

Hubs and Authorities

Social Networks Analysis and Graph Algorithms

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Sources

- D. Easley and J. Kleinberg (2010): Networks, Crowds, and Markets – Chapter 14
- Fei Li's lecture on PageRank (2011)
- Evimaria Terzi's lecture on link analysis (2013)
- URLs in the footer of specific slides

Motivation: rank search results

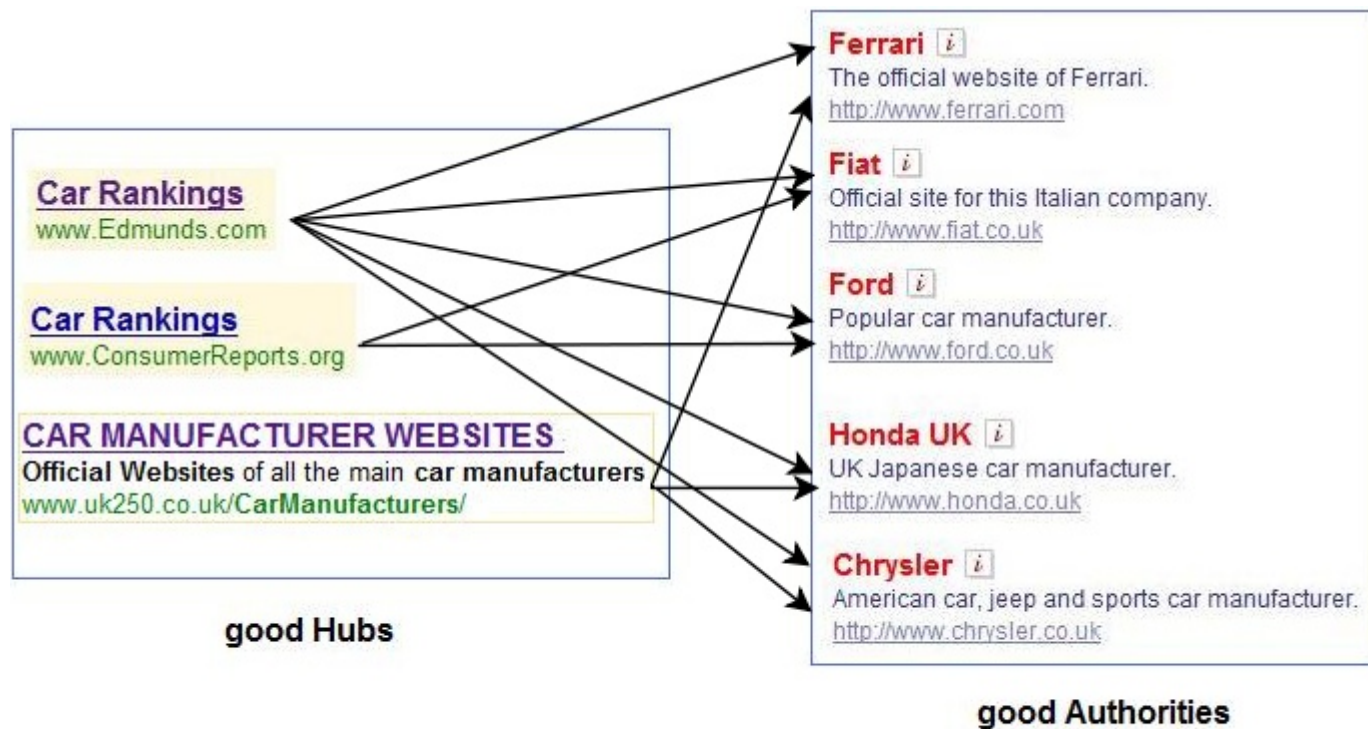
- Demand
 - Information needs are unclear and evolving
- Supply
 - From scarcity to abundance: “filter failure”

Purpose of Link-Based Ranking

- **Static (query-independent)** ranking
- **Dynamic (query-dependent)** ranking
- Applications:
 - Search in social networks
 - Search on the web

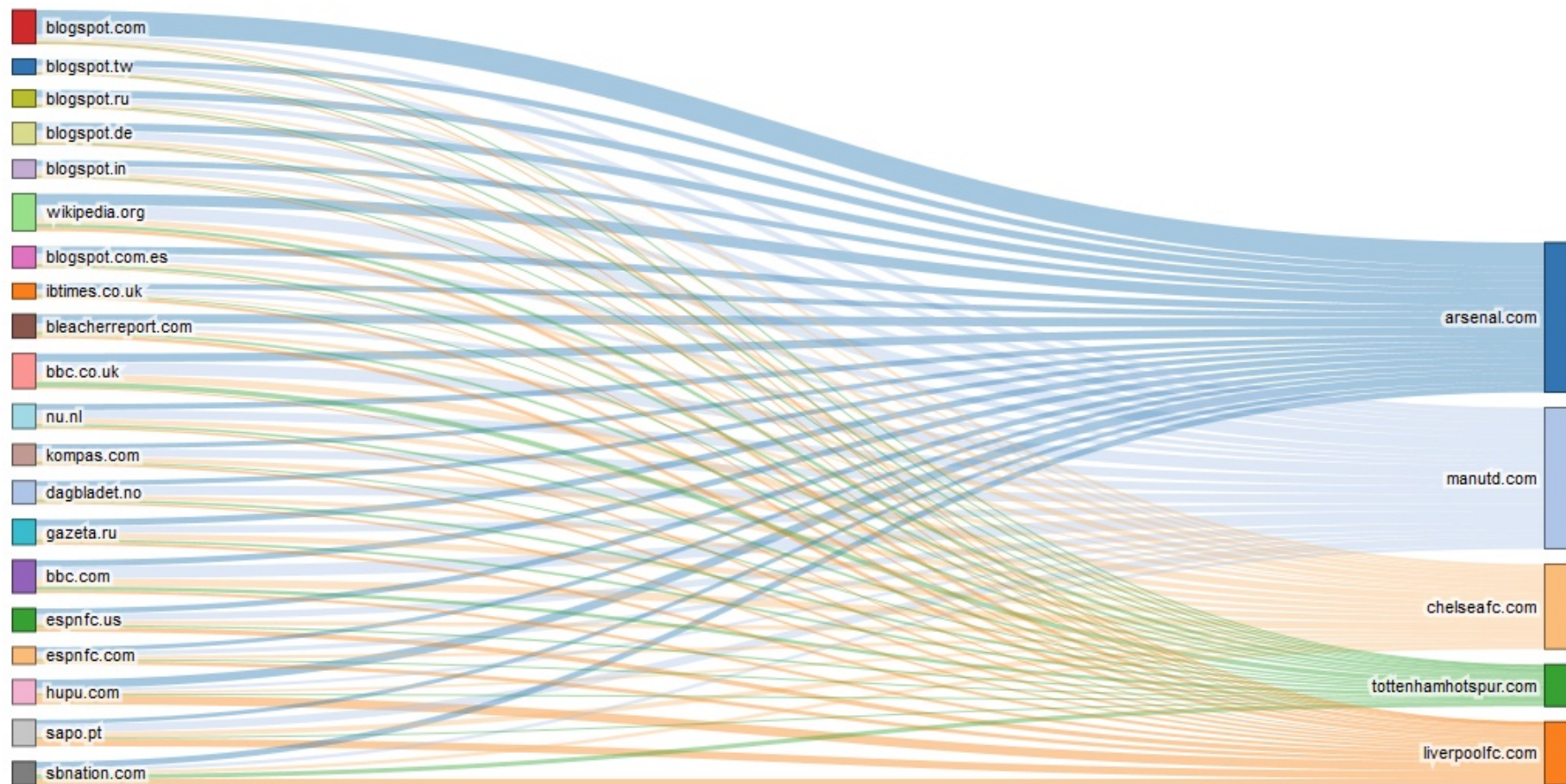
Why computing hubs and authorities?

Example 1: “top automobile makers”



Query: **Top automobile makers**

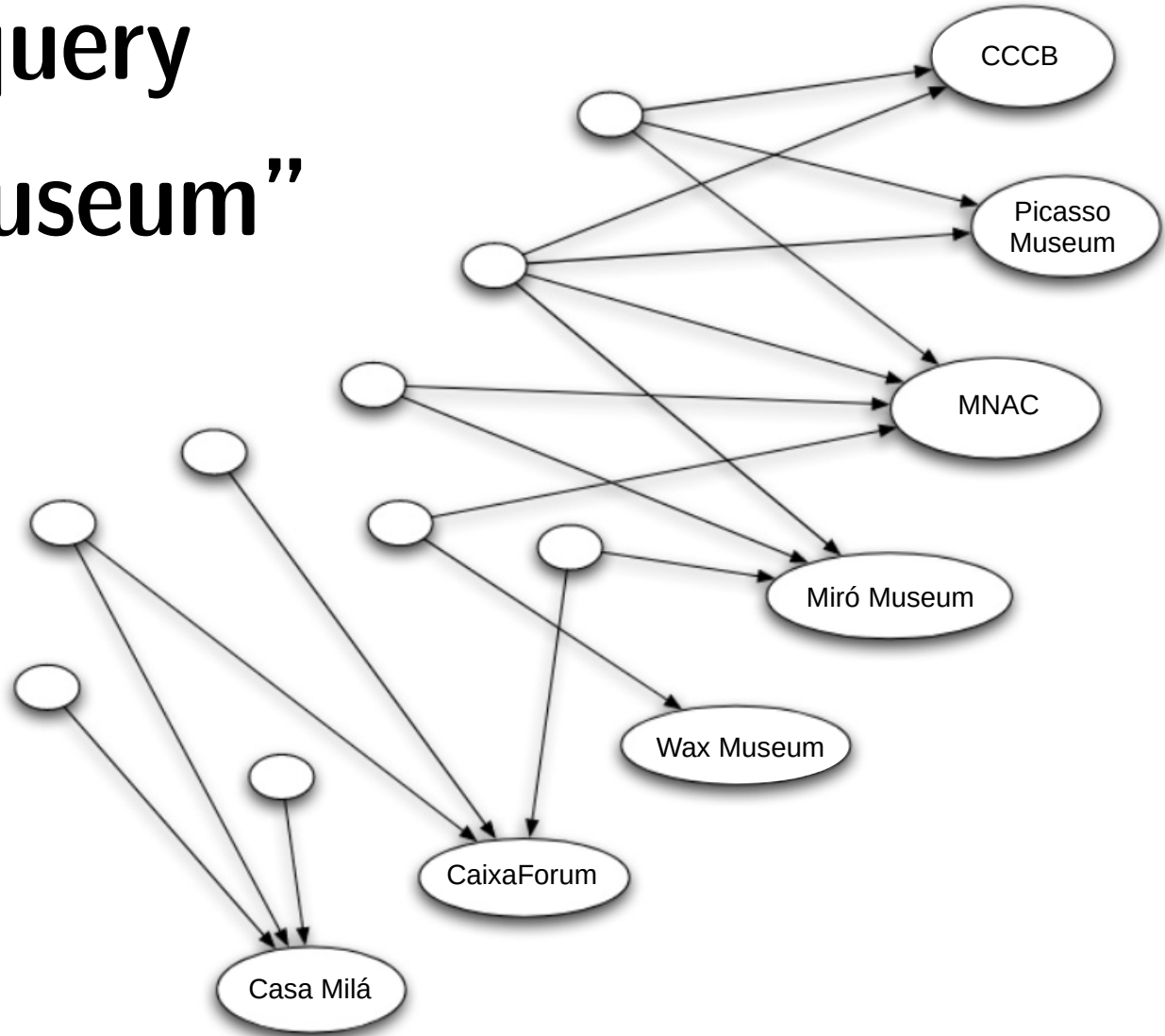
Example 2: UK football teams on the web



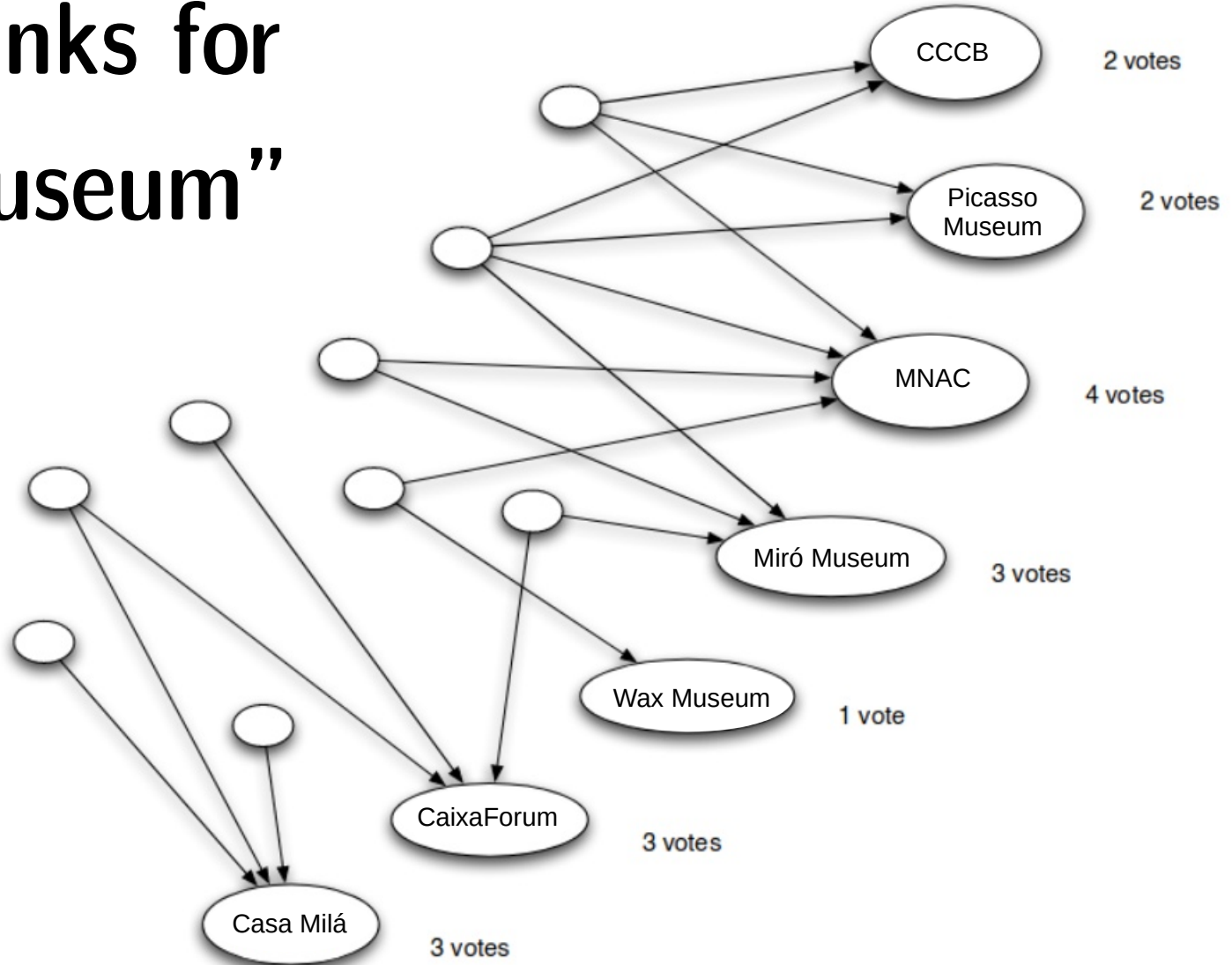
Example: query

“barcelona museum”

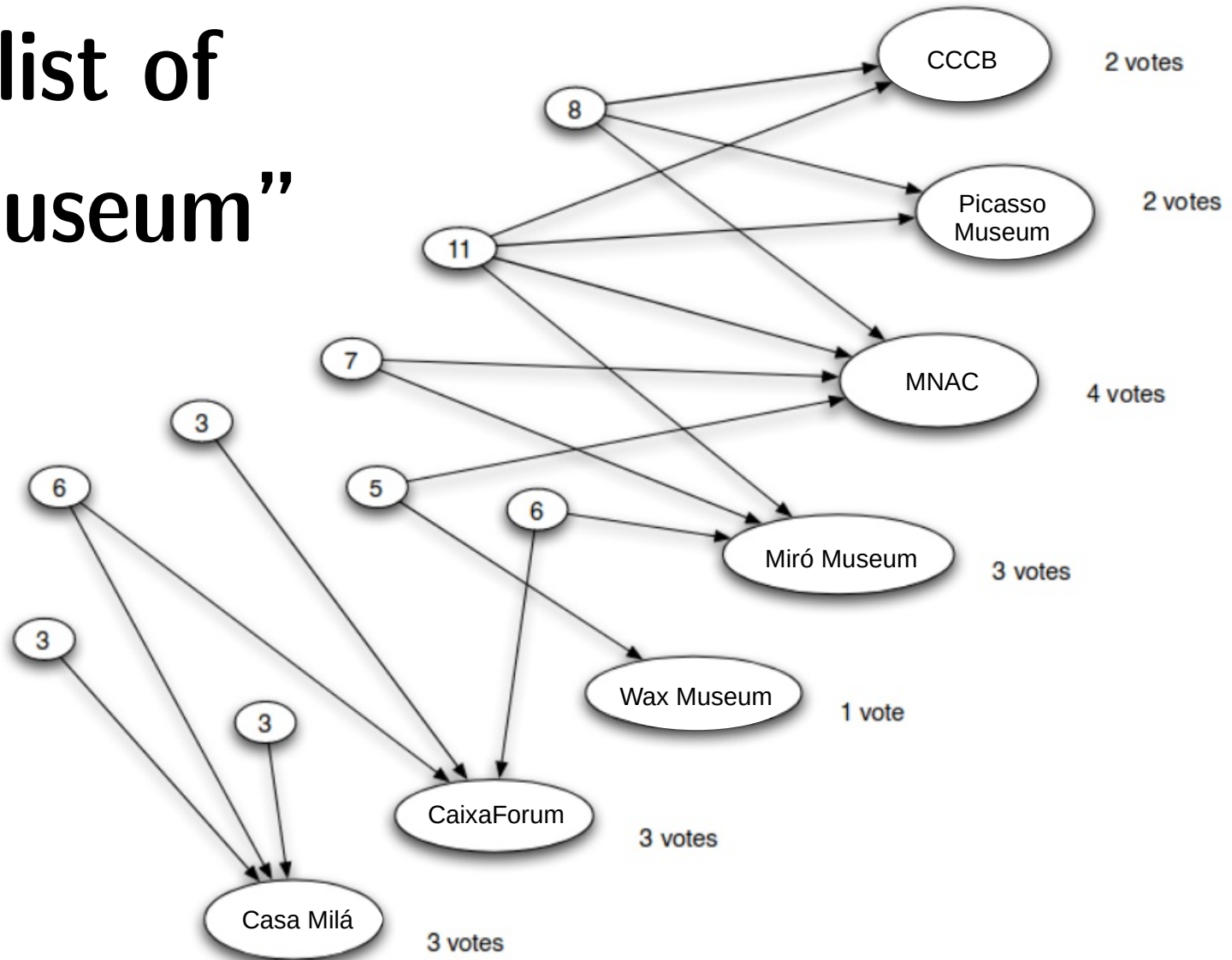
How would you rank
these pages?



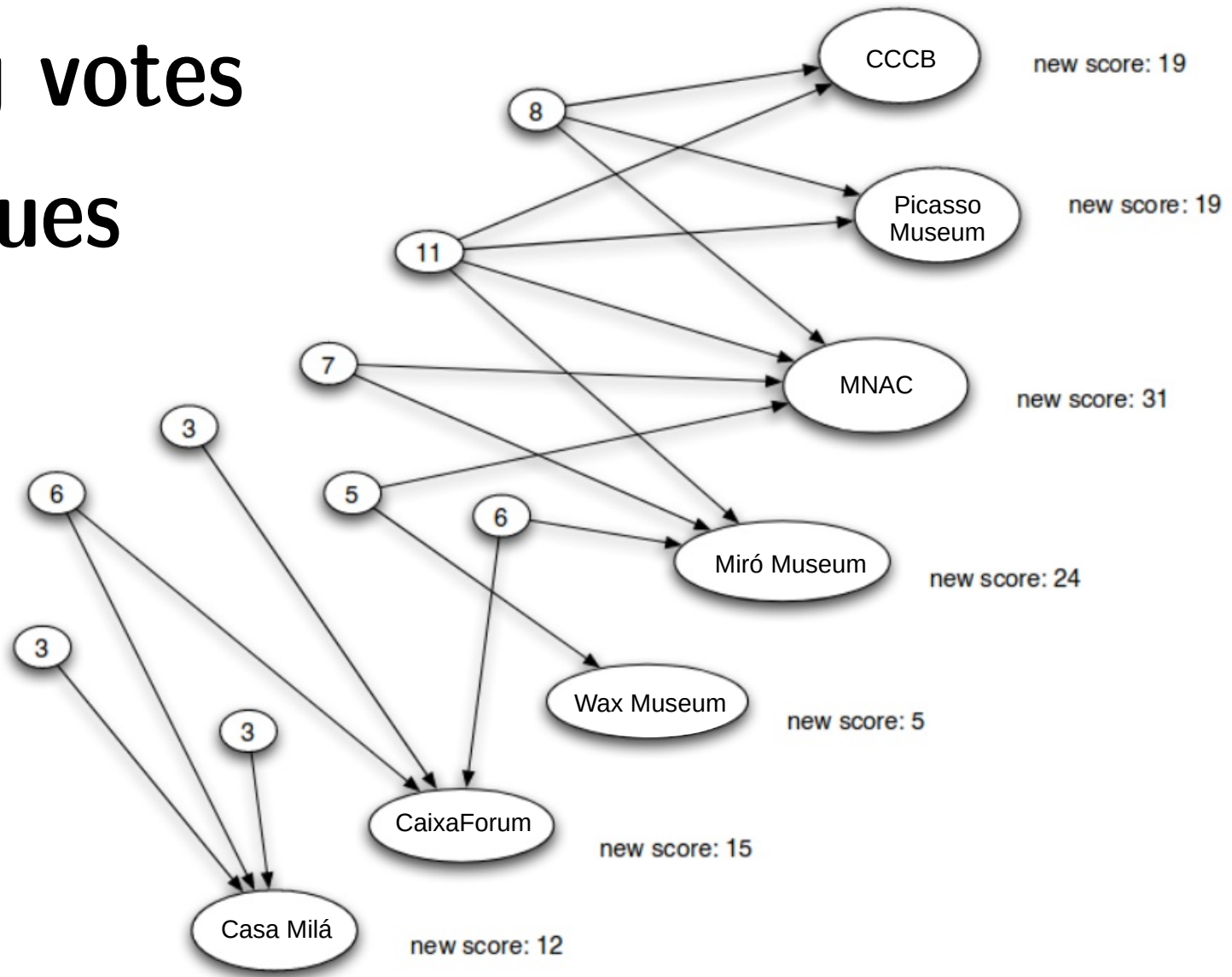
Counting in-links for “barcelona museum”



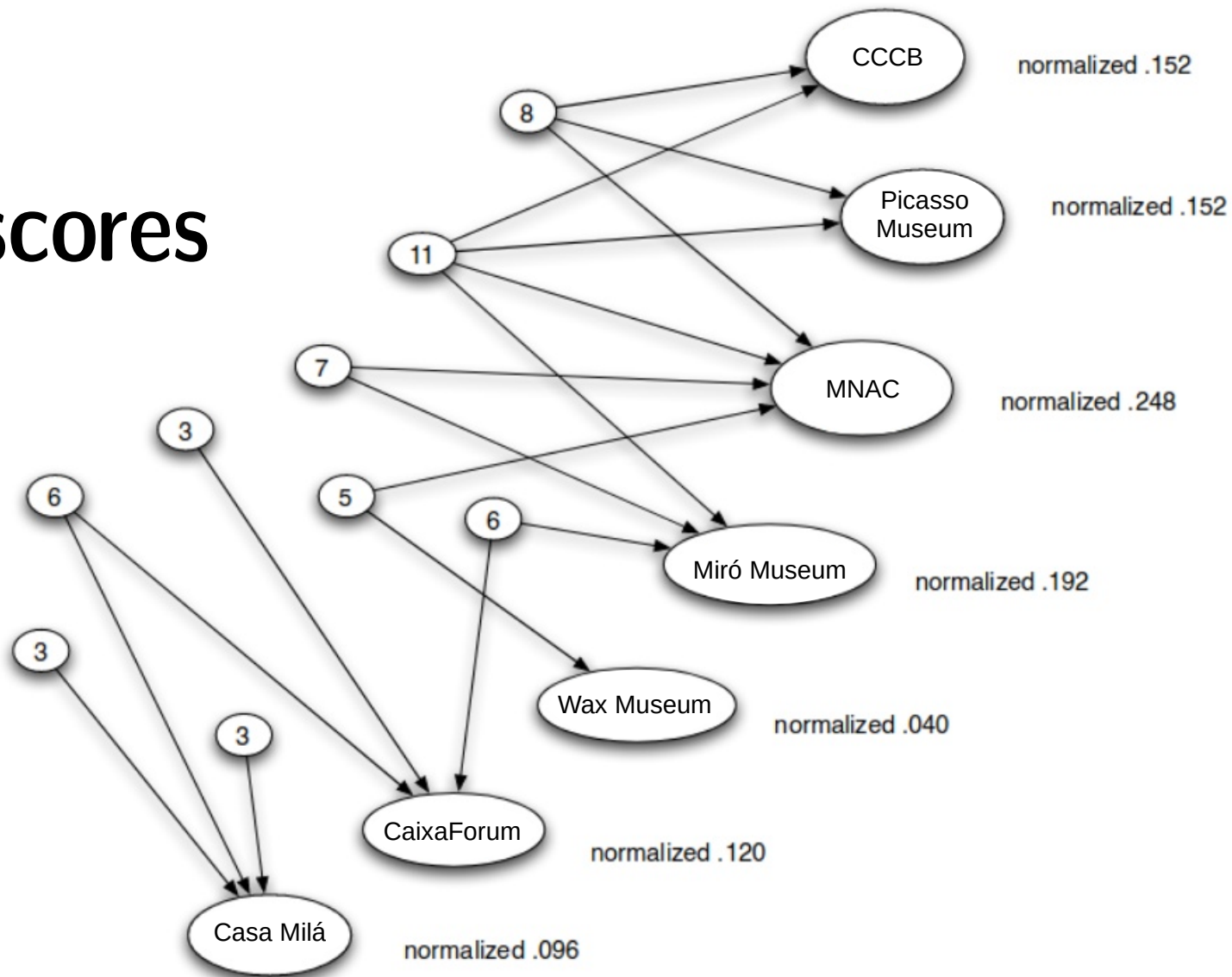
Value of a list of “barcelona museum”



Re-weighting votes by list values



Normalizing scores

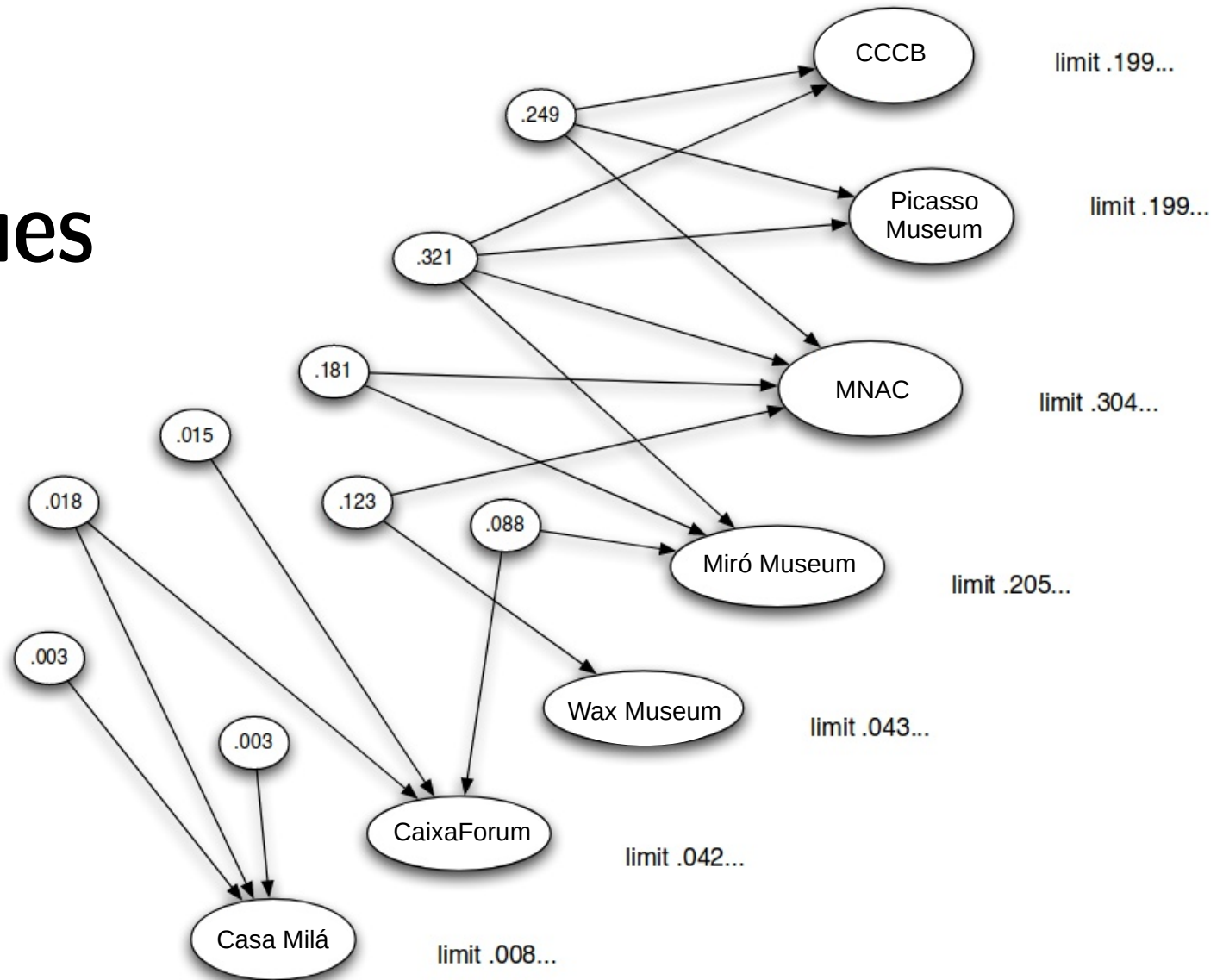


The idea behind Hubs and Authorities

[Kleinberg 1999]

- Highly-recommended items appear in high-value lists
- High-value lists contain highly-recommended items
- **Repeated improvement**
 - Re-calculate scores several times

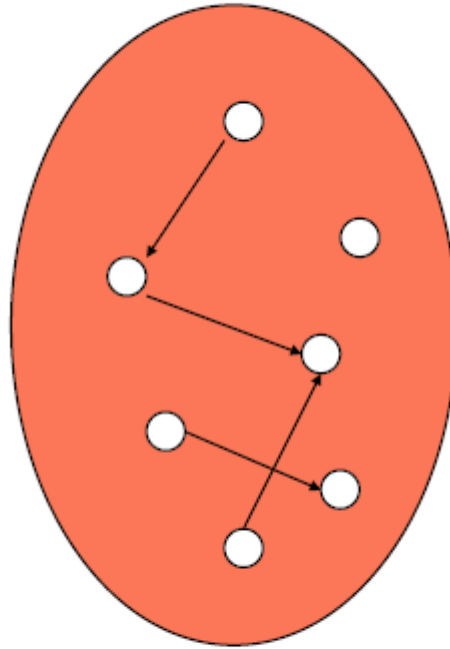
Limit values



This algorithm is called “HITS”

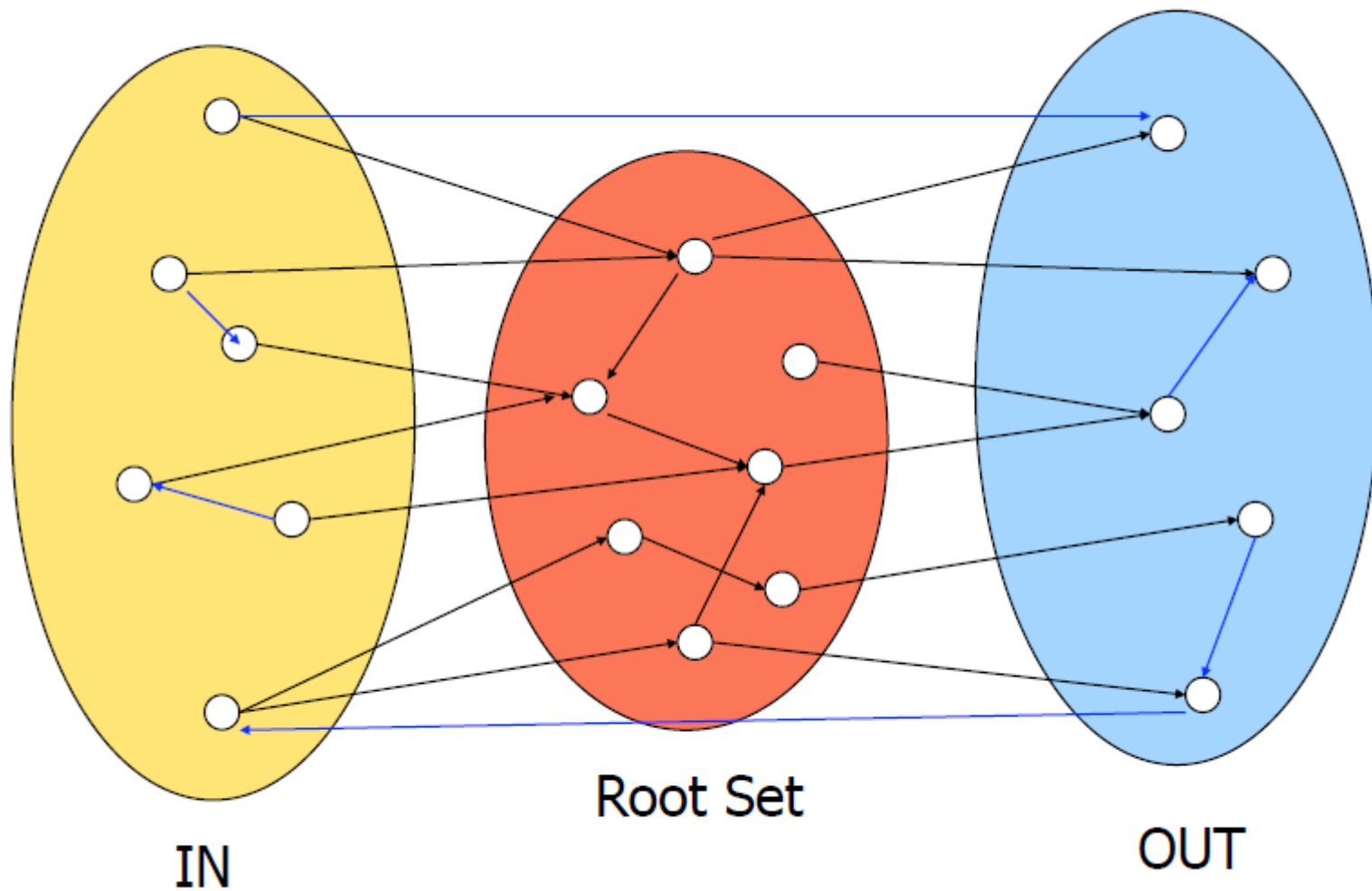
- *Jon M. Kleinberg. 1999. Authoritative sources in a hyperlinked environment. J. ACM 46, 5 (September 1999), 604-632. [[DOI](#)]*
- Query-dependent algorithm
 - Get pages matching the query
 - Expand to 1-hop neighborhood
 - Find pages with good out-links (“hubs”)
 - Find pages with good in-links (“authorities”)

Root set = matches the query



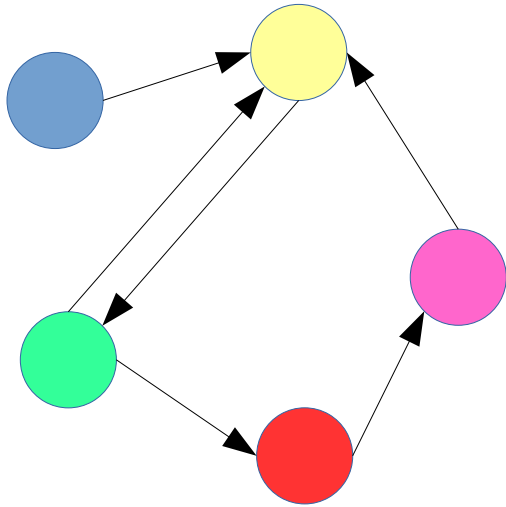
Root Set

Base set S = root set plus 1-hop neighbors

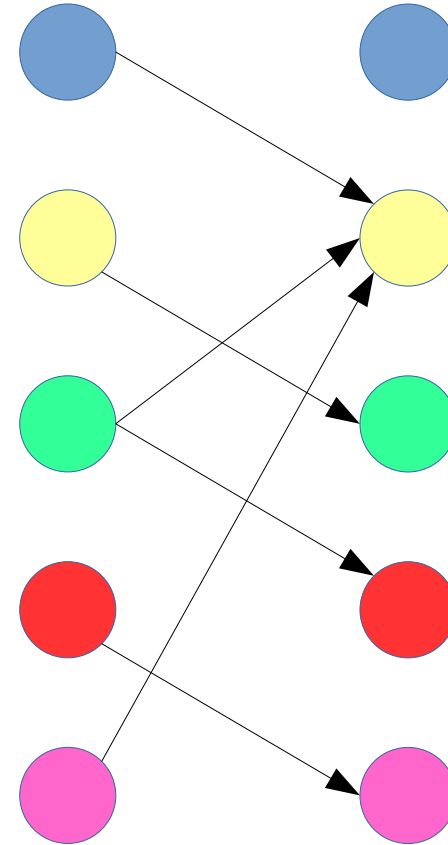
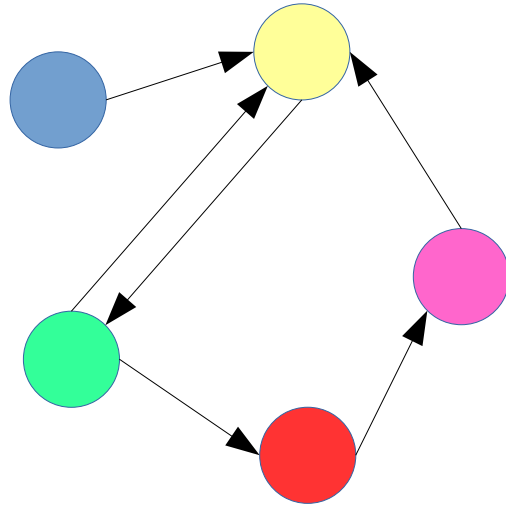


How to compute hubs and authorities

Base graph S of n nodes



Bipartite graph of $2n$ nodes



Bipartite graph of $2n$ nodes

0) Initialization:

$$h_i = \hat{h}_i = 1$$

1) Iteration:

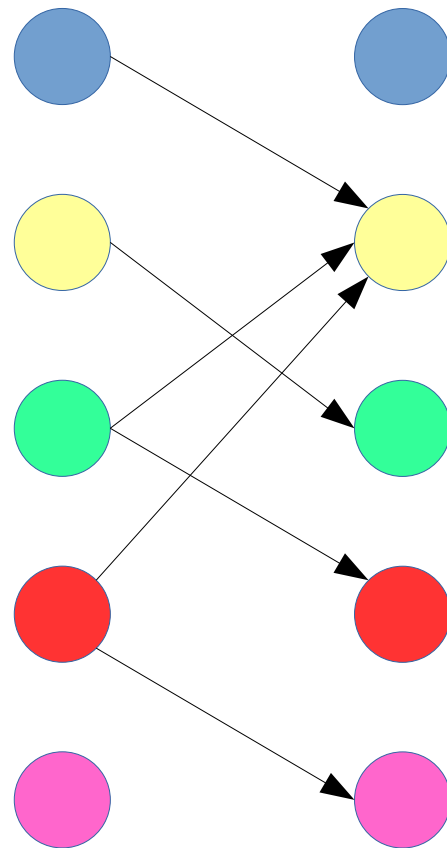
$$a_i = \sum_{j \rightarrow i} \hat{h}_j$$

$$h_i = \sum_{i \rightarrow j} \hat{a}_j$$

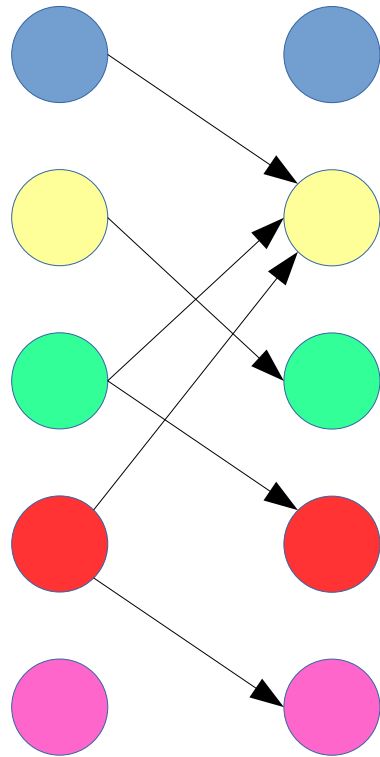
2) Normalization:

$$\hat{a}_i = \frac{a_i}{\sum_j a_j}$$

$$\hat{h}_i = \frac{h_i}{\sum_j h_j}$$



Exercise



$\hat{H}(1)$	$A(1)$	$\hat{A}(1)$	$H(2)$	$\hat{H}(2)$	$A(2)$	$\hat{A}(2)$
1	0					
1	3					
1	1					
1	1					
1	1					

Complete the table

Which one is the largest hub?

Which the largest authority?

Compare rankings

Rank by indegree = rank by auth?

Rank by outdegree = rank by hub?

Spreadsheet links: <https://upfbarcelona.padlet.org/chato/shyq9m6f2g2dh1bw>



What are we computing?

$$a^t = A^T h^{t-1}$$

$$h^t = A a^t$$

$$\text{replacing : } a^t = A^T A a^{t-1}$$

$$\text{after convergence : } a = A^T A a$$

- Vector a is an eigenvector of $A^T A$
- Conversely, vector h is an eigenvector of $A A^T$

Dealing with weighted graphs

(this is an option that does not normalize weights,
one can alternatively normalize them)

$$h_i = \hat{h}_i = 1$$

1) Iteration:

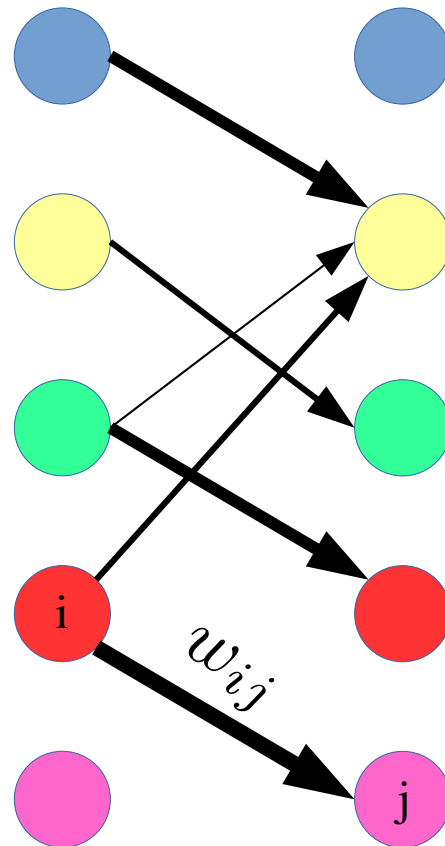
$$a_i = \sum_{j \rightarrow i} (w_{ji} \cdot \hat{h}_j)$$

$$h_i = \sum_{i \rightarrow j} (w_{ij} \cdot \hat{a}_j)$$

2) Normalization:

$$\hat{a}_i = \frac{a_i}{\sum_j a_j}$$

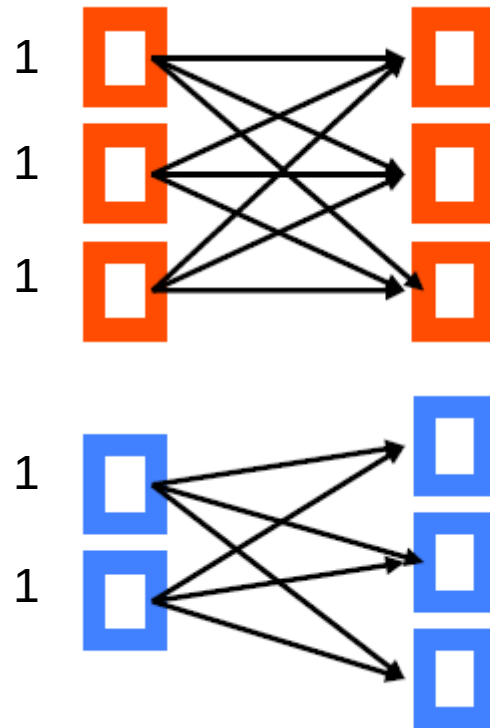
$$\hat{h}_i = \frac{h_i}{\sum_j h_j}$$



Problem: cliques and quasi-cliques

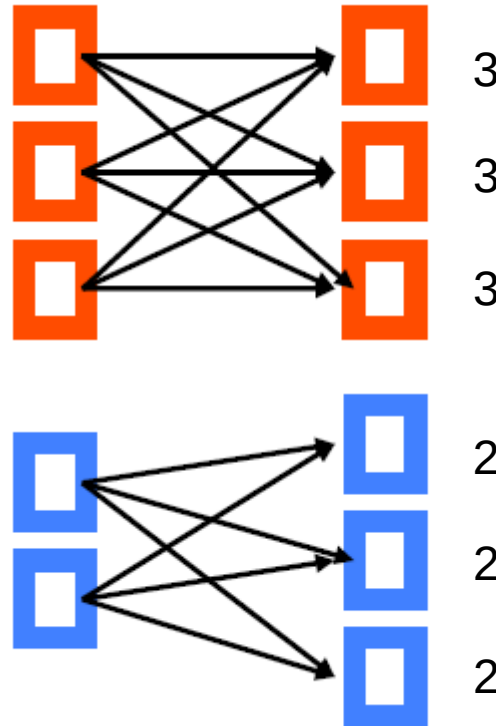
Problem: tightly-knit communities

- Example: a graph made of a $(3,3)$ -clique and a $(2,3)$ -clique



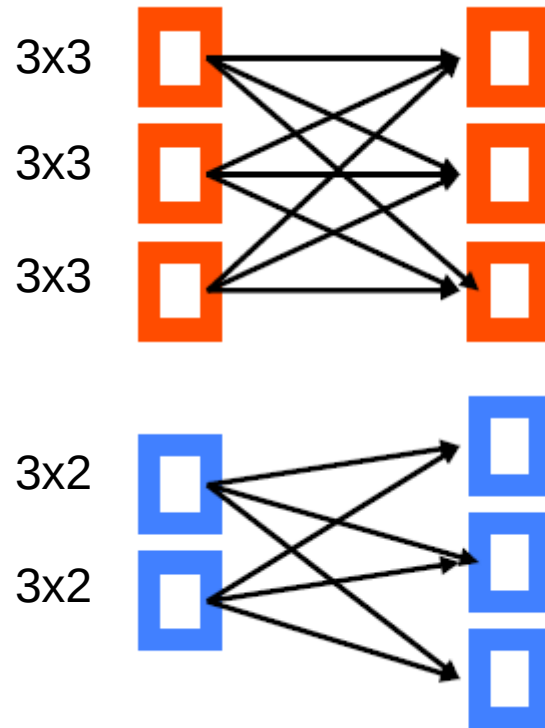
Problem: tightly-knit communities

- Example: a graph made of a $(3,3)$ -clique and a $(2,3)$ -clique



Problem: tightly-knit communities

- Example: a graph made of a $(3,3)$ -clique and a $(2,3)$ -clique

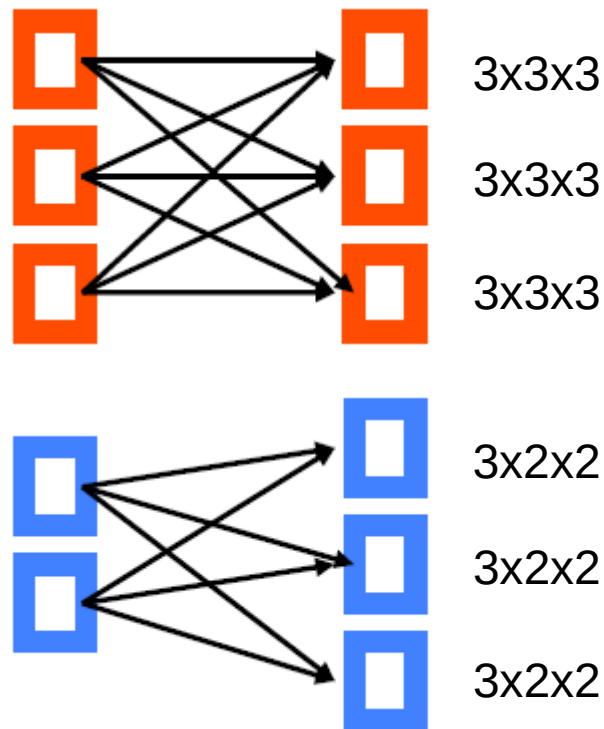


Problem: tightly-knit communities

- Example: a graph made of a (3,3)-clique and a (2,3)-clique

What happens after
n iterations?

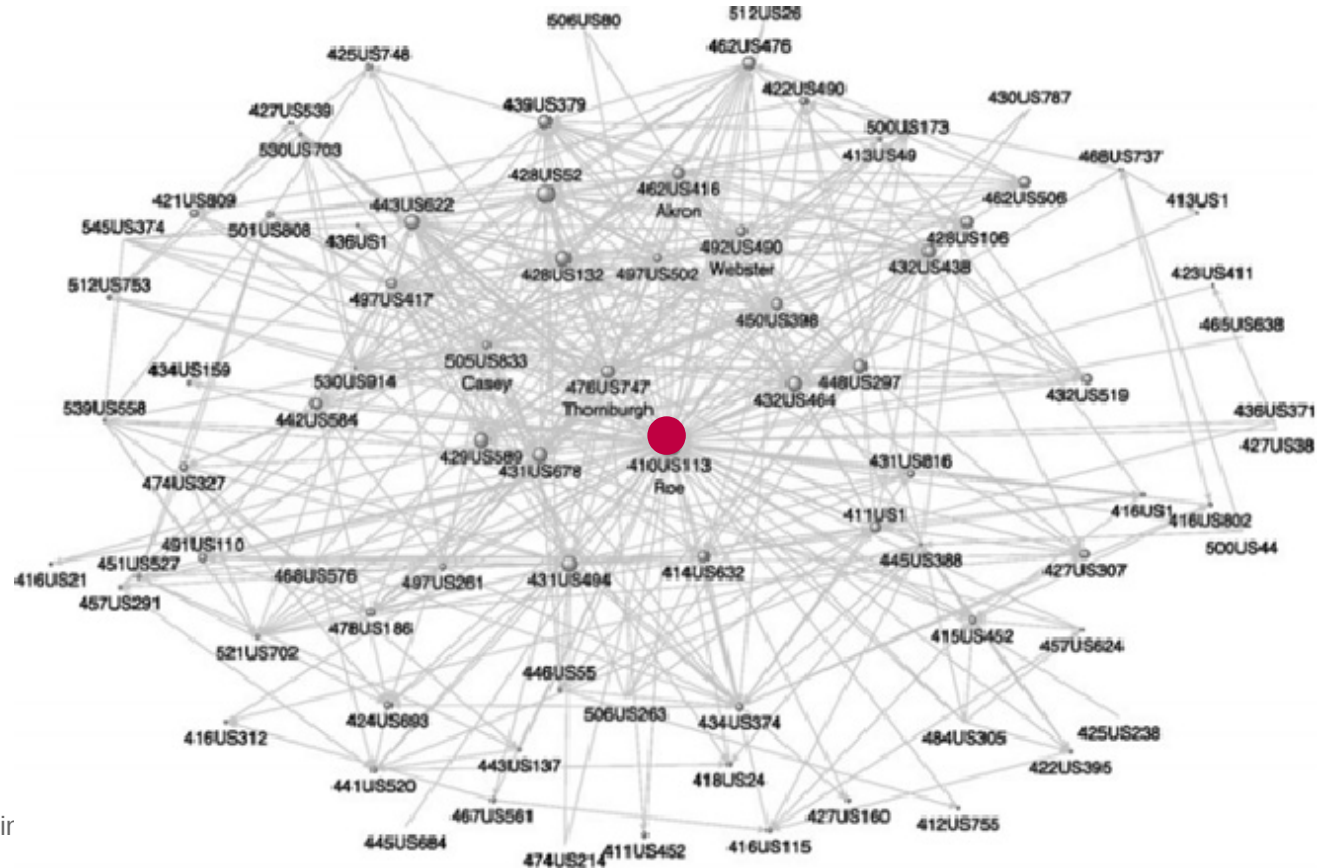
Which community
"wins" (i.e., has the
largest sum of scores)?



A different application of hubs and authorities

The legal precedent network

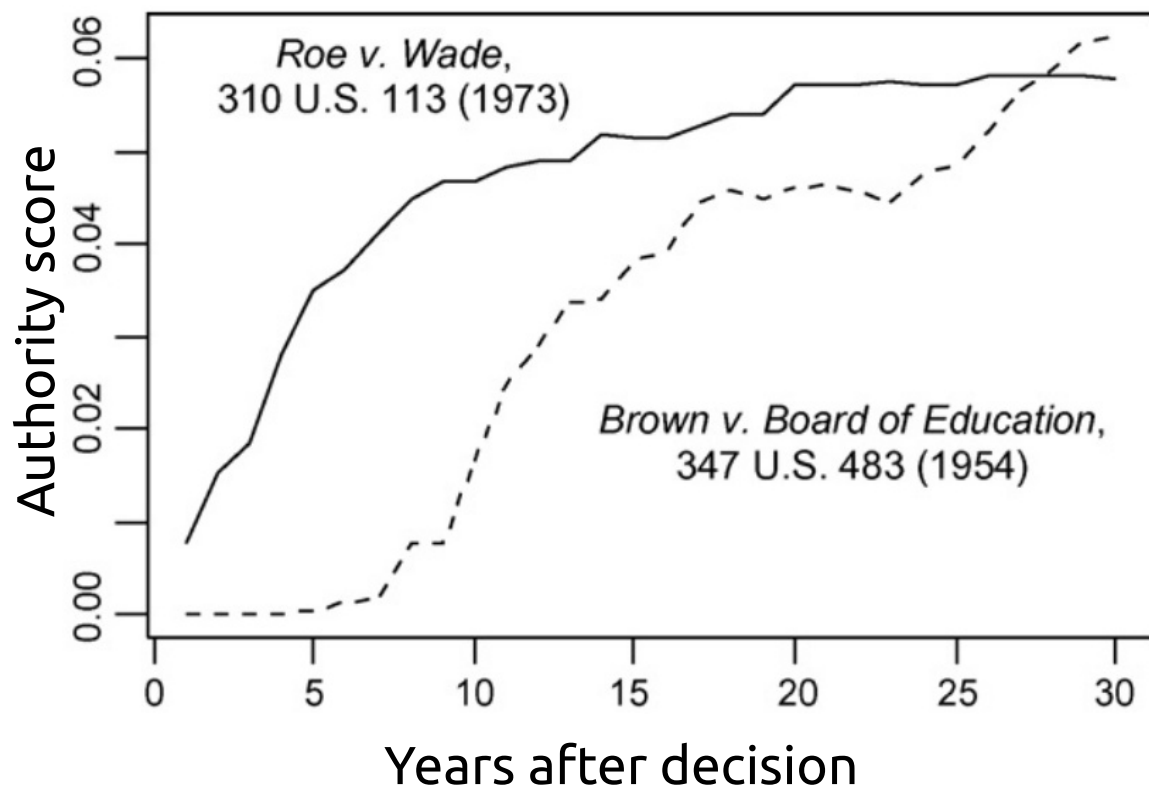
- **Roe v Wade** legalized abortion in the US
- Many cases reference it as a legal precedent
- This is a representation of some of those cases



Hubs and authorities on the legal precedent network

- We can compute authority in this network
- Re-compute every year
- **Different cases acquire authority at different speeds!**

(Roe v Wade legalized abortion, Brown v Board of Education declared race-segregated schools unconstitutional)



Summary

Things to remember

- What is the hubs and authority algorithm
- How to execute it step by step
- Practice with graphs on your own

Practice on your own

- Consider a directed bi-partite graph $G = (V_L \cup V_R, E)$ in which $V_L = \{a, b, c, d\}$ and $V_R = \{1, 2, \dots, 120\}$, and in which all edges go from a node in V_L to a node in V_R :
 - Node a is connected to nodes 1, 2, . . . 120.
 - Node b is connected to nodes 1, 2, . . . 60.
 - Node c is connected to nodes 1, 2, . . . 30.
 - Node d is connected to nodes 1, 2, . . . 15.
- Starting with $\hat{h}(1)(i) = 1$ for $i \in \{a, b, c, d, 1, 2, \dots, 120\}$.
 - 1. Compute $a(1)(i)$ for $i \in \{1, 2, \dots, 120\}$
 - 2. Compute $\hat{a}(1)(i)$ for $i \in \{1, 2, \dots, 120\}$
 - 3. Compute $h(2)(i)$ for $i \in \{a, b, c, d\}$