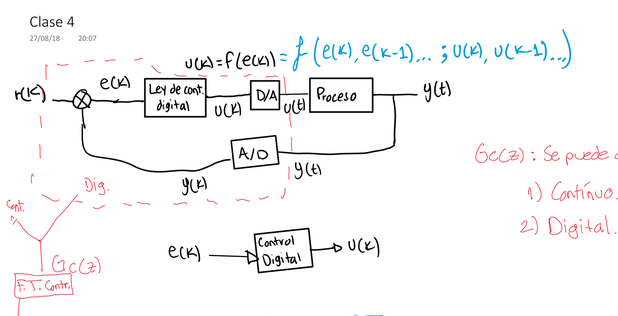
Clase 4

27/08/18 20:07



Gc(z): Se puede obtener de 2 formas:

1) Contínuo.

2) Digital.

Ejemplo: Controlador Digital PI

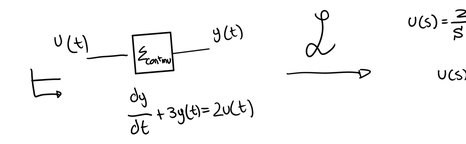
$$u(k)-u(k-1) = (K_p+K_i)e(k) - (K_p)e(k-1)$$

Para pseudocódigo, sería equivalente a:

$$U_{act} = U_{ant} + (K_p+K_i)e_{act} - K_p e_{ant}$$

```
Ki=#;
Kp=#;
ract=0;
Yact=0;
eact=0;
while (1) {
    ract = leer ract;
    Yact = leer A/D; % leer y(t)
    eact = ract - Yact;
    Uact = Uant + (Kp+Ki)eact - (Kp)eant;
    Uant = Uact;
    eant = eact;
    Uact = leer D/A;
}
end;
```

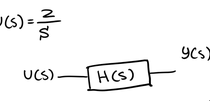
Función de Transferencia Discreta



$$\frac{dy}{dt} + 3y(t) = 2u(t)$$

$\mathcal{L}$ →

$U(s) = \frac{2}{s}$



$Y(s) = H(s)\cdot U(s)$

$H(s) = \frac{2}{s+3}$

$$Y(s) = \frac{2}{s+3}\cdot\frac{2}{s} = \frac{A}{s+3} + \frac{B}{s} = \frac{-4/3}{s+3} + \frac{4/3}{s}$$

A = -4/3

B = 4/3

$\mathcal{L}^{-1}$ →

$$y(t) = \frac{4}{3} - \frac{4}{3}e^{-3t}$$

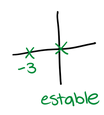
$$y(t) = \frac{4}{3}\left(1-e^{-3t}\right)$$

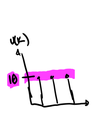
$$Y(s) = \frac{4}{s(s+3)}$$

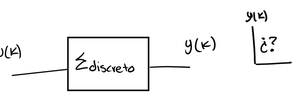
$$y_{ss} = \lim_{s\to 0} sY(s)$$

$$= \lim_{s\to 0} \cancel{s}\left[\frac{4}{\cancel{s}(s+3)}\right]$$

$$y_{ss} = 4/3$$



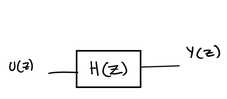




$$y(k) - 0.2y(k-1) = u(k-1)$$

↳ Queremos una solución cerrada!!!

$\mathcal{Z}$ →



H(z) = ?

U(z) = ?

→ $r(z) = H(z)U(z)$

$\mathcal{Z}^{-1}$ → y(k)

$$H(z) = \frac{\mathcal{Z}\{y(k)\}}{\mathcal{Z}\{u(k)\}}\Bigg|_{\text{cond. inicial}=0}$$

① $\mathcal{Z}\{y(k) - 0.2y(k-1) = u(k-1)\}$

$$\Rightarrow \mathcal{Z}\{y(k)\} + \boxed{\mathcal{Z}\{-0.2y(k-1)\}} = \mathcal{Z}\{u(k-1)\}$$

$$\Rightarrow Y(z) - 0.2z^{-1}Y(z) = z^{-1}U(z)$$

$$\Rightarrow Y(z)(1-0.2z^{-1}) = z^{-1}U(z)$$

$$\Rightarrow \frac{Y(z)}{U(z)} = \frac{z^{-1}}{1-0.2z^{-1}} = H(z)$$

$$\Rightarrow H(z) = \frac{1}{z-0.2}$$

Se pueden usar las fórmulas:

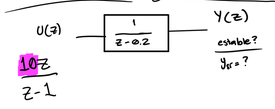
#7) $x(t-KT) = z^{-K}X(z)$

#9) $x(n-k) = z^{-k}x(z)$

Para una función retrasada i, n, T, x veces.

$x(t-KT) = x(k-1)$

Ejemplo: $x(k-3) = z^{-3}x(z)$



$\frac{10z}{z-1}$

estable?

y_{ss} = ?

① estable Y(z):

Polos: z = 1

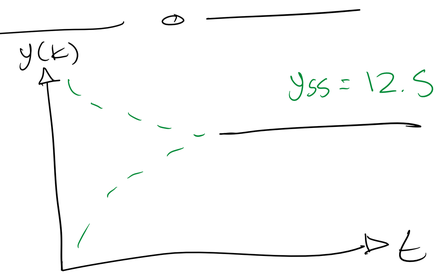
z = 0.2

estable

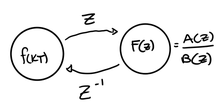
② Método del valor final "discreto" (en Z)

$$y_{ss} = \lim_{k\to\infty} y(k) = \lim_{z\to 1}(1-z^{-1})(Y(z))$$

$$y_{ss} = \lim_{z\to 1}(1-z^{-1})\left[\frac{10z}{(z-0.2)(z-1)}\right] = \frac{10}{0.8} = 12.5$$



*Transformada $\mathcal{Z}^{-1}$ (inversa de z):



$= \frac{A(z)}{B(z)}$

$f(kT) = \mathcal{Z}^{-1}\{F(z)\}$

$$Y(z) = \frac{10z}{(z-0.2)(z-1)} = \frac{A}{z-0.2} + \frac{B}{z-1} = \frac{-2.5}{z-0.2} + \frac{12.5}{z-1}$$

$$A = \frac{10(.2)}{.2-1} = -2.5$$

$$B = \frac{10(1)}{1-0.2} = 12.5$$

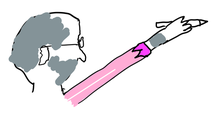
$\mathcal{Z}^{-1}$ →

$$Y(k) = -2.5\frac{z^{-1}}{1-0.2z^{-1}} + 12.5\frac{z^{-1}}{1-z^{-1}}$$

Fórmula 15:

$$\mathcal{Z}\left\{\frac{z^{-1}}{1-az^{-1}}\right\} = \boxed{a^{k-1}\ ;\ k=0,1,2,3...}$$

Es la única fórmula con esta restricción



Para evitar el uso de la #15 se aplica este truco:

1) Paso 1: Fracc. parciales y expansión:

$$\frac{Y(z)}{z} = \frac{10}{(z-0.2)(z-1)} = \frac{A}{z-0.2} + \frac{B}{z-1}$$

A = -12.5

B = 12.5

2) Paso 2: Pasar Z arriba

$$\Rightarrow \frac{Y(z)}{z} = \frac{-12.5}{z-0.2} + \frac{12.5}{z-1}$$

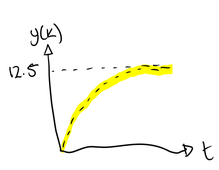
$$\Rightarrow Y(z) = \frac{-12.5z}{z-0.2} + \frac{12.5z}{z-1} = -12.5\frac{1}{1-0.2z^{-1}} + 12.5\frac{1}{1-z^{-1}}$$

Con esta nueva ecuación, ya no se necesita usar la #15

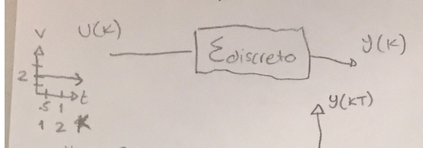
$Y(k) = -12.5(0.2)^k + 12.5$ válida para todo "k"

$Y(k) = 12.5(1-0.2^k)$

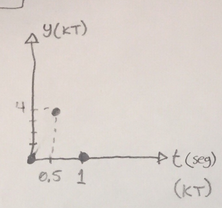
↳ $y_{ss} = \lim_{k\to\infty} y(k) = \lim_{k\to\infty}\left(12.5(1-0.2^k)\right) = 12.5$



Examen Rápido 3




```
Yant=0;
Uant=0;
while(1){
    Uact = leer;
    Yact = -Yant+2Uant;
    Yant = Yact;
    Uant = Uact; % en el caso general, depende de más.
    plot(k*T, Yact) % hold on, etc.
}end
```



$$y(k) + y(k-1) = 2u(k-1)$$

$$y(k) = 2u(k-1) - y(k-1)$$

k = 0

$y(0) = \cancel{2u(-1)} - \cancel{y(-1)}$

k = 1

$y(1) = -y(0) + 2u(0)$

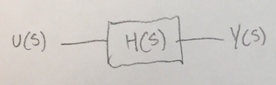
$= 0 + 2(2) = 4$

k = 2

$y(2) = -y(1) + 2u(1)$

$= -4 + 2(2) = 0$

En continuo:



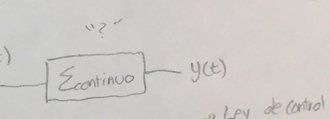
$H(s) = \frac{2}{s+3}$

$\mathcal{L}^{-1}$ → $h(t) = 2e^{-3t}$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{2}{s+3}$$

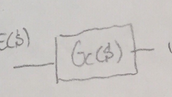
$$sY(s) + 3Y(s) = 2U(s)$$

$\mathcal{L}^{-1}$ → $\underbrace{\frac{dy}{dt}}_{\text{Salida}} + 3y(t) = \underbrace{2u(t)}_{\text{Entrada}}$



PI

$Gc(s) = 5 + \frac{0.5}{s}$ → Ley de control $u(t) = f\,e(t)$



$$Gc(s) = \frac{U(s)}{E(s)} = 5 + \frac{0.5}{s}$$

$$U(s) = \left[5 + \frac{0.5}{s}\right]E(s) = 5E(s) + \frac{0.5}{s}E(s)$$

$\mathcal{L}^{-1}$ → $u(t) = 5e(t) + 0.5\int e(t)dt$