behind point 2 to create screening.

3. Put all points outside the range.

4. See the range very large.

Spatial Uncertainty Hands-on

Variogram and Trend-based Uncertainty Away from a Single Well

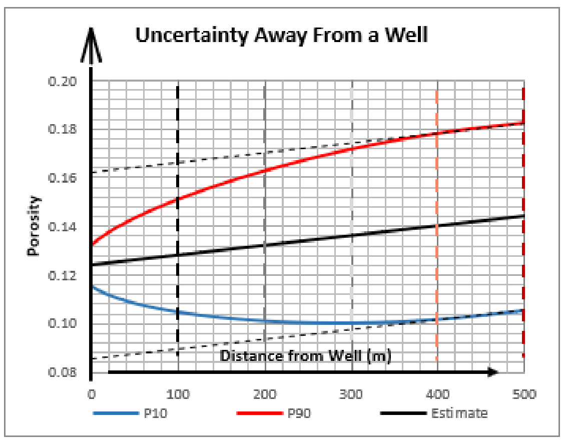
Michael Pyrcz, Geostatistics at Petroleum and Geosystems Engineering, University of Texas at Austin (mpyrcz@austin.utexas.edu)

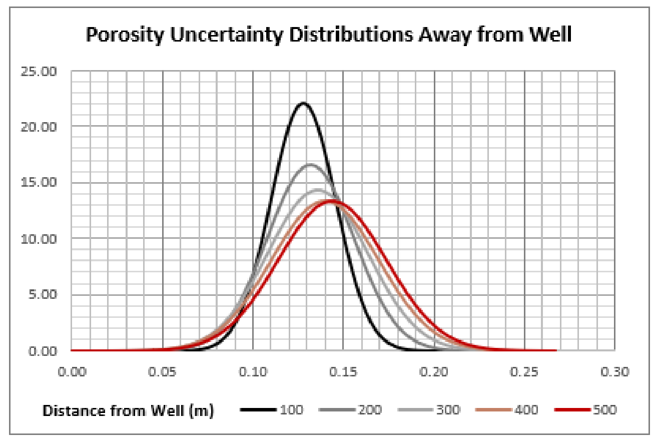
Instructions: set the (1) well porosity value, (2) global porosity variance, (3) trend slope away from the well, and (4) variogram parameterized by the relative nugget effect and spherical range.

Spatial Model	
Well Value	0.124
Global Var.	0.0009
Trend m	0.00004
Nugget	0.05
Spherical	0.95
Range	450

Distance	0	5	10	15	20	25	30	35	40	45	50	55
Estimate	0.124	0.1242	0.1244	0.1246	0.1248	0.125	0.1252	0.1254	0.1256	0.1258	0.126	0.1262
Rel. Var.	5%	7%	8%	10%	11%	13%	14%	16%	18%	19%	21%	22%
St. Dev.	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
P10	0.12	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
P90	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
GlobalP10	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
GlobalP90	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16

Distance	60	65	70	75	80	85	90	95	100	105	110	115	120
Estimate	0.1264	0.1266	0.1268	0.127	0.1272	0.1274	0.1276	0.1278	0.128	0.1282	0.1284	0.1286	0.1288
Rel. Var.	24%	25%	27%	29%	30%	32%	33%	35%	36%	38%	39%	41%	42%
St. Dev.	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
P10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10
P90	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
GlobalP10	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
GlobalP90	0.16	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17





Things to try: change spatial continuity range and observed impact on uncertainty away from well. File: Uncertainty_Away_From_Well_Demo.xlsx

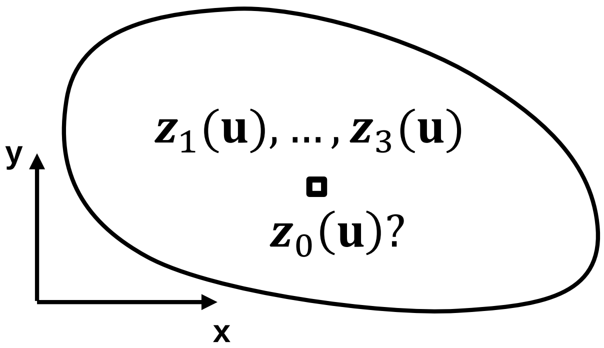
Multivariate Kriging

- Simple kriging may be applied to make estimates given a set of collocated secondary variables at the location to estimate the primary variable.
- This is not spatial estimation, but multivariate estimation!

Covariance between secondary variables **Covariance between secondary and primary variables**

$$\begin{bmatrix} C(\mathbf{z}_1,\mathbf{z}_1) & C(\mathbf{z}_1,\mathbf{z}_2) & C(\mathbf{z}_1,\mathbf{z}_3) \\ C(\mathbf{z}_2,\mathbf{z}_1) & C(\mathbf{z}_2,\mathbf{z}_2) & C(\mathbf{z}_2,\mathbf{z}_3) \\ C(\mathbf{z}_3,\mathbf{z}_1) & C(\mathbf{z}_3,\mathbf{z}_2) & C(\mathbf{z}_3,\mathbf{z}_3) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} C(\mathbf{z}_o,\mathbf{z}_1) \\ C(\mathbf{z}_o,\mathbf{z}_2) \\ C(\mathbf{z}_o,\mathbf{z}_3) \end{bmatrix}$$

redundancy **closeness**



- Given the assumption of Gaussian distributed variables we have a complete model of uncertainty for the primary variable at location $\mathbf{u}$!
- We can back transform for uncertainty in the original variable units.

Multivariate Kriging Demonstration

- Demonstration of multivariate kriging to estimate porosity from secondary data permeability and acoustic impedance.

Kriging-based Multivariate Prediction

Michael Pyrcz, the University of Texas at Austin

This is an example, demonstration of kriging for multivariate prediction, instead of spatial prediction with comparision to multilinear regression.

		mean	0.15	183.71	4203.66	0.00	0.00	0.00	0.00	0.15			
		st.dev.	0.05	345.44	1319.09	1.00	1.00	1.00	0.87	0.04			
									8. Estima	9. Back transfor		10. Calculate P10, P90"	
X	Y	Facies	Porosity	Perm	AI	St_Por	St_Perm	St_AI	St_Por_Es	Por_est	Por_std	P10	P90
100	900	1	0.1002	1.36389	5110.7	-1.0078	-0.5279	0.6876	-0.61957	0.1195	0.025	0.088	0.151
100	800	0	0.1079	12.5768	4671.459	-0.8519	-0.4954	0.3546	-0.35681	0.1326	0.025	0.101	0.164
100	700	0	0.0854	5.98452	6127.548	-1.3057	-0.5145	1.4585	-1.21305	0.09	0.025	0.058	0.122
100	600	0	0.1085	2.44668	5201.638	-0.8416	-0.5247	0.7566	-0.67232	0.1169	0.025	0.085	0.149
100	500	0	0.1025	1.95226	3835.27	-0.962	-0.5262	-0.2793	0.12793	0.1567	0.025	0.125	0.189
100	400	0	0.1106	3.69191	5295.267	-0.7991	-0.5211	0.8275	-0.72657	0.1142	0.025	0.082	0.146
100	300	0	0.0889	1.07358	6744.996	-1.2338	-0.5287	1.9266	-1.57716	0.0718	0.025	0.040	0.104
100	200	0	0.1021	2.39619	5947.338	-0.9695	-0.5249	1.3219	-1.10921	0.0951	0.025	0.063	0.127
100	100	1	0.1375	5.7276	5823.242	-0.2592	-0.5152	1.2278	-1.0349	0.0988	0.025	0.067	0.131
200	900	1	0.1371	14.7713	5621.147	-0.2671	-0.4891	1.0746	-0.91213	0.105	0.025	0.073	0.137
200	800	1	0.126	10.6754	4292.701	-0.4896	-0.5009	0.0675	-0.13584	0.1436	0.025	0.112	0.175
200	700	0	0.1218	3.08583	5397.4	-0.5746	-0.5229	0.905	-0.7867	0.1112	0.025	0.079	0.143
200	600	0	0.0951	0.96257	4619.786	-1.1091	-0.529	0.3155	-0.33216	0.1338	0.025	0.102	0.166
200	500	0	0.0875	1.82327	4949.881	-1.2627	-0.5265	0.5657	-0.52513	0.1242	0.025	0.092	0.156
200	400	0	0.0986	4.57102	5789.623	-1.04	-0.5186	1.2023	-1.01576	0.0998	0.025	0.068	0.132
200	300	0	0.1074	13.5819	7881.899	-0.8621	-0.4925	2.7885	-2.23717	0.039	0.025	0.007	0.071
200	200	0	0.0935	0.4088	6104.849	-1.1413	-0.5306	1.4413	-1.20245	0.0905	0.025	0.059	0.122
200	100	0	0.0799	0.73455	6485.732	-1.4144	-0.5297	1.73	-1.42543	0.0794	0.025	0.048	0.111
300	900	1	0.1115	27.9998	4183.467	-0.7802	-0.4508	-0.0153	-0.06347	0.1472	0.025	0.115	0.179
300	800	1	0.1195	61.0054	5224.544	-0.6205	-0.3552	0.7739	-0.65742	0.1176	0.025	0.086	0.149
300	700	1	0.1342	44.5959	4556.541	-0.3246	-0.4027	0.2675	-0.27401	0.1367	0.025	0.105	0.169
300	600	1	0.1612	190.74	3997.089	0.2179	0.0203	-0.1566	0.124417	0.1566	0.025	0.125	0.188
300	500	1	0.1072	13.3016	5684.915	-0.8675	-0.4933	1.1229	-0.9502	0.1031	0.025	0.071	0.135
300	400	1	0.1117	10.8511	5499.753	-0.7771	-0.5004	0.9826	-0.8429	0.1084	0.025	0.077	0.140
300	300	0	0.1052	6.3122	4715.684	-0.9064	-0.5135	0.3882	-0.38575	0.1312	0.025	0.099	0.163
300	200	0	0.1033	2.81735	6217.196	-0.9461	-0.5237	1.5265	-1.2671	0.0873	0.025	0.055	0.119
300	100	0	0.0991	3.18878	6456.338	-1.0305	-0.5226	1.7078	-1.40703	0.0803	0.025	0.048	0.112
400	900	0	0.0785	1.1098	5440.289	-1.4436	-0.5286	0.9375	-0.81278	0.1099	0.025	0.078	0.142
400	800	0	0.1104	4.11622	5956.295	-0.8017	-0.5199	1.3287	-1.11363	0.0949	0.025	0.063	0.127
400	700	1	0.1228	18.827	4937.337	-0.5538	-0.4773	0.5562	-0.50956	0.125	0.025	0.093	0.157
400	600	1	0.1208	5.97043	4040.573	-0.593	-0.5145	-0.1236	0.009597	0.1508	0.025	0.119	0.183
400	500	1	0.114	132.124	4459.013	-0.7311	-0.1493	0.1936	-0.17455	0.1417	0.025	0.110	0.174
400	400	1	0.0838	5.55878	5953.782	-1.3362	-0.5157	1.3268	-1.11146	0.095	0.025	0.063	0.127
400	300	1	0.118	1.99268	5557.638	-0.6503	-0.5261	1.0265	-0.8811	0.1065	0.025	0.075	0.138
400	200	0	0.1169	6.63131	6735.729	-0.6721	-0.5126	1.9196	-1.56904	0.0722	0.025	0.040	0.104
400	100	0	0.0664	0.03361	6124.087	-1.6869	-0.5317	1.4559	-1.2139	0.0899	0.025	0.058	0.122
500	900	0	0.0934	1.30794	4873.951	-1.1448	-0.528	0.5081	-0.48089	0.1264	0.025	0.095	0.158
500	800	0	0.084	0.7714	5284.836	-1.3326	-0.5296	0.8196	-0.72187	0.1144	0.025	0.083	0.146
500	700	0	0.1098	52.5009	6581.836	-0.8143	-0.3798	1.8029	-1.4567	0.0778	0.025	0.046	0.110
500	600	1	0.122	8.22227	5296.785	-0.5702	-0.508	0.8287	-0.72527	0.1143	0.025	0.082	0.146

Workflow Steps:

0. Standardize the variables, the mean and variance have been corrected to 0 and 1 respectively with affine correction (we should use Gaussian transform for the complete workflow),
1 - 7. Multivariate kriging to calculate kriging estimate and kriging variance.
8. Estimate standardized porosity at each location.
9. Calculate the back transform of the kriging estimate and the kriging variance and 10. the local P10 and P90.

1. Calculate the correlation

	Por	Perm	AI
Por	1.00	0.54	-0.85
Perm	0.54	1.00	-0.49
AI	-0.85	-0.49	1.00

2. Calculate the covariance of the standardized variables

	Por	Perm	AI
Por	1.00	0.54	-0.85
Perm	0.54	1.00	-0.49
AI	-0.85	-0.49	1.00

3. Redudancy Matrix

	Perm	AI
Perm	1	-0.49
AI	-0.49	1

4. Closeness Matrix

	Por
Perm	0.54
AI	-0.85

5. Invert Redundancy Matrix

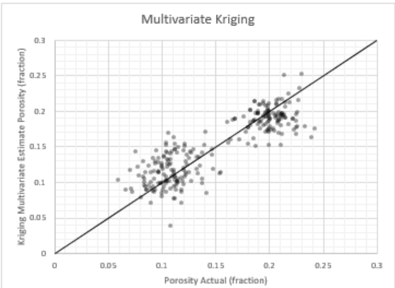
	Perm	AI
Perm	1.31	0.64
AI	0.64	1.31

6. Calculate the Weights

	Weights
Perm	0.17
AI	-0.77

7. Calculate the kriging variance

Kriging Variance 0.25



Comparision to Multilinear Regression

-0.7736	0.1673	-2.06E-16
0.0356	0.0356	0.0310267
0.7507	0.5013	#N/A
388.45	258	#N/A
195.2	64.823	#N/A

b2: slope of fit	-0.774	
se2: standard error of slope	0.036	**se1:** standard er
r2: proportion var. explained	0.75	**sey:** standard err
Fstat: for test of all coefficients	388.4	**d.f.:** d
ssreg: explained variance	195.2	**ssresid:** une

Test Significance of Coefficients

$H_0: b_i = 0$

$H_1: b_i \neq 0$

	tstat b1 = b1/se1		tstat b0 = b0/se0	tcritical
tstat	21.72	*tstat*	4.69	2.25
probability	######	*probability*	######	

	AI	Perm
Result from Hyposthesis tests for coefficients	**Reject H0: Slop**	**Reject H0: Intercept = 0**

The file is https://github.com/GeostatsGuy/ExcelNumericalDemos/blob/master/Kriging_Multivariate_Estimation_Demo.xlsx

Spatial Estimation New Tools

Topic	Application to Subsurface Modeling
Trend Modeling	Decompose variance into deterministic trend and stochastic residual. *30% of porosity variance is described by a linear depth trend and 70% is described by a 3D variogram model.*
Kriging Estimates and Kriging Variances	Kriging provides the best estimate and a measure of estimation variance. *Given a kriging estimate of 13% and kriging variance of 9% and the assumption of a Gaussian distribution we have a complete local distribution of uncertainty for pre-drill porosity.*
Multivariate Kriging	*Multivariate kriging combines secondary information sources while accounting for closeness and redundancy.* *Given secondary data the likelihood distribution for local porosity is mean of 15% and standard deviation of 2.5% with a Gaussian distribution.*

Multivariate Modeling
Spatial Estimation

Lecture outline . . .

- **Trend Modeling**
- **Kriging**