Distances in Scale-Free Networks

Social Networks Analysis and Graph Algorithms

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Contents

- Distance distribution of scale-free networks

Sources

- A. L. Barabási (2016). Network Science – Chapter 04

- URLs cited in the footer of specific slides

Consequences of having extremely large degree nodes
## *(also known as "large hubs")*

Air travel

- You can travel between almost all pairs of European airports directly or (most of the time) with at most one stop

- All you have to do is **go to a well connected airport**

- This is because there are large degree airports
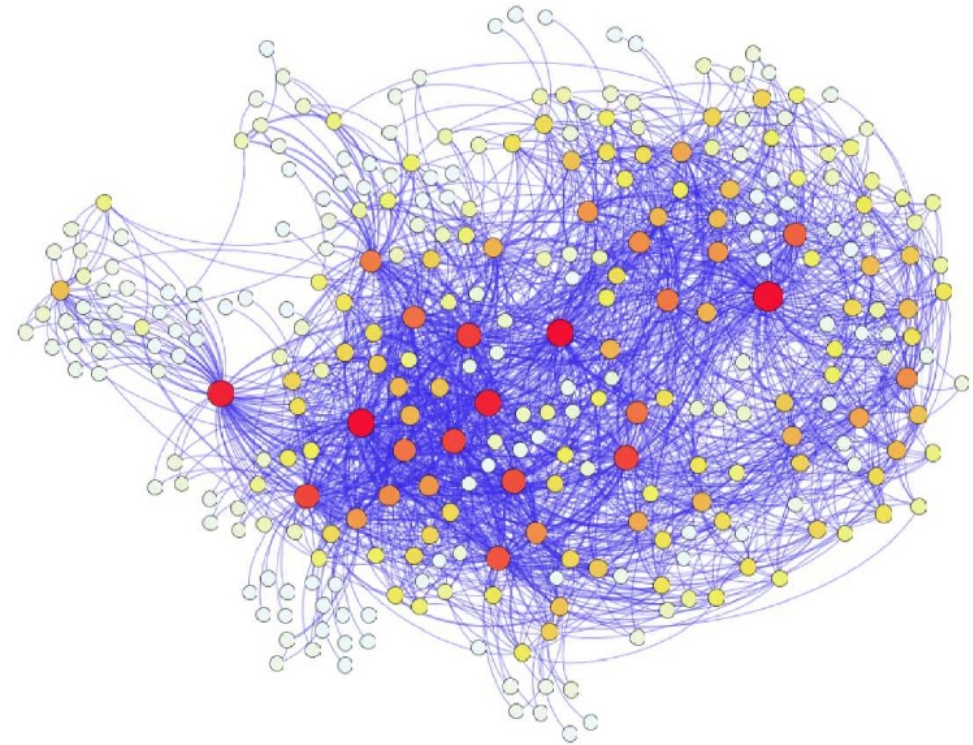
Cardillo, A et al. (2013). Modeling the multi-layer nature of the European Air Transport Network: Resilience and passengers re-scheduling under random failures. Euro. Phys. J. Special Topics, 215(1), 23-33. [DOI]

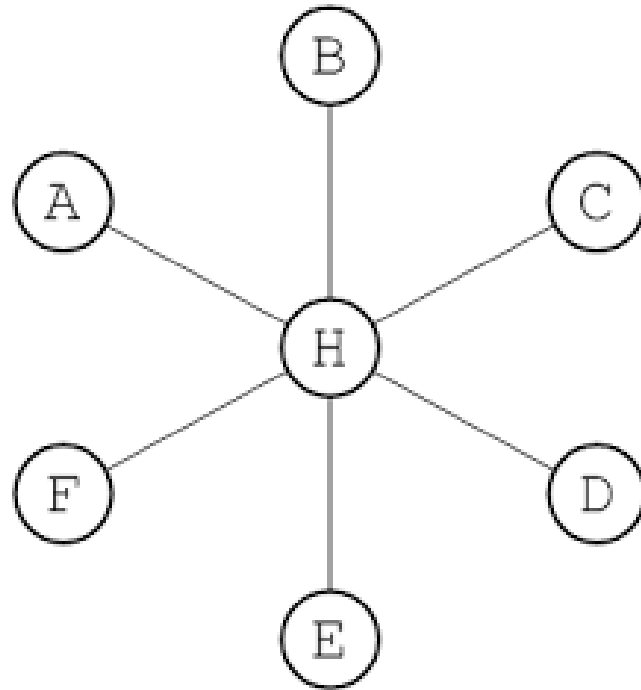# In general, having "hubs" or large degree nodes reduces distances
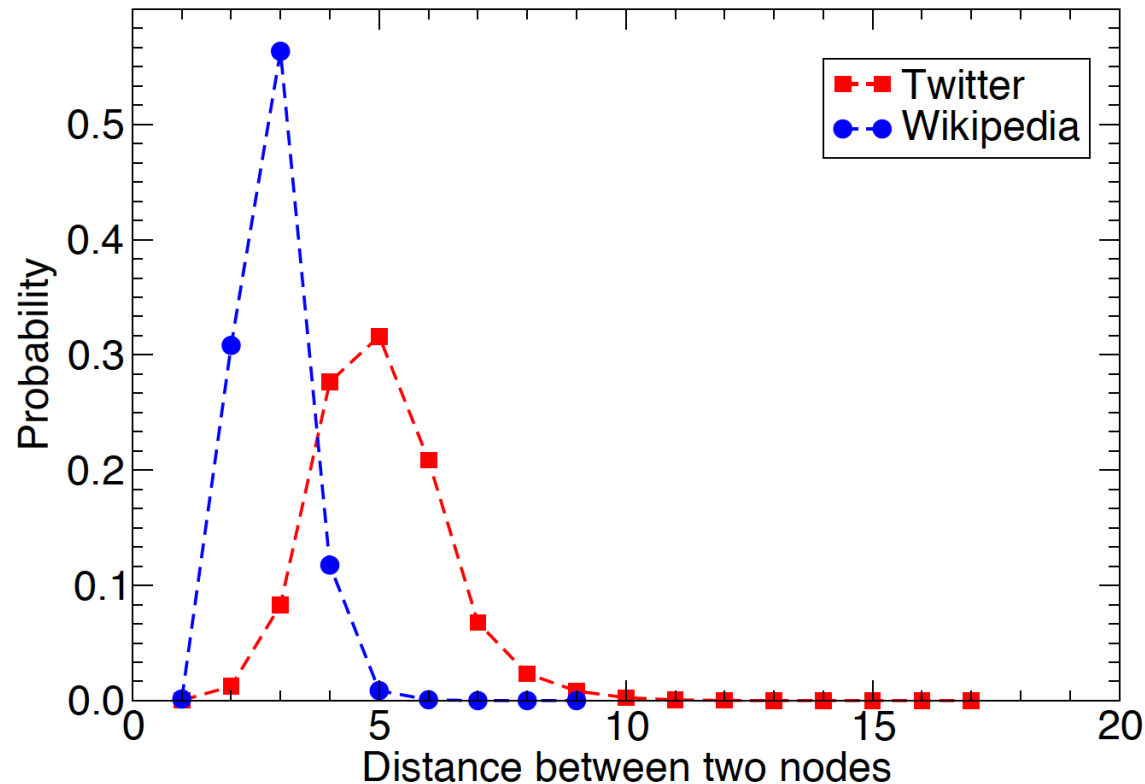
Distance distributions: simulation results

Scale-free networks of increasing size, $<k> = 3$
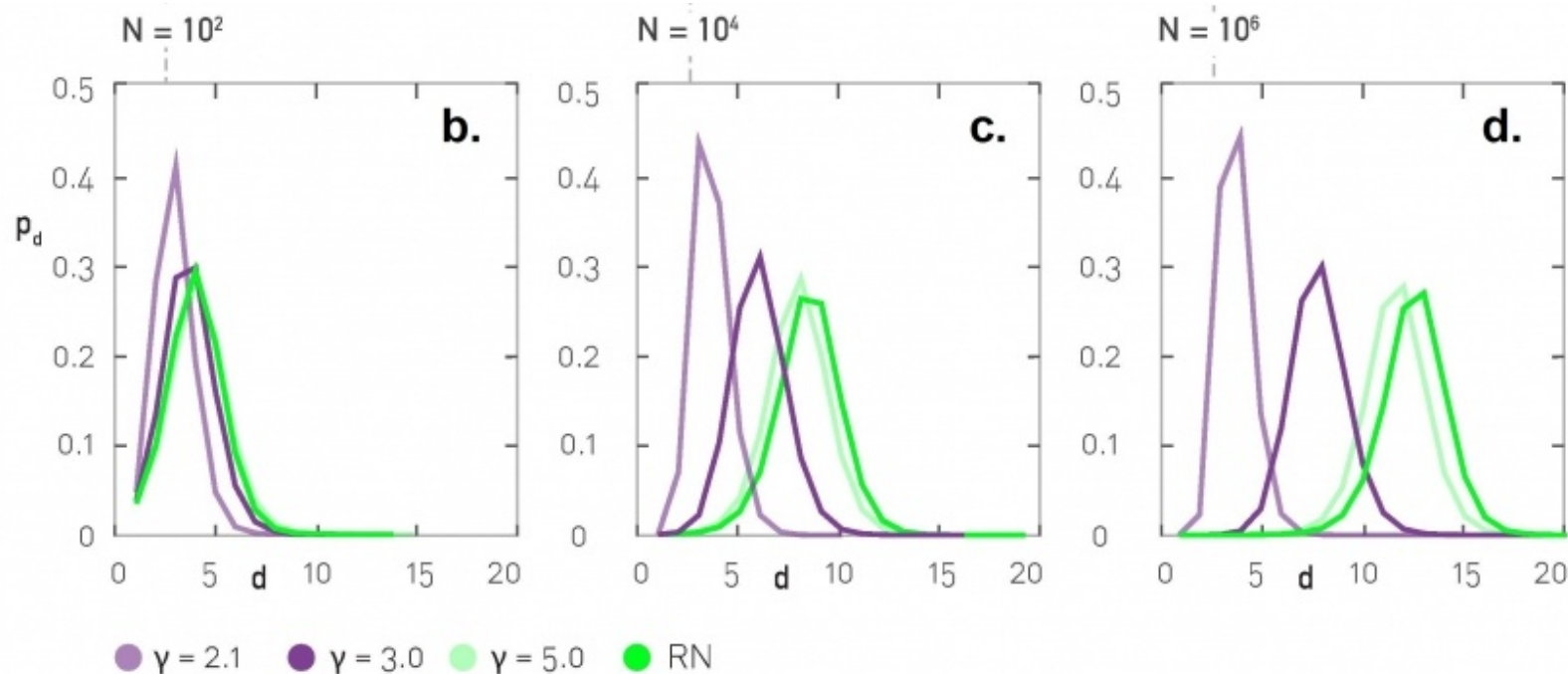
Distance regimes

Anomalous regime $\gamma = 2$

Ultra-small world $2 < \gamma < 3$

- Average distance follows log(log(N))

- Example (humans):

$$N \approx 7 \times 10^9$$

$$\log N \approx 22.66$$

$$\log \log N \approx 3.12$$

Average distance

Scale-free network

$$p_k \propto k^{-\gamma}$$

- Depends on ɣ and N
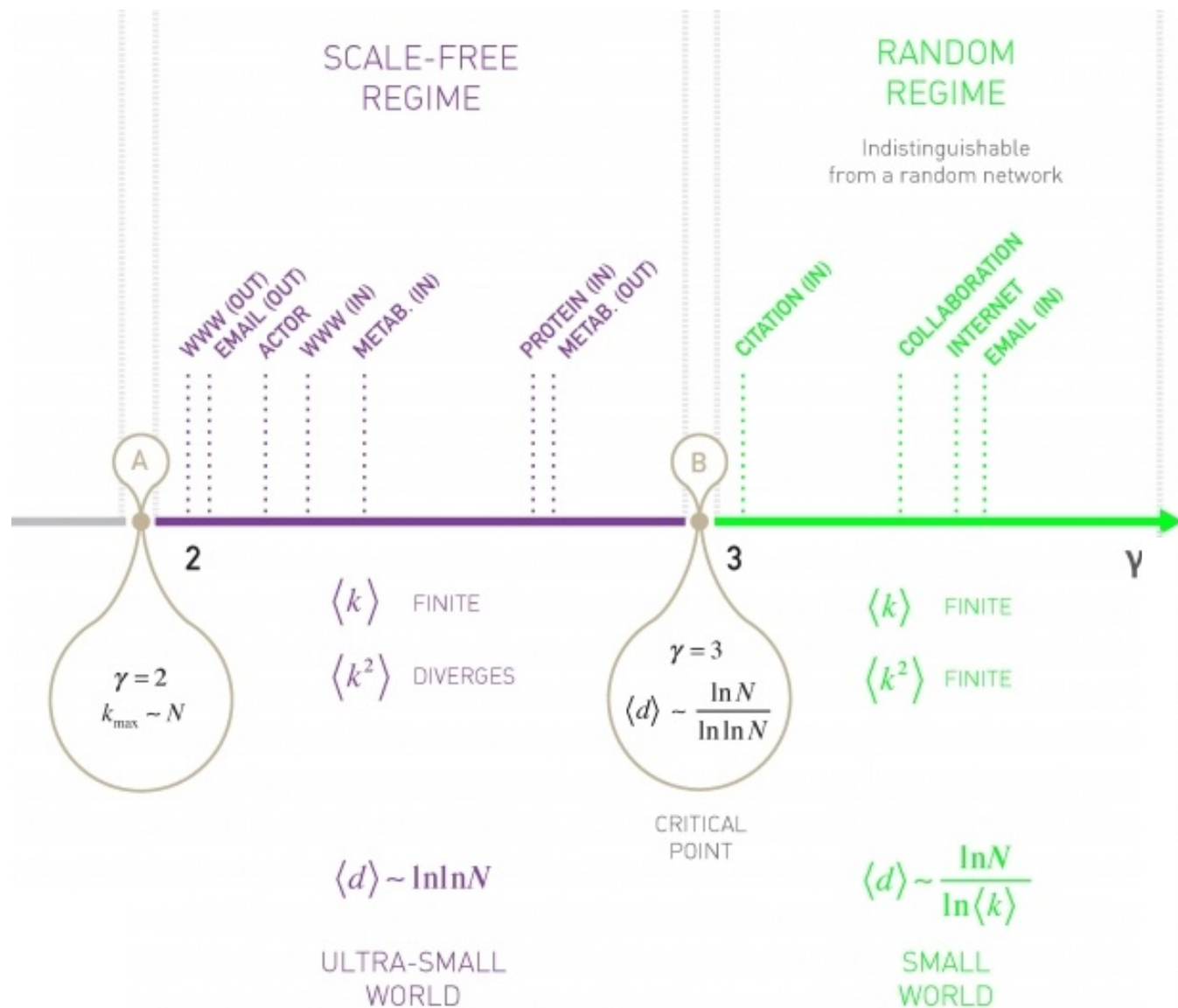
$$\langle d \rangle = \begin{cases} \text{const.} & \text{if } \gamma = 2 \\ \log \log \mathrm{N} & \text{if } 2 < \gamma < 3 \\ \log \mathrm{N} / \log \log \mathrm{N} & \text{if } \gamma = 3 \\ \log \mathrm{N} & \text{if } \gamma > 3 \end{cases}$$

Same as in
ER graphs

Small world $\gamma > 3$

- Average distance follows log(N)

- Similar to ER graphs where it followed log(N)/log($<$k$>$)

The degree distribution exponent plays an important role
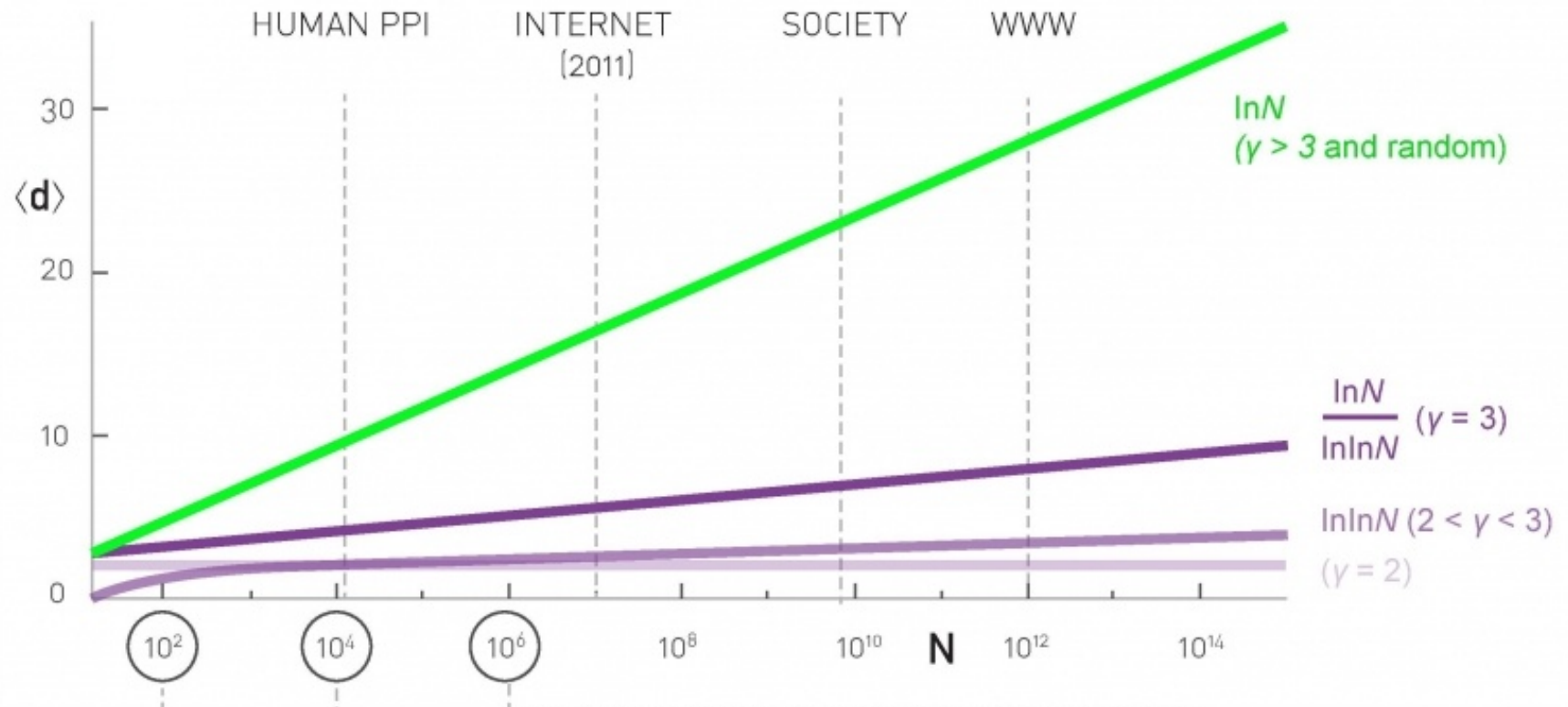
When $\gamma > 3$

- In this case it is hard to distinguish this case from an ER graph
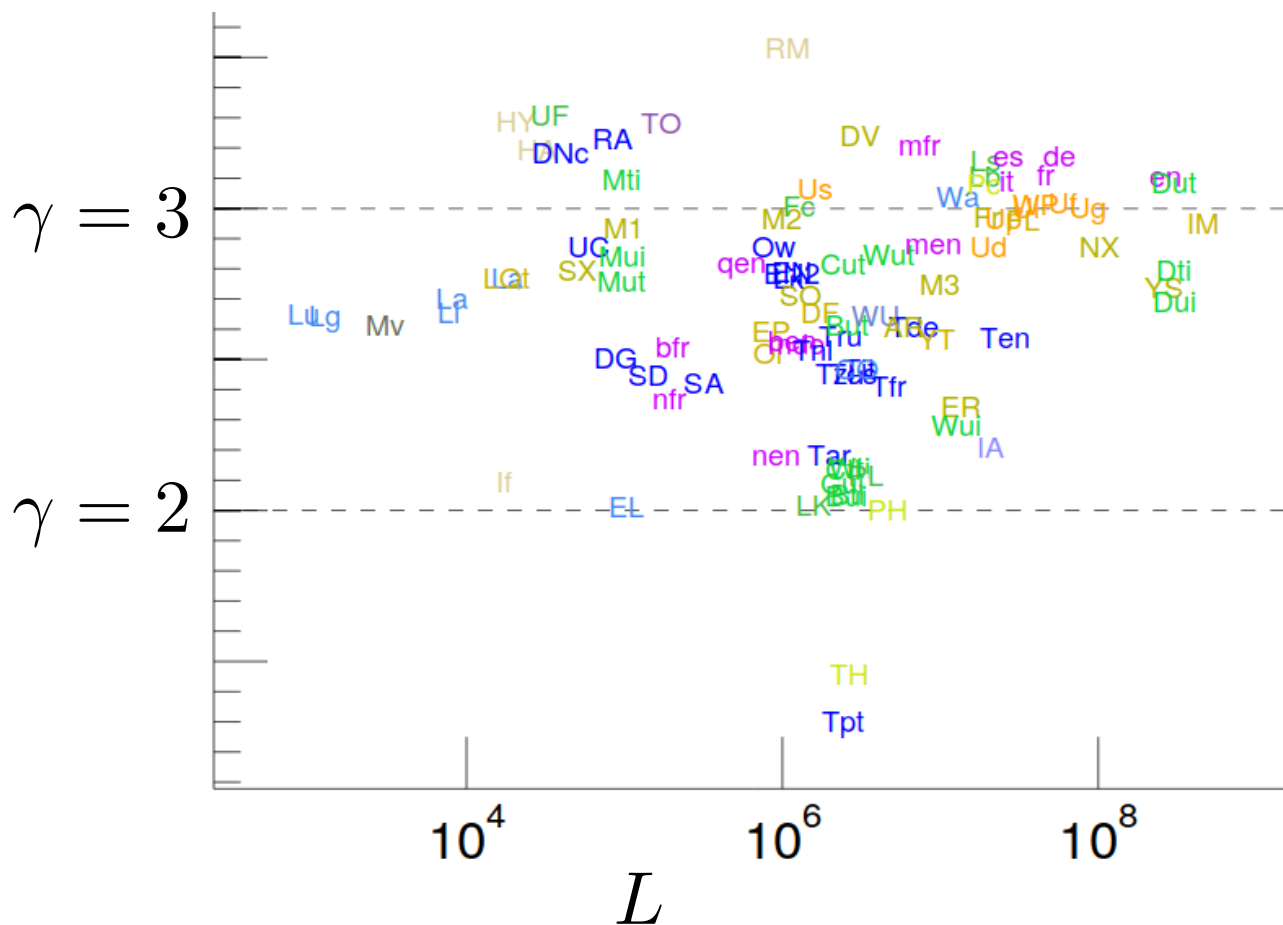
- In most real complex networks (but not all)

$$2 < \gamma < 3$$

When $\gamma > 3$

- Remember $\qquad k_{\mathrm{max}} = k_{\mathrm{min}} N^{\frac{1}{\gamma-1}} \qquad N = \left(\dfrac{k_{\mathrm{max}}}{k_{\mathrm{min}}} \right)^{\gamma-1}$

- Observing the scale-free properties requires that
  $k_{\mathrm{max}} >> k_{\mathrm{min}}$, e.g. $k_{\mathrm{max}} = 10 \ k_{\mathrm{min}}$

- Then if $\gamma = 5, N > 10^8$

- There are not many such networks for which we have available data

Average distance and N

Examples

$\gamma = 3$

$\gamma = 2$

L

EL	Wikipedia elections
LK	Linux kernel mailing list threads
BuI	BibSonomy u-i
BtI	BibSonomy t-i
CuI	CiteULike u-i
If	Infectious
PL	Prosper loans
CtI	CiteULike t-i
WtI	Twitter t-i
nen	Wikinews (en)
Tar	Wikipedia talk, Arabic
WuI	Twitter u-i
ER	Epinions
nfr	Wikinews (fr)
Tfr	Wikipedia talk, French
SD	Slashdot
Tzh	Wikipedia talk, Chinese
Tes	Wikipedia talk, Spanish

Etc.

Summary

Things to remember

- Regimes of distance and connectivity

Practice on your own

- Remember the regimes of a graph given $<k>$

    (It is useful to know this by heart)

- Estimate distance distributions for some graphs