HUGE

# Hello

Fuzzy Logic

July 31, 2018

# Agenda.

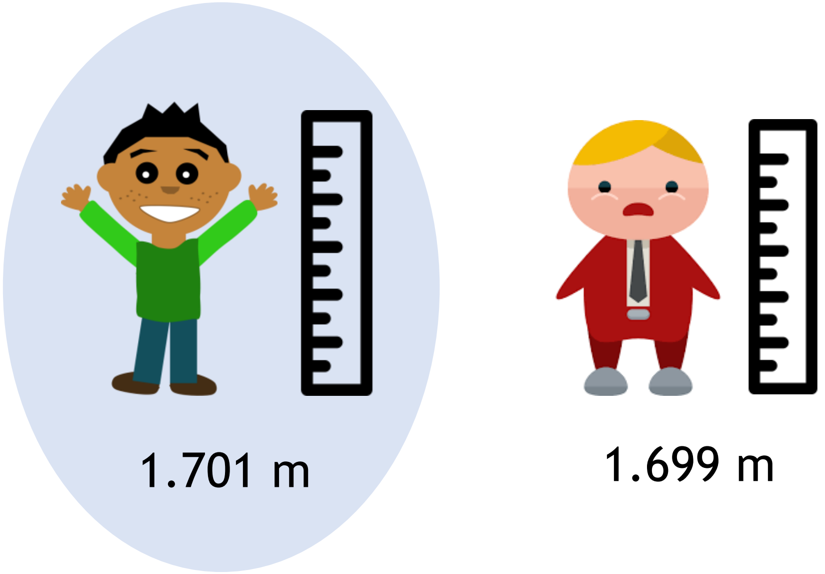
1. Motivation
2. Ideas
3. Fuzzy sets
4. Membership functions
5. Properties

# Motivation

Is it not enough with the classic logic?

No, classic logic does not reflects the nature of the concepts and human thoughts.

e.g.
Let's define tall person if he/she is taller than 1.7m




$$Tall\ people = \{\, height \mid height > 1.7\ m\}$$

# Basic ideas

**Uncertainty: No sure and clear knowledge of any concept.**

**Modeling and representation: Simplification of a real world situation through abstraction.**

**Modeling characteristics:**

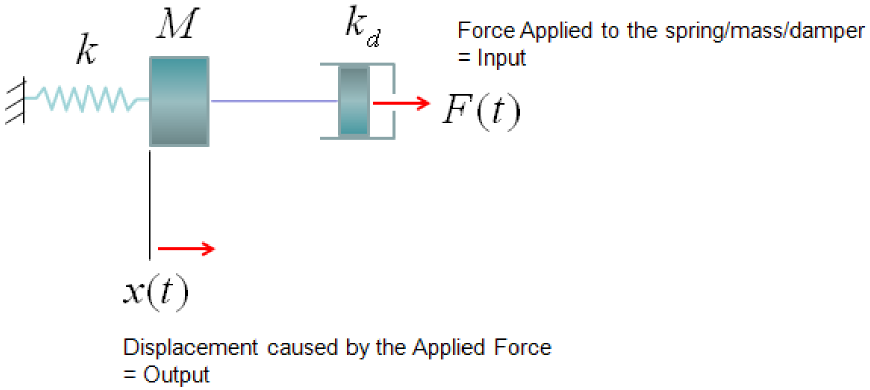
**Formality**

**Optimization**

**Solvable**

**Generality**

**Specificity**



# Types of uncertainty

Total certainty, certainty

Determinism

Face or seal?

True or false?

Randomness

Neural networks are efficient?

Ambiguity: More information allows solving the problem

The blonde woman is tall or low?

Vagueness: Accuracy in definitions

What does A mean? and B? ... variables not specified

Confusion

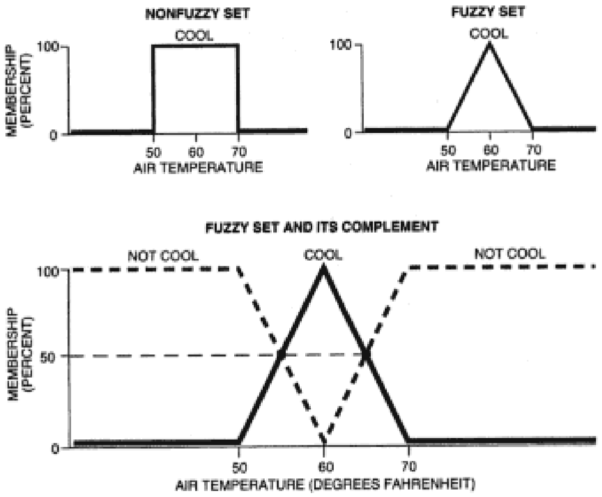
# Some types of modeling

Randomness
Risk

$$P(A) = \frac{N_A}{N}$$

Probability

Ambiguity
More information allows solving the problem
Vagueness-Precision in definitions




Fuzzy sets

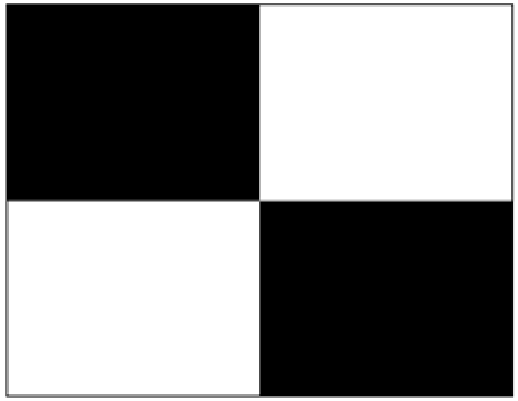
# Classic vs. Fuzzy

**Classical logic:**

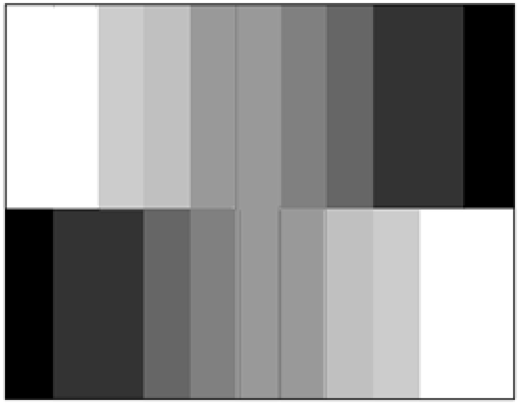
**Two values (0,1) are considered to express true / false.**

**Diffuse logic:**

**It is a multi-valued type of logic. 'possible'.**



Classical logic



Diffuse logic

# Fuzzy sets

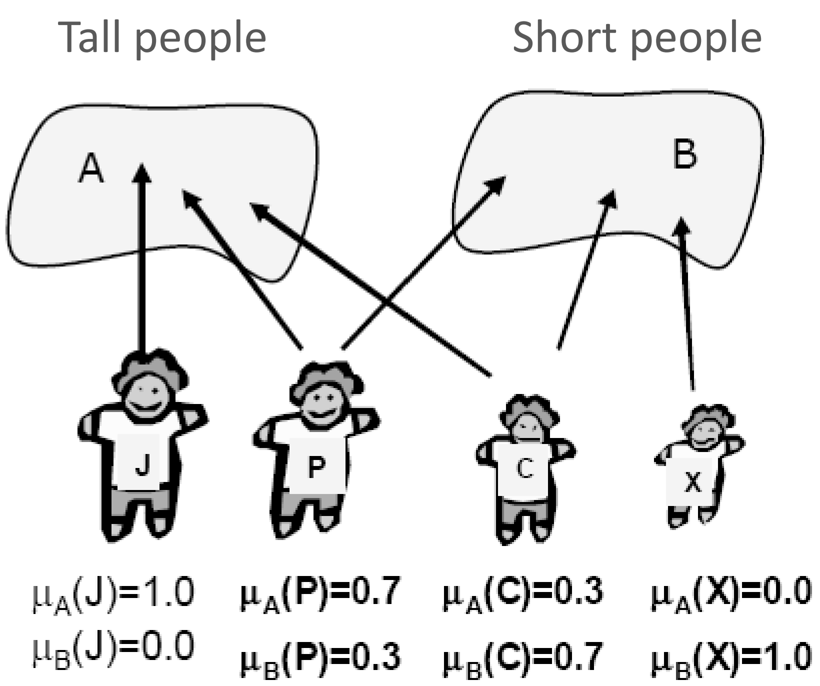
It is a model to describe the meaning of vague or imprecise words.

They are functions that relate a universe of objects.

The membership function that makes this relation for the fuzzy set A is the function

µA (x) gives the degree of belonging of the element x in the diffuse set A.

The fuzzyness describes the vagueness or imprecision of an event, definition or affirmation.



# Membership function

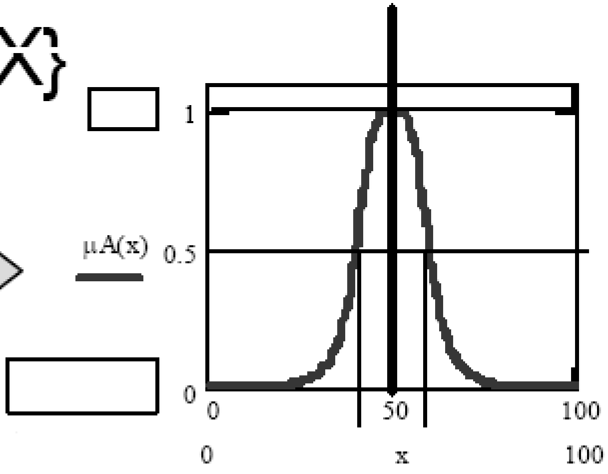
**Assign to each element of the set a degree of belonging between 0 and 1. A fuzzy set A in X is defined by the set of ordered pairs**

$$x, \mu_a(x)$$

**Continuous fuzzy set**

$$A = \{ x, \mu_A(x) | \; x \in X \}$$

$$\mu_A(x) = \frac{1}{1 + \left[ \frac{x - 50}{10} \right]^4}$$



# Membership function representations

**Continuous variables:**

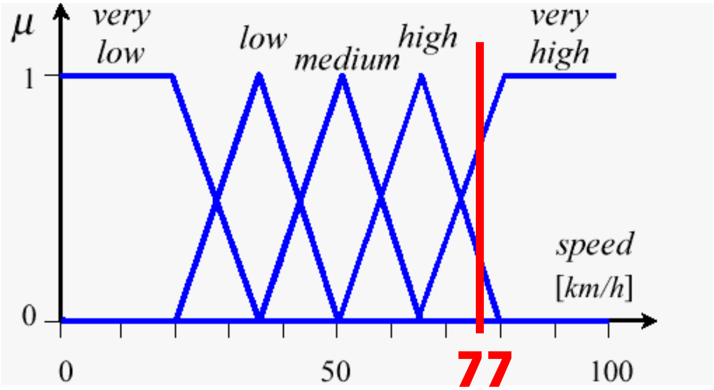
$$A = \int_{x} \frac{\mu_A(x)}{x}$$

**Discrete variables:**

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \ldots \frac{\mu_A(x_n)}{x_n} = \sum_{x \in X} \frac{\mu_A(x_i)}{x_i}$$

**The signs of sum and integral do not mean sum or integration but the union of the pairs ( $x_i$ , $\mu_A(x_i)$ ). In both cases, the horizontal line does not mean division. This is a boundary bar.**

# Representation of fuzzy measurements




**Speed at which we are going (77) a vertical is drawn.**

**Intersection value is taken looking at the vertical axis, there are TWO, the first is at a height of 0.20 and 0.75.**

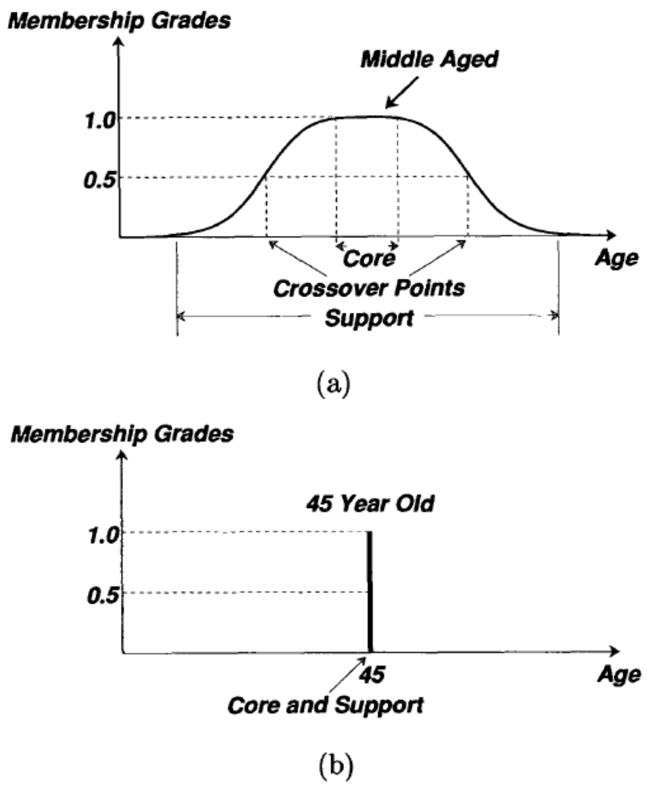
**77 Km/h = 0.2 high speed, 0.75 Very high speed. Thus, the system has a "fuzzy" estimate of the current speed.**

# Membership functions properties

**Center: Average of elements $\mu_x = 1$**

**Fuzzy Singleton: Support is a point F in U with $\mu_F = 1$**

**Height: Height, Greater membership degree… Normal set = 1**



# Membership functions properties

The cardinality of a classical set is defined as the number of elements in the set.

The cardinality of a diffuse set A is the sum of all the membership degrees of all the elements x in A, that is:

$$|A| = \sum_{x \in U} \mu_A(x)$$

$$|A| = \int_{x \in U} \mu_A(x)dx$$

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# Done.