TD Transformations n° 2
Laplace

Exercice n° 2 :

$$3[\sin(2t) - 2t\cos(2t)] + 2\sin(3t)\,e^{-4t} - t^2e^{-2t}$$

- $\sin(2t) \to \frac{2}{p^2+4}$
- $2t\cos(2t) \to -2\frac{d}{dp}\left(\frac{p}{p^2+4}\right) = -2\frac{p^2+4-2p^2}{(p^2+4)^2}$
- $2\sin(3t).e^{-4t} = \frac{6}{(p+4)^2+9}$: $.e^{-at}$ = translation
- $-t^2e^{-2t} = (-1)^2\frac{d^2}{dp^2}\left(\frac{1}{p+2}\right)$:

$$\Rightarrow \underline{\frac{6}{p^2+4}.\frac{4-p^2}{(p^2+4)^2} + \frac{6}{(p+4)^2+9} - \frac{2}{(p+2)^3}}$$

Exercice n° 3 :

$f' \sim pF(p) - f(0^+)$, la voici l'erreur

Exercice n° 4 :

$\to s(t) = H(t) - 3H(t-2) + 2H(t-3)$

$\to u(t) = -H(t) + 2tH(t-1) - 3tH(t-2) + tH(t-3)$

$\to S(p) = \frac{1}{p} - 3.\frac{e^{-2p}}{p} + 2.\frac{e^{-3p}}{p}$

$\to U(p) = -\frac{1}{p} + 2.\frac{e^{-p}}{p^2} - 3\frac{e^{-2p}}{p^2} + \frac{e^{-3p}}{p^2}$

Exercice n° 5 :

- $\frac{5}{p+2} : \underline{5e^{-2t}}$
- $\frac{4p-3}{p^2+4} = \frac{4p}{p^2+4} - \frac{3}{p^2+4} = \underline{4\cos(2t) - \frac{3}{2}\sin(2t)}$
- $\frac{1}{p^2+6p+13} = \frac{1}{(p+3)^2+4} = \underline{\frac{e^{-3t}.\sin(2t)}{2}}$
- $\frac{(p-2)^2}{p^3} = \frac{p^2-4p+4}{p^3} = \frac{1}{p} - \frac{4}{p^2} + \frac{4}{p^3} = \underline{1 - 4t + 2t^2}$

$\frac{5}{p^2+2p} = \frac{A}{p} + \frac{B}{p+2}$

$\rightarrow \frac{5}{p+2} = A + \frac{Bp}{p+2}$ $\quad p = 0 \quad A = 5/2$

$\rightarrow \frac{5}{p} = B \; : p = -2 \quad B = -\frac{5}{2}$

$= \frac{5}{2p} - \frac{5}{2(p+2)} \Leftrightarrow \underline{\frac{5}{2} - \frac{5}{2}e^{-2t}}$

$\frac{4p-3}{p^2+4} = \frac{A}{p-2} + \frac{B}{p+2} \; : A = 5/4 \; ; \; B = 11/4 \quad \frac{5}{4(p-2)} + \frac{11}{4(p+2)} = \underline{\frac{5}{4}e^{2t} + \frac{11}{4}e^{-2t}}$

$\frac{1}{p^2+6p+8} = \frac{1}{(p+3)^2-1} = \frac{1}{(p+2)(p+4)} \Leftrightarrow \underline{\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t}}$

$\frac{(p-e^{-p})^2}{p^3} = \frac{p^2 - 2pe^{-p} + e^{-2p}}{p^3} = \frac{1}{p} - \frac{2e^{-p}}{p^2} + \frac{e^{-2p}}{p^3} = \underline{1 - 2(t-1) + (t-2)^2}$

Esercice no 6:

$y' + 3y = e^{-t}$

$(p+3)Y = \frac{1}{(p+1)} \; : Y = \frac{1}{(p+3)(p+1)}$

$Y = \frac{A}{p+1} + \frac{B}{p+3} \; : A = \frac{1}{2}, \; B = -1/2$

$\underline{y(t) = \frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}}$

$y'' + 6y' + 13y = 1 \Leftrightarrow p^2Y + 6pY + 13Y = \frac{1}{p}$

$\Rightarrow Y = \frac{1}{p(p^2+6p+13)} = \frac{1}{p} \cdot \frac{1}{(p+3)^2+4} = \frac{A}{p} + \frac{B+Cp}{(p+3)^2+4}$

$\Rightarrow 1 = A((p+3)^2+4) + p(B+Cp)$

- $p = 0 \Rightarrow A = 1/13$
- $B = \frac{1}{p} - A\left(p + 6 + \frac{13}{p}\right) - Cp = \frac{1}{p} - \frac{p}{13} - \frac{6}{13} - \frac{1}{p} - Cp$

  $p = 0 \Rightarrow B = -6/13$
- $C = \frac{1}{p^2} - \frac{1}{13} - \frac{6}{p} - \frac{1}{p^2} - \frac{B}{p} = \frac{-1}{13} \quad (B = -6/13)$

  $\Rightarrow C = -1/13$

$Y = \frac{1}{13p} + \frac{-6-p}{(p+3)^2+4} \; : y(t) = \frac{1}{13} - \left(\frac{6}{2}\cdot\sin(2t) + \frac{1}{2}\cos(2t)\right)\cdot e^{-3t}$

· $y' + 2y = e^{-4t} \iff pY + 2Y = \frac{1}{p+4}$

$$Y = \frac{1}{(p+4)(p+2)} = \frac{A}{p+4} + \frac{B}{p+2}$$

· $A = \frac{1}{p+2} - \frac{B(p+4)}{p+2} \;:\; p = -4 \;:\; A = -1/2$

· $B = \frac{1}{p+4} - \frac{A(p+2)}{p+4} \;:\; p = -2 \;:\; B = 1/2$

$$Y = \frac{1}{2(p+2)} - \frac{1}{2(p+4)} = \frac{1}{2}\cdot\left(\frac{1}{(p+2)} - \frac{1}{(p+4)}\right)$$

$$\underline{y(t) = \frac{1}{2}\cdot\left(e^{-2t} - e^{-4t}\right)}$$

· $y'' + 6y' + 8y = 1 \;:\; p^2Y + 6pY + 8Y = \frac{1}{p}$

$$Y = \frac{1}{p(p^2+6p+8)} = \frac{1}{p(p+4)(p+2)} = \frac{A}{p} + \frac{B}{(p+4)} + \frac{C}{(p+2)}$$

$$\to A = \frac{1}{(p+4)(p+2)} - \frac{Bp}{(p+4)} - \frac{Cp}{(p+2)} \;/\; p = 0 \;/\; A = \frac{1}{8}$$

$$\to B = \frac{1}{p(p+2)} - \frac{(p+4)}{8p} - \frac{C(p+4)}{(p+2)} \;/\; p = -4 \;/\; B = \frac{1}{8}$$

$$\to C = \frac{1}{p(p+4)} - \frac{(p+2)}{8p} - \frac{(p+2)}{8(p+4)} \;/\; p = -2 \;/\; C = -\frac{1}{4}$$

$$Y = \frac{1}{8}\cdot\left(\frac{1}{p} + \frac{1}{(p+4)} - \frac{2}{(p+2)}\right)$$

$$\underline{y(t) = \frac{1}{8}\cdot\left(1 + e^{-4t} - 2e^{-2t}\right)}$$