Distances in Scale-Free Networks

Social Networks Analysis and Graph Algorithms

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Contents

Distance distribution of scale-free networks

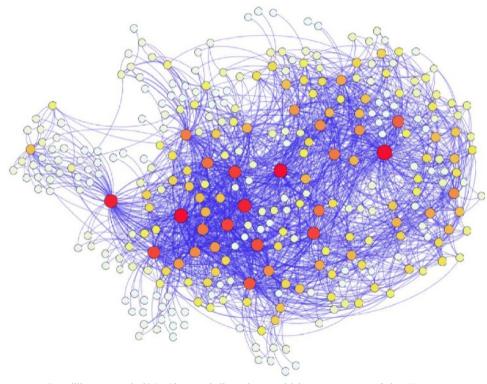
Sources

- A. L. Barabási (2016). Network Science Chapter 04
- URLs cited in the footer of specific slides

Consequences of having extremely large degree nodes (also known as "large hubs")

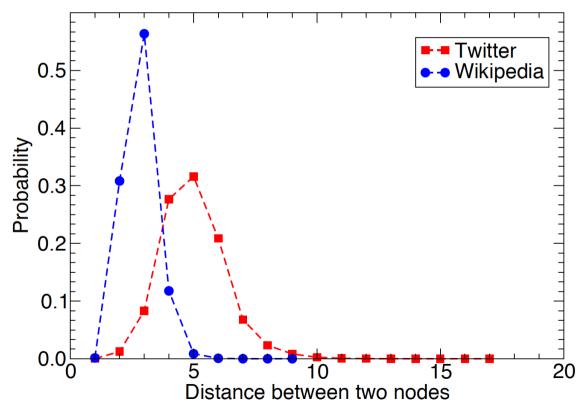
Air travel

- You can travel between almost all pairs of European airports directly or (most of the time) with at most one stop
- All you have to do is go to a well connected airport
- This is because there are large degree airports



Cardillo, A et al. (2013). Modeling the multi-layer nature of the European Air Transport Network: Resilience and passengers re-scheduling under random failures. Euro. Phys. J. Special Topics, 215(1), 23-33. [DOI]

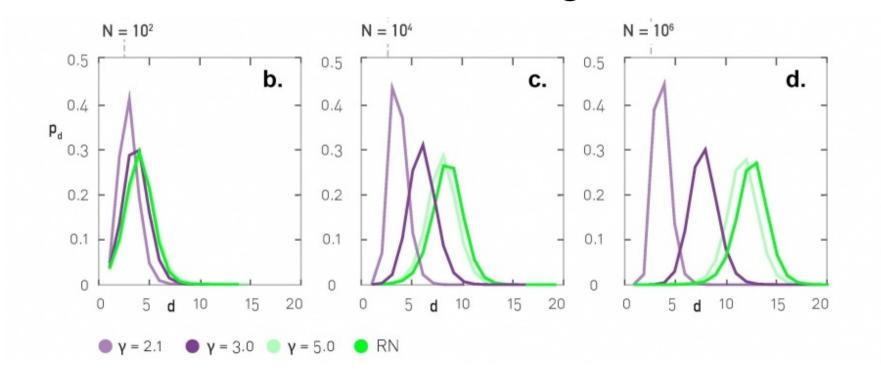
In general, having "hubs" or large degree nodes reduces distances



Distance regimes

Distance distributions: simulation results

Scale-free networks of increasing size, $\langle k \rangle = 3$

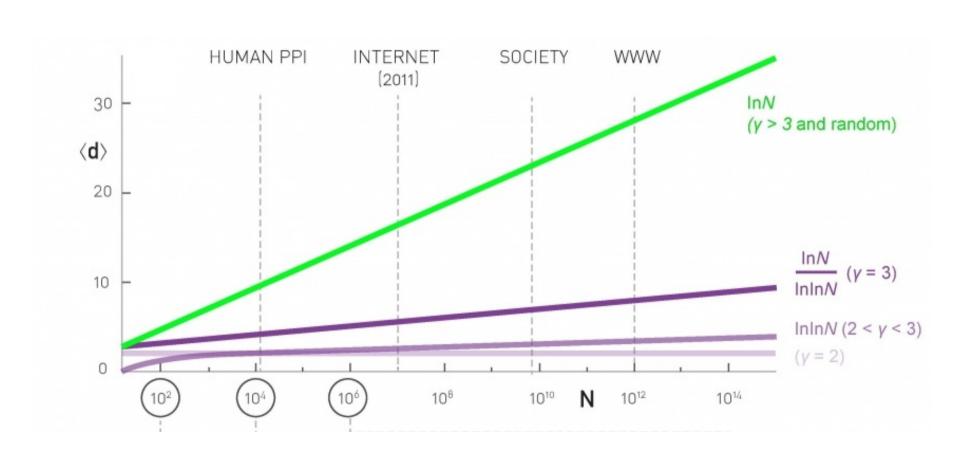


Average distance

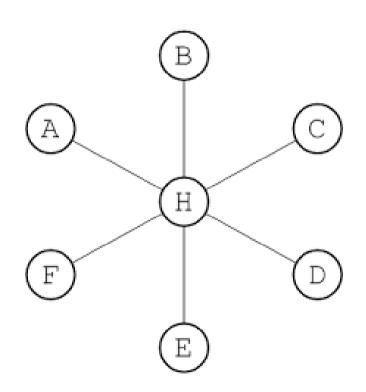
Depends on \(\cong \) and \(\cong \)

$$\langle d \rangle = \begin{cases} \text{const.} & \text{if } \gamma = 2 \\ \log \log \mathbf{N} & \text{if } 2 < \gamma < 3 \\ \log \mathbf{N}/\log \log \mathbf{N} & \text{if } \gamma = 3 \\ \log \mathbf{N} & \text{if } \gamma > 3 \end{cases}$$
 Same as in ER graphs

Average distance and N



Anomalous regime $\gamma=2$



Ultra-small world

$$2 < \gamma < 3$$

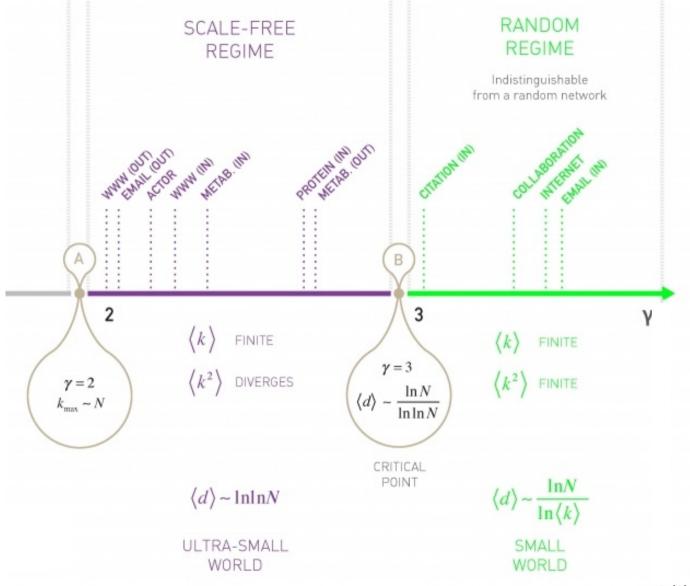
- Average distance follows log(log(N))
- Example (humans):

$$N \approx 7 \times 10^9$$
 $\log N \approx 22.66$
 $\log \log N \approx 3.12$

Small world $\gamma > 3$

- Average distance follows log(N)
- Similar to ER graphs where it followed log(N)/log(<k>)

The degree distribution exponent plays an important role



When $\gamma > 3$

- In this case it is hard to distinguish this case from an ER graph
- In most real complex networks (but not all) $2 < \gamma < 3$

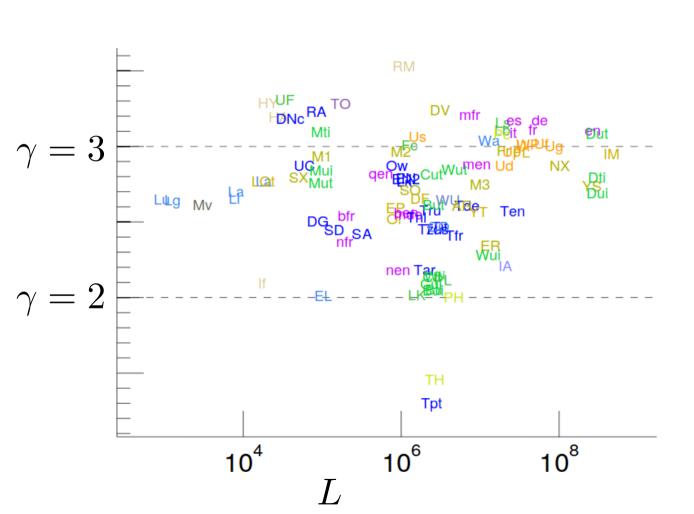
When $\gamma > 3$

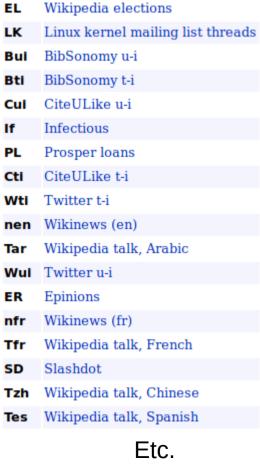
Remember

$$k_{\max} = k_{\min} N^{\frac{1}{\gamma - 1}}$$
 $N = \left(\frac{k_{\max}}{k_{\min}}\right)^{\gamma - 1}$

- Observing the scale-free properties requires that $k_{\text{max}} >> k_{\text{min}}\text{, e.g. }k_{\text{max}} = 10\ k_{\text{min}}$
- Then if $\gamma=5, N>10^8$
- There are not many such networks for which we have available data

Examples





Summary

Things to remember

Regimes of distance and connectivity

Practice on your own

- Remember the regimes of a graph given <k> (It is useful to know this by heart)
- Estimate distance distributions for some graphs