This is a test

Case One Algorithm

Step 1

$z'' = rz$

poles of r

Γ (set of poles)

pole c_1 of order 1

$$\begin{cases} [\sqrt{r}]_{c_1} = 0 \\ \alpha_{c_1}^{+} = 1 \\ \alpha_{c_1}^{-} = 1 \end{cases}$$

pole c_2 of order 2

$$\begin{cases} [\sqrt{r}]_{c_2} = 0 \\ \alpha_{c_2}^{+} = \frac{1}{2} + \frac{1}{2}\sqrt{1+4b} \\ \alpha_{c_2}^{-} = \frac{1}{2} - \frac{1}{2}\sqrt{1+4b} \end{cases}$$

Where b is the coefficient of $\frac{1}{(x-c_2)^2}$ in the partial fractions expansion of r.

pole c_3

of order ≥ 4 (must be even order). Let v be half the order value.

$$\begin{cases} [\sqrt{r}]_{c_3} = \sum_{i=2}^{v} \frac{a_i}{(x-c_3)^i} \\ \alpha_{c_1}^{+} = \frac{1}{2}\left(\frac{b}{a_v} + v\right) \\ \alpha_{c_1}^{-} = \frac{1}{2}\left(-\frac{b}{a_v} + v\right) \end{cases}$$

This is the sum of terms of $\frac{1}{(x-c)^i}$ in the Laurent series expansion of $\sqrt{r}$ (not r) at the pole c_3. The sum of terms is for $2 \leq i \leq v$

b is the coefficient of the term $\frac{1}{(x-c_3)^{v+1}}$ in r (found from the partial fraction decomposition of r) minus the coefficient of same term but in $[\sqrt{r}]_{c_3}$.

This a is the coefficient of $\frac{1}{(x-c_3)^v}$ in the above Laurent sum.

Order of r at ∞

Order is > 2

$$\begin{cases} [\sqrt{r}]_{\infty} = 0 \\ \alpha_{\infty}^{+} = 0 \\ \alpha_{\infty}^{-} = 1 \end{cases}$$

Order is 2

$$\begin{cases} [\sqrt{r}]_{\infty} = 0 \\ \alpha_{\infty}^{+} = \frac{1}{2} + \frac{1}{2}\sqrt{1+4b} \\ \alpha_{\infty}^{-} = \frac{1}{2} - \frac{1}{2}\sqrt{1+4b} \end{cases}$$

This b is the coefficient of $\frac{1}{x^2}$ in the Laurent series expansion of r at ∞. If $r = \frac{s}{t}$ where $\gcd(s,t) = 1$ then b can be more easily found using $b = \frac{\text{lcoeff}(s)}{\text{lcoeff}(t)}$ where lcoeff means the leading coefficient.

Order is $-2v \leq 0$ which must be even since case 1.

$$\begin{cases} [\sqrt{r}]_{\infty} = \sum_{i=0}^{v} a_i x^v \\ \alpha_{\infty}^{+} = \frac{1}{2}\left(\frac{b}{a} - v\right) \\ \alpha_{\infty}^{-} = \frac{1}{2}\left(-\frac{b}{a} - v\right) \end{cases}$$

This is the sum of terms x^i for $0 \leq i \leq v$ in the Laurent series for $\sqrt{r}$ (not r) at ∞

This b is the coefficient of x^{v-1} in $r = \frac{s}{t}$ (Which is found using long division) minus the coefficient of same term but in $\left([\sqrt{r}]_{\infty}\right)^2$.

This a is the coefficient of a_v in the above Laurent series sum.

Step 2

For each family $s = (s(c))_{c \in \Gamma \cup \infty}$ where $s(c)$ is $+$ or $-$ let

$$d = \alpha_{\infty}^{s(\infty)} - \sum_{c \in \Gamma} \alpha_c^{s(c)}$$

If family found which produced d an integer and positive then find

$$\omega = \sum_{c \in \Gamma} \left(s(c)[\sqrt{r}]_c + \frac{\alpha_c^{s(c)}}{x-c} \right) + s(\infty)[\sqrt{r}]_{\infty}$$

Step 3

Find polynomial $p(x)$ of degree d which satisfies $p'' + 2\omega p' + (\omega' + \omega^2 - r)p = 0$. Then the solution to $z'' = rz$ is given by

$$z = p e^{\int w \, dx}$$

case_1.ipe Nasser M. Abbasi 2/1/2022