a) $\begin{cases} T(n) = T(n/2) + \Theta(1) \\ T(1) = 1 \end{cases}$

$$T(n) = T(n/2) + 1$$

$$T(n) = \left(T(n/2^2) + 1\right) + 1 = T(n/2^2) + 2$$

$$T(n) = \left(T(n/2^3) + 1\right) + 2 = T(n/2^3) + 3$$

$$\vdots$$

$$T(n/2^k) + k \quad \text{onde } k = \log n$$

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$$T(1) + \log n = 1 + \log n = \Theta(\log n)$$

b) $\begin{cases} T(n) = T(n/2) + \Theta(n) \\ T(1) = 1 \end{cases}$

$n = 2^k$

$= T(2^{k-1}) + 2^k$

$= T(2^{k-2}) + 2^{k-1} + 2^k$

$= T(2^{k-3}) + 2^{k-2} + 2^{k-1} + 2^k$

$\vdots$

$= T(1) + 2 + 2^2 \dots 2^{k-2} + 2^{k-1} + 2^k$

$= 1 + 2 + 2^2 \dots\dots 2^k$

$= \sum_{i=0}^{k} 2^i = 2^{k+1} - 1 = \cancel{2}^{\cancel{\log} n} \times 2 - 1$

$= 2n - 1 \quad \Theta(n)$

c) $\begin{cases} T(n) = 2T\left(\frac{n}{2}\right) + \Theta(1) \\ T(1) = 1 \end{cases}$

$K = \log n$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

$$= 2\left(2T\left(\frac{n}{2^2}\right) + 1\right) + 1 = 2^2 T\left(\frac{n}{2^2}\right) + 2$$

$$= 2^2\left(2T\left(\frac{n}{2^3}\right) + 1\right) + 2 = 2^3 T\left(\frac{n}{2^3}\right) + 3$$

$$\rightarrow 2^k T\left(\frac{n}{2^k}\right) + K$$

$$= 2^{\log n} T(1) + \log n$$

$$= n + \log n = \Theta(n)$$

d) $\begin{cases} T(n) = 2T(n/2) + \Theta(n) \\ T(1) = 1 \end{cases}$

$$T(n) = 2T(n/2) + n$$

$$= 2\left(2T(n/2^2) + \frac{n}{2}\right) + n = 2^2 T(n/2^2) + 2n$$

$$= 2^2\left(2T(n/2^3) + \frac{n}{2^2}\right) + 2n = 2^3 T(n/2^3) + 3n$$

$$\rightarrow 2^k T(n/2^k) + kn \qquad k = \log n$$

$$\cancel{2^{\log n}}\, T(1) + n \log n$$

$$n + n \log n = \Theta(n \log n)$$