Modelling a product-mix determination problem

J. Haddock

Department of Industrial Engineering, Clemson University, Clemson, SC 29631, USA

M. Rodriguez

President and General Manager, Cadillac Uniform and Linen Supply, Bayamon, PR 00619, USA
(Received July 1984; revised May 1985)

This study presents a combined linear programming package and a user-oriented program for its data input employed in product-mix determination. An optimal product-mix is to be determined for a rapidly changing manufacturing environment. The study consists of developing an interactive computer program for the solution of a product-mix problem. The user-oriented program has been developed so that a person familiar with the plant can determine the 'best' mix which should be produced. This study provides a quantitative tool to aid in the decision-making process when time is limited and the production process dynamic.

Key words: mathematical models, linear programming, product-mix

The proportionality and additivity assumptions in linear programming (LP) guarantee that the objective function and constraint functions are linear.[1] The certainty assumption is that all the parameters of the model are known constants. These assumptions are the major limitations of LP in solving real problems. However, there are certain applications where LP can provide a better solution than any other available method. One such case is using LP to solve product-mix determination problems. In typical manufacturing applications, time standards are used as deterministic quantities, thus making related calculations and estimates deterministic with the above assumptions implicitly stated. Furthermore, the above statement is even more applicable when automated manufacturing is considered, because in reality operation times are much closer to being deterministic. In addition, the percentage of defects produced from each work station is typically smaller and also fairly deterministic.

The product-mix determination problem involves determining the optimal production level of different products given a set of capacity restrictions. This study presents a combined linear programming package (LINPRO) and a user-oriented program for its data input employed in product-mix determination. Although the interactive program is described, emphasis is given to the model development.

Objectives

An optimal product-mix is to be determined for a rapidly changing manufacturing environment. The management of the plant has to make decisions which meet company requirements in a limited time. The present decision-making process is mainly a combination of past experience and simple calculations. This project provides a tool for determining the best allocation of resources in order to maximize an activity as defined below.

Problem statement

The study consists of developing an interactive computer program which combines a user-oriented computer program and an optimization package for product-mix determination. The input-data program was designed so that the user does not have to know computer programming or the LP technique; he/she must be familiar with the manufacturing process or have someone familiar with the plant who supplies the data. Logically some data must be identified before running the program, but most of it is already available in the *Cost Bill and Current Standard Report*. The data is described later. This data input program, after creating the LP model, automatically runs LINPRO, so that LINPRO works as a 'black-box' to the user. The output data, provided by LINPRO, would then be used to set the

0307-904X/85/05370-05/$03.00

production goals for each product in the plant. The LP formulation can be classified as a product-mix-determination problem. Several LP formulations exist in the literature.

Model development

The model's objective is to maximize the activity. At the beginning of each fiscal year a proposed budget is submitted by each plant to the headquarters. This budget is based on the man-hours required to produce the proposed units (the number of units for each different product). The actual total man-hours required for production is called the 'activity' which is compared with the proposed man-hours. Maximizing the actual activity is equivalent to maximizing the employment level and is also used to justify overhead costs.

Figure 1 presents the flow diagram for the production system studied. This diagram describes the flow of raw material, sub-assemblies, and final product through the different work stations in the plant. In the diagram the squares represent the work sections and the arrows the flow direction of the raw material, the sub-assemblies, and the final product. The manufacturing process is applied to more than 80 different products. For simplicity, 13 different products were chosen as representative. This will not affect the scope of the study because product differences are minimal.

The decision variables are the production levels (in thousands of units) of the different products. This means that the model contains n decision variables, Q_i for $i = 1, \ldots, 13$. The restrictions are of four types: demand per period, man-hours per period, machine-hours per period, and shifts per period; where the user has the option of selecting the period length. The man-hours per period restrictions are associated with: injection moulding No 2. The machine-hour per period restrictions are associated with: and extrusion, blow moulding, curing, sub-assembly No 1, and heat treatment and performance testing. The shifts per period restrictions are associated with: closure assembly, final assembly, packing, final packing and manning.

Table 1 presents the different operations involved in manufacturing each product: X means that the operation is performed on the product, otherwise a blank is present. The production standards are in hours/1000 units. Only direct hours are considered.

Model

The proposed activity for any given period is A. The number of man-hours required to produce 1000 units of each code (i.e. product number) are defined as c_i (for $i = 1, \ldots, n$) for each product. The a_{ij} are defined as the

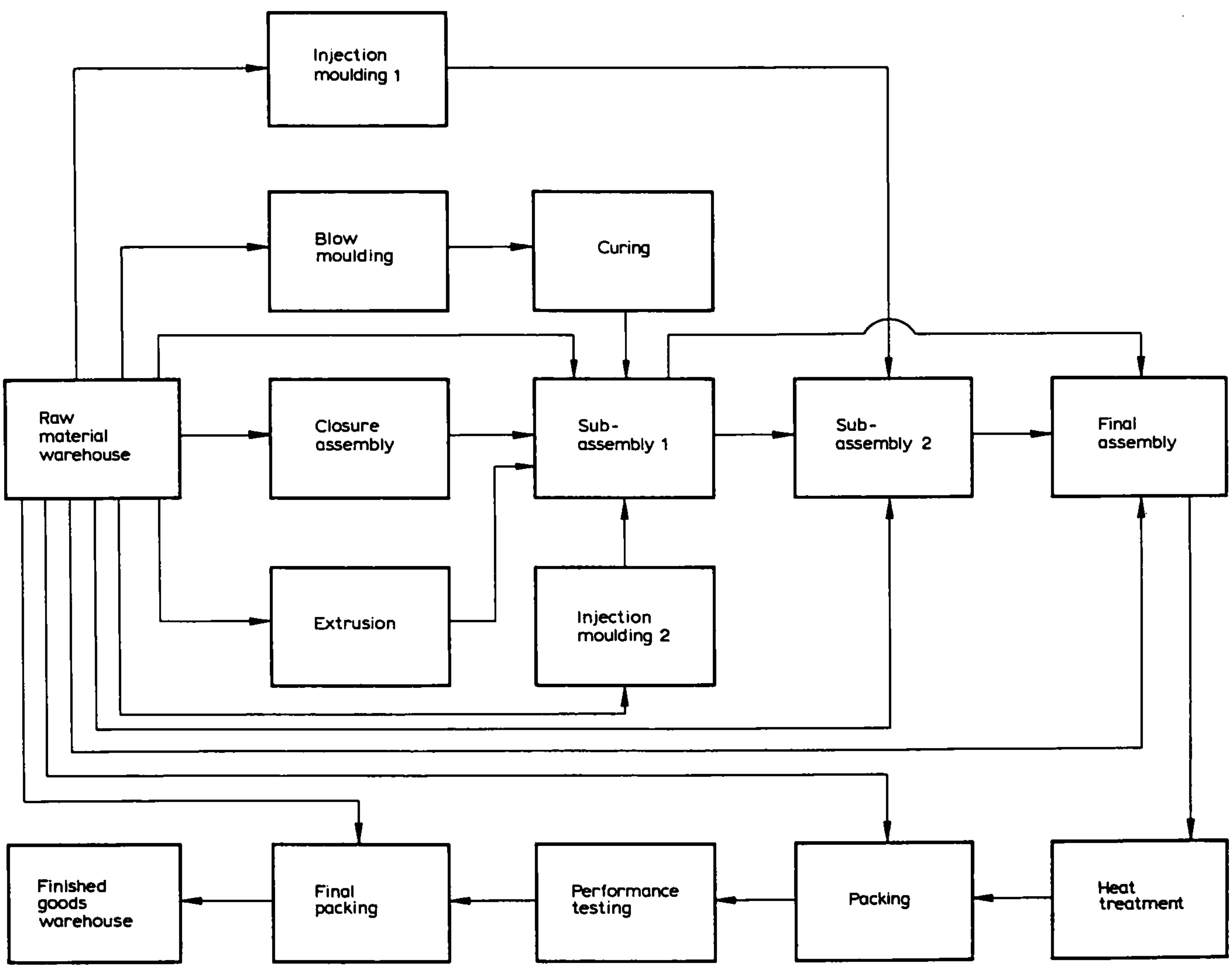

Figure 1 Flow diagram

Table 1 Operations in the manufacturing process

Product	Blow moulding	Extrusion	Curing	Injection moulding 2	Closure assembly 1	Sub-assembly 1	Injection moulding 1
1	x	x	x			x	x
2	x	x	x			x	x
3	x	x	x		x	x	
4	x	x	x	x		x	
5	x	x	x	x		x	x
6	x	x	x	x	x	x	
7	x	x	x	x		x	
8	x	x	x	x		x	x
9	x	x	x	x	x	x	
10	x	x	x	x		x	
11	x	x	x	x		x	x
12	x	x	x	x	x	x	x
13	x	x	x				

Product	Sub-assembly 2	Final assembly	Wrapping	Performance and heat treatment	Final packing	Manning
1	x	x	x	x	x	x
2	x	x	x	x	x	x
3	x	x	x	x	x	x
4	x	x	x	x	x	x
5	x	x	x	x	x	x
6	x	x	x	x	x	x
7	x	x	x	x	x	x
8	x	x	x	x	x	x
9	x	x	x	x	x	x
10	x	x	x	x	x	x
11	x	x	x	x	x	x
12	x	x	x	x	x	x
13	x	x	x	x	x	x

product time standard (in hours/1000 units) for product i in restriction j.

The objective function for any given year could be defined as:

$$\text{Max } Z = (c_1Q_1 + c_2Q_2 + \ldots + c_nQ_n)/A \qquad (1)$$

The sum within the parenthesis represents the actual activity. This is the total number of man-hours required to produce the optimal mix. Although it is not necessary in order to obtain the optimal solution, this actual activity is divided by the proposed activity to obtain the objective function value relative to the activity proposed. In other words, if the objective function value of the optimal solution is a fraction (i.e. less than 1), the proposed activity is not achievable. An objective function value greater than 1 means that the actual production could be greater than planned. Both situations would provide the 'best' solution.

Management defines its production levels in terms of direct labour hours (man-hours employed). Thus, maximizing man-hours utilized, in this context, is equivalent to maximizing production. This should not be confused with over utilization of a labour force in total. The labour force and work content for each product are consistent and remain unchanged. In essence, production efficiencies are fixed.

As previously mentioned, the model is composed of three types of constraints. The user may also specify the demand projections for each product.

Demand (market) constraints (projections)

The level of demand for each product is represented as M_i for $i = 1, \ldots, n$. Therefore the demand constraints are:

$$Q_i \leqslant M_i \qquad i = 1, \ldots, n \qquad (2)$$

Machine-hours per period

The constraints for machine-hours per period are defined as:

$$\sum_{i=1}^{n} HM_{ij}Q_i \leqslant B_j \qquad j = 1, \ldots, 4 \qquad (3)$$

The sum is over all the products (n) for all the constraints; one for each station. HM_{ij} represents the required machine-hours in station j to produce a thousand good units of product i. HM_{ij} is computed as a_{ij} multiplied by the number of machines at station j. B_j is defined as the number of machine-hours per year at station j and is computed as: B_j = hours/shift x shifts/day x days/year x number of machines.

The heat treatment and performance testing restrictions require additional explanation. This is a combined process for which the constraints consider both processes simultaneously. The constraints are defined as:

$$\sum_{i=1}^{n} [H_{is}/U_{is} + H_{ip}/U_{ip}]\, Q_i \leqslant B_j \qquad j = 5 \qquad (4)$$

Here H_{is} represents the standard time, required to test the performance of the ith product, and H_{ip} represents the standard time for heat treatment: both are in hours/load. U_{is} and U_{ip} are defined as units/load of product i for heat treatment and performance testing, respectively. A load is a batch of units.

Note that the coefficients of the Q_i's represent the combined hours of heat treatment and performance testing required to process a thousand units of product i. Both processes include loading and unloading the machines. In these restrictions the production standards are for 'product class', which is discussed in the next section.

Shifts per period

Defining the shifts per period constraints is more complicated than defining the previous constraints. The stations comprise a progressive assembly line. Several operators work simultaneously at these stations which are balanced in units per eight-hour shift (PE_{ij}). PE_{ij} represents the standard production in 1000 units per shift for product i at station j. U_i is defined as the total usage. The total usage is related to the defective products from each station. The total number of units produced is equal to the number of good units produced multiplied by the total usage. The constraints are defined as:

$$\sum_{i=1}^{n} (U_i/PE_{ij})\, Q_i \leqslant D_j \qquad j = 1, \ldots, 6 \tag{5}$$

D_j is expressed in shifts/period and is computed as: D_j = shifts/period x days/year. The coefficients in this equation represent the number of shifts required to produce 1000 non-defective units of product i.

For final assembly, wrapping and final packing one more factor is taken into account. There are five different classes among the n studied. The production standards vary for each of these classes. However, the constraint is formulated in the sameway for all the stations in this category (shifts/period). The difference is reflected in the calculation of the coefficients.

The manning restriction is not for a particular station, but for the entire process. These constraints (there is one for each of two different shifts, A and B) model the fact that only certain combinations of products, depending on their class, are possible. The constraints establish the maximum number of shifts A and B available per period to produce the possible product combinations.

Other restrictions

The remaining restrictions are for man-hours per year. These restrictions proved not limiting in practice. They are related to stations where the operators do not depend on machines (i.e. manual operations). This means that the real constraints would be in space and people, which have never been limiting in this particular plant. However, if management wants to model different scenarios with limiting values for these stations the system provides this capability.

The restrictions are defined as:

$$\sum_{i=1}^{n} (HH_{ij}Q_i) \leqslant E_j \qquad j = 1, 2, 3 \tag{6}$$

HH_{ij} represents the man-hours required to produce enough assemblies to complete 1000 non-defective units of product i. The E_j is computed as: E_j = shifts/period x shifts/day x days/year x number of operators.

Model summary

$$\text{Max } Z = (c_1Q_1 + c_2Q_2 + \ldots + c_nQ_n)/A \tag{1}$$

subject to:

$$Q_i \leqslant M_i \text{ for } i = 1, \ldots, n \tag{2}$$

$$\Sigma\, HM_{ij}Q_i \leqslant B_j \text{ for } j = 1, \ldots, 4 \tag{3}$$

$$\Sigma\, [H_{is}/U_{is} + H_{ip}/U_{ip}]\, Q_i \leqslant B_j \text{ for } j = 5 \tag{4}$$

$$\Sigma\, (U_i/PE_{ij})\, Q_i \leqslant D_j \text{ for } j = 1, \ldots, 6 \tag{5}$$

$$\Sigma\, (HH_{ij}Q_i) \leqslant E_j \text{ for } j = 1, 2, 3 \tag{6}$$

$$Q_i \geqslant 0 \text{ for all } i \tag{7}$$

Data-input program

A user-oriented computer program was developed so that a person familiar with the plant could determine the optimal mix which should be produced. The program is self-explanatory when executed. It simply asks for the data necessary to run LINPRO and stores it in a file. The program is in BASIC and runs in an interactive form with the user. After sets of data are entered, the program asks the user to verify that the data is correct and provides an option for redefining each particular set of data. This program then transfers the data file to LINPRO for determining the optimal product mix.

The computer program is very straightforward. However, its significance is not trivial. Many formulations of real problems fail to be implemented because the user is not the person who developed the model and does not feel comfortable in dealing with it. Furthermore, even when the user and analyst are one and the same, handling the data and keeping the model updated is too time-consuming and often not worth pursuing.

Further considerations

The management of the plant may find it necessary, in the future, to introduce minimum production levels for certain products. This could happen if quantities of certain products were already committed. Should this happen, the only change in the decision support system (DSS) would occur in the data input program.

A more important consideration would be the interface program between the user and the LP package. If the number of restrictions (stations) or products grew the data input would get tedious and time consuming. The best approach for solving this problem would be to combine a data base management program with the LP package. This would allow the user to change only certain data elements without running the entire data input program. In this particular case, most of the data (i.e. time standards) will not change from run to run. The only frequent changes would occur in the demand constraints.

At present, the system does not provide much flexibility to the decision-maker (the plant manager) in establishing demand levels and activity coefficients. This makes unnecessary a sophisticated DSS that permits the decision-maker to produce different scenarios either for sensitivity analysis or to make other decisions. Moreover, the solution provided by the actual system would be the best solution, making unnecessary the consideration of other alternatives. Another less efficient approach would be to modify the data input program in order to produce partial changes in the data and/or produce different scenarios for the decision-maker.

Conclusions

In the past, it was difficult to determine the optimal product mix in this plant, especially when short turnaround was required. The rapidly changing nature of the decision process made solutions more difficult because of the potential for changes with incomplete information. This also makes it very difficult to keep a model updated and usable.

This study has provided a quantitative tool to aid in the decision-making process when time is limited and the production process dynamic. A significant advantage of this model is that it is easy to execute and update.

References

1 Hillier, F. S. and Lieberman, G. J. 'Introduction to operations research', Holden-Day (3rd edn) 1980

2 Alter, S. L. 'Decision support systems, current practice and continuing challenges', Addison-Wesley, Reading, Mass., 1980

3 Ben-Bassat, M. and Freedy, A. 'Knowledge requirements and management in expert decision support systems for (military) situation assessment', *IEEE Transactions on Systems, Man, and Cybernetics* 1982, **SMC-12** (4)

4 Blanning, R. W. 'What is happening in DSS?', *Interfaces* 1983, **13** (5), 71

5 Farwell, D. C. 'A model based approach to decision support system flexibility', *Interfaces* 1982, **12** (5)

6 Garish, B. 'A decision support system for managing the transportation needs of a large corporation', *AIIE Transactions*, March 1981

7 Holsapple, C. W. and Whinston, A. B. 'A decision dupport system for area-wide water quality planning', *Socio-Econ. Plan. Sci.* 1976, **10**, 265

8 Mori, F., Tsuji, T. and Sato, T. 'A conversational decision support system for resource allocation without explicit objective function', *National Computer Conference*, 1980

9 Nof, S. Y. and Gurecki, R. 'MDSS: Manufacturing decision support system', *AIIE Spring Conference*, Atlanta, GA, May 1980

10 Phillips, D. T., Ravindran, A. and Solberg, J. J. *Operations Research: Principles and Practice*, Prentice Hall, 1976

11 Rodriguez, M. 'Determinacion de la mezcla optima de productos: Una aplicacion de programacion lineal', *Master's Thesis*, University of Puerto Rico, Rio Piedras, P. R., 1981

12 Sprague, R. H. and Watson, H. J. 'A decision support system for banks', *OMEGA* 1976, **4** (6)

13 Turban, E. 'DSS: A new frontier for industrial engineering', *Comput. & Indus. Engng.* 1983, 7 (1), 41