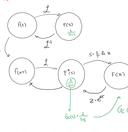
Clase 11
2020-11-11

Diseño de control digital mediante la discretización de controladores continuos.



Tenemos un $G(s)$

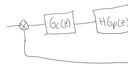
$$G(s)\Big|_{s=\frac{1}{T}\ln(z)} = G(z)$$

$$\Rightarrow G(s)=\frac{10}{s}$$

$$\Rightarrow G(s)\Big|_{s=\frac{1}{T}\ln z} = \frac{10}{\frac{1}{T}\ln z} = \frac{10T}{\ln z}$$

$$\Rightarrow G(z)=\frac{10}{\frac{1}{T}\ln z}$$

$$\Rightarrow G(z)=\frac{10T}{\ln z}$$



La clave es aproximar la transformación no lineal (exacta) a funciones que nos permiten obtener un $G_c(z)$ "estabilizable".

exacta: $s=\frac{1}{T}\ln z \approx f(z)$ — método Euler / Método Tustin

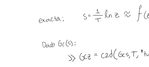
Dado $G_c(s)$:

$\Rightarrow$ Gcz = c2d(Gcs,T,'metodo');

...Veamos un ejemplo para obtener las 2 aproximaciones de $s=\frac{1}{T}\ln z$

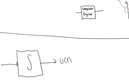
Ejemplo: Considerar un controlador proporcional integral.

$$G_c(s)=\frac{1}{s} \rightarrow \frac{U(s)}{E(s)}=\frac{1}{s}$$

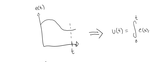


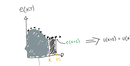
$$\Rightarrow u(t)=\int_0^t e(t)dt$$

$$u(t)=u(t_0)+\int_{t_0}^{t} e(t)dt$$



$$\Rightarrow u(t_1)=u(t_0)+\int_{t_0}^{t_1} e(t)dt$$





$$U((k+1)T)=U(kT)+Te(kT)$$

$\mathcal{L}$ (parametro):

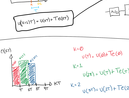
$$U(kT)=U(k-1)T)+Te((k+1)T)$$

$\mathcal{Z}$:

$$U(z)=z^{-1}U(z)+Tz^{-1}E(z)$$

$$(1-z^{-1})U(z)=Tz^{-1}E(z)$$

$$\frac{U(z)}{E(z)}=\frac{Tz^{-1}}{1-z^{-1}}=\frac{T}{z-1} \rightarrow G_c(z)=\frac{T}{z-1}$$



$$\Rightarrow G_c(z)=\frac{T}{z-1}$$

$$s=\frac{1}{T}\ln(z)=? \approx \frac{z-1}{T}$$

$$\boxed{G_c(s)\Big|_{s=\frac{z-1}{T}}=G_c(z)}$$

Ejemplo: Obtener un controlador digital para el sig. controlador continuo

$$G_c(s)=K_p+\frac{K_i}{s}$$

$$\Rightarrow G_c(s)\Big|_{s=\frac{z-1}{T}}=G_c(z)$$

$$\Rightarrow G_c(z)=K_p+\frac{K_i}{\left(\frac{z-1}{T}\right)}=K_p+\frac{K_iT}{z-1}$$

Método Tustin

$$\Rightarrow A=\left[e(kT)+e((k+1)T)\right]\frac{T}{2}$$

$$\Rightarrow U((k+1)T)=U(kT)+\frac{T}{2}\left[e(kT)+e((k+1)T)\right]$$

$\mathcal{L}$ FT:

$$U(kT)=U((k-1)T)+\frac{T}{2}\left[e((k-1)T)+e(kT)\right]$$

$$U(z)=z^{-1}U(z)+\frac{T}{2}\left[z^{-1}E(z)+E(z)\right]$$

$$(1-z^{-1})U(z)=\frac{T}{2}\left[z^{-1}+1\right]E(z)$$

$$\frac{U(z)}{E(z)}=\left[\frac{T}{2}\right]\left[z^{-1}+1\right]\left[\frac{1}{1-z^{-1}}\right]=\frac{T(z^{-1}+1)}{2(1-z^{-1})}=\frac{T(z+1)}{2(z-1)}=G_{pc}(z)$$

$$G_c(s)=\frac{1}{s} \longrightarrow s=\frac{2}{T}\frac{z-1}{z+1} \longrightarrow G_c(z)=\frac{T}{2}\frac{z+1}{z-1}$$

$$\downarrow$$

$$s=\frac{1}{T}\ln z$$

Aproximación de Tustin

$$G_c(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}=G_c(z)$$

Repasar técnicas de Control Continuo:

Asignación Dinámica:

① Para el sig. proceso:

$$G_p(s)=\frac{10}{100s+1}$$

Diseñar un cont. continuo t.:

- $E_{ss}=0$ ante escalón de rel. de tipo escalón.
- Tiempo de estabilización en lazo cerrado igual al mismo del proceso.

① Proponer $H_r(s)$:

$$H_r(s)=\frac{z^*}{s+1}$$

si: $z^*=1$ ∴ $E_{ss}=0$

$$H_r(s)=\frac{1}{s+1}$$

Para el proceso:

$$T_s^*: 4T_s^*=4(100)=400\text{ seg}$$

$$T_r^*=\left(\frac{1}{2}\right)T_s^*=\frac{400}{2}=200\text{ seg}$$

$$T_d^*=4T_r^*=\frac{T_s^*}{4}=\frac{400}{4}=50\text{ seg}$$

$$H_r(s)=\frac{1}{50s+1}$$

② Sacar $G_p(s)$:

$$G_c(s)=\frac{1}{\frac{10}{100s+1}}\cdot\frac{\frac{1}{100s+1}}{1-\frac{1}{50s+1}}=\frac{100s+1}{10}\cdot\frac{1}{50s+1-1}$$

$$G_c(s)=\frac{100s+1}{10(50s)}=\frac{100s+1}{500s}=\frac{100s}{500s}+\frac{1}{500s}=\underbrace{\frac{1}{5}}_{K_p}+\underbrace{\frac{K_i}{s}}_{\frac{1}{500s}}$$