

Distances in Scale-Free Networks

Social Networks Analysis and Graph Algorithms

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Contents

- Distance distribution of scale-free networks

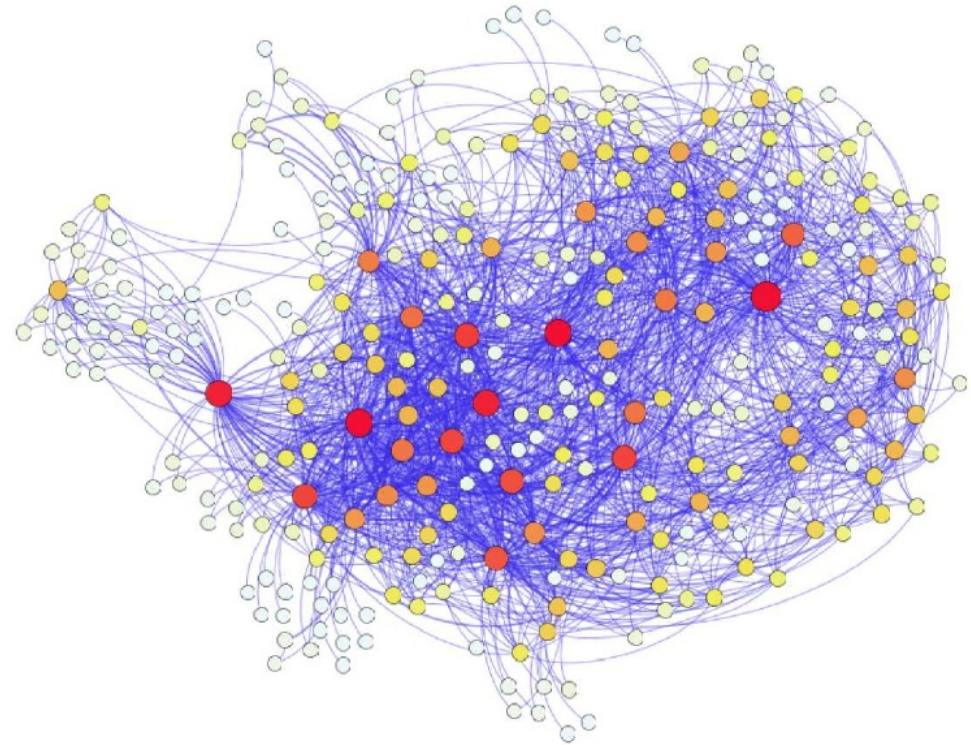
Sources

- A. L. Barabási (2016). Network Science – Chapter 04
- URLs cited in the footer of specific slides

Consequences of having
extremely large degree nodes
(also known as “large hubs”)

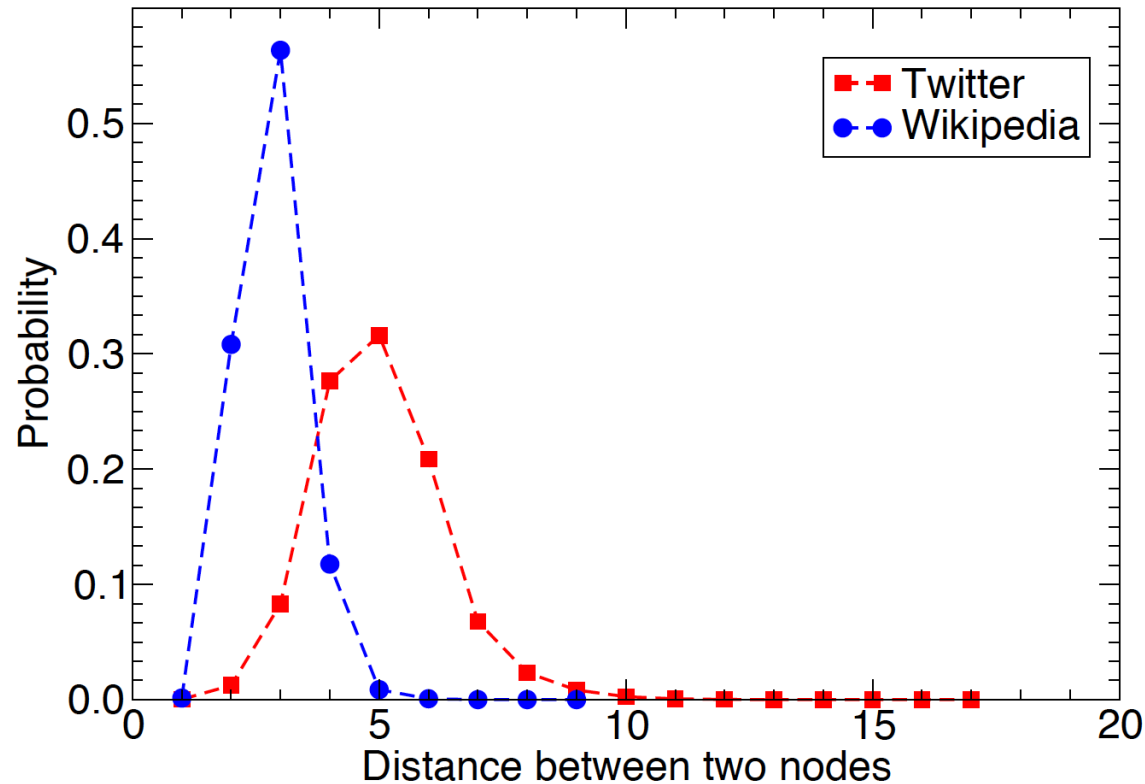
Air travel

- You can travel between almost all pairs of European airports directly or (most of the time) with at most one stop
- All you have to do is **go to a well connected airport**
- This is because there are large degree airports



Cardillo, A et al. (2013). Modeling the multi-layer nature of the European Air Transport Network: Resilience and passengers re-scheduling under random failures. Euro. Phys. J. Special Topics, 215(1), 23-33. [\[DOI\]](#)

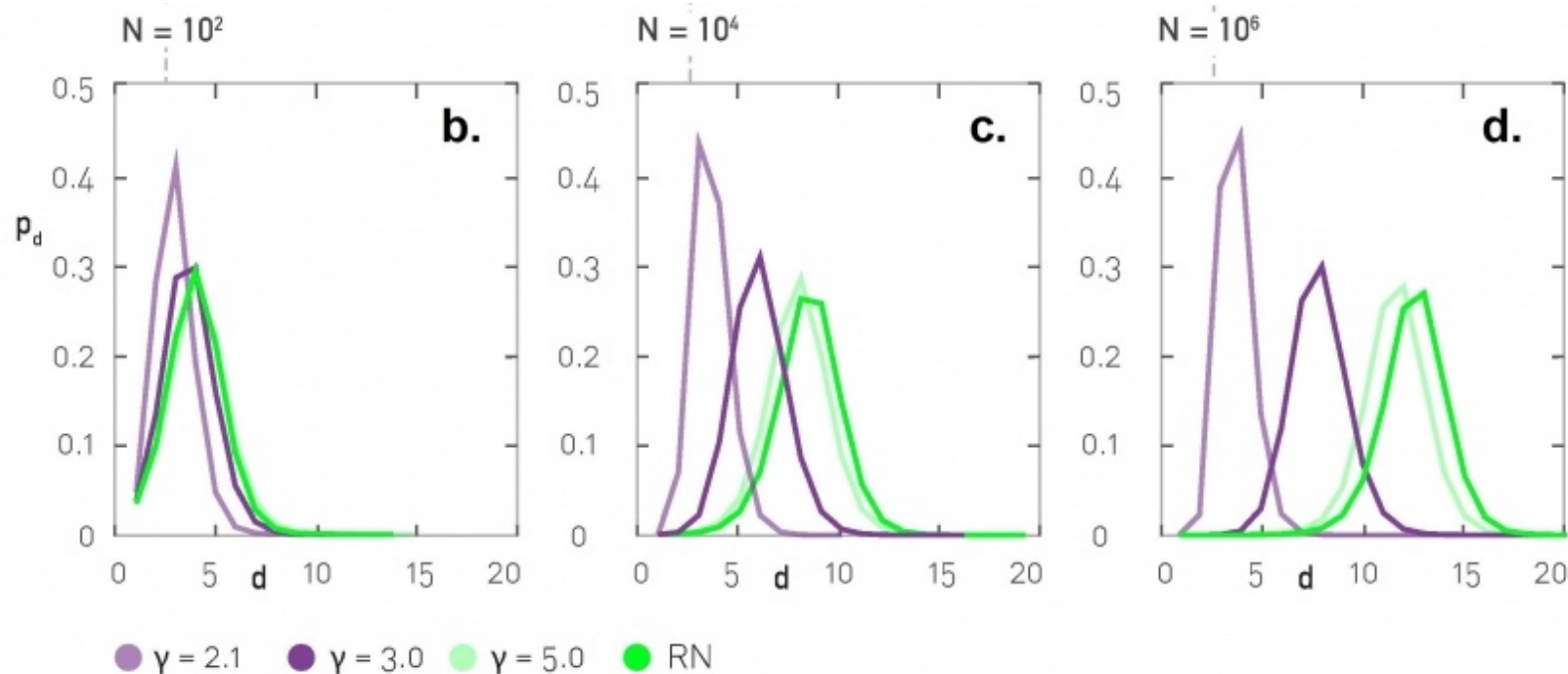
In general, having “hubs” or large degree nodes reduces distances



Distance regimes

Distance distributions: simulation results

Scale-free networks of increasing size, $\langle k \rangle = 3$



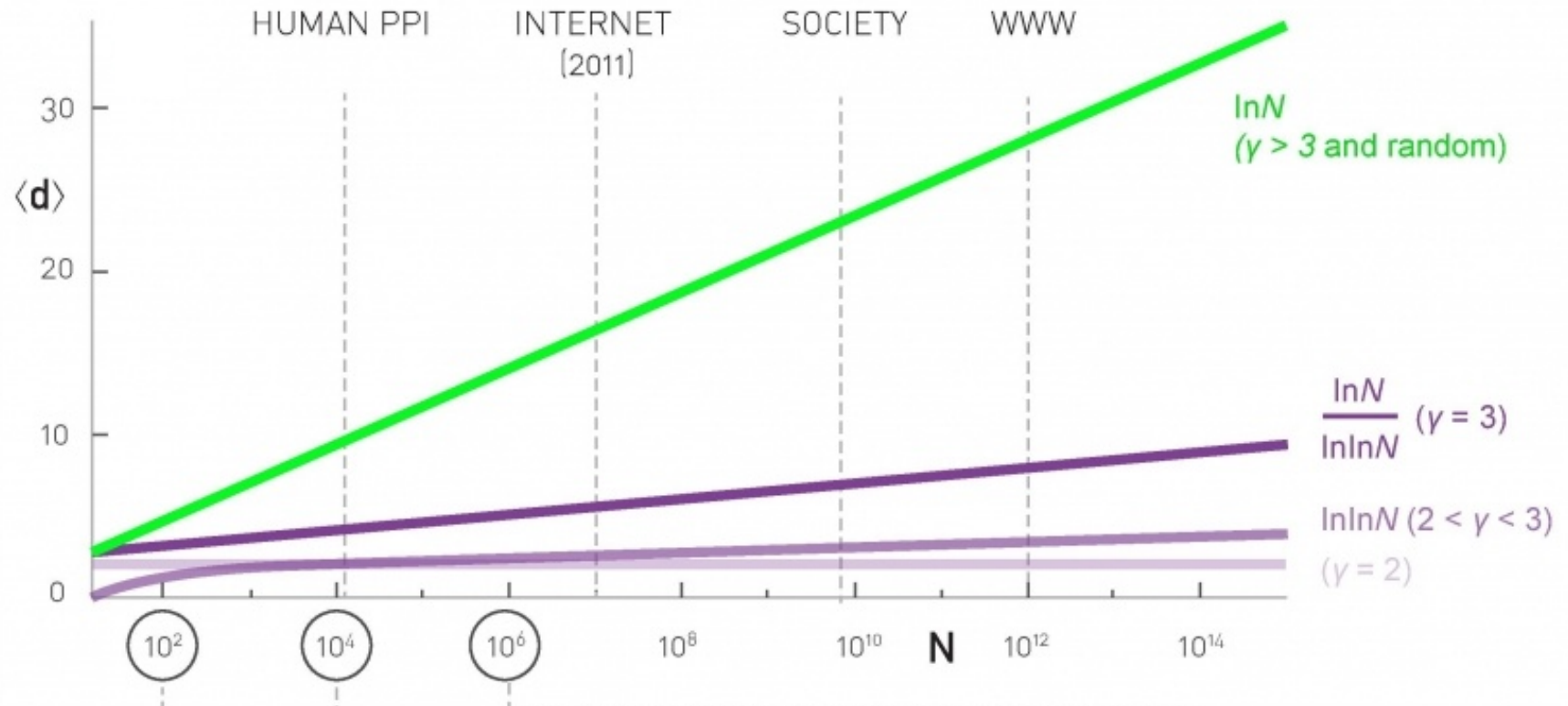
Average distance

- Depends on γ and N

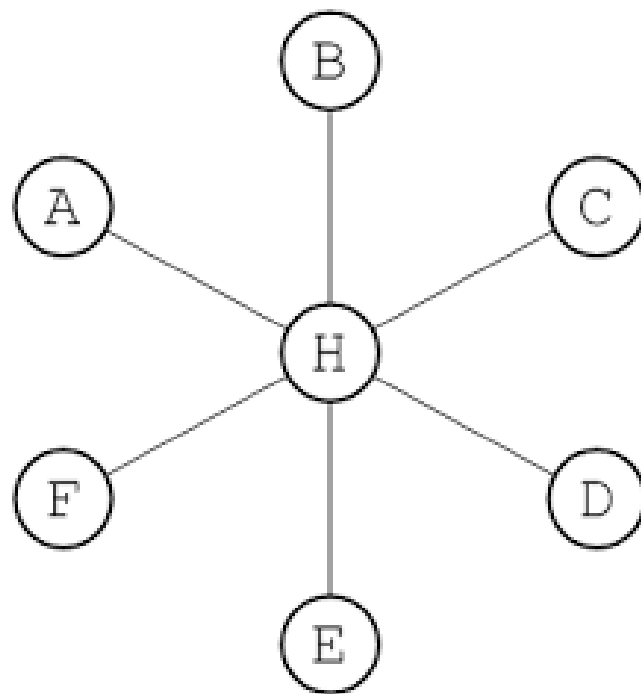
$$\langle d \rangle = \begin{cases} \text{const.} & \text{if } \gamma = 2 \\ \log \log N & \text{if } 2 < \gamma < 3 \\ \log N / \log \log N & \text{if } \gamma = 3 \\ \log N & \text{if } \gamma > 3 \end{cases}$$

← Same as in
ER graphs

Average distance and N



Anomalous regime $\gamma = 2$



Ultra-small world $2 < \gamma < 3$

- Average distance follows $\log(\log(N))$
- Example (humans):

$$N \approx 7 \times 10^9$$

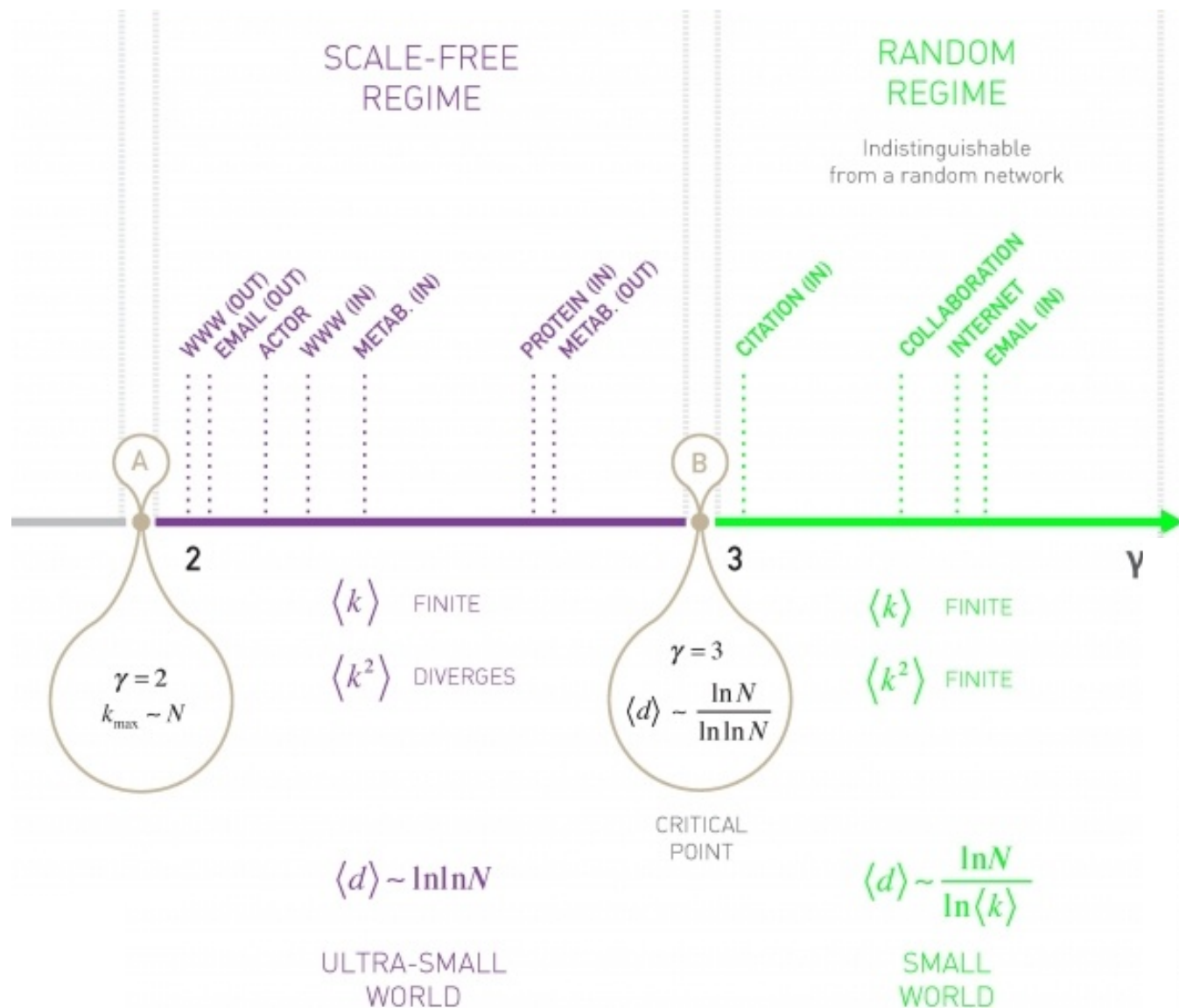
$$\log N \approx 22.66$$

$$\log \log N \approx 3.12$$

Small world $\gamma > 3$

- Average distance follows $\log(N)$
- Similar to ER graphs where it followed $\log(N)/\log(\langle k \rangle)$

The degree distribution exponent plays an important role



When $\gamma > 3$

- In this case it is hard to distinguish this case from an ER graph
- In most real complex networks (but not all)

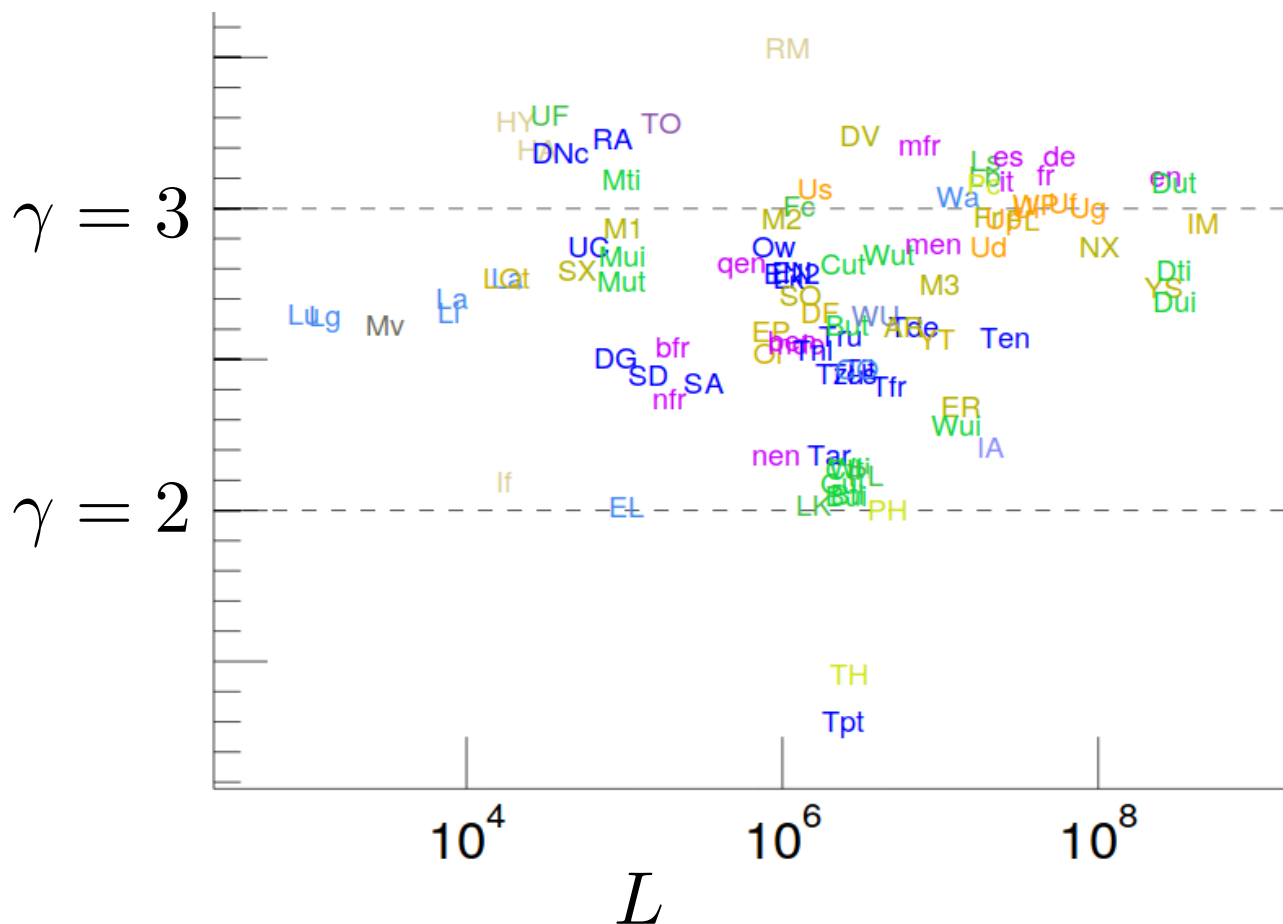
$$2 < \gamma < 3$$

When $\gamma > 3$

- Remember $k_{\max} = k_{\min} N^{\frac{1}{\gamma-1}}$ $N = \left(\frac{k_{\max}}{k_{\min}} \right)^{\gamma-1}$
- Observing the scale-free properties requires that $k_{\max} \gg k_{\min}$, e.g. $k_{\max} = 10 k_{\min}$
- Then if $\gamma = 5$, $N > 10^8$
- There are not many such networks for which we have available data

Examples

<http://konect.uni-koblenz.de/statistics/prefatt>



EL	Wikipedia elections
LK	Linux kernel mailing list threads
Bul	BibSonomy u-i
Bti	BibSonomy t-i
Cul	CiteULike u-i
If	Infectious
PL	Prosper loans
Cti	CiteULike t-i
Wti	Twitter t-i
nen	Wikinews (en)
Tar	Wikipedia talk, Arabic
Wul	Twitter u-i
ER	Epinions
nfr	Wikinews (fr)
Tfr	Wikipedia talk, French
SD	Slashdot
Tzh	Wikipedia talk, Chinese
Tes	Wikipedia talk, Spanish

Etc.

Summary

Things to remember

- Regimes of distance and connectivity

Practice on your own

- Remember the regimes of a graph given $\langle k \rangle$
(It is useful to know this by heart)
- Estimate distance distributions for some graphs