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Dynamical dark energy models and the ΛCDM cosmological model

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ABSTRACT

Context. A breakthrough in modern cosmology was the discovery of an accelerating universe. It indicated the presence of a new dominant component with negative pressure that causes the acceleration: Dark Energy. While a cosmological constant universe is appealing because of its simplicity, it poses the problem of the magnitude of the cosmological constant. Because of that, we explore new models of dark energy.

Aims. We study the cosmology of different dark energy models with dynamical equation of state, both at a level of background cosmology and linear matter perturbations. In order to differentiate between models, we introduce the so-called statefinder parameters r, s, which depend on the third derivate with respect the scale factor.

Methods. We integrate numerically both background parameters and the matter growth parameters for several dark energy equations of state: Linear model $w(z) = w_0 + w_1 z$, Linder Ansatz $w(z) = w_0 + w_1 \frac{z}{1+z}$ and an oscillating $H(z)$ model. We neglect radiation, we use a prior of $\Omega_{m,0} = 0.3$ and we assume flat geometry of the universe.

Results. We show the comparison between the standard cosmological model with equation of state $w = -1$ and different dark energy parametrizations. All models agree with an accelerating universe at the present time. $w(z) = w_0 + w_1 z$ parametrization differs significantly from the ΛCDM model, while $w(z) = w_0 + w_1 \frac{z}{1+z}$ parametrization approaches our standard cosmological model. At high redshift, oscillating model evolves identically as ΛCDM, while Linear model diverges for cosmological perturbations.

Conclusions. We conclude that $w(z) = w_0 + w_1 \frac{z}{1+z}$ model fits better ΛCDM model and behaves well at high redshift contrary to Linear model that diverges. Oscillating dark energy approaches better ΛCDM at cosmological perturbation level. Future observational surveys will provide significant insights into cosmological probes and new measurements of dark energy properties. New information about this parameters may be obtained, leading to a better understanding of dark energy's nature.

Key words. dark energy – cosmology

1. Introduction

Current observational data provide strong evidence that we live in a spatially flat universe expanding at an accelerating rate. Einstein's field equations with radiation and non-relativistic matter only cannot lead to accelerating solutions. We need to introduce a new exotic component with a sufficient negative pressure called Dark Energy (DE) that dominates the energy density today. Current constraints indicate that this new component contributes 70% to the total energy density with an equation of state $w = -1$. This has led to think that dark energy is just Einstein's cosmological constant Λ.

The ΛCDM model, based on cold dark matter and a cosmological constant, is the most simple and economical one which is in a very good agreement with observational data. Despite the tremendous observational progress, it suffers from the cosmological constant problem and there are no new insights into the physics behind this mysterious component. These are the basic incentives to explore other possibilities like dynamical equations of state models.

Many dark energy models predict very similar expansion histories and therefore, all of them are still in agreement with current data. In order to discriminate between them, we study new parameters that depends of the third derivative of the cosmic time, the statefinder parameters:

$$r = \frac{\dddot{a}}{aH^3} \tag{1}$$

$$s = \frac{r-1}{3(q-\frac{1}{2})} \tag{2}$$

where $a(t)$ is the scale factor, which describes the universe expansion, where the present value is $a(t_0) = 1$. $H = \dot{a}/a$ is the Hubble parameter and $q = -\ddot{a}/aH^2$ is the decelerating parameter.

We can extend the analysis studying the growth of linear perturbations through the growth function $\delta = \delta\rho/\rho$. The combination of the statefinders parameters and the description of the evolution of matter perturbations can provide significant insight into properties of dark energy.

All these quantities can be computed for a given dark energy model and their values can be extracted from future observations. Observational surveys will help answer questions about the distribution of dark matter in the universe, its expansion history and how the large scale structure of the universe did form through cosmological probes like weak gravitational lensing and baryonic acoustic oscillations. They will allow us to map the dark universe and improve our constraints on dark energy.

Our goal is to study the cosmology of several dark energy models with dynamical equation of state and compare them to the ΛCDM cosmological model.

This work is organized as follows: In §2, we provide a theoretical basis on cosmology, in §3 we present different dark energy models, in §4 we show our numerical results and in §5 we summarize our work.

2. Theoretical Framework

2.1. Background Evolution

The study of the universe starts with the cosmological principle: on large scales, the universe is homogeneous and isotropic. Under this assumption, the metric, which describes the space-time geometry, can be written in the Friedman-Lemaître-Robertson-Walker (FLRW) form:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \tag{3}$$

where r, θ and ϕ are the comoving spatial coordinates, t is time and k is the curvature of space. Evidence suggests that $k \approx 0$.

Using the flat FLRW metric in Einstein's equations of general relativity, we obtain the Friedmann equations which describe the evolution of the scale factor:

$$H^2 = \frac{8\pi G\rho}{3} \tag{4}$$

$$\dot{H} = -4\pi G(\rho + p) \tag{5}$$

where G is the Newton's constant, ρ the total energy density and p the total pressure.

We define the critical energy density $\rho_{crit} \equiv 3H^2/8\pi G$. Then, the total energy density can be described relative to its critical value:

$$\Omega = \frac{\rho}{\rho_{crit}} \tag{6}$$

where $\Omega = \Omega_m + \Omega_r + \Omega_{de} = 1$ and the indices m, r, de represent the matter, radiation and dark energy component respectively.

Due to conservation of energy and assuming no interaction between the fluid components, we can derive the continuity equation for each fluid:

$$\dot{\rho} + 3H\rho\,(1 + w) = 0 \tag{7}$$

where w its the equation of state defined as $w \equiv p/\rho$.

The simplest description it is to assume $w = constant$, this is the case of matter $w_m = 0$ and radiation $w_r = 1/3$. The ΛCDM model it is the simplest model, with dark energy equation of state parameter $w = -1$, that can describe the majority of observations of the universe. Constant equations of state do not describe scalar fields dark energy or modified gravity models and therefore, dynamical dark energy models are the most general way to describe dark energy. In this paper, we will assume some parametrization of the dark energy equation of state parameter as a function of redshift $1 + z = 1/a$. We can solve the continuity equation for an arbitrary dark energy equation of state:

$$\rho_{\rm de}(z) = \rho_{\rm de,0}\exp\left(3\int_0^z \frac{1 + w\,(z')}{1 + z'}dz'\right) \tag{8}$$

where $\rho_{de,0}$ it is the present-day energy density of dark energy.

The dimensionless expansion rate of the universe $E(z) = H(z)/H_0$, where H_0 is the present value of the Hubble parameter, can be written as:

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + (1 - \Omega_{m,0} - \Omega_{r,0})\,F(z)} \tag{9}$$

where $\Omega_{r,0} = 9x10^{-5}$, $(1 - \Omega_{m,0} - \Omega_{r,0}) = \Omega_{de,0}$, with $\Omega_{m,0}, \Omega_{r,0}, \Omega_{de,0}$ the present values of the normalized densities of matter, radiation and dark energy respectively, and $F(z)$ is given by:

$$F(z) = \exp\left(3\int_0^z \frac{1 + w\,(z')}{1 + z'}dz'\right) \tag{10}$$

For matter and radiation, the normalized energy density is given by:

$$\Omega_m = \frac{\Omega_{m,0}(1+z)^3}{E(z)^2} \tag{11}$$

$$\Omega_r = \frac{\Omega_{r,0}(1+z)^4}{E(z)^2} \tag{12}$$

With all these cosmological equations we can derive a new expression for the deceleration and first statefinder parameter:

$$q(z) = -1 + (1+z)\frac{E'(z)}{E(z)} \tag{13}$$

$$r(z) = q(z)(2q(z) + 1) + (1+z)q'(z) \tag{14}$$

2.2. Cosmological Perturbations

To explain the large scale structure that we see in the universe, we need to introduce small energy density perturbations $\delta = \delta\rho_m/\rho_m$. The evolution of these fluctuations is given by:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0 \tag{15}$$

We can express the fluctuations and the equation of state in terms of the scale factor, which leads to:

$$\delta'' + \left(\frac{3}{a} + \frac{E'(a)}{E(a)}\right)\delta' = \frac{3}{2}\frac{\Omega_{m,0}}{a^5E(a)^2}\delta \tag{16}$$

where the prime denotes differentiation with respect the scale factor.

It is useful to introduce new quantities that contains very important sensitivity to dark energy parameters:

$$f \equiv \frac{d\,ln\,\delta}{d\,ln\,a} = \Omega_m^\gamma \tag{17}$$

where f is the growth rate and γ is the growth index.

3. Equation of state parametrizations

Being the ratio of pressure to energy density, the equation of state parameter w provides a useful phenomenological description of dark energy. A general function of redshift $w(z)$ would be the most general way to describe dark energy. In this section we present different dark energy models where the equation of state evolves with the universe. We do not know the physics behind dark energy and therefore, we explore models beyond the cosmological constant using explicit parametrization of the equation of state parameter $w(z)$. All of these functions have many free parameters that we need to measure. Even measuring a few is challenging and therefore the most popular models contain only a few. We use free parameter values computed in a previous work [1]. Table 1 shows the best fit parameters for each model using SNIa data.

3.1. ΛCDM model

Is the simplest model characterized by a sigle parameter with equation of state $w = -1$. The dimensionless Hubble parameter is given by:

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + 1 - \Omega_{m,0}} \tag{18}$$

3.2. Linear Ansatz

First order Taylor expansion of $w(z)$ around zero:

$$w(z) = w_0 + w_1 z \tag{19}$$

which leads to the dimensionless Hubble parameter:

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0})(1+z)^{3(1+w_0-w_1)}e^{3w_1 z}} \tag{20}$$

3.3. Linder Ansatz

This model behaves like Linear model at low redshift but it is bounded at high redshift:

$$w(z) = w_0 + w_1 \frac{z}{1+z} \tag{21}$$

this leads to the dimensionless Hubble parameter:

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + (1-\Omega_{m,0})(1+z)^{3(1+w_0+w_1)}e^{3w_1(\frac{1}{1+z}-1)}} \tag{22}$$

3.4. Oscilatting Ansatz

We can also start directly from the dimensionless Hubble parameter:

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + a_1\cos(a_2 z^2 + a_3) + (1 - a_1\cos(a_3) - \Omega_{m,0})} \tag{23}$$

4. Results

In this section we show our results for several dark energy models with a given parametrization. Since we are not interested in Cosmic Microwave Background anisotropies, we integrate the equations numerically using *Mathematica*.

We compute all the quantities of interest using the parameters shown in Table 1. The parameters are the results of fitting SNIa data to several cosmological models and $H(z)$ ansatze, assuming flat geometry of the universe and a prior of $\Omega_{m,0} = 0.3$. We adopt these assumptions. Furthermore, we can neglect the $\Omega_{r,0}$ term studying times after matter-radiation equality $\sim z = 10^3$, but dark energy shows a very different behavior once radiation became smaller. In the standard cosmological model, dark energy density overcomes matter density at recent times (Fig. 1). This behavior can be seen in Fig. 3, that shows the deceleration parameter versus redshift. All models agree with an accelerating universe at the present time. We can see that Linear and Oscilatting Ansatz models deviates significantly from the ΛCDM model. The first and second statefinder parameters r and s versus redshift are shown in Fig. 4 and 5 for all four models. We see in Fig. 4 that Linder model approaches ΛCDM model, while Linear and OA models deviates at high redshift. In Fig. 5 we see more difference between Linder and ΛCDM model, while Linear and OA models diverges at $z \sim 0.5$.

For matter perturbations, we integrate equation (16) from $a \sim 0.01$ to the present time. To set the appropriate initial conditions, we solve equation (15) analytically in a matter dominated universe, finding that $\delta \sim a$. We see in Fig. 6 that Linear model has an extreme behavior at low scale factor, which is to be expected since Linear model its a Taylor expansion valid at low redshift, while OA model approaches ΛCDM model at high redshift. We show the solution of f and γ in Fig. 7 and Fig. 8. We see that the growth rate increases with redshift for all models. The growth index for the ΛCDM model is $\gamma \sim 0.55$ as it should be, while the OA model approaches better to the ΛCDM model.

Table 1. Best fit parameters for Linder, Linear and OA models.

Models	Best Fit Parameters
Linder	$w_0 = -1.29,\ w_1 = 2.84$
Linear	$w_0 = -1.25,\ w_1 = 1.97$
Oscilatting Ansatz	$a_1 = -3.36,\ a_2 = 2.12,$ $a_3 = -0.06\pi$

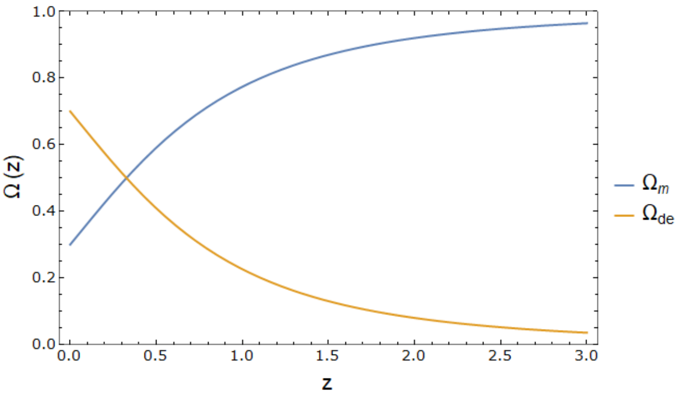

Fig. 1. Omega matter (blue line) and Omega dark energy (orange line) versus redshift for the ΛCDM model.

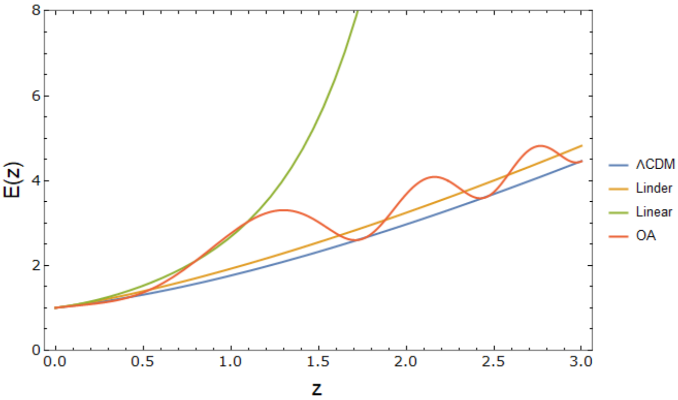

Fig. 2. Dimensionless Hubble parameter versus redshift for the ΛCDM (blue line), Linder (orange line), Linear (green line) and Oscilating Ansatz (red line) models.

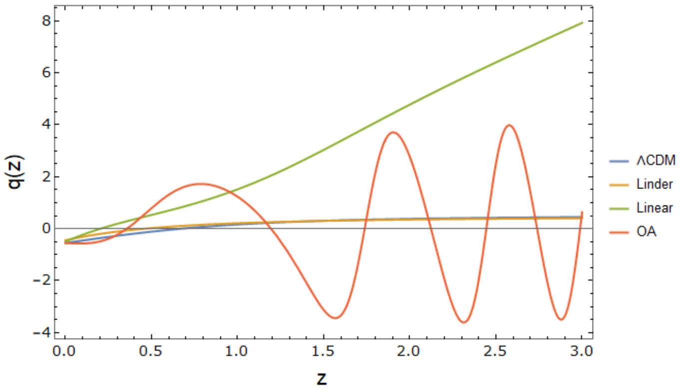

Fig. 3. Deceleration parameter versus redshift for the ΛCDM (blue line), Linder (orange line), Linear (green line) and Oscilating Ansatz (red line) models.

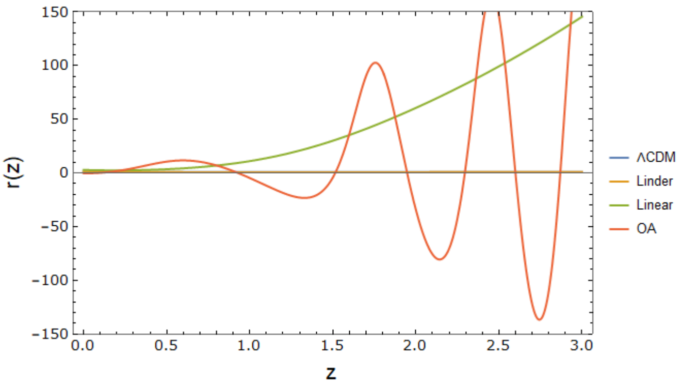

Fig. 4. First statefinder parameter r versus redshift for the ΛCDM (blue line), Linder (orange line), Linear (green line) and Oscilating Ansatz (red line) models.

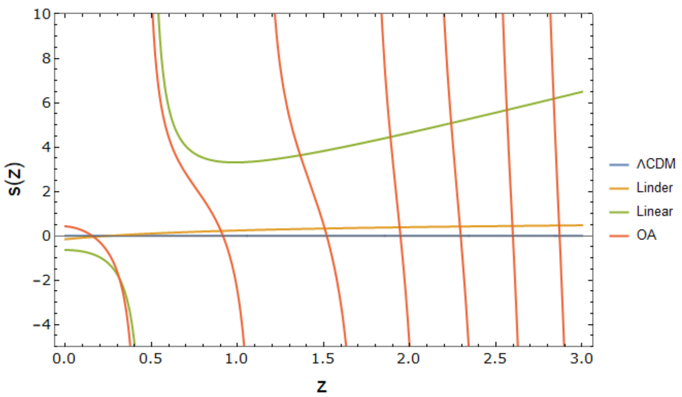

Fig. 5. Second statefinder parameter s versus redshift for the ΛCDM (blue line), Linder (orange line), Linear (green line) and Oscilating Ansatz (red line) models.

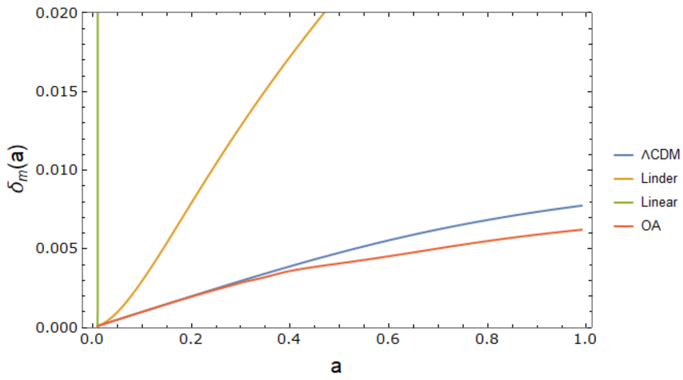

Fig. 6. Matter density perturbation versus the scale factor for the ΛCDM (blue line), Linder (orange line), Linear (green line) and Oscilating Ansatz (red line) models.

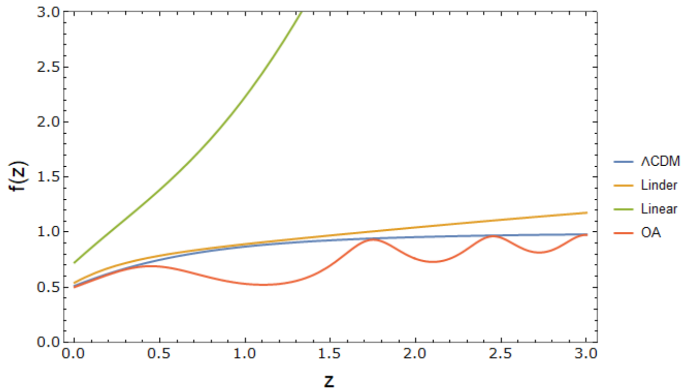

Fig. 7. Growth rate f versus redshift for the ΛCDM (blue line), Linder (orange line), Linear (green line) and Oscilating Ansatz (red line) models.

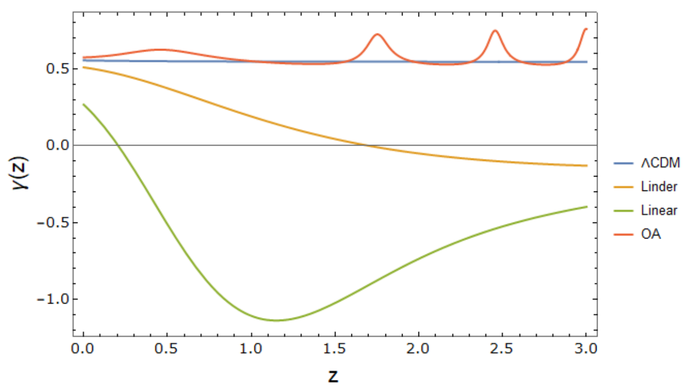

Fig. 8. Growth index γ versus redshift for the ΛCDM (blue line), Linder (orange line), Linear (green line) and Oscilating Ansatz (red line) models.

5. Conclusions

We have discussed three different dark energy parametrizations both at background and linear perturbations level and we compared them with the standard cosmological model. We used the parameters that better fits to SNIa data. Using *Mathematica*, we have computed the Hubble and decelerating parameter, as well as the statefinder parameters r, s. We have integrated numerically the evolution of linear perturbation equations to obtain both growth rate f and growth index γ.

At a background evolution level, we can only see an agreement between Linder model and ΛCDM model since both shows similar expansion histories. While the statefinder parameters for the standard cosmological model are trivial $r = 1, s = 0$, this is not true for the three other models. While all models agree on the present value of the deceleration parameter $q(0)$ and the first statefinder parameter $r(0)$, we can establish significant differences regarding the present value of the second statefinder parameter $s(0)$ at the present time: $s(0)$ changes between positive, zero and negative values.

At a cosmological perturbation level, we can see a better agreement between the oscillating model and ΛCDM model. Matter density perturbations match between OA and ΛCDM models at high redshift, while the Linear model diverges in both matter perturbations and growth rate f.

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References

[1] S. Nesseris and L. Perivolaropoulos. Comparison of cosmological models using recent supernova data. *Phys. Rev. D*, 70, 043531 (2004).

[2] Grigoris Panotopoulos and Ángel Rincón. Growth index and statefinder diagnostic of oscillating dark energy. *Phys. Rev. D*, 97, 103509 (2018).

[3] Grigoris Panotopoulos and Ángel Rincón. Model comparison using supernovae and Hubble parameter data. *European Physical Journal Plus*, 134, 354 (2019).

[4] S. Nesseris and L. Perivolaropoulos. Testing ΛCDM with the growth function δ(a): Current constraints. *Phys. Rev. D*, 77, 023504 (2008).

[5] Juan Magaña, Víctor H. Cárdenas, and Verónica Motta. Cosmic slowing down of acceleration for several dark energy parametrizations. *J. Cosmology Astropart. Phys.*, 2014, 017 (2014).

[6] Imanol Albarran, Mariam Bouhmadi-López, and João Morais. Cosmological perturbations in an effective and genuinely phantom dark energy Universe. *Physics of the Dark Universe*, 16, 94 (2017).

[7] Dragan Huterer and Daniel L. Shafer. Dark energy two decades after: observables, probes, consistency tests. *Reports on Progress in Physics*, 81, 016901 (2018).

[8] Lavrentios Kazantzidis and Leandros Perivolaropoulos. Evolution of the f σ_8 tension with the Planck15 /Λ CDM determination and implications for modified gravity theories. *Phys. Rev. D*, 97, 103503 (2018).

[9] Mehdi Rezaei, Mohammad Malekjani, Spyros Basilakos, Ahmad Mehrabi, and David F. Mota. Constraints to dark energy using PADE parameterisations. *arXiv e-prints*, page arXiv:1706.02537 (2017).

[10] David Polarski and Radouane Gannouji. On the growth of linear perturbations. *Physics Letters B*, 660, 439 (2008).

[11] Ahmad mehrabi and Spyros Basilakos. Dark energy reconstruction based on the Padé approximation; an expansion around the ΛCDM. *European Physical Journal C*, 78, 889 (2018).

[12] M. R. Setare and E. N. Saridakis. Quintom dark energy models with nearly flat potentials. *Phys. Rev. D*, 79, 043005 (2009).