K-Core Decomposition

Social Networks Analysis and Graph Algorithms

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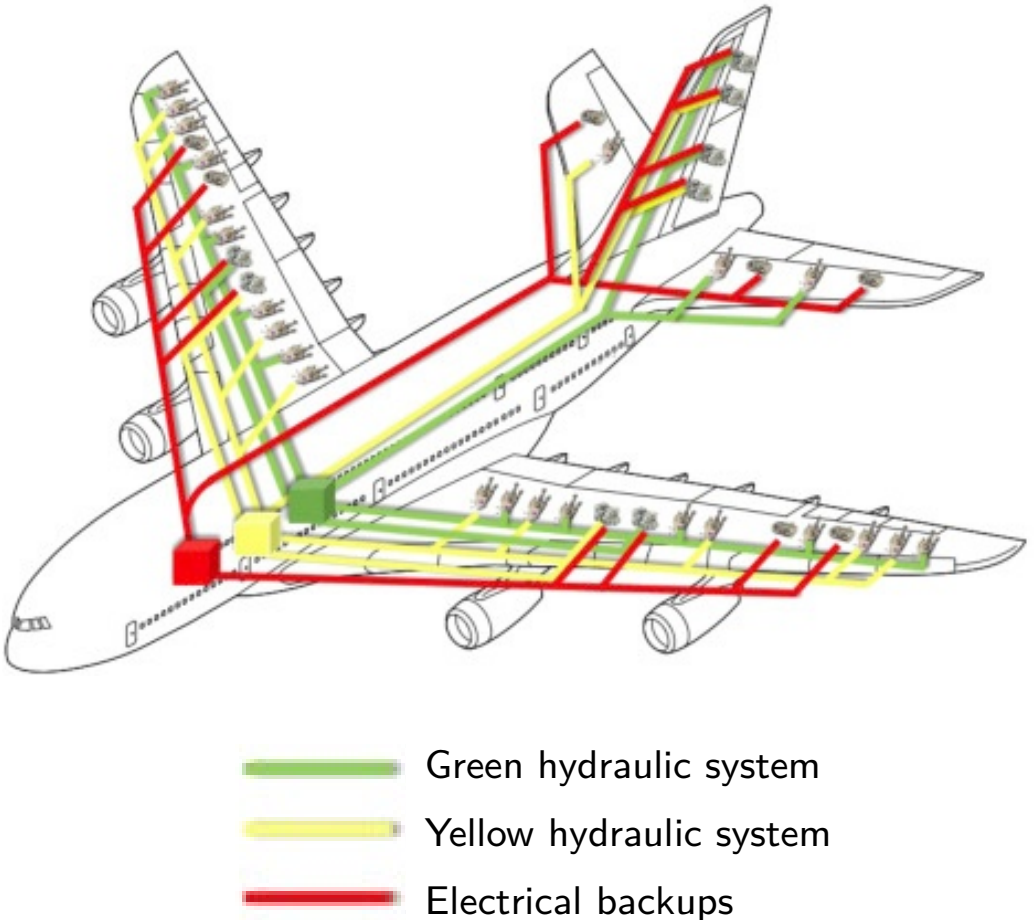
Sources

- A. L. Barabási (2016). Network Science – Chapter 09
- A. Beutel, L. Akoglu, C. Faloutsos (2015). Tutorial at KDD
- A. Frieze, A. Gionis, C. Tsourakakis (2013). "Algorithmic techniques for modeling and mining large graphs (AMAzING)" Tutorial at KDD
- V. E. Lee, N. Ruan, R. Jin, C. Aggarwal (2010). A survey of algorithms for dense sub-graph discovery. Chapter 10 of "Managing and Mining Graph Data"
- URLs cited in the footer of slides

Robustness

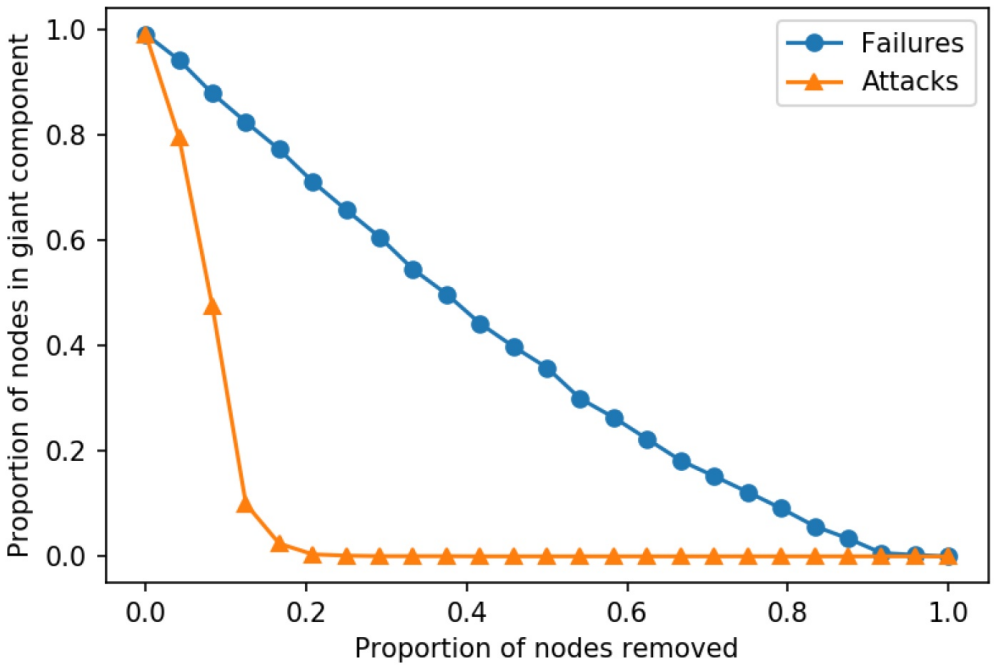
Robustness to failure

- A system is **robust** if the failure of some of its components does not affect its function
- Aircrafts, for instance, have:
  - Separate primary and back-up flight instruments (airspeed, altimeter, ...);
  - 3+ independent hydraulic systems
  - Primary and emergency landing gear;
  - Multiple sources of power



Robust networks maintain connectedness

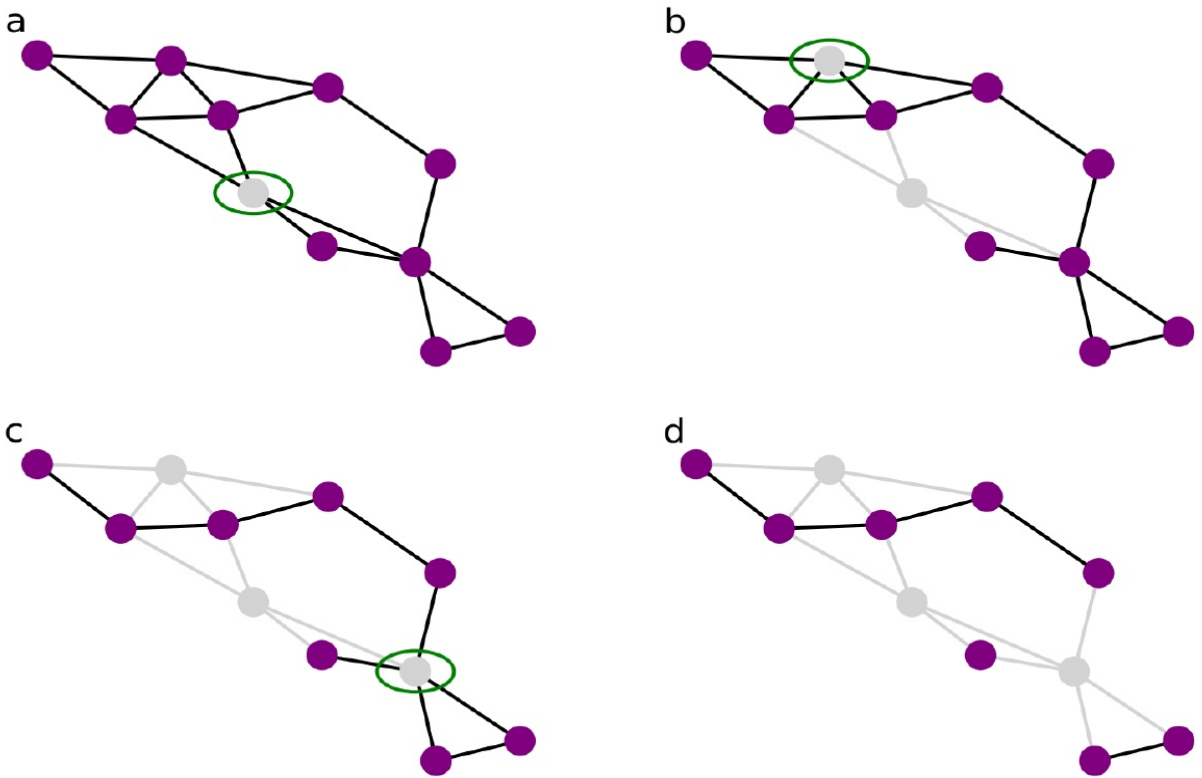
The size of the giant component in this network of flights is reduced slowly by random removals ("**failures**"), but reduced quickly by removing high-degree edges ("**attacks**")



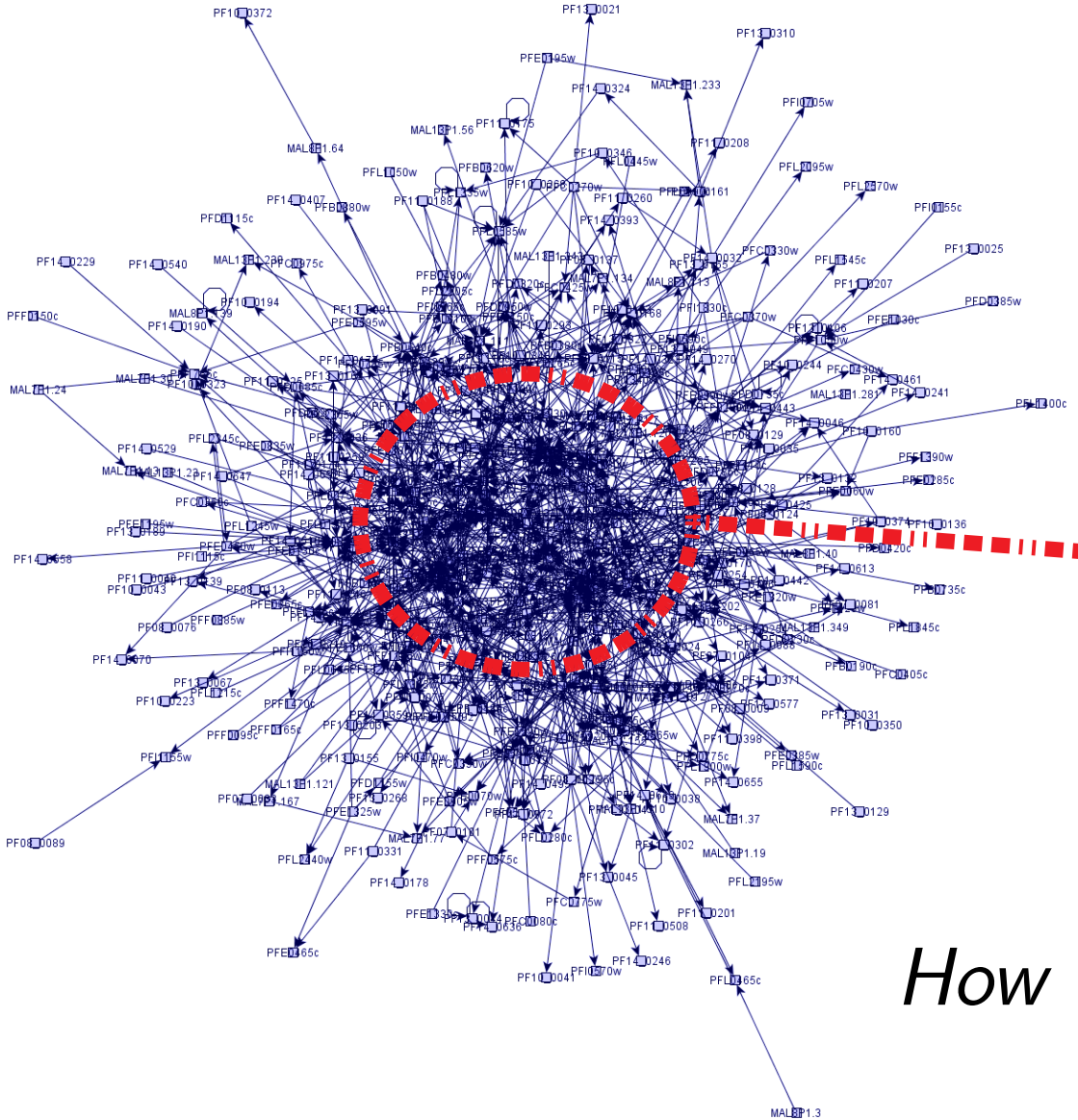
Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Targeted removal of nodes

The targeted removal of 4 nodes **disconnects** this graph into three components



Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.



Many networks look like “hairballs”

They have a **core** and a **periphery**

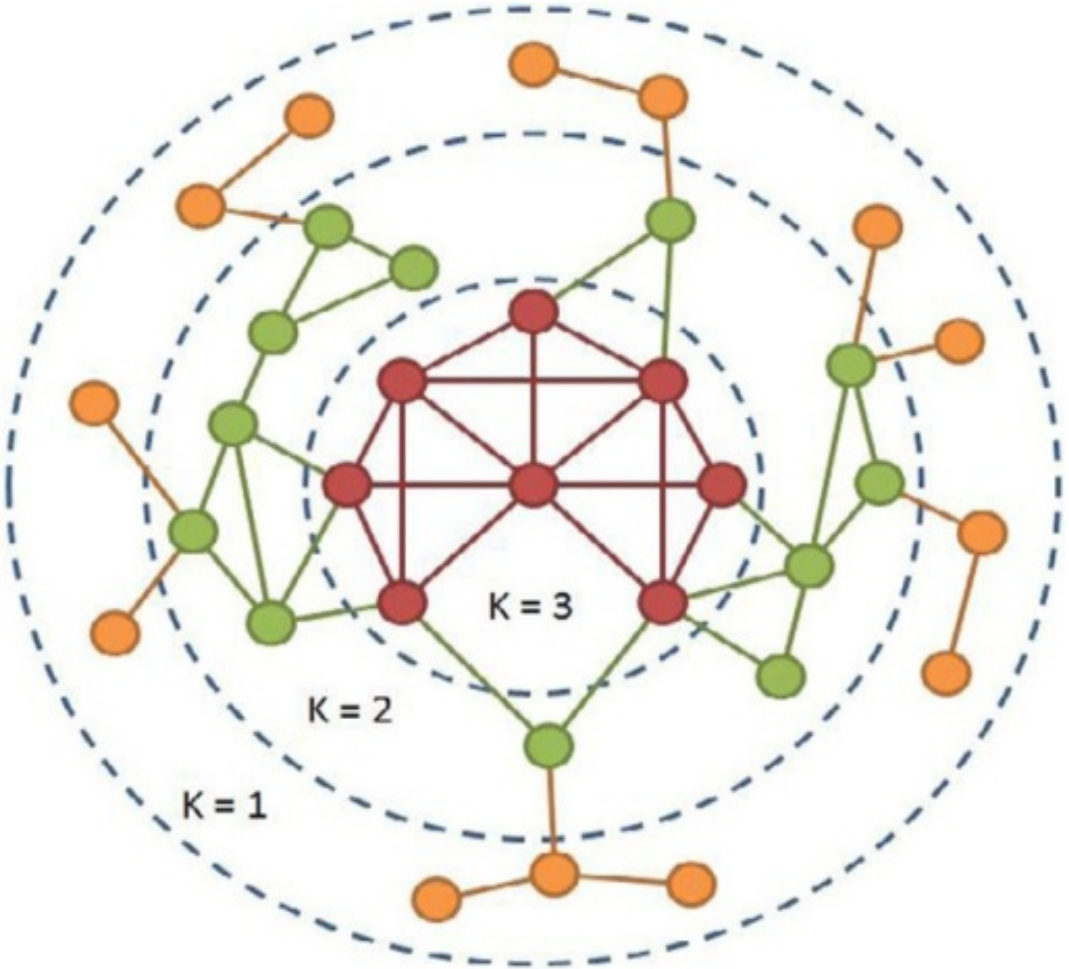
How can we find the core?

k-core decomposition
is a method to decompose
a graph into *layers*

k-core decomposition

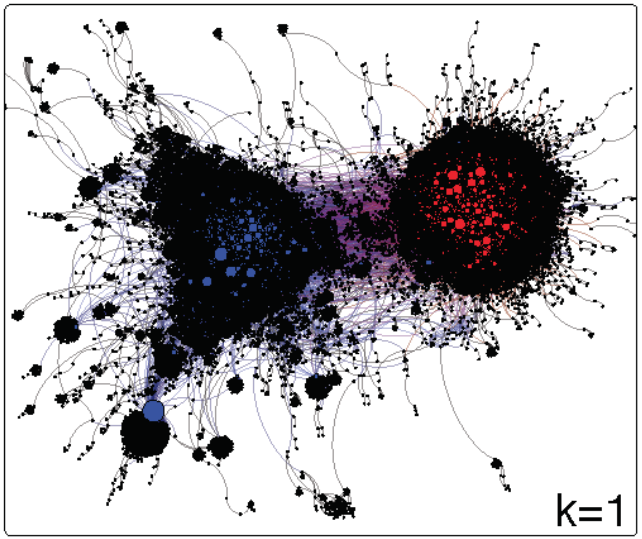
- Remove all nodes having degree ≤ 1 until there are no such nodes
  - Those are in the 1-core
- Remove all nodes having degree ≤ 2 until there are no such nodes
  - Those nodes are in the 2-core
- Remove all nodes having degree ≤ 3 until there are no such nodes
  - Those nodes are in the 3-core
- Etc.

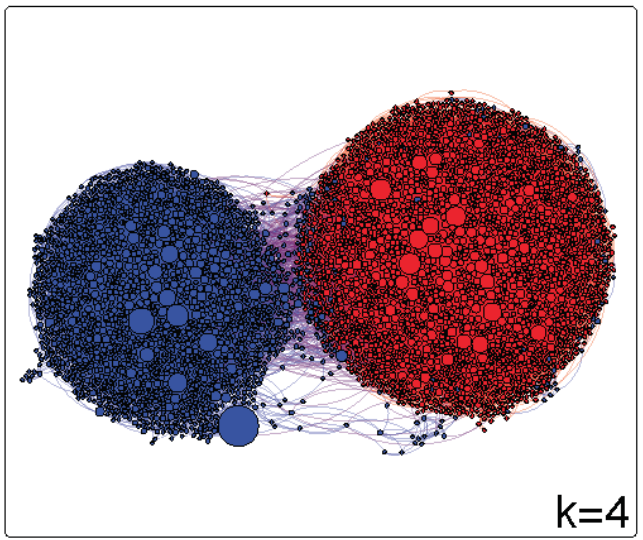
Example 1

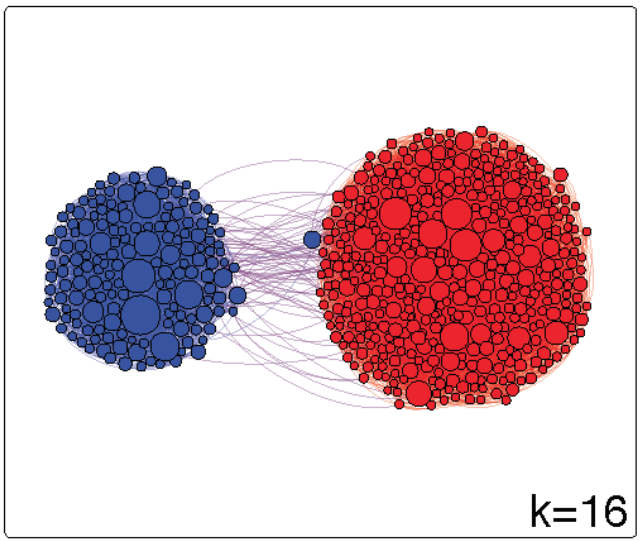


https://openi.nlm.nih.gov/detailedresult.php?img=3368241_fnagi-04-00013-g0001&req=4

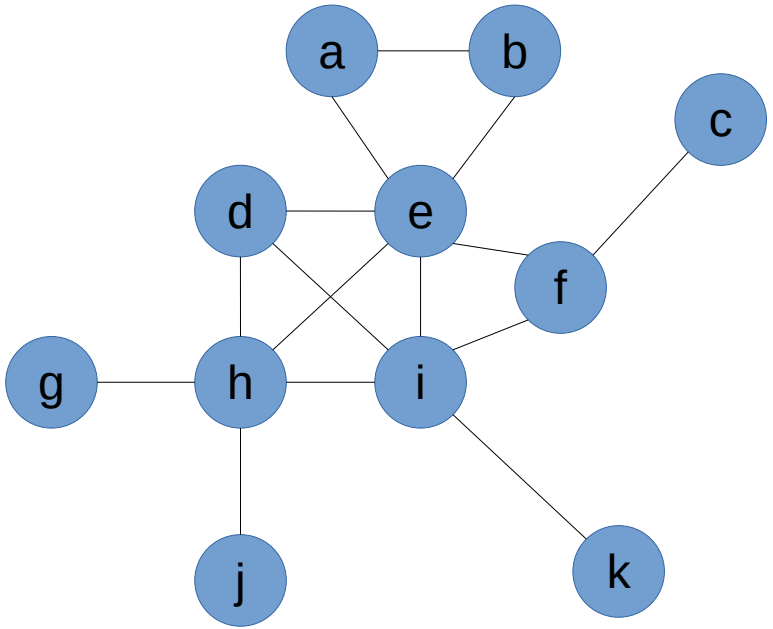
Example 2







Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.



Exercise

For each node in the graph, indicate the max k-core to which it belongs

(Mark each node with a number, upload an image of the result.)

[Source]

Pin board: https://upfbarcelona.padlet.org/chato/tt19-k-core-decomposition-575mnnke44c5ybyl

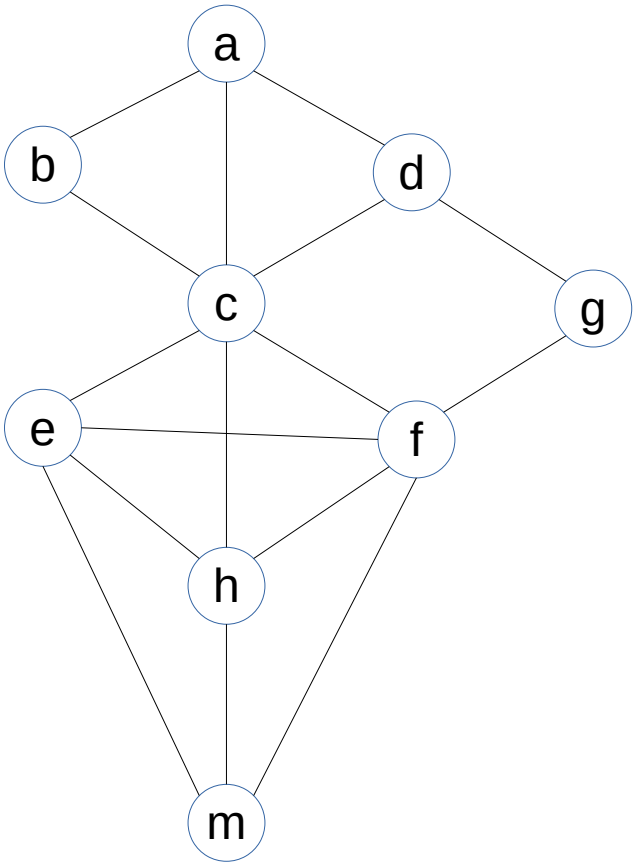
Summary

Things to remember

- What does it mean for a network to be robust?
- What is the k-core decomposition
- How to compute it on a graph

Practice on your own

Find the 3-core of this graph



Solution by Vivekanand Khyade (start at 01:23)
https://youtu.be/8sNZ5d8eNC8?t=83