Betweenness

Social Networks Analysis and Graph Algorithms

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Sources

- D. Easly and J. Kleinberg (2010). Networks, Crowds, and Markets – Section 3.6B
- A. L. Barabási (2016). Network Science – Section 9.3
- P. Boldi and S. Vigna (2014). Axioms for Centrality in *Internet Mathematics*
- Esposito and Pesce: Survey of Centrality 2015.
- URLs cited in the footer of slides

Types of centrality measure

- **Non-spectral**
  - Degree
  - Closeness and harmonic closeness
  - Betweenness
- Spectral
  - HITS
  - PageRank

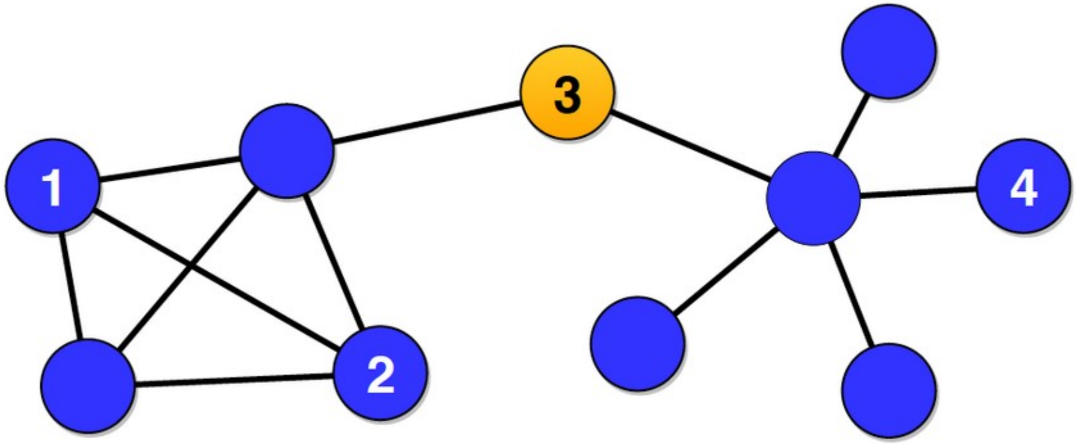
Betweenness

Definitions

The **betweenness of an edge** is the number of shortest paths that cross that edge

The **betweenness of a node** is the number of shortest paths that cross that node

Example 1



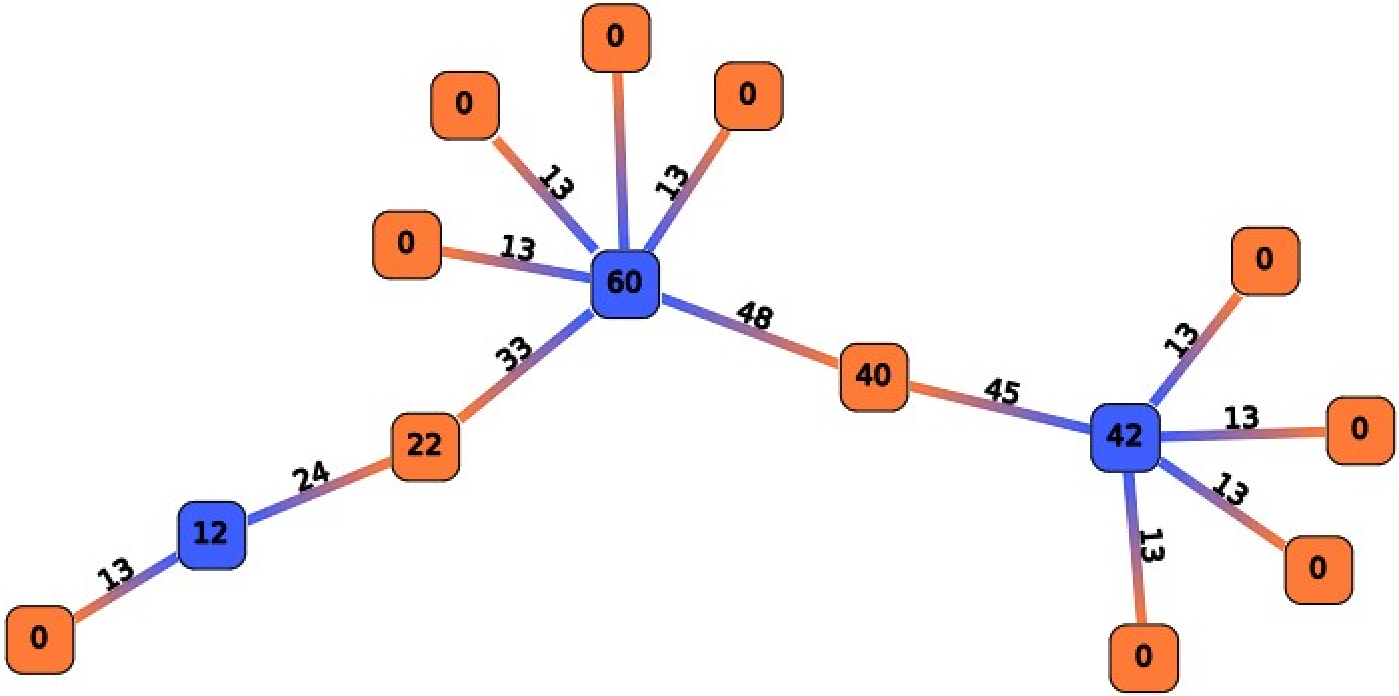
There are 20 shortest paths that cross through node 3. Why?

The shortest path between nodes 1 and 2 does not cross node 3, but the shortest path between nodes 1 and 4 does cross node 3.

Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Example 2

Here, nodes and edges are labeled with their betweenness.



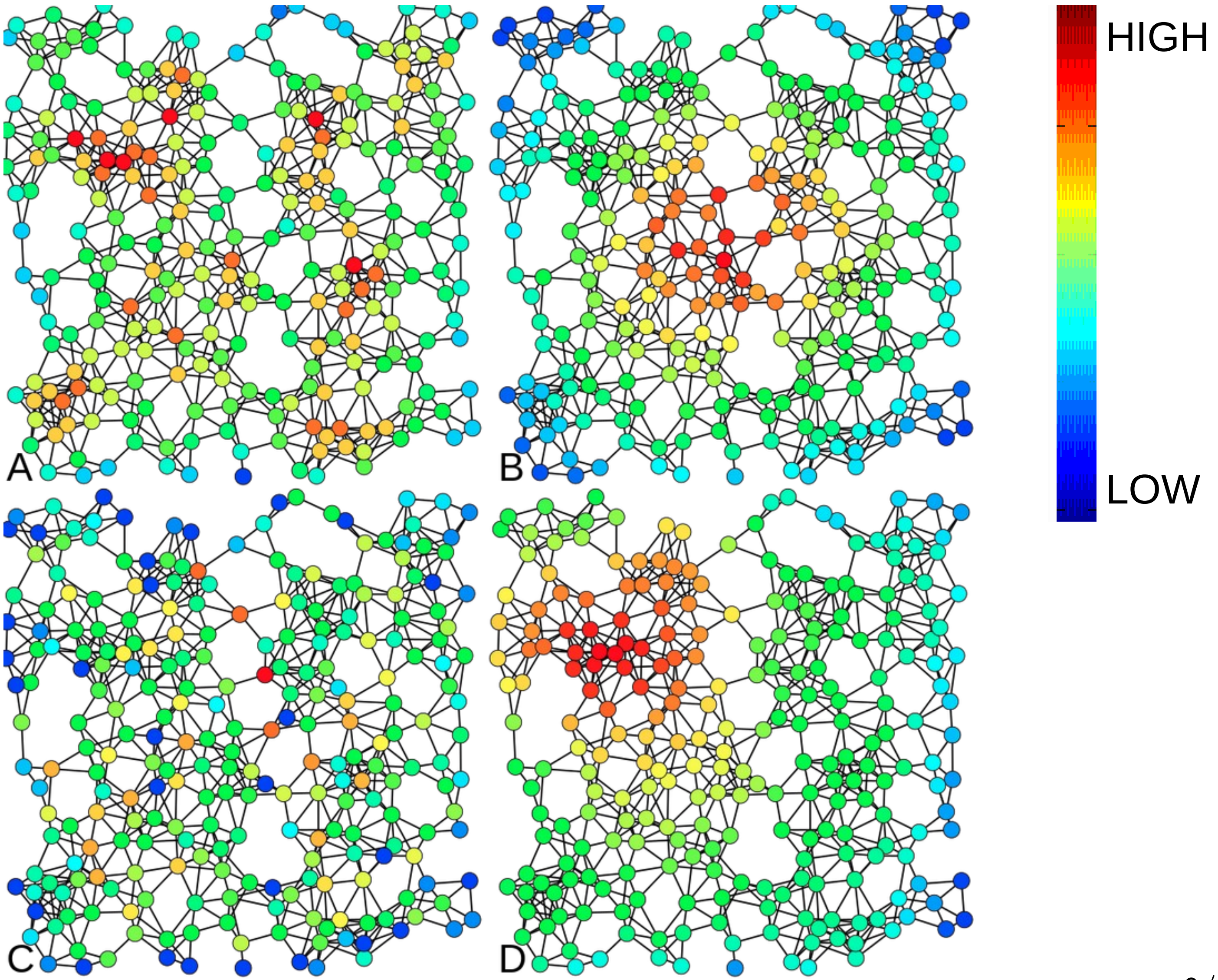
http://rnav.labri.fr/rNAV2.0_public_version/online_doc/_images/ComputeBetweennessCentrality.png

A: Degree

B: Closeness

C: Betweenness

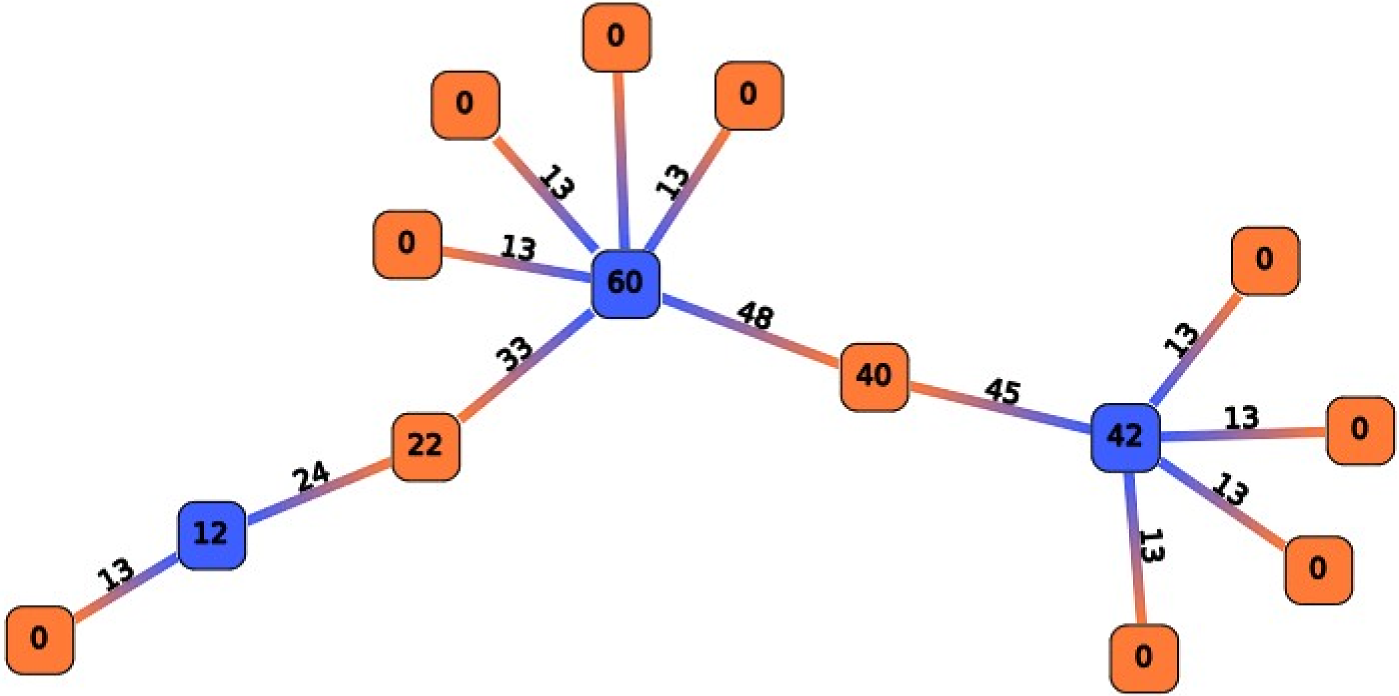
D: PageRank



Edge Betweenness

An **edge** has high betweenness if it is part of many shortest-paths

... how to compute this efficiently?



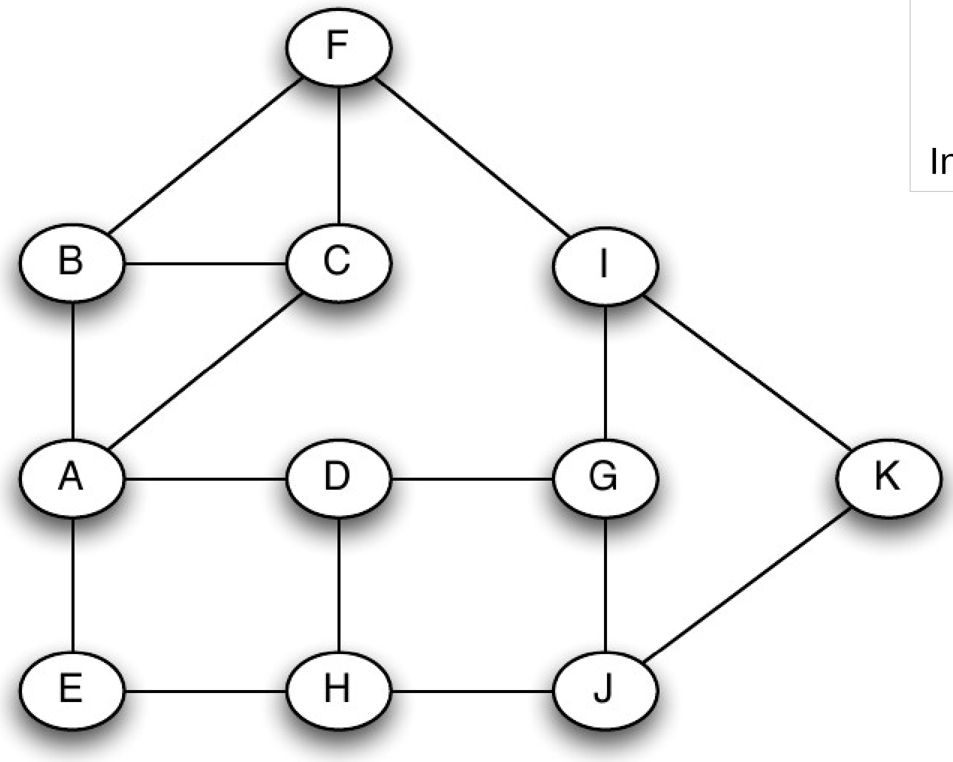
http://rnav.labri.fr/rNAV2.0_public_version/online_doc/_images/ComputeBetweennessCentrality.png

Algorithm [Brandes, Newman]

- For every node u in V
  - Layer the graph performing a BFS from u
  - For every node v in V, $v \neq u$, sorted by layer
    - Assign to v a number $s(v)$ indicating how many shortest paths from u arrive to v
  - For every node v in V, $v \neq u$, sorted by reverse layer
    - Score to distribute $= 1 +$ score from children
    - Add score to parent edges in proportion to $s(v)$
- In the end divide all edge scores by two

Ulrich Brandes. A faster algorithm for betweenness centrality. Journal of Mathematical Sociology, 25:163–177, 2001.
Mark E. J. Newman. Scientific collaboration networks: II. Shortest paths, weighted networks, and centrality. Physical Review E, 64:016132, 2001.

Example



For every node u in V

- Layer the graph performing a BFS from u
- For every node v in V, v≠u, sorted by layer
  - Assign to v a number s(v) indicating how many shortest paths from u arrive to v
- For every node v in V, v≠u, sorted by reverse layer
  - Score to distribute = 1 + score from children
  - Add score to distribute to parent edges in proportion to s(v)

In the end divide all edge scores by two

Example

For every node u in V
- Layer the graph performing a BFS from u
- **For every node v in V, v≠u, sorted by layer**
  - **Assign to v a number s(v) indicating how many shortest paths from u arrive to v**
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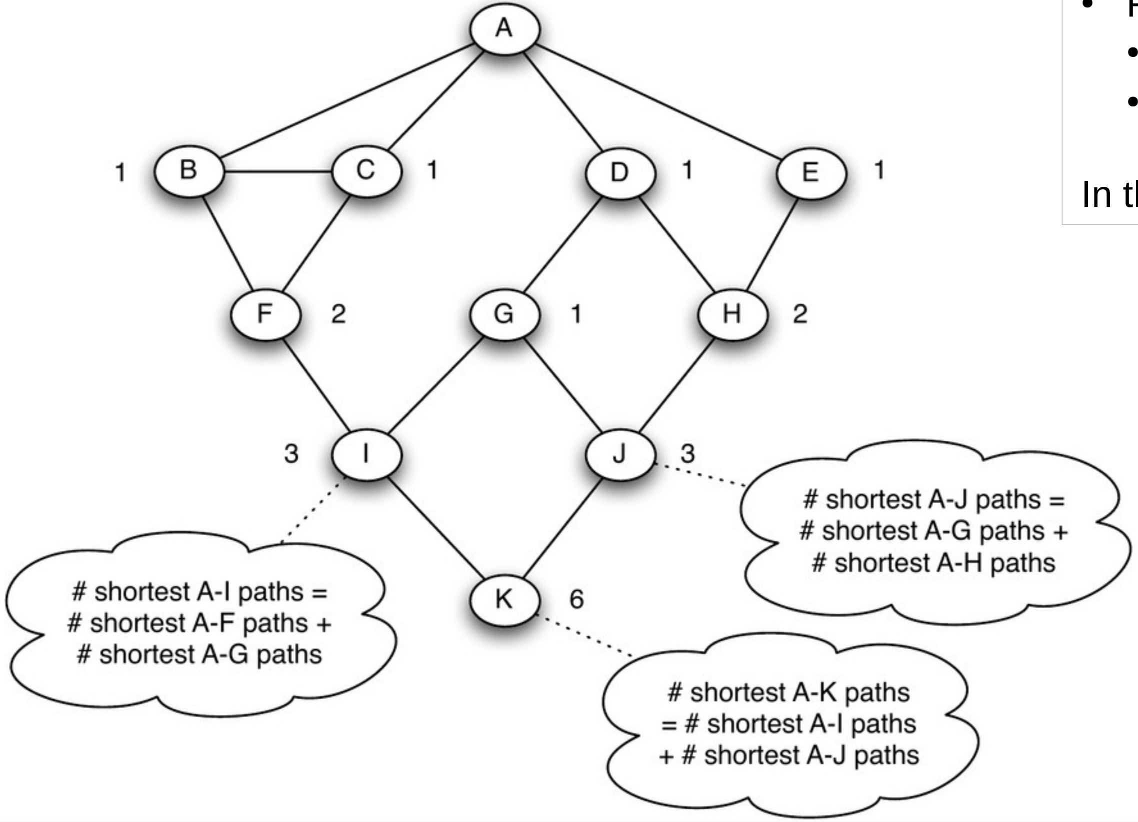


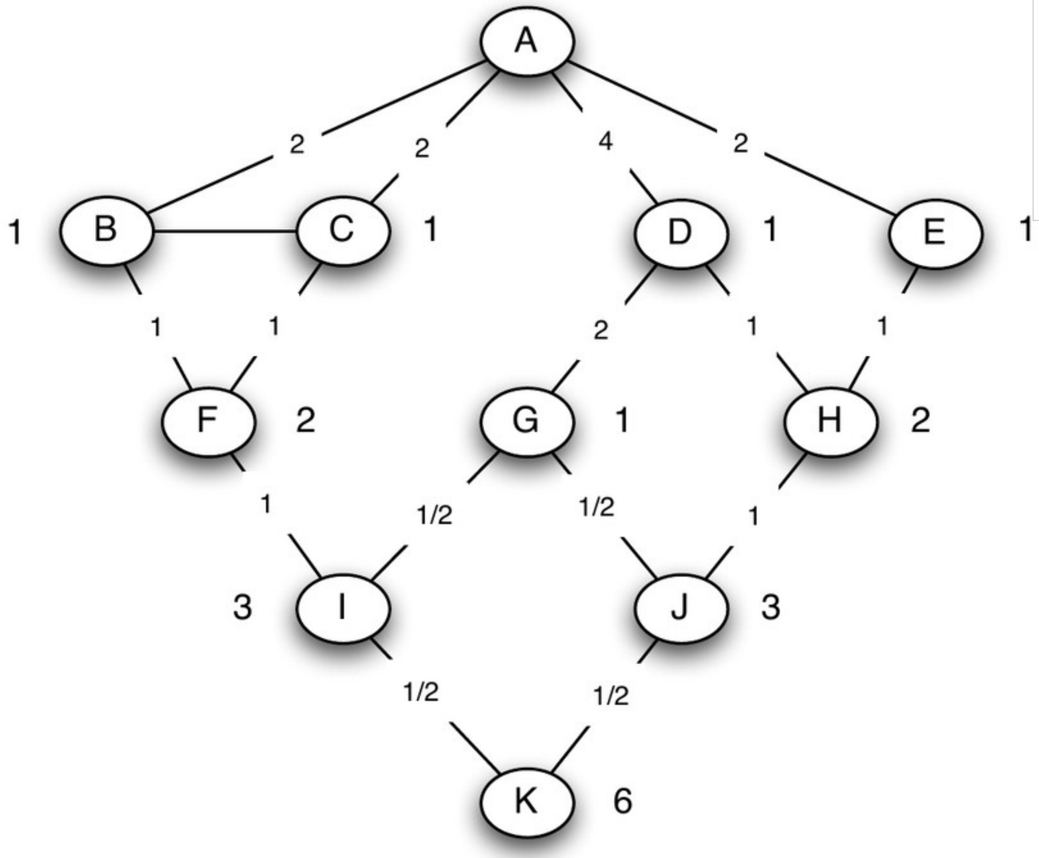
All nodes in layer 1 get s(v)=1

Remaining nodes: simply add s(.) of their parents

Example

For every node u in V
- Layer the graph performing a BFS from u
- For every node v in V, v≠u, sorted by layer
  - Assign to v a number s(v) indicating how many shortest paths from u arrive to v
- **For every node v in V, v≠u, sorted by rev. layer**
  - **Score to distribute = 1 + score from children**
  - **Add score to distribute to parent edges in proportion to s(v)**

In the end divide all edge scores by two



Nodes without children distribute a score of 1

Other nodes distribute 1 + whatever they receive from their children

Result

For every node u in V
- Layer the graph performing a BFS from u
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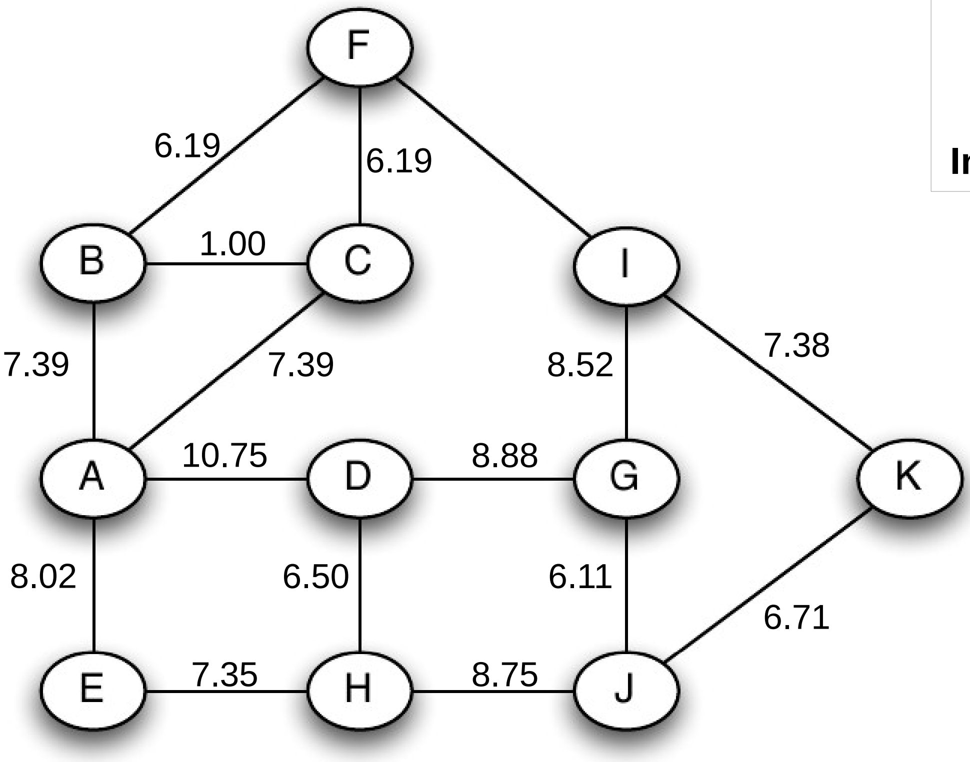
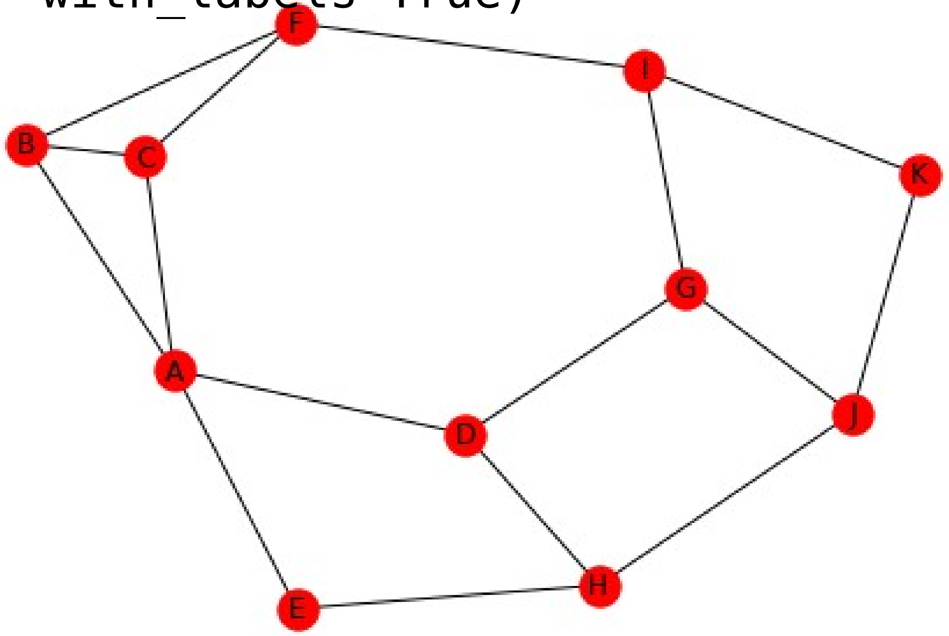


Computed using NetworkX (edge betweenness)

NetworkX code

```
import networkx as nx
g = nx.Graph()
g.add_edge("A", "B")
g.add_edge("A", "C")
g.add_edge("A", "D")
g.add_edge("A", "E")
g.add_edge("B", "C")
g.add_edge("B", "F")
g.add_edge("C", "F")
g.add_edge("D", "G")
g.add_edge("D", "H")
g.add_edge("E", "H")
g.add_edge("F", "I")
g.add_edge("G", "I")
g.add_edge("G", "J")
g.add_edge("H", "J")
g.add_edge("I", "K")
g.add_edge("J", "K")
nx.edge_betweenness(g, normalized=False)
```

```
nx.draw_spring(g,
with_labels=True)
```



Exercise

Try to compute it by inspection first

Then use the algorithm;
you should get the same results

For every node u in V
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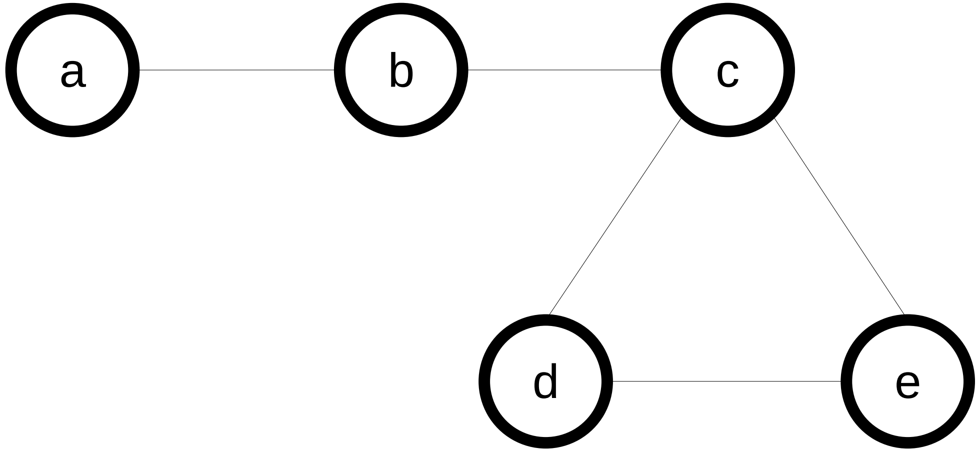


Fractional values?

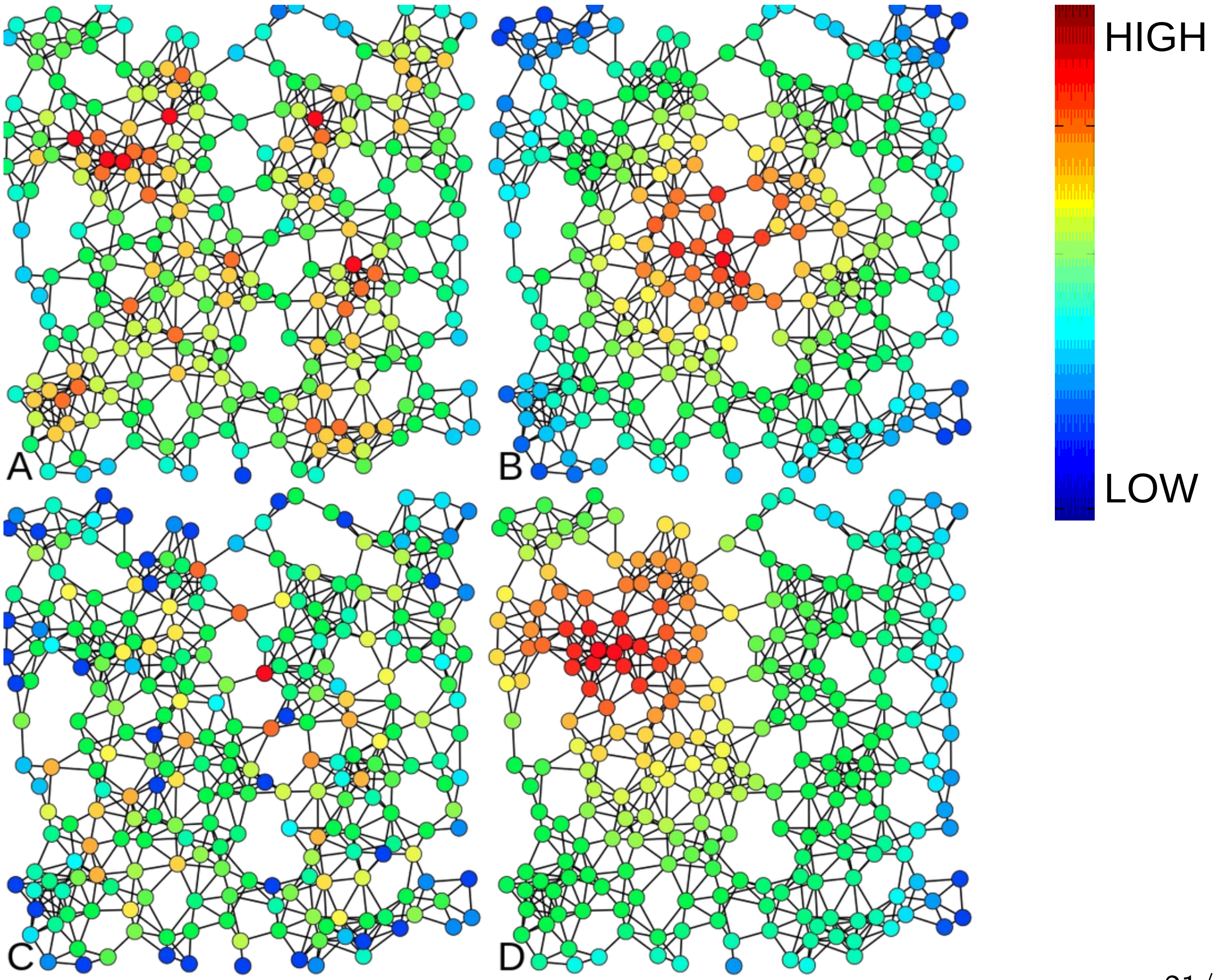
- In a graph with cycles, you may get **fractional values** of the edge betweenness for an edge

A: Degree

B: Closeness

C: Betweenness

D: PageRank



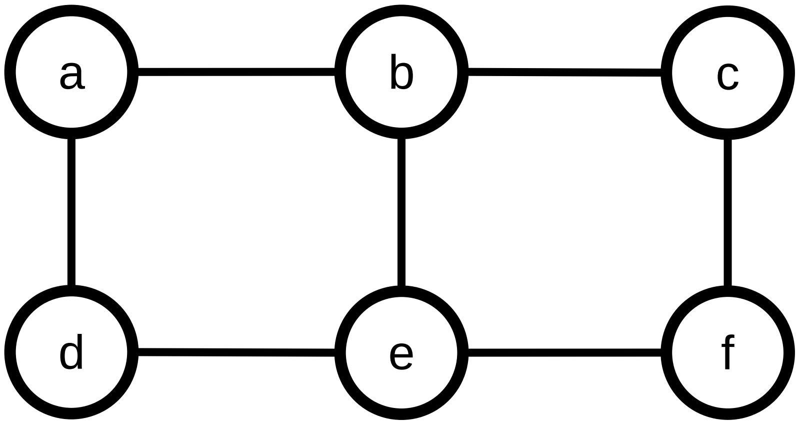
Summary

Things to remember

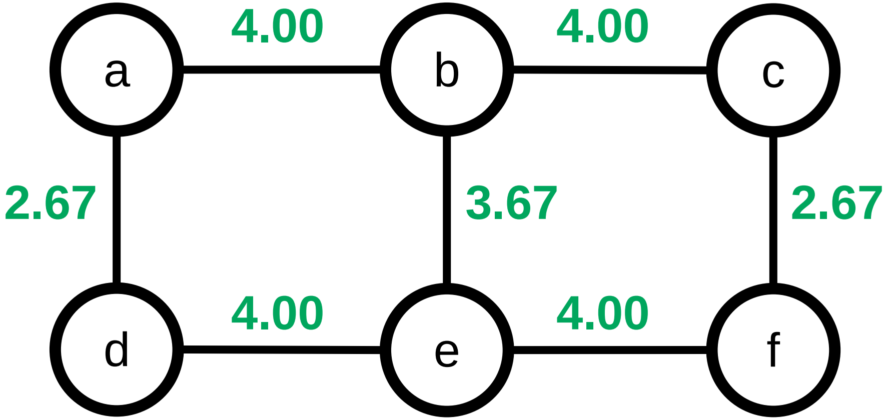
- Closeness and harmonic closeness
- Node and edge betweenness
- Practice running the Brandes-Newman algorithm on small graphs
- Write code to execute the Brandes-Newman algorithm

Practice on your own

- Compute edge betweenness on this graph



Practice on your own (cont.)



If you don't get this result, check:

https://www.youtube.com/watch?v=uYjWbp8VC7c

Two constructive problems

1. Sketch a graph of N nodes in which a node, which you should mark with an asterisk (*), should have betweenness approximately equal to N and closeness approximately $1/N$ for large N. Explain briefly.

2. Sketch a graph of N nodes in which a node, which you should mark with an asterisk (*), should have betweenness approximately equal to N and closeness approximately $2/N^2$ for large N. Explain briefly.

Do not use a concrete N. Use a general N, for instance by using the ellipsis ($\ldots$) to denote multiple nodes.