

Betweenness

Introduction to Network Science

Carlos Castillo

Topic 18

Sources

- [Networks, Crowds, and Markets Ch 3.6B](#)
- Barabási 2016 Section 9.3.2
- P. Boldi and S. Vigna: [Axioms for Centrality](#) in Internet Mathematics 2014.
- Esposito and Pesce: [Survey of Centrality](#) 2015.
- C. Castillo: [Other centrality slides](#) 2016

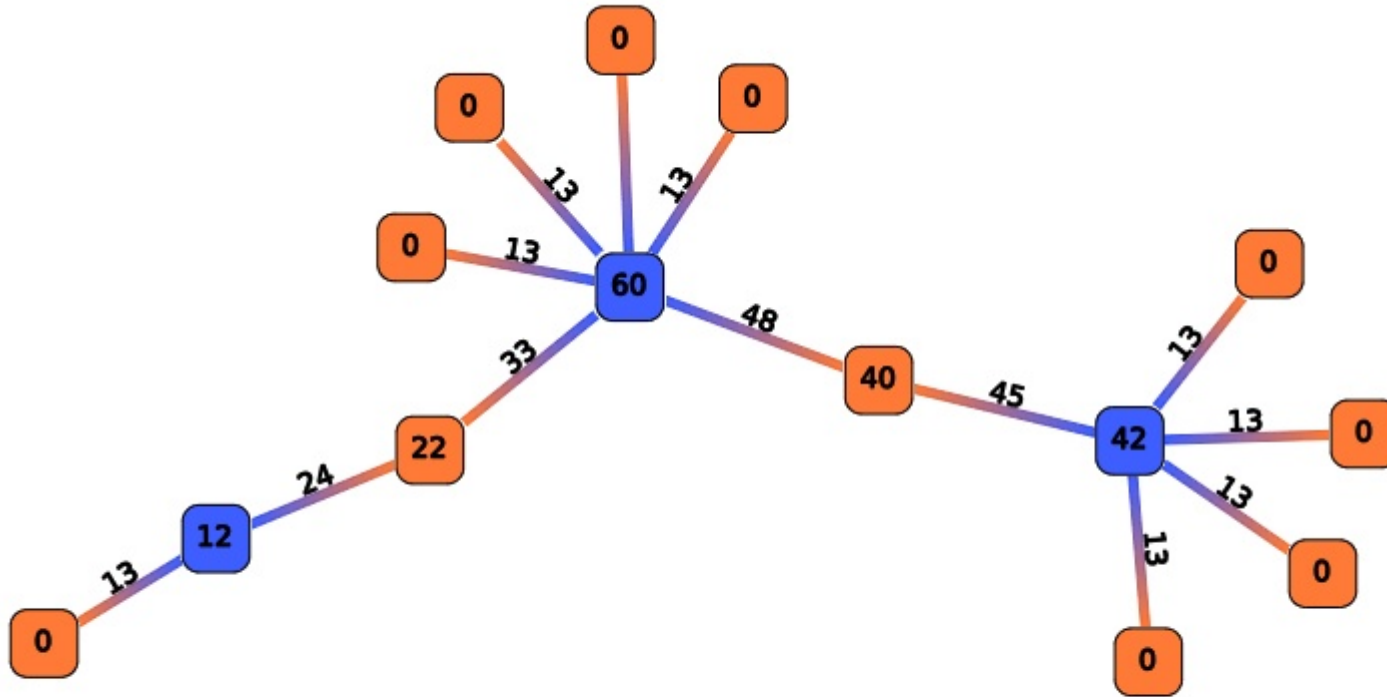
Types of centrality measure

- Spectral
 - HITS
 - PageRank
- **Non-spectral**
 - Degree
 - Closeness and harmonic closeness
 - Betweenness

Betweenness

Node and Edge Betweenness

A node/edge has high betweenness if it participates in many shortest-paths

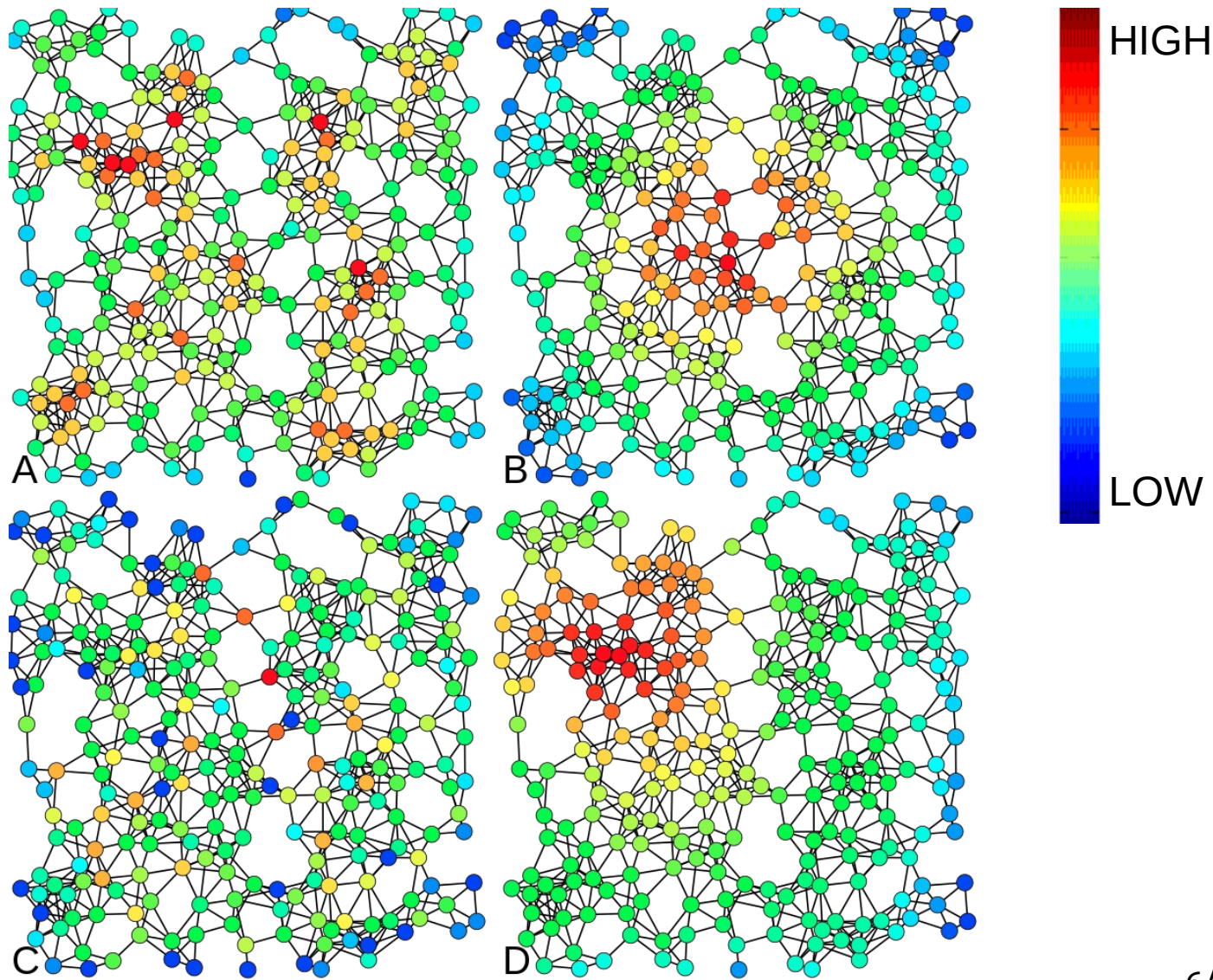


A: Degree

B: Closeness

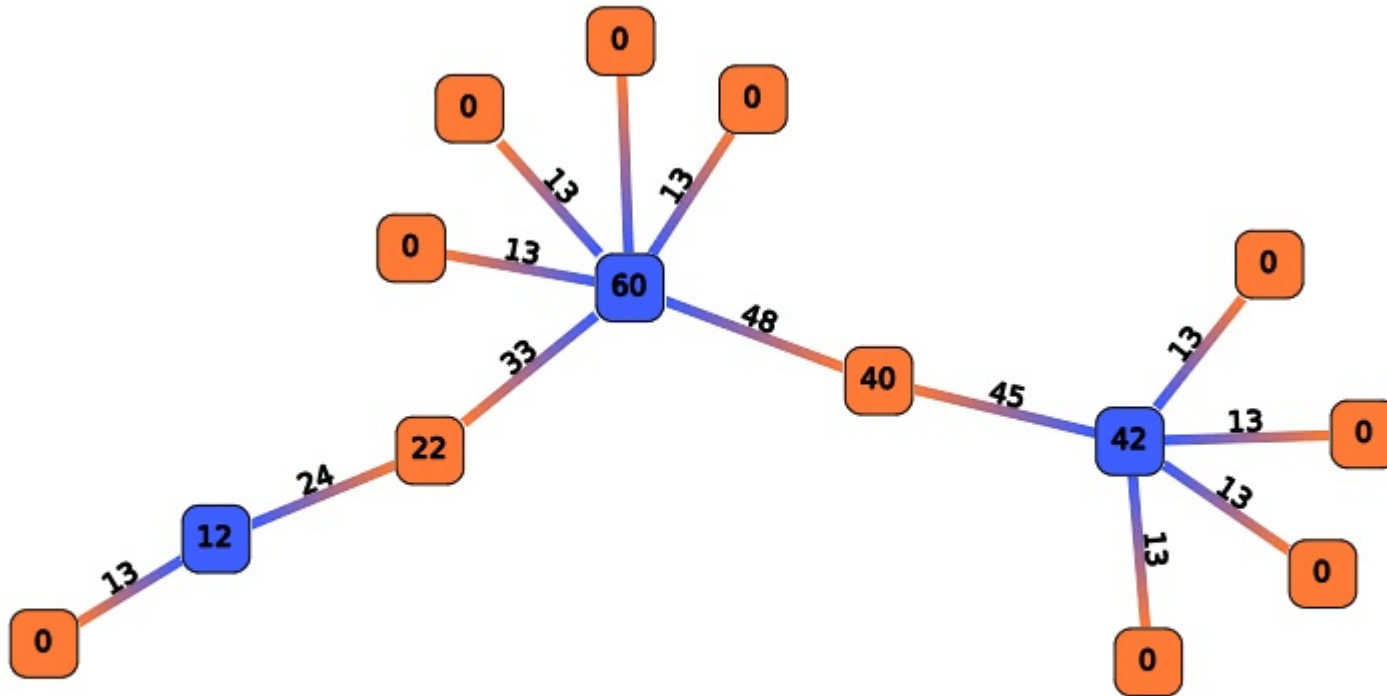
C: Betweenness

D: PageRank



Edge Betweenness

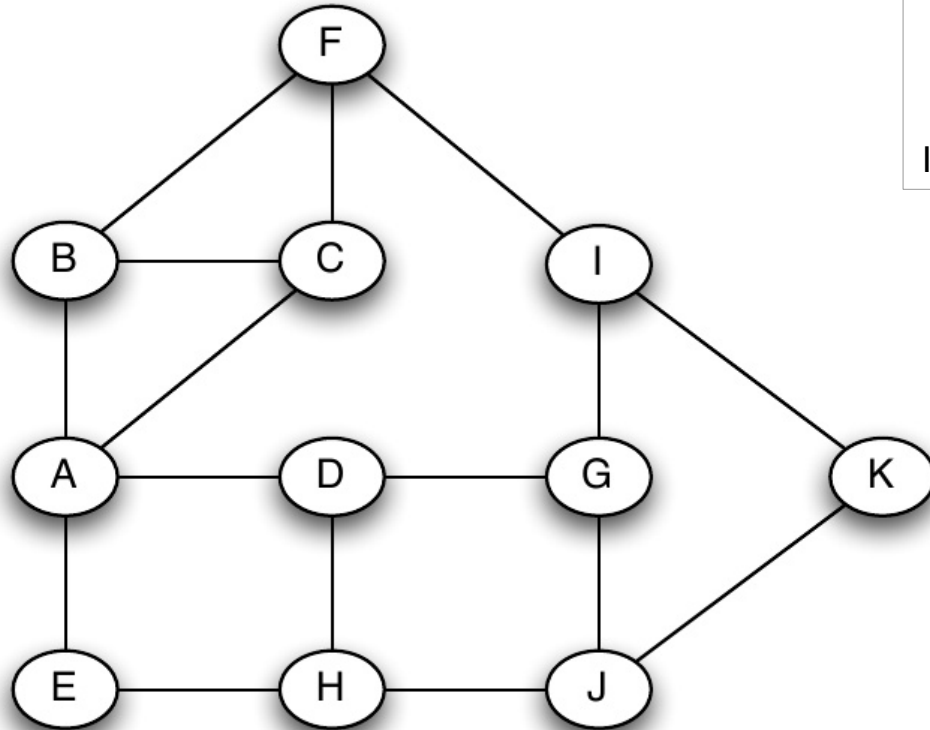
An **edge** has high betweenness if it is part of many shortest-paths ... how to compute this efficiently?



Algorithm [Brandes, Newman]

- For every node u in V
 - Layer the graph performing a BFS from u
 - For every node v in V , $v \neq u$, sorted by layer
 - Assign to v a number $s(v)$ indicating how many shortest paths from u arrive to v
 - For every node v in V , $v \neq u$, sorted by reverse layer
 - Score to distribute = 1 + score from children
 - Add score to parent edges in proportion to $s(v)$
- In the end divide all edge scores by two

Example

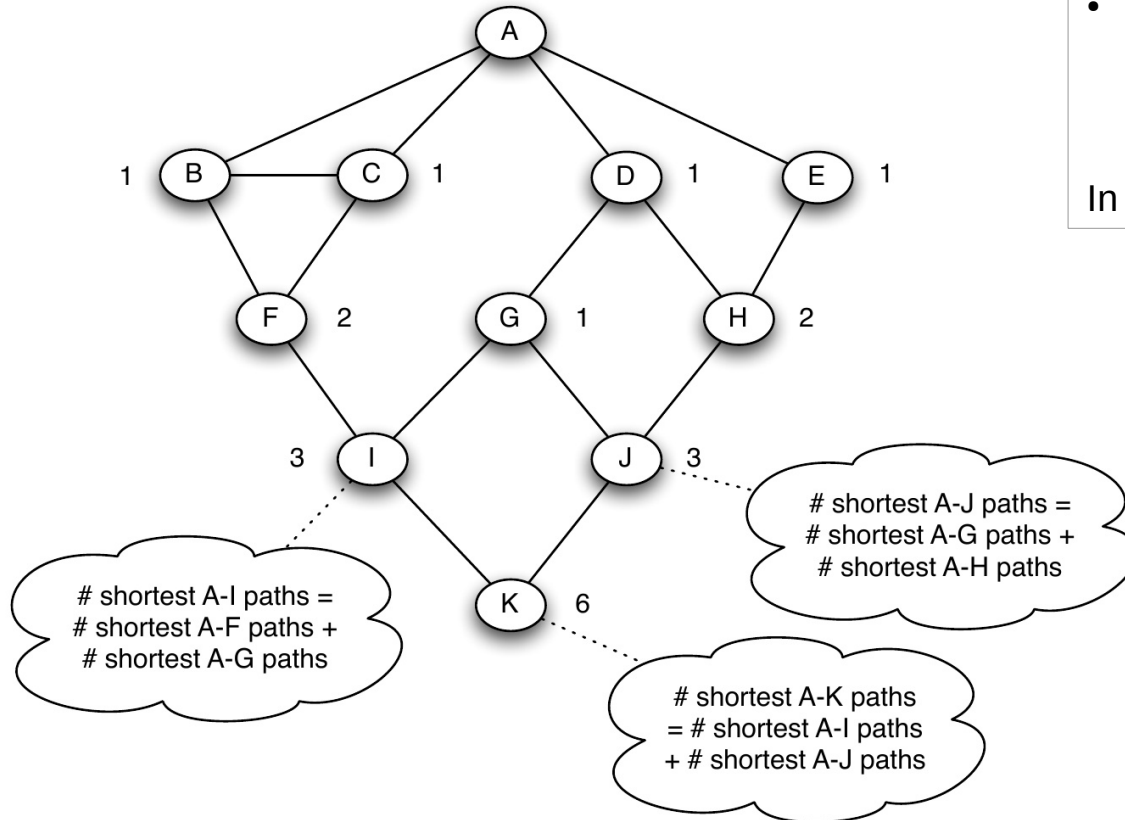


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Example



For every node u in V

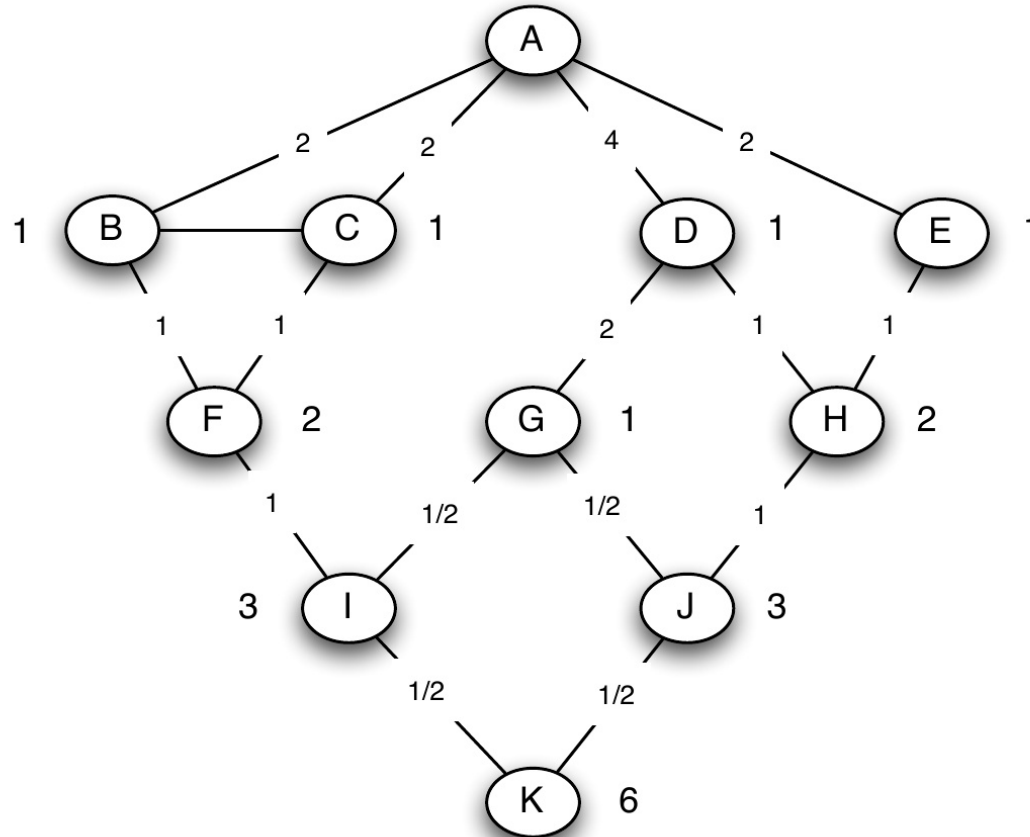
- Layer the graph performing a BFS from u
- **For every node v in V , $v \neq u$, sorted by layer**
 - **Assign to v a number $s(v)$ indicating how many shortest paths from u arrive to v**
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 - Score to distribute = $1 + \text{score from children}$
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In the end divide all edge scores by two

All nodes in layer 1 get $s(v)=1$

Remaining nodes: simply add $s(\cdot)$ of their parents

Example



For every node u in V

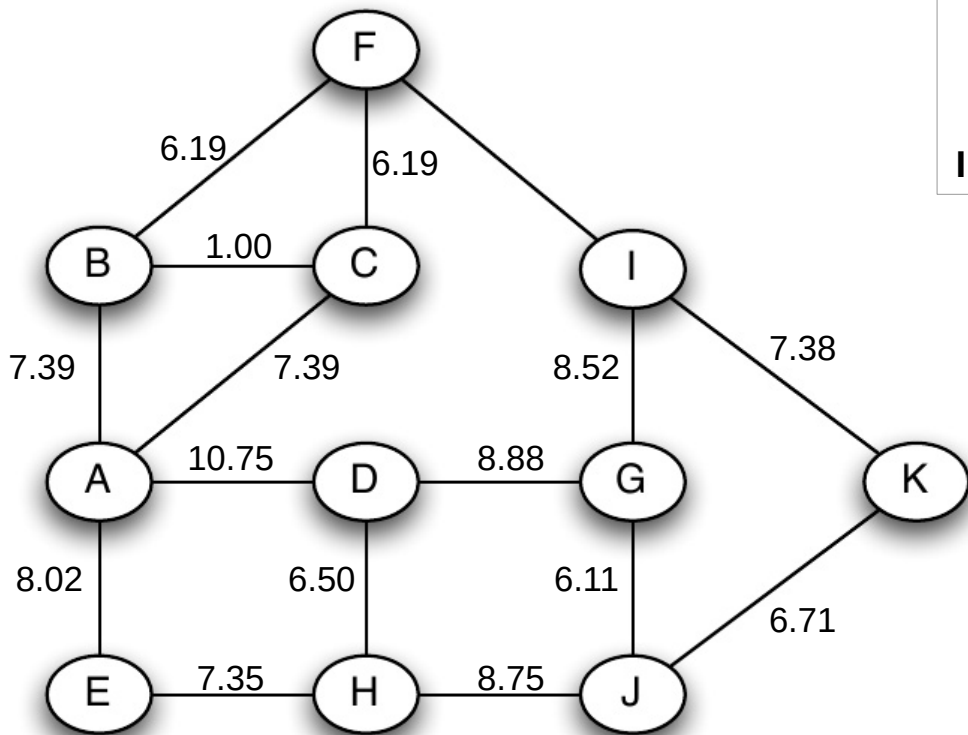
- Layer the graph performing a BFS from u
- For every node v in V , $v \neq u$, sorted by layer
 - Assign to v a number $s(v)$ indicating how many shortest paths from u arrive to v
- **For every node v in V , $v \neq u$, sorted by rev. layer**
 - **Score to distribute = 1 + score from children**
 - **Add score to distribute to parent edges in proportion to $s(v)$**

In the end divide all edge scores by two

Nodes without children distribute a score of 1

Other nodes distribute 1 + whatever they receive from their children

Result



For every node u in V

- Layer the graph performing a BFS from u
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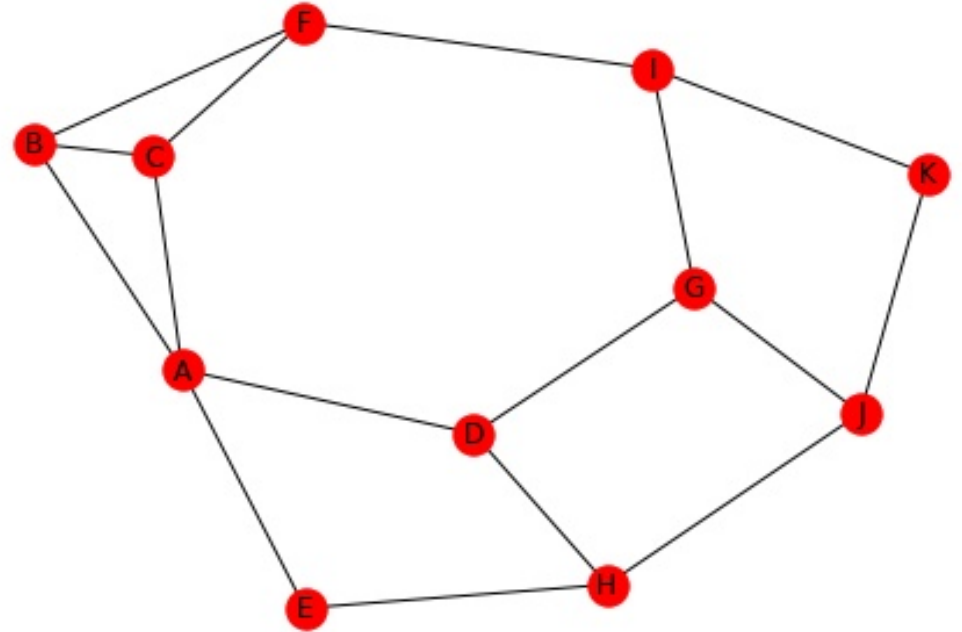
In the end divide all edge scores by two

Computed using NetworkX
(edge betweenness)

NetworkX code

```
import networkx as nx
g = nx.Graph()
g.add_edge("A", "B")
g.add_edge("A", "C")
g.add_edge("A", "D")
g.add_edge("A", "E")
g.add_edge("B", "C")
g.add_edge("B", "F")
g.add_edge("C", "F")
g.add_edge("D", "G")
g.add_edge("D", "H")
g.add_edge("E", "H")
g.add_edge("F", "I")
g.add_edge("G", "I")
g.add_edge("G", "J")
g.add_edge("H", "J")
g.add_edge("I", "K")
g.add_edge("J", "K")
nx.edge_betweenness(g, normalized=False)
```

```
nx.draw_spring(g, with_labels=True)
```



Exercise

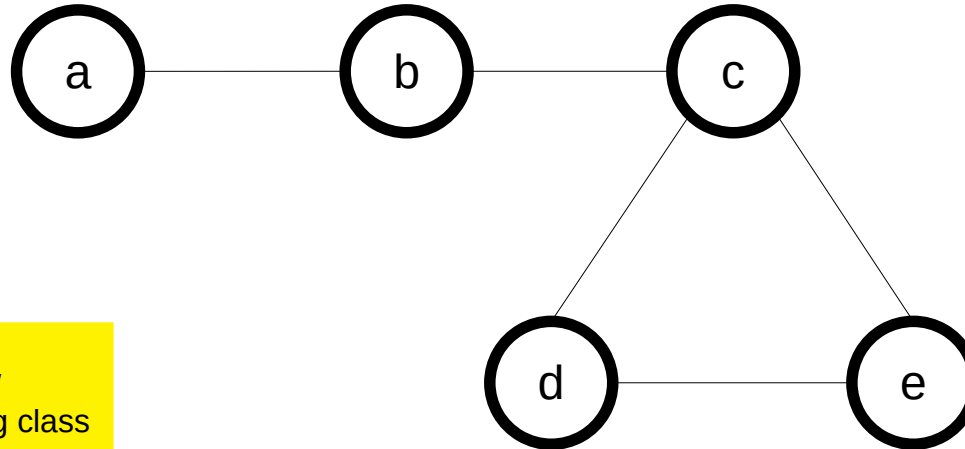
Try to compute it by inspection first

*Then use the algorithm;
you should get the same results*

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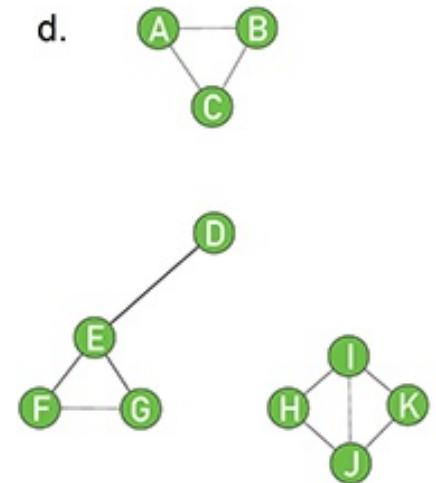
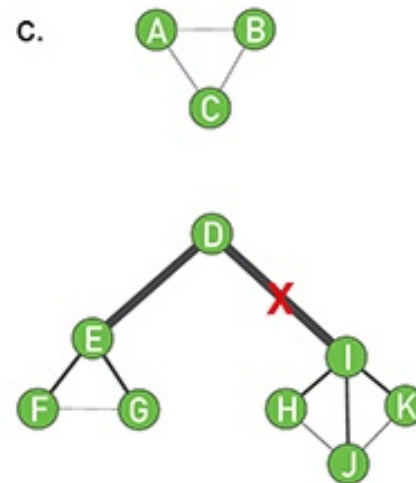
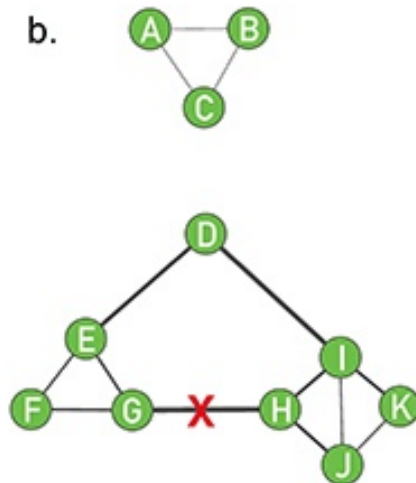
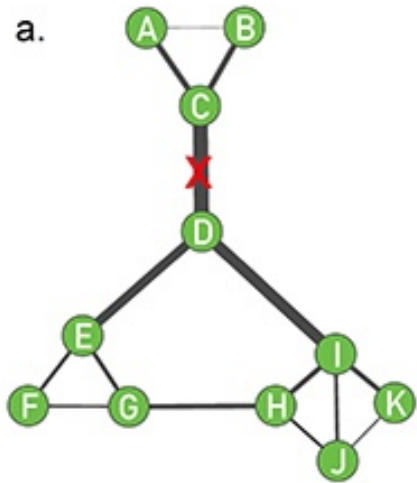
Answer in Nearpod Draw-it
<https://nearpod.com/student/>
Access to be provided during class

Fractional values?

- In a graph with cycles, you may get **fractional values** of the edge betweenness for an edge

Application: the Girvan-Newman algorithm

- Repeat:
 - Compute edge betweenness
 - Remove edge with larger betweenness



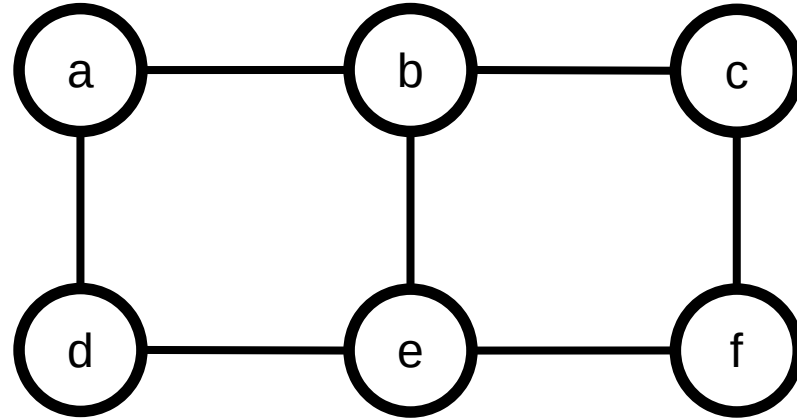
Summary

Things to remember

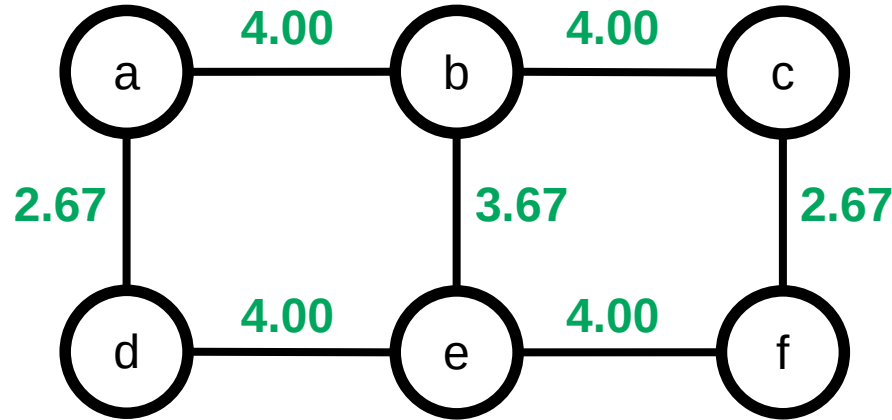
- Closeness and harmonic closeness
- Node and edge betweenness
- Practice running the Brandes-Newman algorithm on small graphs
- Write code to execute the Brandes-Newman algorithm

Practice on your own

- Compute edge betweenness on this graph



Practice on your own (cont.)



If you don't get this result, check:

<https://www.youtube.com/watch?v=uYjWbp8VC7c>

Two constructive problems

1. Sketch a graph of N nodes in which a node, which you should mark with an asterisk (*), should have betweenness approximately equal to N and closeness approximately $1/N$ for large N . Explain briefly.
2. Sketch a graph of N nodes in which a node, which you should mark with an asterisk (*), should have betweenness approximately equal to N and closeness approximately $2/N^2$ for large N . Explain briefly.

Do not use a concrete N . Use a general N , for instance by using the ellipsis (...) to denote multiple nodes.