Traffic Model and Performance Analysis for Cellular Mobile Radio Telephone Systems with Prioritized and Nonprioritized Handoff Procedures

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Abstract—**A traffic model and analysis for cellular mobile radio telephone systems with handoff are described. Three schemes for call traffic handling are considered. One is nonprioritized and two are priority oriented. Fixed channel assignment is considered. In the nonprioritized scheme the base stations make no distinction between new call attempts and handoff attempts. Attempts which find all channels occupied are cleared. In the first priority scheme considered, a fixed number of channels in each cell are reserved exclusively for handoff calls. The second priority scheme employs a similar channel assignment strategy, but, additionally, the queueing of handoff attempts is allowed. Appropriate analytical models and criteria are developed and used to derive performance characteristics. These show, for example, blocking probability, forced termination probability, and fraction of new calls not completed, as functions of pertinent system parameters. General formulas are given and specific numerical results for nominal system parameters are presented.**

I. Introduction

THE PERFORMANCE of cellular mobile radio telephone systems in which cell size is relatively small and the handoff procedure has an important effect is investigated in this paper. Spectrally efficient mobile radio service for a large number of customers can be provided by cellular systems [1], [2]. The service area is divided into cells. Users communicate via radio links to base stations in the cells. Channel frequencies are reused in cells that are sufficiently separated in distance so that mutual interference is beneath tolerable levels.

Channel frequencies for mobile radio systems are allocated to base stations to be used in each cell by various channel assignment schemes. In fixed channel assignment (FCA) a group of channels is assigned to each base station. Different groups of channels are assigned to each cell according to definite rules. In dynamic channel assignment (DCA) no fixed relationship exists between the channel frequencies and the cells. Any channel can be used in any cell if no interference constraints are violated. In hybrid channel assignment (HCA) some channels are fixed assigned to cells, and others are assigned dynamically [1], [3]–[11]. Throughout this paper we assume that FCA is used.

When a new call is originated and attempted in a cell, one of the channels assigned to the base station of the cell is used for communication between the mobile user and the base station (if any channel is available for the call). If all the channels assigned to the base station are in use, the call attempt is assumed to be blocked and cleared from the system (blocked calls cleared (BCC)). When a new call gets a channel, it keeps the channel until the call is completed in the cell or the mobile moves out of the cell. When the call is completed in the cell, the channel is released and becomes available to serve another call. When the mobile crosses a cell boundary into an adjacent cell while the call is in progress, the call requires a new base station and channel frequency to continue. The procedure of changing channels is called "handoff." If no channel is available in the new cell into which the mobile moves, the handoff call is forced to terminate before completion. Simulation studies of handoff schemes have appeared in the literature [4]. For convenience in subsequent discussion we define the cell into which the mobile is moving and desires a handoff as the *target* cell for that handoff. Furthermore, we call the cell which the mobile is *leaving*, the *source* cell of the handoff attempt.

The required co-channel interference constraint is expressed as the ratio of the distance D between the centers of nearest neighboring cells that simultaneously can use the same channel to the cell radius R. This ratio, sometimes called the co-channel reuse ratio, is related to the number of cells per cluster N_C by $N_C = (D/R)^2/3$ [1]. When N_C is chosen from cochannel interference considerations, the capacity (e.g., erlangs carried per unit area at given performance level) of the mobile radio system depends on the cell radius. The cell radius R should be small for a high capacity system since this allows more frequency reuse in a given service area. On the other hand, in small cell systems, there are increased numbers of cell boundary crossings by mobiles. The average call duration (or channel holding time) in a cell $\bar{T}_H$ becomes less than the average unencumbered message duration $\bar{T}_M$. Also the distribution of channel holding time is different from that of message duration. In addition to the new call attempts, handoff call attempts are generated. The handoff attempt rate depends

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on cell radius as well as other system parameters. An important effect that should be considered is that some fraction of handoff attempts will be unsuccessful. Some calls will be forced to terminate before message completion.

In this paper we develop analytical models to investigate these effects and to examine the relationships between performance characteristics and system parameters. We wish to state at the outset that the analysis is approximate and contains simplifying assumptions made for the sake of analytical tractability. An *exact* analysis does not appear to be feasible. The informed reader will recognize the complexity of the problem. *Even* simulation studies of mobile communications traffic that have appeared in the technical literature, contain many simplifying assumptions, especially regarding distributions of certain random quantities which are needed to model physical reality. We feel that the underlying assumptions made in the development presented here are not unreasonable in view of the complexity of the real problem and the strength and tractability of the resulting analysis. A simulation study of the proposed handoff procedures is underway.

II. Traffic Model

A. Calling Rates

The basic system model assumes that the new call origination rate is uniformly distributed over the mobile service area. We denote the average number of new call originations per second per unit area as Λ_a. A very large population of mobiles is assumed, thus the average call origination rate is for practical purposes independent of the number of calls in progress. A hexagonal cell shape is also assumed for the system because it has some definite advantages over other possible shapes [1]. The cell radius R for a hexagonal cell is defined as the maximum distance from the center of a cell to the cell boundary. With the cell radius R, the average new call origination rate per cell Λ_R is

$$\Lambda_R=\frac{3\sqrt{3}}{2}R^2\Lambda_a. \tag{1}$$

Additionally, handoff attempts are made, with an average handoff attempt rate per cell denoted Λ_{Rh}. This rate will be related to other system parameters. The ratio γ_o of handoff attempt rate to new call origination rate (per cell) is

$$\gamma_o \triangleq \frac{\Lambda_{Rh}}{\Lambda_R}. \tag{2}$$

If a fraction P_B of new call originations is blocked and cleared from the system, the average rate at which new calls are carried is

$$\Lambda_{Rc}=\Lambda_R(1-P_B). \tag{3}$$

Also, if a fraction P_{fh} of handoff attempts fails, the average rate at which handoff calls are carried is

$$\Lambda_{Rhc}=\Lambda_{Rh}(1-P_{fh}). \tag{4}$$

The ratio γ_c of the average carried handoff attempt rate to the average carried new call origination rate is defined

$$\gamma_c \triangleq \frac{\Lambda_{Rhc}}{\Lambda_{Rc}}=\gamma_o\frac{(1-P_{fh})}{1-P_B)}. \tag{5}$$

B. Channel Holding Time in a Cell

The channel holding time T_H in a cell is defined as the time duration between the instant that a channel is occupied by a call and the instant it is released by either completion of the call or a cell boundary crossing by the mobile. This is a function of system parameters such as cell size, speed and direction of mobiles, etc. The distribution of T_H is investigated in this section.

We let the random variable T_M denote the unencumbered message duration, that is, the time an assigned channel would be held if no handoff is required. The random variable T_M is assumed to be exponentially distributed with the mean value $\bar{T}_M(\triangleq 1/\mu_M)$. Because of handoff, the distribution of this random variable will generally differ from that of the channel holding time. We assume that the velocity of a mobile is a random variable but remains constant during the mobile travel in a cell. The speed in a cell is assumed to be uniformly distributed on the interval $[0, V_{\max}]$. Specifically, the probability density functions (pdf) of V and T_M are

$$f_{T_M}(t)=\begin{cases}\mu_M e^{-\mu_M t}, & \text{for } t\geq 0\\ 0, & \text{otherwise}\end{cases} \tag{6}$$

$$f_V(v)=\begin{cases}\dfrac{1}{V_{\max}}, & \text{for } 0\leq v\leq V_{\max}\\ 0, & \text{otherwise}\end{cases}. \tag{7}$$

When a mobile crosses a cell boundary, the model assumes that vehicular speed and direction change. The direction of travel is also assumed to be uniformly distributed and independent of speed.

We define the random variable T_n as the time (from the onset of a call) for which a mobile resides in the cell to which the call is originated. The time for which a mobile resides in the cell in which the call is handed off is denoted T_h. In the Appendix we develop a mathematical model and expressions for the pdfs $f_{T_n}(t)$ and $f_{T_h}(t)$.

When a call is originated in a cell and gets a channel, the call holds the channel until the call is completed in the cell or the mobile moves out of the cell. Therefore, the channel holding time T_{Hn} is either the unencumbered message duration T_M or the time T_n for which the mobile resides in the cell, whichever is less. For a call which has been handed off successfully, the channel is held until the call is completed in the cell or the mobile again moves out of the cell before call completion. Because of the memoryless property of the exponential distributions, the remaining message duration of a call after handoff has the same distribution as the unencumbered message duration. In this case the channel holding time T_{Hh} is either the remaining message duration T_M or mobile residing time T_h in the cell; whichever is less. The random variables

T_{Hn} and T_{Hh} are therefore given by

$$T_{Hn} = \min\ (T_M,\ T_n)$$
$$T_{Hh} = \min\ (T_M,\ T_h). \tag{8}$$

The cumulative distribution functions (cdf) of T_{Hn} and T_{Hh} can be expressed as

$$F_{T_{Hn}}(t) = F_{T_M}(t) + F_{T_n}(t)(1 - F_{T_M}(t))$$
$$F_{T_{Hh}}(t) = F_{T_M}(t) + F_{T_h}(t)(1 - F_{T_M}(t)). \tag{9}$$

The distribution of channel holding time can be written as

$$\begin{aligned} F_{T_H}(t) &= \frac{\Lambda_{Rc}}{\Lambda_{Rc} + \Lambda_{Rhc}} F_{T_{Hn}}(t) + \frac{\Lambda_{Rhc}}{\Lambda_{Rc} + \Lambda_{Rhc}} F_{T_{Hh}}(t) \\ &= \frac{1}{1+\gamma_c} F_{T_{Hn}}(t) + \frac{\gamma_c}{1+\gamma_c} F_{T_{Hh}}(t) \\ &= F_{T_M}(t) + \frac{1}{1+\gamma_c} (1 - F_{T_M}(t))(F_{T_n}(t) + \gamma_c F_{T_h}(t)). \end{aligned} \tag{10}$$

From (6),

$$F_{T_H}(t) = \begin{cases} 1 - e^{-\mu_M t} + \dfrac{e^{-\mu_M t}}{1+\gamma_c} (F_{T_n}(t) + \gamma_c F_{T_h}(t)), & \text{for } t \geq 0 \\ 0, & \text{elsewhere.} \end{cases} \tag{11}$$

The complementary distribution function (or survivor function) $F^C_{T_H}(t)$ is

$$F^C_{T_H}(t) = 1 - F_{T_H}(t) = \begin{cases} e^{-\mu_M t} - \dfrac{e^{-\mu_M t}}{1+\gamma_c} (F_{T_n}(t) + \gamma_c F_{T_h}(t)), & \text{for } t \geq 0 \\ 1, & \text{elsewhere.} \end{cases} \tag{12}$$

The probability density function (pfd) of T_H is found by differentiating (11). Thus

$$f_{T_H}(t) = \mu_M e^{-\mu_M t} + \frac{e^{-\mu_M t}}{1+\gamma_c} [f_{T_n}(t) + \gamma_c f_{T_h}(t)] - \frac{\mu_M e^{-\mu_M t}}{1+\gamma_c} [F_{T_n}(t) + \gamma_c F_{T_h}(t)]. \tag{13}$$

For the following analysis the distribution of T_H will be approximated to a negative exponential distribution with mean $\bar{T}_H (\triangleq 1/\mu_H)$ to calculate values of various system characteristics. We will choose one function from the family of negative exponential distribution function which fits best to the distribution of T_H by comparing survivor function $F^C_{T_H}(t)$ and $\exp(-\mu_H t)$. Because a negative exponential distribution function is represented by its mean value, we choose $\bar{T}_H (\triangleq 1/\mu_H)$ which satisfies the following condition:

$$\int_0^\infty (F^C_{T_H}(t) - e^{-\mu_H t})\ dt = 0. \tag{14}$$

To prove the fairness, the "goodness of fit" for this approximation is measured by

$$G = \frac{\int_0^\infty |F^C_{T_H}(t) - e^{-\mu_H t}|\ dt}{2 \int_0^\infty F^C_{T_H}(t)\ dt} \tag{15}$$

where G indicates the normalized difference between two functions and is on the interval [(0, 1)]. A value of $G = 0$ specifies an exact fit and value of $G = 1$ indicates no correlation.

III. Probabilities and Performance Criteria

To clarify subsequent discussion, it is convenient to explain at this point the meaning of certain probabilities which arise in the development and calculation of appropriate system performance characteristics. For analytical tractability, we develop our initial model considering only the availability of radio links between one mobile party and the nearest base station. Blocking that is internal to the land system connecting base stations is ignored for the present. For mobile-to-mobile calls, blocking and forced terminations are considered only for links from one of the mobiles to the base station. Similar simplifying assumptions have appeared in the technical literature even for *simulation* studies of mobile communication systems [4]. Some aspects of the more general case will be discussed subsequently.

The probability that a new call does not enter service because of unavailability of channels is called the blocking probability P_B. A call which is not blocked, of course, enters service, but its ultimate fate has two possible outcomes. One is that the call is completed satisfactorily (when the message exchange is ended and the channel is no longer needed). The other is that the call is forced to terminate prematurely because the mobile experiences an unsuccessful handoff attempt prior to completion. We denote the probability that a call is ultimately forced into termination (though not blocked) by P_F. This represents the *average fraction of new calls which are not blocked* but which are *eventually uncompleted.*

To calculate P_F, *it is convenient to define another probability* P_{fh}. This denotes the probability that a given handoff attempt fails. It represents the *average fraction of handoff attempts* which are *unsuccessful.*

Not all calls which are initially assigned to a channel will require handoff. We characterize the handoff demand using two probabilities P_N and P_H which can be related to other system parameters.

The probability P_N that a new call which is not blocked will require at least one handoff before completion because of the mobile crossing the cell boundary is

$$P_N = \Pr\{T_M > T_n\} = \int_0^\infty [1 - F_{T_M}(t)] f_{T_n}(t)\, dt$$
$$= \int_0^\infty e^{-\mu_M t} f_{T_n}(t)\, dt. \tag{16}$$

The probability P_H that a call which has already been handed off successfully will require another handoff before completion is

$$P_H = \Pr\{T_M > T_h\} = \int_0^\infty [1 - F_{T_M}(t)] f_{T_h}(t)\, dt$$
$$= \int_0^\infty e^{-\mu_M t} f_{T_h}(t)\, dt. \tag{17}$$

Let us define the integer random variable K as the number of times that a nonblocked call is successfully handed off during its lifetime. Since the whole service area is much larger than the cell size, the event that a mobile moves out of the mobile service area during the call is very rare. A nonblocked call will have no successful handoffs if it is completed in the cell in which it was first originated or if it is forced to terminate on the first handoff attempt. It will have exactly k successful handoffs if all of the following events occur: 1) it is not completed in the cell in which it was first originated; 2) it succeeds in the first handoff attempt; 3) it requires and succeeds in $k - 1$ additional handoffs; 4) it is either completed before needing the next handoff or it is not completed but fails on the $(k + 1)$st handoff attempt. The probability function for K is therefore given by

$$\Pr\{K=0\} = (1-P_N) + P_N P_{fh}$$
$$\Pr\{K=k\} = P_N(1-P_{fh})(1-P_H+P_H P_{fh}) \cdot \{P_H(1-P_{fh})\}^{k-1}, \qquad k = 1, 2, \cdots. \tag{18}$$

From this, the mean value of K is found to be

$$\bar{K} = \sum_{k=0}^{\infty} k \Pr\{K=k\} = \frac{P_N(1-P_{fh})}{1-P_H(1-P_{fh})}. \tag{19}$$

If the entire service area has M cells, the total average new call attempt rate which is not blocked is $M\Lambda_{Rc}$, and the total average handoff call attempt rate is $\bar{K}M\Lambda_{Rc}$. Assuming that these traffic components are equally distributed among all cells, we find

$$\gamma_c = \frac{\bar{K}M\Lambda_{Rc}}{M\Lambda_{Rc}} \equiv \bar{K}. \tag{20}$$

To proceed further, it is convenient at this point to specify in greater detail the mathematical analysis required to determine P_B (the fraction of new calls blocked) and P_{fh} (the fraction of handoff attempts that fail). These quantities depend on the scheme used to manage handoffs.

IV. Channel Assignment Schemes

When no priority is given to handoff call attempts over new call attempts, no difference exists between these call attempts; the probabilities of blocking and handoff attempt failure are the same. However, the occurrence of a call being forced to terminate is considerably less desirable from the user's viewpoint than is the occurrence of blocking. The probability of forced termination can be decreased by giving priority (for channels) to handoff attempts (over new call attempts). In this section, two priority schemes are described, and the expressions for P_B and P_{fh} are derived. A subset of the channels allocated to a cell is to be exclusively used for handoff calls in both priority schemes. In the first priority scheme, a handoff call is terminated if no channel is immediately available in the target cell. In the second priority scheme, the handoff call attempt is held in queue until either a channel becomes available for it, or the received signal power level becomes lower than the receiver threshold level.

A. Priority Scheme I

Priority is given to handoff attempts by assigning C_h channels exclusively for handoff calls among the C channels in a cell. The remaining $C - C_h$ channels are shared by both new calls and handoff calls. A new call is blocked if the number of available channels in the cell is less than or equal to C_h when the call is originated. A handoff attempt is unsuccessful if no channel is available in the target cell. We assume that both new and handoff call attempts are generated according to a Poisson point process with mean rates per cell of Λ_R and Λ_{Rh}, respectively. As discussed previously, the channel holding time T_H in a cell is approximated to be exponentially distributed with mean $\bar{T}_H (\triangleq 1/\mu_H)$. We define the state E_j of a cell such that a total of j calls is in progress for the base station of that cell. Let P_j represent the steady-state probability that the base station is in state E_j; the probabilities can be determined in the usual way for birth-death processes [12]. The pertinent state-transition diagram is shown in Fig. 1. From Fig. 1, the "rate up = rate down" state equations are

$$P_j = \begin{cases} \dfrac{\Lambda_R + \Lambda_{Rh}}{\mu_H} P_{j-1}, & \text{for } j = 1, 2, \cdots, C - C_h \\[2ex] \dfrac{\Lambda_{Rh}}{\mu_H} P_{j-1}, & \text{for } j = C - C_h + 1, \cdots, C. \end{cases} \tag{21}$$

Using (21) recursively, along with the normalization condition

$$\sum_{j=0}^{\infty} P_j = 1,$$

the probability distribution $\{P_j\}$ is easily found as follows:

$$P_0 = \left[\sum_{k=0}^{C-C_h} \frac{(\Lambda_R + \Lambda_{Rh})^k}{k!\,\mu_H^k} + \sum_{k=C-C_h+1}^{C} \frac{(\Lambda_R + \Lambda_{Rh})^{C-C_h} \Lambda_{Rh}^{k-(C-C_h)}}{k!\,\mu_H^k} \right]^{-1} \tag{22}$$

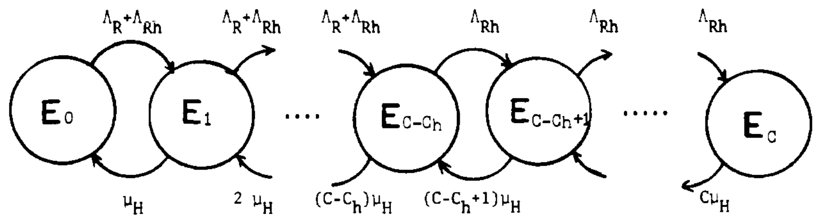

Fig. 1. State-transition diagram for Priority Scheme I.

$$P_j = \begin{cases} \dfrac{(\Lambda_R+\Lambda_{Rh})^j}{j!\mu_H^j} P_0, & \text{for } j=1, 2, 3, \cdots, C-C_h \\[2ex] \dfrac{(\Lambda_R+\Lambda_{Rh})^{C-C_h}\Lambda_{Rh}^{j-(C-C_h)}}{j!\mu_H^j} P_0, & \\[2ex] & \text{for } j=C-C_h+1, \cdots, C \end{cases} \tag{23}$$

The probability of blocking for a new call is the sum of the probabilities that the state number of the base station is larger than or equal to $C - C_h$. Hence

$$P_B = \sum_{j=C-C_h}^{C} P_j. \tag{24}$$

The probability of handoff attempt failure P_{fh} is the probability that the state number of the base station is equal to C. Thus

$$P_{fh} = P_c. \tag{25}$$

B. Priority Scheme II

In Priority Scheme II, we assume that the same channel-sharing method as that of Priority Scheme I is used, except that queueing of *handoff* attempts is allowed if necessary. No queueing of *new* call attempts takes place. To analyze this scheme, it is necessary to consider the handoff procedure in more detail. When a mobile moves away from the base station, the received power generally decreases. When the received power gets lower than a handoff threshold level, the handoff procedure is initiated. The handoff area has been defined as the area in which the average received power level of a mobile receiver from the base station is between the handoff threshold level and the receiver threshold level [13]. If the handoff attempt finds all channels in the target cell occupied, we consider that it can be queued. If any channel is released while the mobile is in the handoff area, the next queued handoff attempt is accomplished successfully. If the received power level from the source cell's base station falls below the receiver threshold level prior to the mobile being assigned a channel in the target cell, the call is forced into termination. When a channel is released in the cell, it is assigned to the next handoff call attempt waiting in the queue (if any). If more than one handoff call attempt is in the queue, the first-come-first-served queueing discipline is used. We assume that the queue size at the base station is unlimited. Fig. 2 shows a schematic representation of the flow of call attempts through a base station.

The time for which a mobile is in the handoff area depends on system parameters such as the speed and direction of mobile travel and the cell size. We define this as the *dwell time* of a mobile in the handoff area and denote it by the random variable T_Q. For simplicity of analysis, we assume that this dwell time is exponentially distributed with mean $\bar{T}_Q(\triangleq 1/\mu_Q)$.

Let us define the state of a base station E_j such that the sum of the number of channels being used in the cell and the number of handoff call attempts in the queue is j. For those states whose state number j is less than or equal to C, the state transition relation is the same as for scheme I.

We define the random variable X as the elapsed time from the instant a handoff attempt joins the queue (i.e., the mobile enters the handoff area toward a target cell in which all channels are occupied) to the first instant that a channel is released in the fully occupied target cell. For state numbers less than C, X is equal to zero. Succinctly, X is the minimum remaining holding time of those calls in progress in the fully occupied target cell. When a handoff attempt joins the queue for a given target cell, other handoff attempts may already be in queue (each of which is associated with a particular mobile). When any of these first joined the queue, the *time* that it could remain on queue *without succeeding* is denoted by T_Q (according to our previous definition). We define the random variable T_i, to be the *remaining* dwell time for that attempt which is in the ith queue position when another handoff attempt joins the queue. Under the memoryless assumptions here, the distributions of all T_i and T_Q are identical. Let $N(t)$ be the state number of the system at time t. From the description of this scheme and the properties of the exponential distribution it follows that

$$\begin{aligned} \Pr\ &\{N(t+h)=C+k-1 \mid N(t)=C+k\} \\ &= \Pr\ \{X \le h \text{ or } T_1 \le h \text{ or } \cdots T_k \le h\} \\ &= 1-\Pr\ \{X>h \text{ and } T_1>h \text{ and } \cdots T_k>h\} \\ &= 1-\Pr\ \{X>h\} P_r\{T_1>h\} \cdots P_r\{T_k>h\} \\ &= 1-\exp\ [-(C\mu_H+k\mu_Q)h] \end{aligned} \tag{26}$$

since the random variables X, T_1, T_2, $\cdots$, T_k are independent. From (26) we see that it follows the birth-and-death process and resulting state transition diagram is as shown in Fig. 3.

In the usual way for birth–death processes, the probability

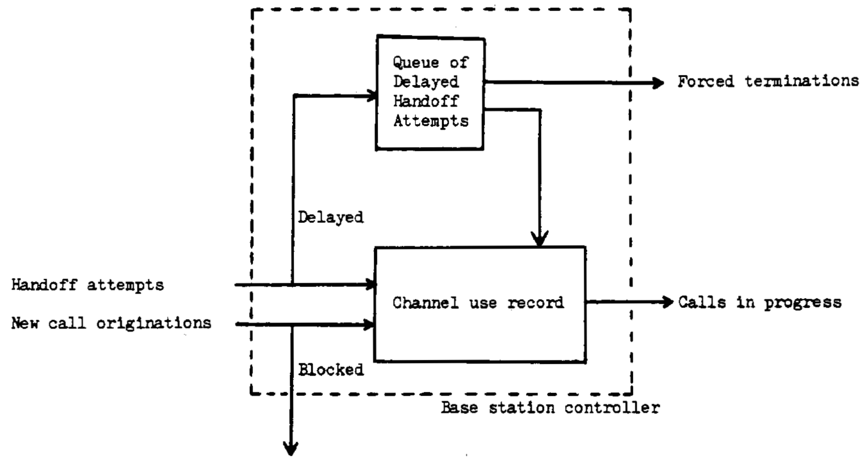

Fig. 2. Call flow diagram for Priority Scheme II.

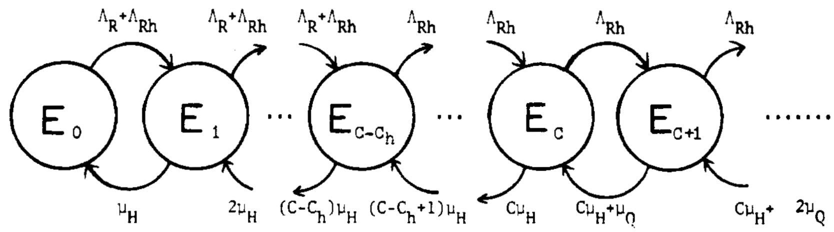

Fig. 3. State-transition diagram for Priority Scheme II.

distribution $\{P_j\}$ is easily found to be

$$P_0=\left[\sum_{k=0}^{C-C_h}\frac{(\Lambda_R+\Lambda_{Rh})^k}{k!\mu_H^k}+\sum_{k=C-C_h+1}^{C}\frac{(\Lambda_R+\Lambda_{Rh})^{C-C_h}\Lambda_{Rh}^{k-(C-C_h)}}{k!\mu_H^k}+\sum_{k=C+1}^{\infty}\frac{(\Lambda_R+\Lambda_{Rh})^{C-C_h}\Lambda_{Rh}^{k-(C-C_h)}}{C!\mu_H^C\prod_{i=1}^{k-C}(C\mu_H+i\mu_Q)}\right]^{-1} \tag{27}$$

$$P_j=\begin{cases}\dfrac{(\Lambda_R+\Lambda_{Rh})^j}{j!\mu_H^j}P_0, & \text{for } 1\le j\le C-C_h\\[2ex] \dfrac{(\Lambda_R+\Lambda_{Rh})^{(C-C_h)}\Lambda_{Rh}^{k-(C-C_h)}}{j!\mu_H^j}P_0, & \text{for } C-C_h+1\le j\le C\\[2ex] \dfrac{(\Lambda_R+\Lambda_{Rh})^{(C-C_h)}\Lambda_{Rh}^{k-(C-C_h)}}{C!\mu_H^C\prod_{i=1}^{j-C}(C\mu_H+i\mu_Q)}P_0, & \text{for } j\ge C+1\,.\end{cases} \tag{28}$$

The probability of blocking P_B is the sum of the probabilities that the state number of the base station is larger than or equal to $C - C_h$. Hence

$$P_B=\sum_{j=C-C_h}^{\infty}P_j. \tag{29}$$

A given handoff attempt which joins the queue will be successful if both of the following events occur before the mobile moves out of the handoff area: 1) all of the attempts which joined the queue earlier than the given attempt have been disposed; 2) a channel becomes available when the given attempt is at the first position in the queue.

Thus the probability of a handoff attempt failure can be calculated as the average fraction of handoff attempts whose mobiles leave the handoff area prior to their coming into the first queue position and getting a channel. Noting that arrivals which find k attempts in queue enter position $k + 1$, this can be concisely stated mathematically as

$$P_{fh}=\sum_{k=0}^{\infty}P_{C+k}\ \Pr\{\text{attempt fails given it enters the queue in position } k+1\} \tag{30}$$

or

$$P_{fh}\triangleq\sum_{k=0}^{\infty}P_{C+k}P_{fh|k} \tag{31}$$

in which $P_{fh|k}$ in (31) is defined as the rightmost term in (30). Since handoff success for those attempts which enter the queue in position $k + 1$ requires coming to the head of the queue and getting a channel, we have, under the memoryless conditions assumed in this development,

$$(1-P_{fh|k})=\left[\prod_{i=1}^{k} P(i|i+1)\right] \cdot \Pr\ \{\text{get channel in first position}\} \quad (32)$$

in which $P(i|i+1)$ represents the probability that an attempt in position $i + 1$ moves to position i before its mobile leaves the handoff area.

An attempt in position $i + 1$ will either be cleared from the system or will advance in queue to the next (lower) position. It will advance if the remaining dwell time of its mobile exceeds either 1) at least one of the remaining dwell times T_j, $j = 1, 2, \cdots, i$, for any attempt ahead of it in the queue, or 2) the minimum remaining holding time X of those calls in progress in the target cell. Thus

$$1-P(i|i+1)=\Pr\ \{T_{i+1}\le X,\ T_{i+1}\le T_j,\ j=1, 2, \cdots, i\}, \qquad i=1, 2, \cdots. \quad (33)$$

Since the mobiles move independently of each other and of the channel holding times, the joint probability in (33) can be expressed as a product. Then because of the memoryless property, we find

$$1-P(i|i+1)=\int_0^{\infty} e^{-C\mu_H\tau}\mu_Q e^{-\mu_Q\tau}\,d\tau \cdot \left[\int_0^{\infty} e^{-\mu_Q\tau}\mu_Q e^{-\mu_Q\tau}\,d\tau\right]^i$$

$$=\left(\frac{\mu_Q}{C\mu_H+\mu_Q}\right)\left(\frac{1}{2}\right)^i, \quad \text{for } i=1, 2, \cdots. \quad (34)$$

The handoff attempt at the head of the queue will get a channel (succeed) if its remaining dwell time T_1 exceeds X. Thus

$$\Pr\ \{\text{get channel in first position}\}=\Pr\ \{T_1>X\}$$

$$=\int_0^{\infty} e^{-C\mu_H\tau}\mu_Q e^{-\mu_Q\tau}\,d\tau=\frac{\mu_Q}{C\mu_H+\mu_Q}. \quad (35)$$

The sequence of (27), (28) and (30)–(35) defines, for computational purpose, all quantities needed to calculate p_{fh} for Priority Scheme II.

Equations (1), (2), (5), (12)–(14), (16), (17), (19), (20), (22)–(25), (27)–(35) form a set of simultaneous nonlinear equations which can be solved for system variables when parameters are given. For example, given R, $\bar{T}_M$, $V_{\max}$, C, C_h, Λ_a, the quantities P_B, P_{fh}, P_N, P_H, T_n, T_h, γ_c, μ_H, Λ_R, Λ_{Rh} can be considered unknowns. Beginning with an initial guess for the unknowns, the equations were solved numerically using the method of successive substitution.

C. Probabilities of Forced Termination, Blocking, and Noncompleted Calls

Various performance characteristics can then be readily calculated for each priority scheme. Of particular interest are the fraction of new call attempts which are blocked, completed, and forced into termination due to unsuccessful handoff. If the cell radius is large (compared to speed × mean holding time), the chance for a mobile to cross a cell boundary during a call duration is small. In this case the probability of blocking P_B is the major indication of system traffic performance. When the cell radius is small, however, a higher probability exists that a mobile crosses a cell boundary during the call duration. Also, the mean channel holding time of a call in a cell is smaller. Under these circumstances, nonblocked calls on the average experience more handoffs. As a result a greater chance of forced termination exists due to unsuccessful handoff in a call's lifetime. In this case the probability P_F of forced termination, and the probability P_{fh} that a handoff attempt fails are also important performance measures.

From the user's point of view the probability P_F that a call which is not blocked is *eventually* forced into termination can be more significant than P_{fh}. A call which is not blocked will be eventually forced into termination if it succeeds in each of the first $(l - 1)$ handoff attempts which it requires but fails on the lth. Therefore,

$$P_F=\sum_{l=1}^{\infty} P_{fh}[P_N(1-P_{fh})^{l-1}P_H^{l-1}]=\frac{P_{fh}P_N}{1-P_H(1-P_{fh})} \quad (36)$$

where P_N and P_H are the probabilities of handoff demand of new and handoff calls, as defined previously.

The *fraction* of a *new call attempt* P_{nc} which will *not* be *completed* because of either blocking or unsuccessful handoff is also used as a major parameter for system performance. This probability P_{nc} can be expressed as

$$P_{nc}=P_B+P_F(1-P_B)=P_B+\frac{P_{fh}P_N(1-P_B)}{1-P_H(1-P_{fh})} \quad (37)$$

where the first and second terms represent the effects of blocking and handoff attempt failure, respectively. In (37) we can guess roughly that when cell size is large, probabilities of cell crossing P_N and P_H will be small and the second term of (37) (i.e., effect of cell crossing) will be much smaller than the first term (i.e., effect of blocking). However, when the cell size is decreased, P_N and P_H will increase, and the relative magnitude of the second term will increase. The noncompleted call probability P_{nc} can be considered as a unified measure of both blocking and forced termination effects.

Another measure of system performance which may be interesting is the weighted sum of P_B and P_F

$$\text{CF}=(1-\alpha)P_B+\alpha P_F \quad (38)$$

where α is in the interval [(0, 1)] and indicates the relative importance of the blocking and forced termination effects. For some application P_F may be more important than P_B from the user's point of view, and relative cost α can be assigned using the system designer's judgment.

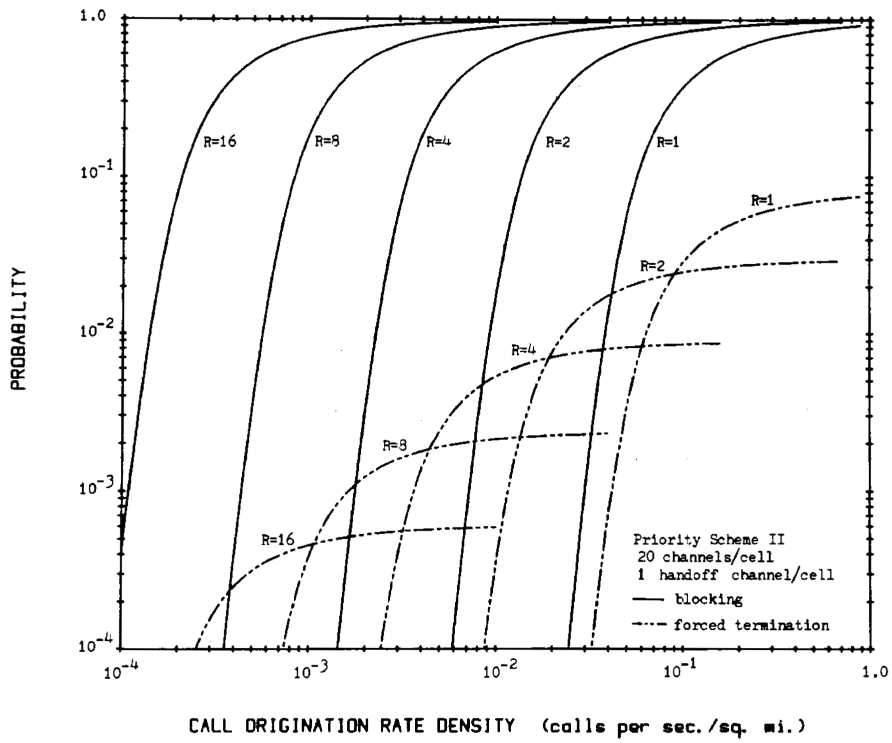

Fig. 4. Blocking and forced termination probabilities (Priority Scheme II, 20 channels/cell, 1 handoff channel/cell).

V. Performance Characteristics

Numerical results were obtained for all schemes discussed here. Generally, it was found that for given system parameters, Priority Scheme II allowed significantly smaller forced termination probabilities for given blocking probability. Most of the figures presented here therefore describe scheme II. The average of the unencumbered message duration $\bar{T}_M = 120$ s and the maximum speed of a mobile $V_{\max} = 60$ mi/h are used for the calculations.

The effect of cell radius on P_B and P_F can be seen in Fig. 4 which shows these as functions of (new) call origination rate per unit area Λ_a. A total of 20 channels per cell ($C = 20$) with one handoff channel per cell ($C_h = 1$) was assumed. Priority Scheme II was used for this figure, and mean dwell time for handoff attempt $\bar{T}_Q$ was assumed to be $\bar{T}_H/10$. It was found that P_F is much smaller than P_B and that the difference between them decreases as cell size decreases. As expected for larger R the effect of handoff attempts and forced terminations on system performance is smaller.

Fig. 5 shows P_B and P_F as functions of Λ_a for different values of C_h with cell radius $R = 2$ mi and $C = 20$ channels. The effects of priority given to handoff calls over new calls are shown. When more priority is given to handoff calls by increasing C_h, P_F decreases by orders of magnitude with only small to moderate increase in P_B. This exchange is important because (as was mentioned previously) forced terminations are usually considered much less desirable than blocked calls.

Fig. 6 shows the cost function CF as function of C_h for various values of weighting factor α for the system with cell radius $R = 2$ mi, $C = 20$ channels/cell, and $\Lambda_a = 0.01$ calls/mi^2. A greater number of handoff channels C_h is required to minimize the cost function CF when α is large, i.e., P_F has more weight than P_B. For most of the range of α, the required numbers of C_h, which minimize the cost function CF, are small because P_B has the major effect on CF.

The changes of P_B and P_F as functions of C_h for various values of C are shown in Fig. 7 where Λ_a is chosen as the value such that P_B is equal to 0.01 with $C_h = 0$ for the cases of C. It can be seen that if the total number of channels per cell C is increased, the loss (increase) in the blocking probability P_B is less as the number of handoff channels is increased; but the same order of magnitude reduction in forced termination probability P_F is attained.

Blocking and forced termination probabilities for the two priority schemes are shown in Fig. 8 as functions of call origination rate density Λ_a. The forced termination probability P_F is smaller for scheme II than for scheme I, but almost no difference exists in blocking probability P_B. We get this superiority of Priority Scheme II by queueing the delayed handoff attempts for the dwell time of the mobile in the handoff area.

The noncompleted call probability P_{nc} is shown as a function of call arrival rate density for various values of R in Fig. 9. For a system with fixed cell radius R, the noncompleted call probability increases rapidly with increasing new call origination rate density.

For various Λ_a, Fig. 10 shows the effect (on P_{nc}) of priority given to handoff calls by increasing C_h. The noncompleted call probability P_{nc} increases with increasing C_h. This shows that, with cell radius $R = 2.0$ mi, the major reason for the noncompletion of a call is blocking rather than forced termination.

Fig. 11 shows the required cell size as functions of Λ_a such that $P_{nc} = 0.02$ for various values of C. When the D/R ratio

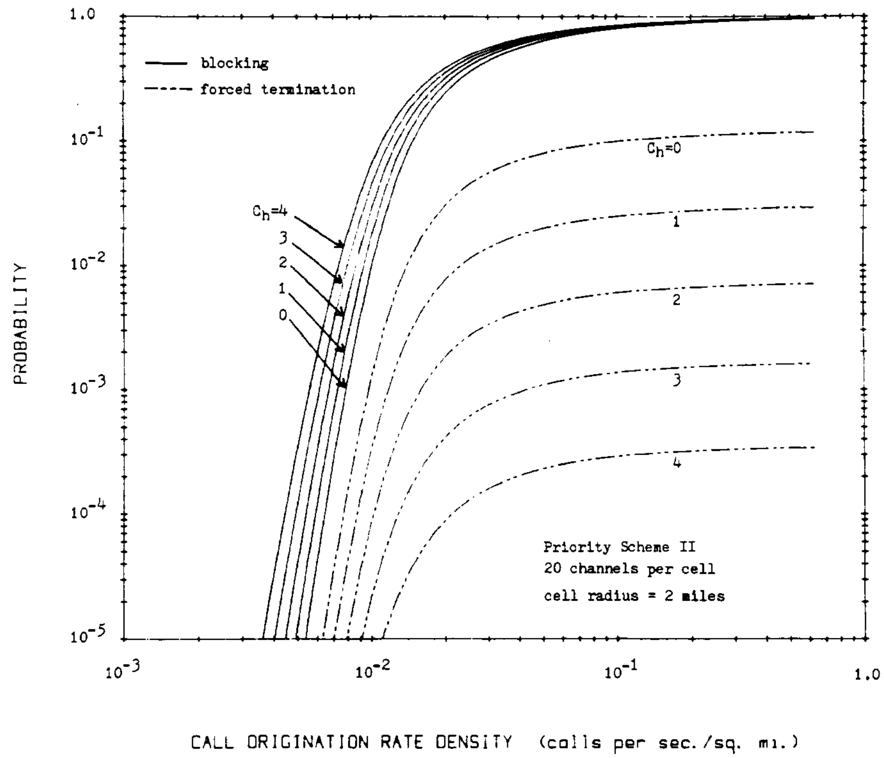

Fig. 5. Blocking and forced termination probabilities for systems with different priority (Priority Scheme II, 20 channels/cell, cell radius = 2 mi).

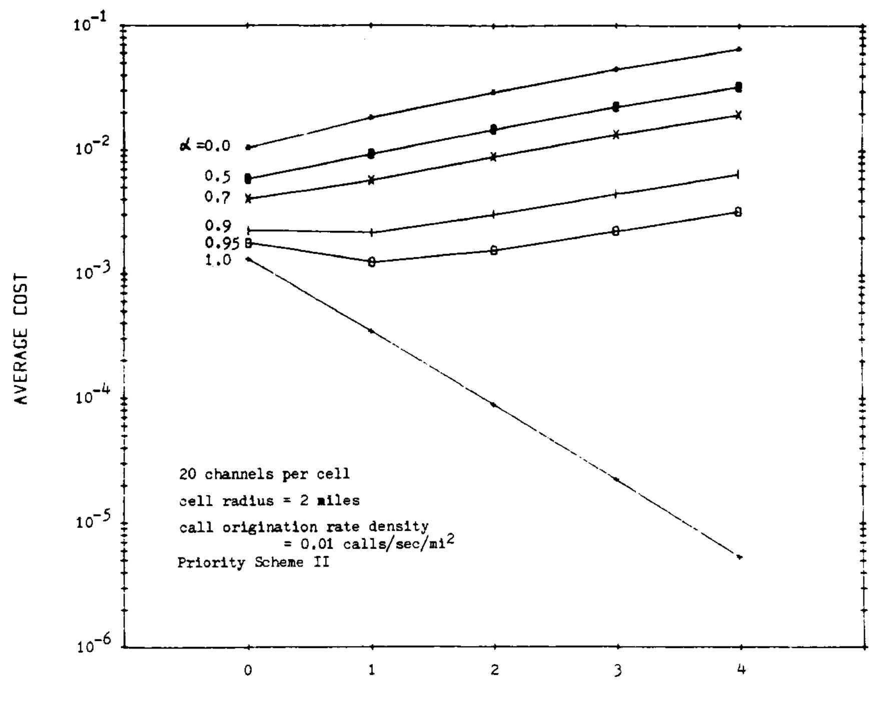

Fig. 6. Average cost of noncompletion (Priority Scheme II, 20 channels/cell, cell radius = 2 mi, call origination rate density = 0.01 (calls/s)/mi^2).

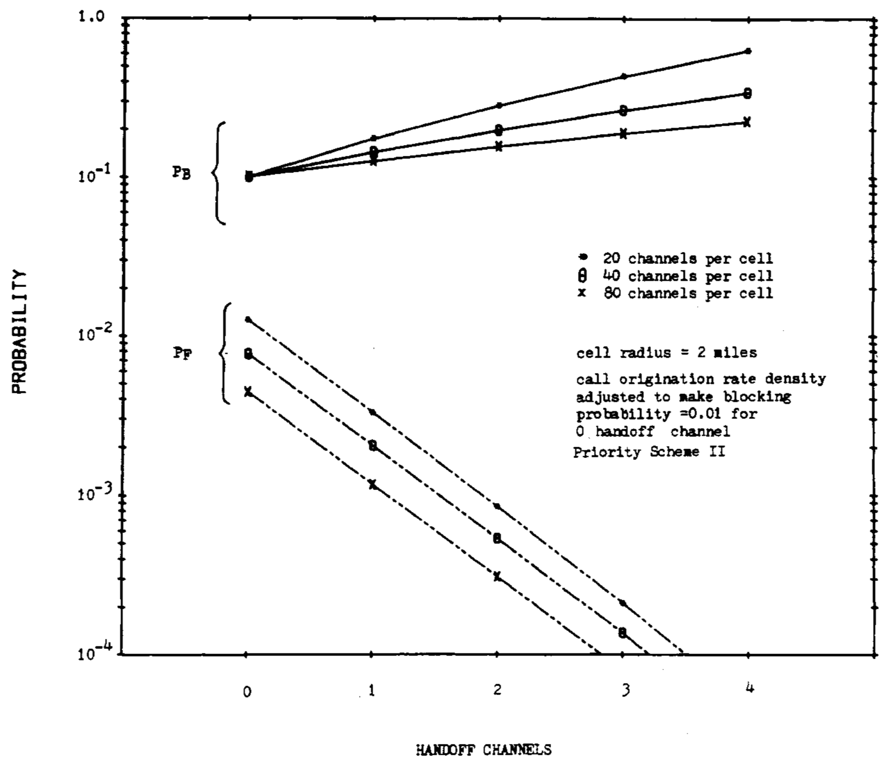

Fig. 7. Blocking and forced termination probabilities (Priority Scheme II, cell radius = 2 mi, call origination rate density adjusted to make blocking probability = 0.01 for 0 handoff channel).

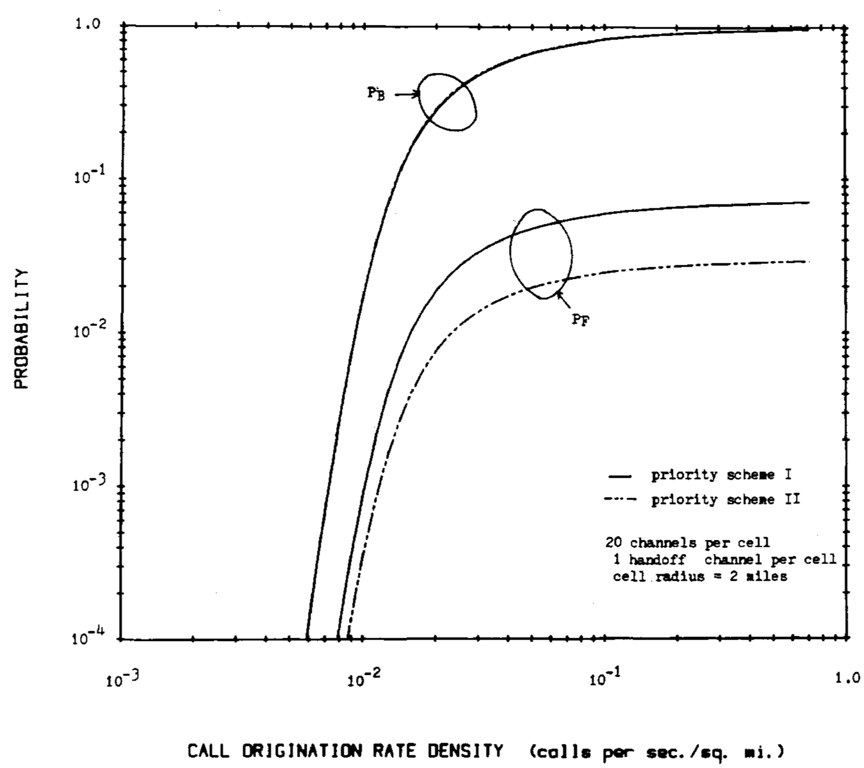

Fig. 8. Blocking and forced terminations for priority schemes (20 channels/cell, 1 handoff channel/cell, cell radius = 2 mi).

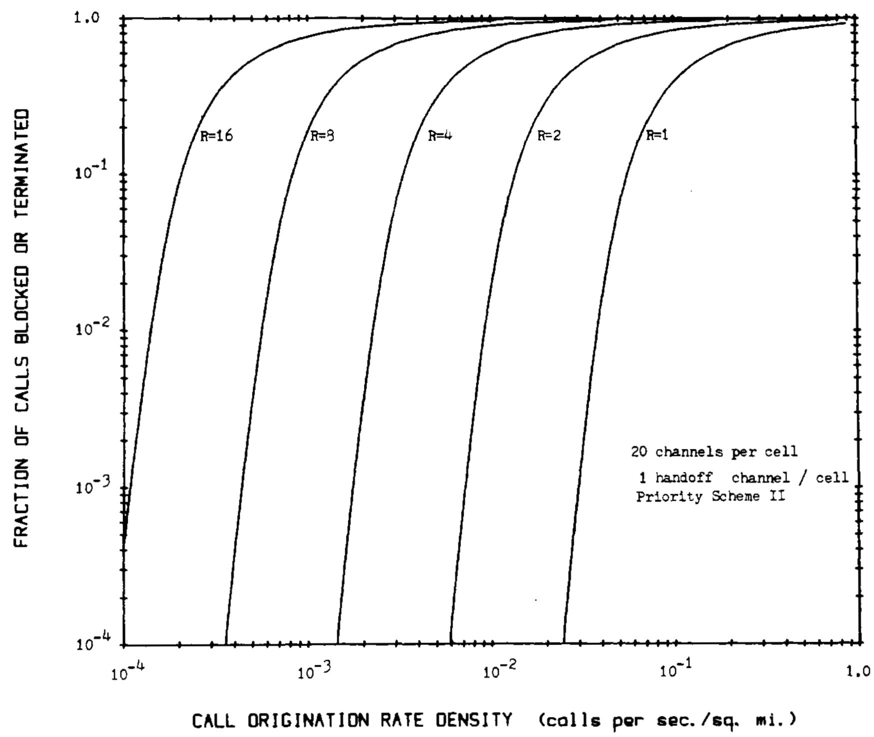

Fig. 9. Fraction of new calls uncompleted (Priority Scheme II, 20 channels/cell, 1 handoff channel/cell).

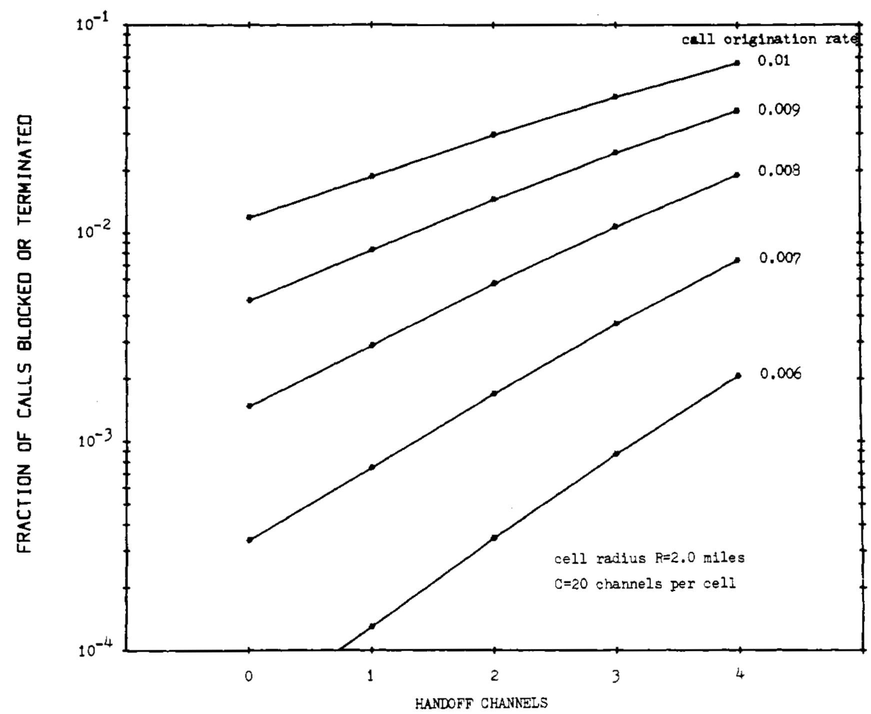

Fig. 10. Effect of reserved channels on uncompleted calls (20 channels/cell, cell radius = 2 mi).

and the number of channels per cell are determined from co-channel interference constraints, the spectrum bandwidth allocated to the system, and the modulation method; the required cell size can be determined for the required call arrival rate density Λ_a from this kind of graph.

The mean channel holding time in a cell $\bar{T}_H$ is expected to decrease with decreasing cell size. Fig. 12 shows this quantitatively. Notice that $\bar{T}_H$ becomes smaller with smaller cell size, but sensitivity to change in cell size is smaller for larger cells. As cell size increases the limiting factor is the unencumbered holding time of a call, that is, the holding time that a call would use if there were no forced termination.

Earlier in the paper we approximated the cumulative distribution function of the channel holding time in a cell (see (14)). The goodness-of-fit G of this approximation, defined as (15), is shown in Table I for various cell sizes. We see that G

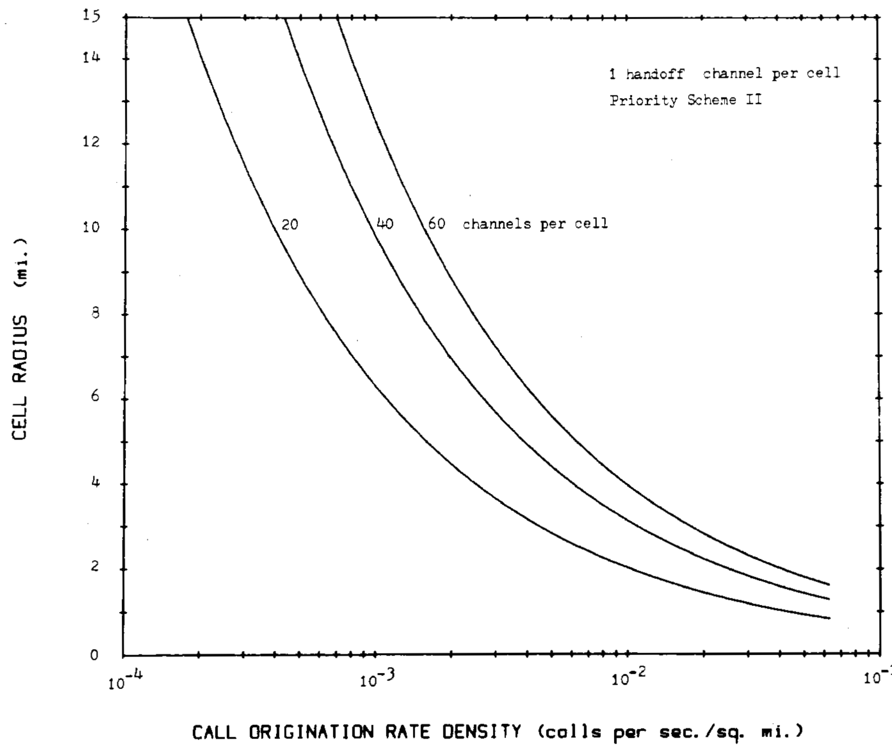

Fig. 11. Cell size for call completion probability = 0.98 (Priority Scheme II, 1 handoff channel/cell).

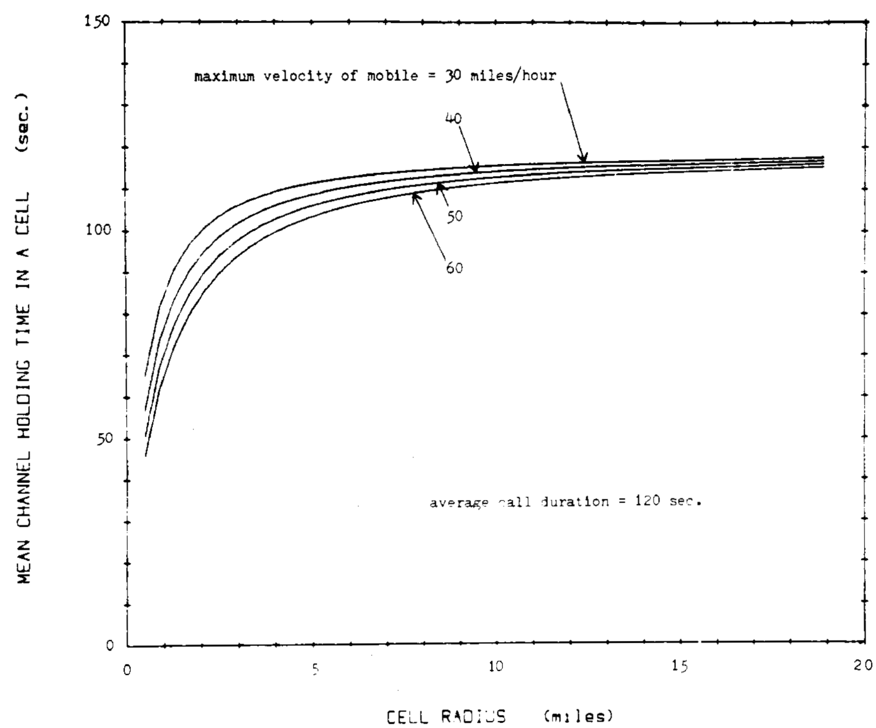

Fig. 12. Effect of cell size on channel holding time (average call duration = 120 s).

is very small for all ranges of cell radius R. This supports the use of the approximation in our calculations.

VI. Further Discussion

As mentioned earlier, we ignored blocking which is internal to the land network, and we considered only the availability of radio channels between one mobile party to a call, and the nearest base station. These assumptions are common in analyses and simulations of mobile systems [4], [11]. For mobile-to-land or land-to-mobile calls in systems whose *internal blocking* is negligible, the blocking of new calls and the failure of handoff attempts would indeed occur only at the mobile party to a call. Therefore, the performance characteristics obtained in the previous sections apply directly to those situations. However, for mobile-to-mobile calls, the call is blocked if either party to the call is blocked. The analysis is

TABLE I
GOODNESS-OF-FIT OF FUNCTION APPROXIMATION

Cell Radius, R	G
1.0	0.020220
2.0	0.000120
4.0	0.000003
6.0	0.000094
8.0	0.000121
10.0	0.000107
12.0	0.000086
14.0	0.000066
16.0	0.000053

somewhat more complicated, but some rough extensions of the foregoing results can be obtained easily.

The mobiles of both parties generally move independently of each other. For cellular systems with small cell size, the case that the mobiles of both parties to a call are in the same cell is very small. If this case is ignored, then the blocking probability P'_B of the mobile-to-mobile call is

$$P'_B = 1-(1-P_B)^2 = 2P_B - P_B^2 \tag{39}$$

where P_B is the blocking probability of one mobile party to a call.

Similarly, a nonblocked mobile-to-mobile call is forced into termination if *either* mobile party to the call fails in a handoff attempt at a cell boundary. Because of this, calls can be terminated when one of the mobile parties is *in* a cell (not at the cell boundary). Therefore, the average channel holding time in a cell $\bar{T}_H$ can be less than that obtained by our more through (but also more restrictive) analysis. However, if the handoff attempt failure probability is very small, those effects may be ignored. With this assumption, the handoff failures of both mobile parties to a call are considered independent of each other. Then the probability of forced termination P'_F of the nonblocked mobile-to-mobile calls is found as

$$P'_F = 1-(1-P_F)^2 = 2P_F - P_F^2 \tag{40}$$

where P_F is the forced termination probability of one mobile party to a call.

VII. CONCLUSION

A traffic model for mobile radio telephone systems with cellular structure, frequency reuse, and handoff has been considered. The probability of blocking P_B of new call attempts as well as the probability of forced termination P_F of nonblocked calls were plotted as functions of call origination rate density. As expected, forced termination probability P_F for smaller cell systems is more significant. We found that P_F is decreased by a significantly larger order of magnitude than the increase of P_B when more priority is given to handoff calls by increasing the number of handoff channels.

Two prioritized handoff procedures were considered. In Priority Scheme I, a number of channels is used exclusively for handoff calls while the remaining channels are used for both new calls and handoff calls. Blocked calls are cleared from the system immediately. In Priority Scheme II, handoff call attempts can be queued for the time duration in which a mobile dwells in the handoff area between cells. Channels are shared in the same way as in Priority Scheme I. It was found that P_F is lower for Scheme II while there is essentially no difference for P_B over the interesting range of parameters.

The noncompletion probability P_{nc} that a new call attempt is not completed by either blocking or forced termination was defined as one of the system performance measures. It was found that P_B is the major component of P_{nc}, even for small cell systems, with $R = 2.0$ mi. Because of this P_{nc} is increased when more priority is given to handoff calls. A weighted sum of P_B and P_F (cost function CF) was defined and used as another measure of system performance. The value of CF depends on the weighting factor α. As expected more priority is required to decrease CF when the weighting between P_F and P_B is shifted to the former. The required cell radius is shown as a function of call origination rate density for numbers of channels per cell C and values of P_{nc}. This graph is useful to determine the cell size from system parameters after the D/R ratio is chosen from co-channel constraints requirements. It is believed that the model and analysis in this paper can provide useful tools for designing and predicting the performance of cellular mobile radio telephone systems.

APPENDIX

PROBABILITY DISTRIBUTIONS OF RESIDING TIME IN A CELL

The probability distributions of the residing times T_n and T_h are to be investigated. The random variable T_n is defined as the time (duration) that a mobile resides in the cell *in which its call originated.* Also T_h is defined as the time a mobile resides in a cell *to which its call is handed off.*

In mobile radio telephone systems the boundary between cells are determined by average received signal power levels from adjacent base stations. However, the received signal power levels vary from time to time because of shadowing and fading effects, even though the transmitting signal power is constant and distance from base station is fixed. Therefore, the actual cell boundary is not critically fixed, and the handoff area may be defined between the cells in which the received signal power level is lower than handoff threshold level and higher than receiver threshold level [13]. For this reason we approximate the hexagonal cell shape as a circle to simplify analysis.

When the hexagonal cell radius is R, the cell is approximated by a circle which has same area. Then the radius R_{eq} of the approximating circle is

$$R_{eq} = \sqrt{\frac{3\sqrt{3}}{2\pi}}\, R \approx 0.91R. \tag{41}$$

The relation between R and R_{eq} is shown in Fig. 13. The base station is assumed to be at the center of a cell and is indicated by a letter B in the figure. The location of a mobile in a cell, which is indicated by a letter A in the figure, is represented by its distance r and direction ϕ from the base station as shown. To find the distributions of T_n and T_h, we assumed that the

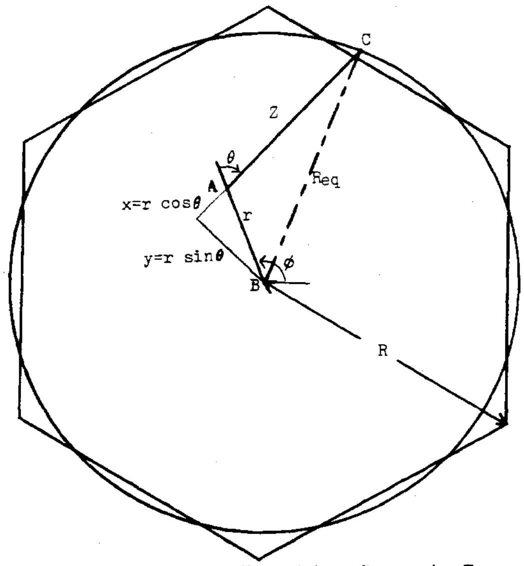

Fig. 13. Illustration of distance from point A in cell (where call is originated), to point C on cell boundary (where mobile exits from cell).

mobiles are spread evenly over the area of the cell. Then r and ϕ are random variables with pdf's

$$f_r(r) = \begin{cases} 2r/R_{eq}^2, & \text{for } 0 \le r \le R_{eq} \\ 0 & \text{elsewhere} \end{cases} \quad (42)$$

$$f_\phi(\phi) = \begin{cases} 1/2\pi, & \text{for } 0 \le \phi \le 2\pi \\ 0 & \text{elsewhere.} \end{cases} \quad (43)$$

We assume that a mobile travels in any direction with equal probability, and its direction remains constant during its travel in the cell. If we define the direction of mobile travel by the angle θ (with respect to a vector from the base station to the mobile), as shown in the figure, the distance Z from the mobile to boundary of approximating circle is

$$Z = \sqrt{R_{eq}^2 - (r \sin \theta)^2} - r \cos \theta. \quad (44)$$

Because ϕ is evenly distributed in a circle, Z is independent of ϕ and from the symmetry we can consider the random variable θ is in interval $[0, \pi]$ with pdf

$$f_\theta(\theta) = \begin{cases} 1/\pi, & \text{for } 0 \le \theta \le \pi \\ 0, & \text{elsewhere.} \end{cases} \quad (45)$$

If we define new random variables x, y as

$$x = r \cos \theta$$
$$y = r \sin \theta,$$

then

$$Z = \sqrt{R_{eq}^2 - y^2} - x$$
$$W = x.$$

Since the mobile is assumed to be equiprobably located in the approximating circle

$$f_{XY}(x, y) = \begin{cases} 2/\pi R_{eq}^2, & \text{for } -R_{eq} \le x \le R_{eq}, \\ & 0 \le x^2 + y^2 \le R_{eq}^2, \\ & 0 \le y \le R_{eq} \\ 0, & \text{elsewhere} \end{cases}$$

From (42), (44), and (45), the joint density function of Z and W can be found by standard methods

$$f_{ZW}(z, w) = \frac{|z+w|}{\sqrt{R_{eq}^2 - (z+w)^2}} f_{XY}(x, y)$$
$$= \frac{2}{\pi R_{eq}^2} \frac{|z+w|}{\sqrt{R_{eq}^2 - (z+w)^2}},$$
$$\text{for } 0 \le z \le 2R_{eq}, \; -\frac{1}{2} z \le w \le -z + R_{eq}.$$

The pdf of the distance Z is then

$$f_Z(z) = \int_{-z/2}^{R_{eq}-z} \frac{2}{\pi R_{eq}^2} \frac{(z+w)}{\sqrt{R_{eq}^2 - (z+w)^2}} \, dw, \quad \text{for } 0 \le z \le 2R_{eq}$$

$$= \begin{cases} \dfrac{2}{\pi R_{eq}^2} \sqrt{R_{eq}^2 - \left(\dfrac{z}{2}\right)^2}, \\ \qquad \text{for } 0 \le z \le 2R_{eq} \\ 0, \quad \text{elsewhere.} \end{cases} \quad (46)$$

We assume that the speed V of a mobile is constant during its travel in the cell and random variable which is uniformly distributed on the interval $[0, V_{max}]$ with pdf

$$f_V(v) = \begin{cases} 1/V_{max}, & \text{for } 0 \le v \le V_{max} \\ 0, & \text{elsewhere.} \end{cases}$$

Then the time T_n is expressed by

$$T_n = \frac{Z}{V}$$

with pdf

$$f_{T_n}(t) = \int_{-\infty}^{\infty} |w| f_Z(tw) f_V(w)\, dw$$

$$= \begin{cases} \dfrac{2}{V_{max}\pi R_{eq}^2} \displaystyle\int_0^{V_{max}} w \sqrt{R_{eq}^2 - \left(\frac{tw}{2}\right)^2}\, dw, & \text{for } 0 \le t \le \dfrac{2R_{eq}}{V_{max}} \\ \dfrac{2}{V_{max}\pi R_{eq}^2} \displaystyle\int_0^{2R_{eq}/t} w \sqrt{R_{eq}^2 - \left(\frac{tw}{2}\right)^2}\, dw, & \text{for } t \ge \dfrac{2R_{eq}}{V_{max}} \end{cases}$$

$$= \begin{cases} \dfrac{8R_{eq}}{3V_{max}\pi t^2} \left[1 - \sqrt{\left\{1 - \left(\dfrac{tV_{max}}{2R_{eq}}\right)^2\right\}^3}\right], & \text{for } 0 \le t \le \dfrac{2R_{eq}}{V_{max}} \\ \dfrac{8R_{eq}}{3V_{max}\pi t^2}, & \text{for } t \ge \dfrac{2R_{eq}}{V_{max}}. \end{cases} \tag{47}$$

The cdf of T_n is

$$F_{T_n}(t) = \int_{-\infty}^{t} f_{T_n}(x)\, dx$$

$$= \begin{cases} \dfrac{2}{\pi} \arcsin\left(\dfrac{V_{max}t}{2R_{eq}}\right) - \dfrac{4}{3\pi} \tan\left[\dfrac{1}{2} \arcsin\left(\dfrac{V_{max}t}{2R_{eq}}\right)\right] + \dfrac{1}{3\pi} \sin\left[2 \arcsin\left(\dfrac{V_{max}t}{2R_{eq}}\right)\right], & \text{for } 0 \le t \le \dfrac{2R_{eq}}{V_{max}} \\ 1 - \dfrac{8R_{eq}}{3\pi V_{max}} \dfrac{1}{t}, & \text{for } t \ge \dfrac{2R_{eq}}{V_{max}}. \end{cases} \tag{48}$$

To find the distribution of T_h, we note that when a handoff call is attempted, it is always generated at the cell boundary, which is taken as the boundary of the approximating circle. Therefore, to find T_h one must recognize that the mobile will move from one point on the boundary to another. The direction of a mobile when it crosses the boundary is indicated by the angle θ between the direction of the mobile and the direction from the mobile to the center of a cell as shown in Fig. 14. If we assume that the mobile moves with any direction with equal probability, the random variable θ has pdf as

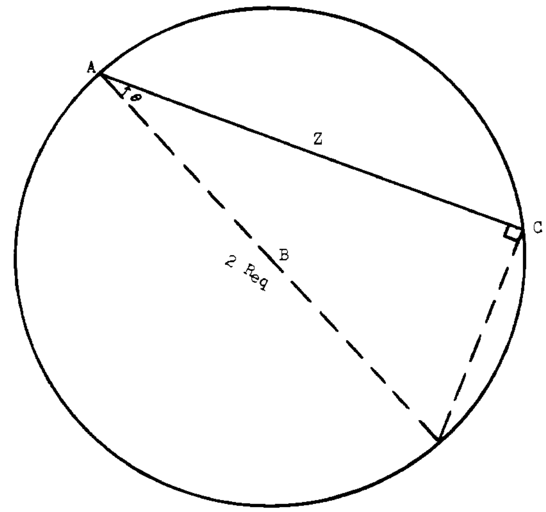

Fig. 14. Illustration of distance from point A on cell boundary (where mobile enters cell), to point C on cell boundary (where mobile exits from cell).

$$f_\theta(\theta) = \begin{cases} \dfrac{1}{\pi}, & \text{for } -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2} \\ 0, & \text{elsewhere.} \end{cases}$$

The distance Z is as shown in Fig. 14

$$Z = 2R_{eq} \cos\theta \tag{49}$$

with cdf

$$F_Z(z) = \Pr\{Z \le z\}$$

$$= \begin{cases} 0, & \text{for } z < 0 \\ 1 - \dfrac{2}{\pi} \arccos\left(\dfrac{z}{2R_{eq}}\right), & \text{for } 0 \le z \le 2R_{eq} \\ 1, & \text{for } z > 2R_{eq}. \end{cases} \tag{50}$$

The pdf of Z is

$$f_Z(z) = \frac{d}{dz} F_Z(z)$$

$$= \begin{cases} \dfrac{1}{\pi} \dfrac{1}{\sqrt{R_{eq}^2 - \left(\dfrac{z}{2}\right)^2}}, & \text{for } 0 \le z \le 2R_{eq} \\ 0, & \text{elsewhere.} \end{cases} \tag{51}$$

The time in the cell T_h is the time that a mobile travels the distance Z with speed V, then

$$T_h = \frac{Z}{V}. \tag{52}$$

With the same assumption about V, the pdf of T_h is

$$f_{T_h}(t) = \int_0^\infty |w| f_Z(tw) f_V(w)\, dw$$

$$= \begin{cases} \dfrac{1}{\pi V_{\max}} \displaystyle\int_0^{V_{\max}} \frac{w}{\sqrt{R_{eq}^2 - \left(\dfrac{tw}{2}\right)^2}}\, dw, \\ \qquad \text{for } 0 \le t \le \dfrac{2R_{eq}}{V_{\max}} \\ \dfrac{1}{\pi V_{\max}} \displaystyle\int_0^{2R_{eq}/t} \frac{w}{\sqrt{R_{eq}^2 - \left(\dfrac{tw}{2}\right)^2}}\, dw, \\ \qquad \text{for } t \ge \dfrac{2R_{eq}}{V_{\max}} \end{cases}$$

$$= \begin{cases} \dfrac{4R_{eq}}{\pi V_{\max}} \dfrac{1}{t^2} \left[1 - \sqrt{1 - \left(\dfrac{V_{\max} t}{2R_{eq}}\right)^2}\,\right], \\ \qquad \text{for } 0 \le t \le \dfrac{2R_{eq}}{V_{\max}} \\ \dfrac{4R_{eq}}{\pi V_{\max}} \dfrac{1}{t^2}, \qquad \text{for } t \ge \dfrac{2R_{eq}}{V_{\max}} \end{cases} \tag{53}$$

and the cdf of T_h is

$$F_{T_h}(t) = \int_{-\infty}^t f_{T_h}(x)\, dx$$

$$= \begin{cases} 0, & \text{for } t < 0 \\ \dfrac{2}{\pi} \arcsin\left(\dfrac{V_{\max} t}{2R_{eq}}\right) - \dfrac{2}{\pi} \tan\left[\dfrac{1}{2} \arcsin\left(\dfrac{V_{\max}}{2R_{eq}}\right)\right], \\ \qquad \text{for } 0 \le t \le \dfrac{2R_{eq}}{V_{\max}} \\ 1 - \dfrac{4R_{eq}}{\pi V_{\max}} \dfrac{1}{t}, & \text{for } t > \dfrac{2R_{eq}}{V_{\max}}. \end{cases} \tag{54}$$

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