Homophily and triangles

Social Networks Analysis and Graph Algorithms

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Contents

- *This is all about friends, and friends of friends*
- Homophily
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Sources

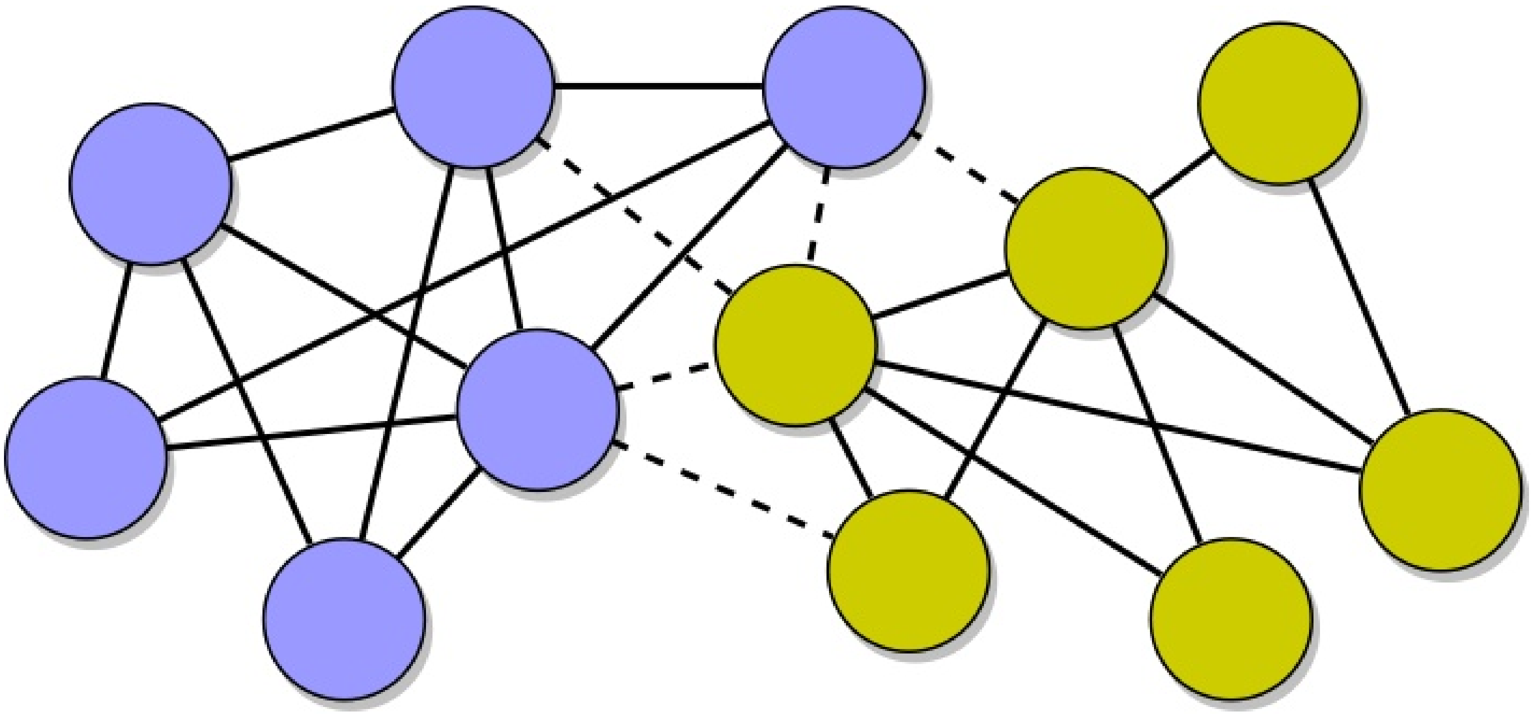
- A. L. Barabási (2016). Network Science – Chapter 02
- F. Menczer, S. Fortunato, C. A. Davis (2020). A First Course in Network Science – Chapter 02
- URLs cited in the footer of specific slides

Homophily

Who is a friend? [Homophily]

- In social networks, nodes have characteristics that influence their preferences
  - Age, gender identity, ethnicity, sexual preference, location, topics of interest, artistic sensitivities, ...
- People tend to befriend those who are like them: that is called **homophily**

“Birds of a feather flock together”



Source: Menczer, Fortunato, David: A First Course on Networks Science. Cambridge, 2020.

Quantifying homophily

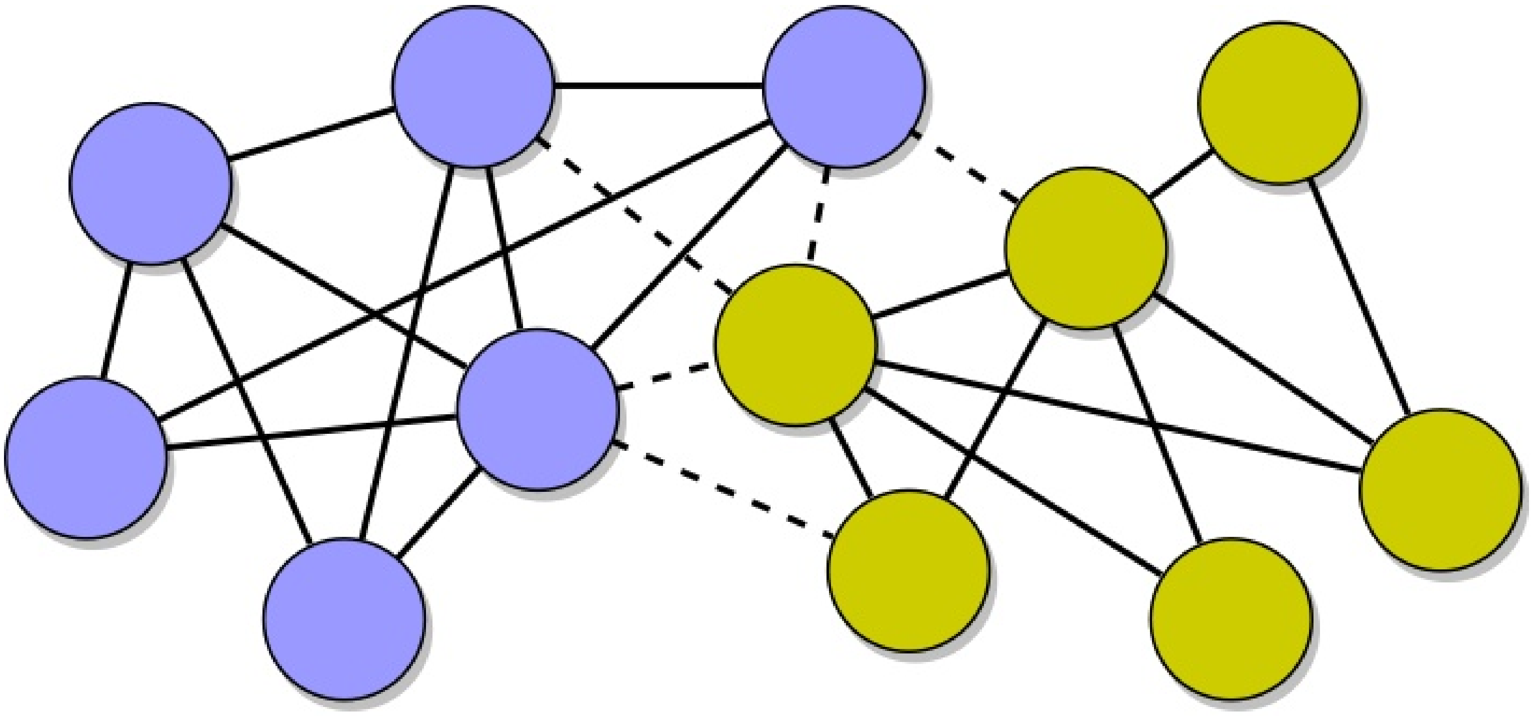
- Let G be a graph of N nodes: N_a “yellow” and N_b “blue”

  $$N = N_a + N_b$$

- Let G have L undirected links (including self loops), of which L_{aa} connect yellow to yellow, L_{ab} connect yellow to blue, and L_{bb} connect blue to blue

  $$L = L_{aa} + L_{ab} + L_{bb} \qquad L_a = L_{aa} + L_{ab} \qquad L_b = L_{bb} + L_{ab}$$

$$N_a = 6,\ N_b = 6,\ L_a = 14,\ L_b = 16,\ L_{ab} = 5,$$



Source: Menczer, Fortunato, David: A First Course on Networks Science. Cambridge, 2020.

Expected links across groups

If yellow nodes have L_a links placed at random (incl. self loops), how many should go to a blue node?

$$L_a \left(\frac{N_b}{N} \right)$$

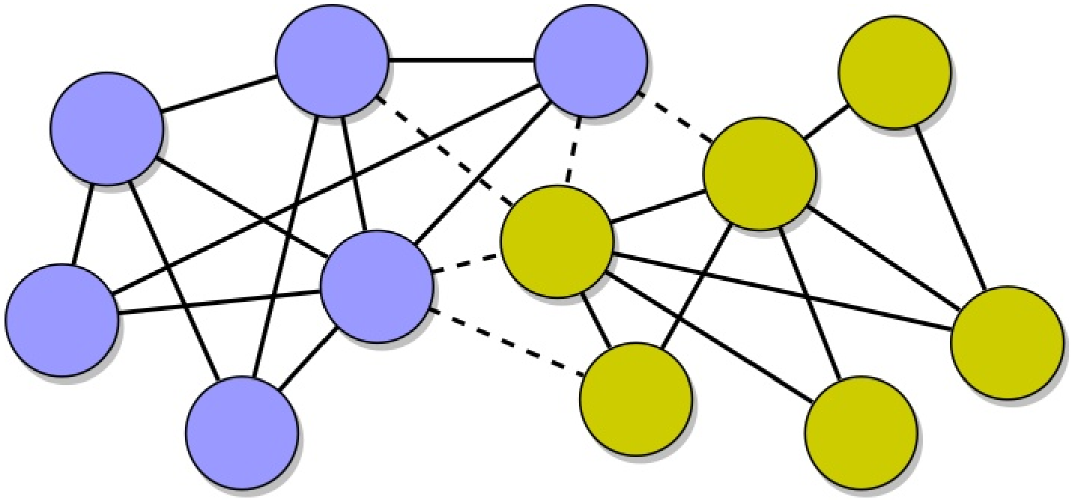
Quantifying homophily of a group

- We compare observed against the **expected number of links crossing to the other group**

$<1 \Rightarrow$ heterophily $\quad$ $1 \Rightarrow$ neutral $\quad$ $>1 \Rightarrow$ homophily

$$\mathrm{Homophily}(a) = \frac{L_{ab}}{L_a\left(\frac{N_b}{N}\right)} \qquad \mathrm{Homophily}(b) = \frac{L_{ab}}{L_b\left(\frac{N_a}{N}\right)}$$

$$\text{Homophily}(a) = 1 - \frac{L_{ab}}{L_a\left(\frac{N_b}{N}\right)} = 1 - \frac{5}{14\left(\frac{6}{12}\right)} = \frac{2}{7}$$

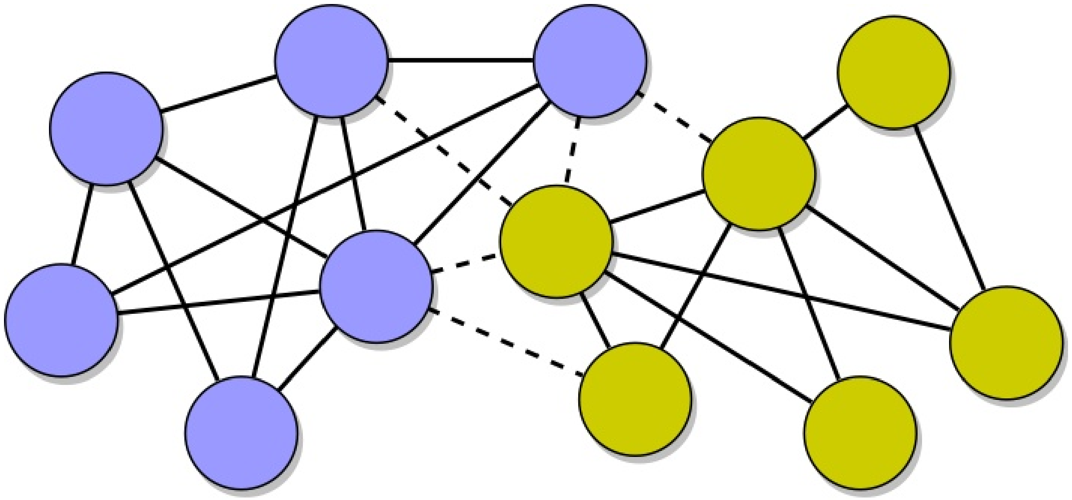


Yellow nodes are homophilic

$N_a = 6$, $N_b = 6$, $L_a = 14$, $L_b = 16$, $L_{ab} = 5$,

Source: Menczer, Fortunato, David: A First Course on Networks Science. Cambridge, 2020.

$$\text{Homophily}(b) = 1 - \frac{L_{ab}}{L_b\left(\frac{N_a}{N}\right)} = 1 - \frac{5}{16\left(\frac{6}{12}\right)} = \frac{3}{8}$$



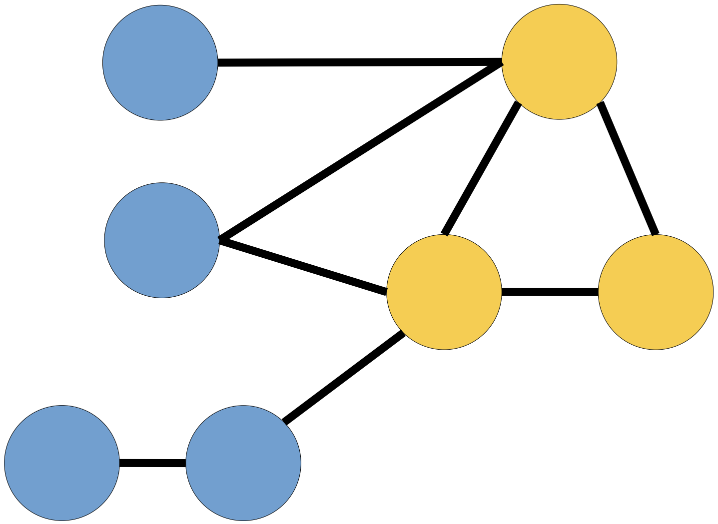
Blue nodes are homophilic

$N_a = 6$, $N_b = 6$, $L_a = 14$, $L_b = 16$, $L_{ab} = 5$,

Source: Menczer, Fortunato, David: A First Course on Networks Science. Cambridge, 2020.

Exercise

Compute homophily of both groups and indicate if each group is homophilic, heterophilic, or neutral



$$\text{Homophily}(a) = \frac{L_{ab}}{L_a \left(\frac{N_b}{N}\right)}$$

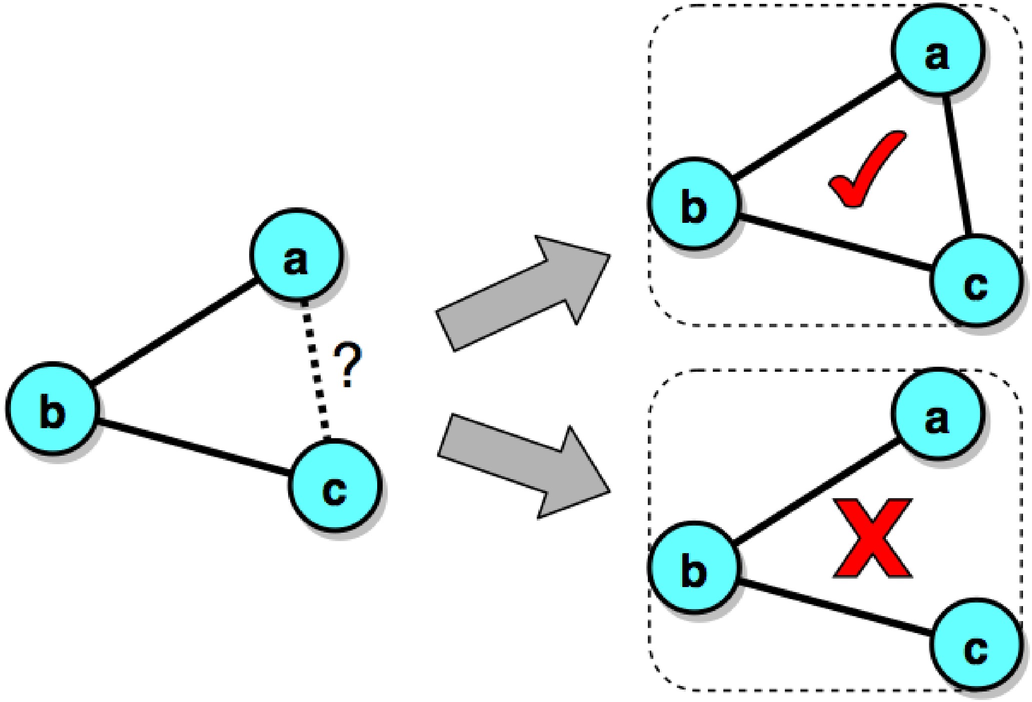
$$\text{Homophily}(b) = \frac{L_{ab}}{L_b \left(\frac{N_a}{N}\right)}$$

Pin board: https://upfbarcelona.padlet.org/chato/iig2u83qdzk4y7xc

Clustering coefficient

Who is a friend? [Triangle closure]

A prevalent way in which we form friendships is by befriending **friends of friends**

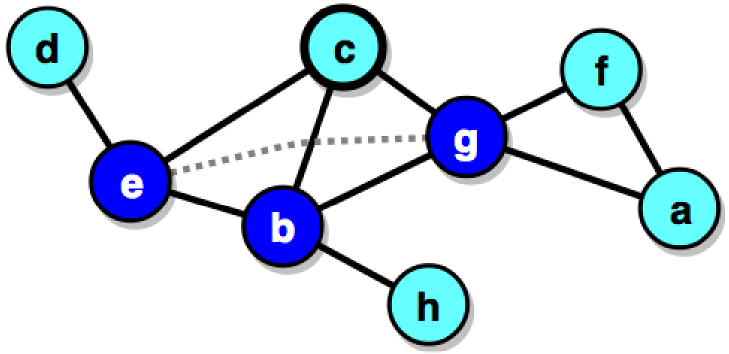


Source: Menczer, Fortunato, David: A First Course on Networks Science. Cambridge, 2020.

Tendency to form triangles

- Other processeses of link formation encourage the closing of “V”’s into triangles
  - You’re more likely to follow an account u because you see content posted by u and re-posted by an account v that you already follow
- Let’s quantify this

Example 1

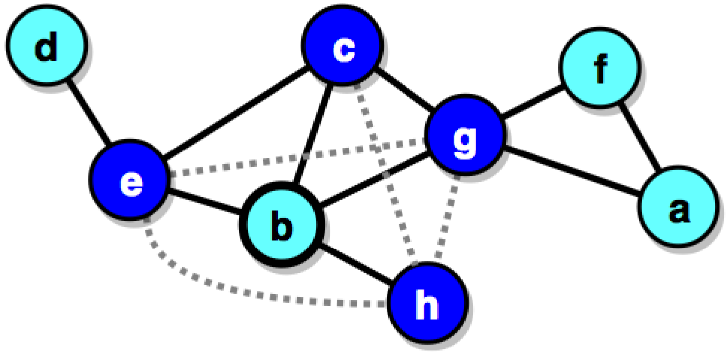


Node c has 3 neighbors: a, b, g

They form two triangles out of the possible 3 (the missing one is drawn with a dotted line)

Source: Menczer, Fortunato, David: A First Course on Networks Science. Cambridge, 2020.

Example 2



Node b has 4 neighbors: e, c, g, h

They form two triangles out of the possible 6 (the missing ones are drawn with a dotted line)

Source: Menczer, Fortunato, David: A First Course on Networks Science. Cambridge, 2020.

Remember

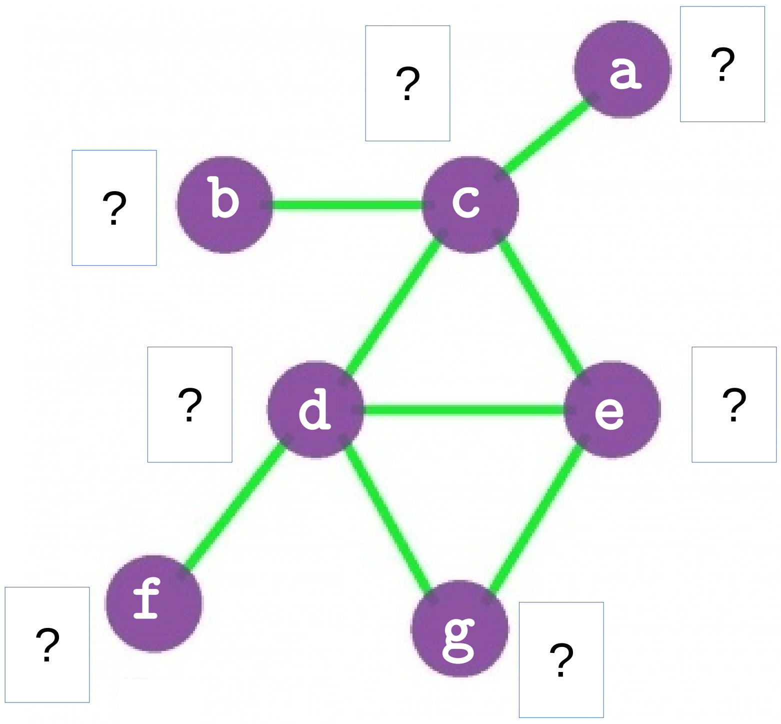
The maximum number of links between k nodes is

$$\frac{k(k-1)}{2}$$

Local clustering coefficient

- The **local clustering coefficient** C_i is a property of a node i
- Let L_i represent the number of links among neighbors of node i

$$C_i = \frac{L_i}{\frac{k_i(k_i-1)}{2}} = \frac{2L_i}{k_i(k_i-1)} \qquad C_i \triangleq 0 \text{ if } k_i \leq 1$$



Exercise

What is the local clustering coefficient of each node?

$$C_i = \frac{2L_i}{k_i(k_i - 1)}$$

$$C_i \triangleq 0 \text{ if } k_i \leq 1$$

Pin board: https://upfbarcelona.padlet.org/chato/v0apheshv2l4hbot

Average clustering coefficient (*“global clustering coefficient”*)

The **average clustering coefficient** is a property of the entire graph

$$\langle C \rangle = \frac{1}{N} \sum_{i=1}^{N} C_i$$

Sometimes this is called the *curvature* of a graph

Summary

Things to remember

- How to quantify if a group is
  - Homophilic
  - Heterophilic
  - Neutral
- Local and global clustering coefficient

Practice on your own

- Determine if the set $\{C, D, G\}$ is homophilic or heterophilic
- Calculate local clustering coefficient of each node in this graph