Closeness

Social Networks Analysis and Graph Algorithms

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Sources

- Networks, Crowds, and Markets Ch 3.6B
- Barabási 2016 Section 9.3.2
- P. Boldi and S. Vigna: Axioms for Centrality in Internet Mathematics 2014.
- Esposito and Pesce: Survey of Centrality 2015.
- Filippo Menczer, Santo Fortunato, and Clayton A. Davis. A First Course in Network Science. Cambridge University Press, 2020 (chapter 2).

Types of centrality measure

- **Non-spectral**
  - Degree
  - Closeness and harmonic closeness
  - Betweenness
- Spectral
  - HITS
  - PageRank

Is u a well-connected person?

- Degree: u has many connections
- **Closeness:** u is close to many people
  - Average distance from u is small
- **Betweenness:** many connections pass through u
  - Large number of shortest paths pass through u
- PageRank: *u is connected to the well-connected*

Closeness

Closeness

- Distance between two nodes is $d(u,v)$
- **Closeness** is the reciprocal of the sum of distances

$$\mathrm{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u,v)}$$

- Some graphs are not connected, in that case $d(u,v)$ can be ∞; assuming $1/\infty = 0$ one can define the **harmonic closeness**:

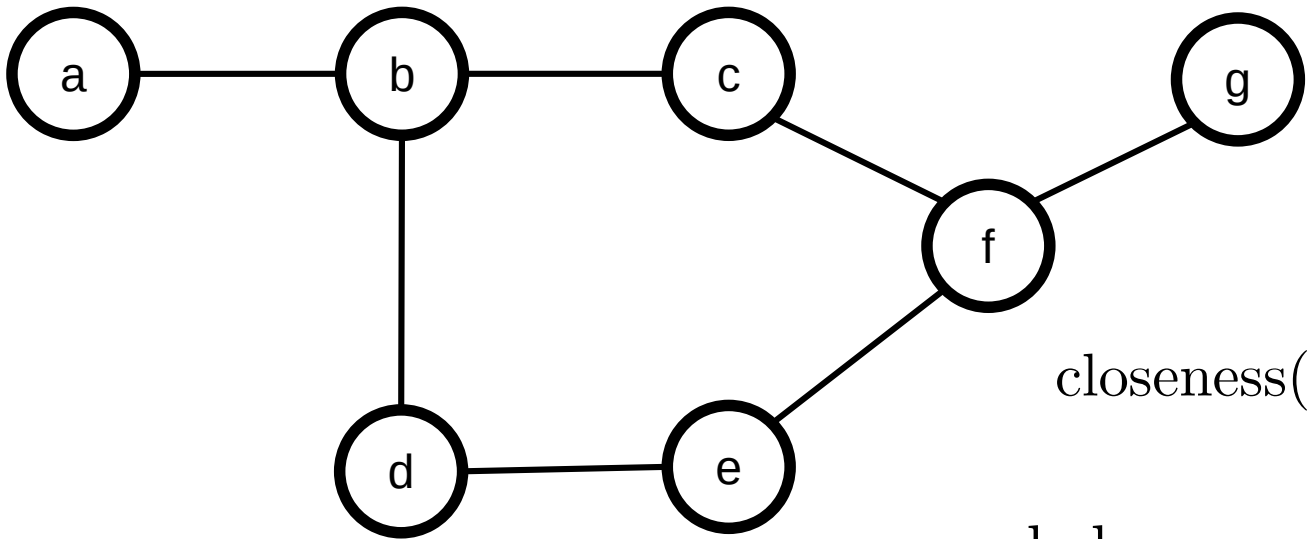
$$\mathrm{hcloseness}(u) = \sum_{v \neq u} \frac{1}{d(u,v)}$$

Exercise

Answer in
Google Spreadsheet

Compute closeness and harmonic closeness for all the nodes

d(u,v) = 1 if v is a neighbor of u



$$\text{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u,v)}$$

$$\text{hcloseness}(u) = \sum_{v \in V, v \neq u} \frac{1}{d(u,v)}$$

Summary

Things to remember

- Closeness and harmonic closeness definitions
- Try to compute them on your own on a graph
- Practice drawing examples of graphs in which a chosen node has high degree but low closeness, or viceversa