1 Background

One of the most fundamental interactions in natural environments is the *predator-prey* relationship. Let us assume two species: the predator (x) and the prey (y). If the species were isolated, each would evolve according to the unconstrained growth model we have already discussed. However, in the presence of the other species, the number is influenced by their interaction. In the *Lokta-Volterra* model, the joint evolution of these two species is given by the two equations

$$\begin{aligned} \frac{y_{n+1}-y_n}{t_{n+1}-t_n} &= k_y y_n - k_{xy} x_n y_n \\ \frac{x_{n+1}-x_n}{t_{n+1}-t_n} &= k_{yx} y_n x_n - k_x x_n \end{aligned} \tag{1}$$

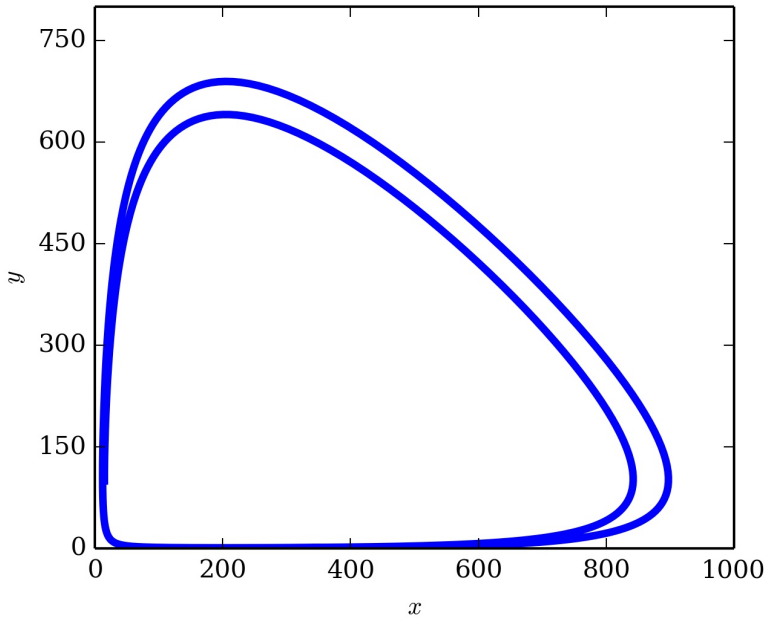
with k_x, k_y, k_{xy} and k_{yx} being constants.

2 Problems

(P1) Assume $x_0 = 15$ and $y_0 = 100$ and take $k_y = 2.0$, $k_{yx} = 0.01$, $k_x = 1.06$ and $k_{xy} = 0.01$. Also, assume that the time difference between observations is 3 days

(P2) Make a plot of the evolution of the population of the prey for 10 months

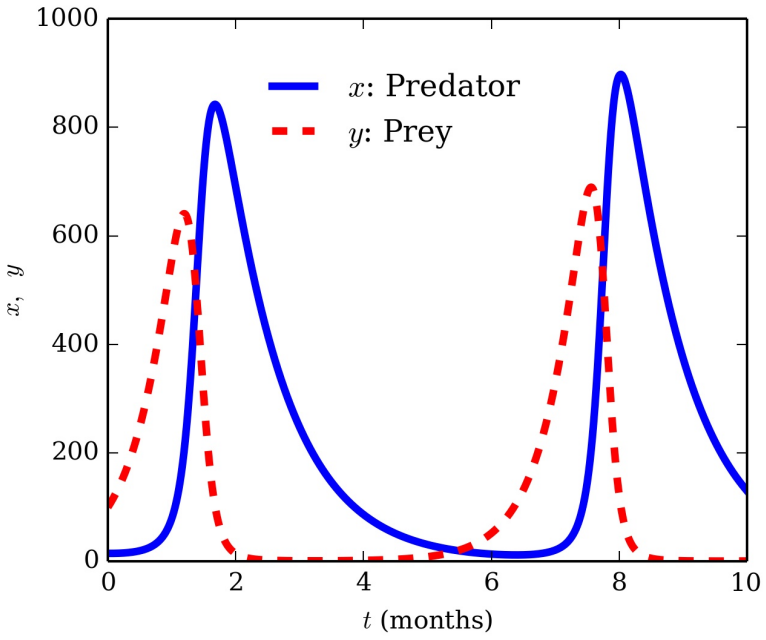
(P3) Make a plot of x vs y for the same period of time



3 Solutions

(P1)
```
import numpy as np
import matplotlib.pyplot as plt

tmax = 10.0
dt = 0.01

n = int(tmax / dt)
x = np.zeros(n)
y = np.zeros(n)

x[0] = 15
y[0] = 100

ky = 2.0
kx = 1.06
kxy = 0.01
kyx = 0.01
dt = 0.01
```

(P2)
```
for i in range(0, n - 1):

    t[i + 1] = t[i] + dt
    x[i + 1] = x[i] + dt * (kyx * y[i] * x[i] - kx * x[i])
    y[i + 1] = y[i] + dt * (ky * y[i] - kxy * x[i] * y[i])

plt.plot(t, x, 'b-')
plt.plot(t, y, 'r-')
plt.show()
```



(P3)
```
plt.plot(x, y, 'b-')
plt.show()
```