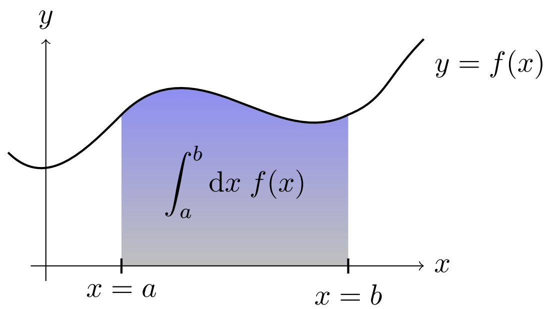
1 Background

An important operation in mathematics is calculating areas under a given curve, this operation is called integration. If the curve is defined as the graph of a function f, then the area delimited by the x axis and the lines $x = 0$ and $x = b$ is mathematically expressed as

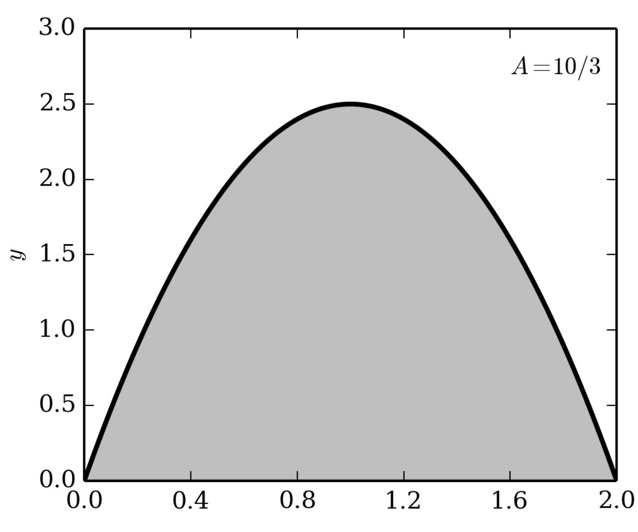
$$A = \int_a^b \mathrm{d}x\ f(x)$$



In this session we will study how to find numerical approximation to the value A.

2 Problems

(P1) Design an algorithm to find the area under the curve $y = 2.5x(2-x)$ in the interval $x \in [0, 1]$. (hint: use a random process to generate points within the shown rectangle)



(P2) Imagine that the are under f can be approximated by a rectangle. Use this to calculate

$$\int_2^{10} \frac{\mathrm{d}x}{\ln x}$$

(P3) Refine the rectangle into $n = 100$ rectangles and calculate

$$\int_2^{10} \frac{\mathrm{d}x}{\ln x}$$

(P4) Integration, being as important as it is, has several highly efficient implementations available for us. Among them are the ones found in `scipy`.

```
import scipy.integrate as integrate

def function(x):
    return 1 / np.log(x)

result = integrate.quad(function, 2, 10)
print result
print mpmath.li(10, offset=True)
```

where we have used the fact that

$$\mathrm{Li}(x) = \int_2^{x} \frac{\mathrm{d}t}{\ln t}$$

(P5) **Advanced Problem:** Design al algorithm to implement the method of Gaussian quadrature. Use to create a function that calculates $\mathrm{Li}(x)$