Distances in Scale-Free Networks

Social Networks Analysis and Graph Algorithms

Prof. Carlos Castillo — https://chato.cl/teach

Universitat Pompeu Fabra Barcelona

Contents

- Distance distribution of scale-free networks

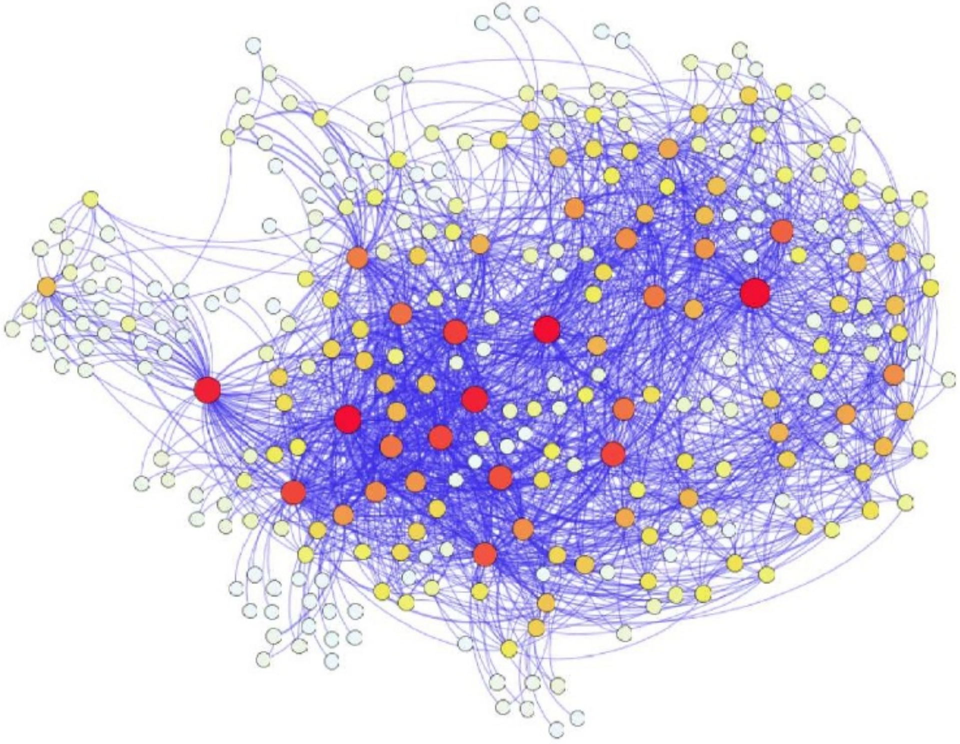
Sources

- A. L. Barabási (2016). Network Science – Chapter 04
- URLs cited in the footer of specific slides

Consequences of having extremely large degree nodes *(also known as "large hubs")*

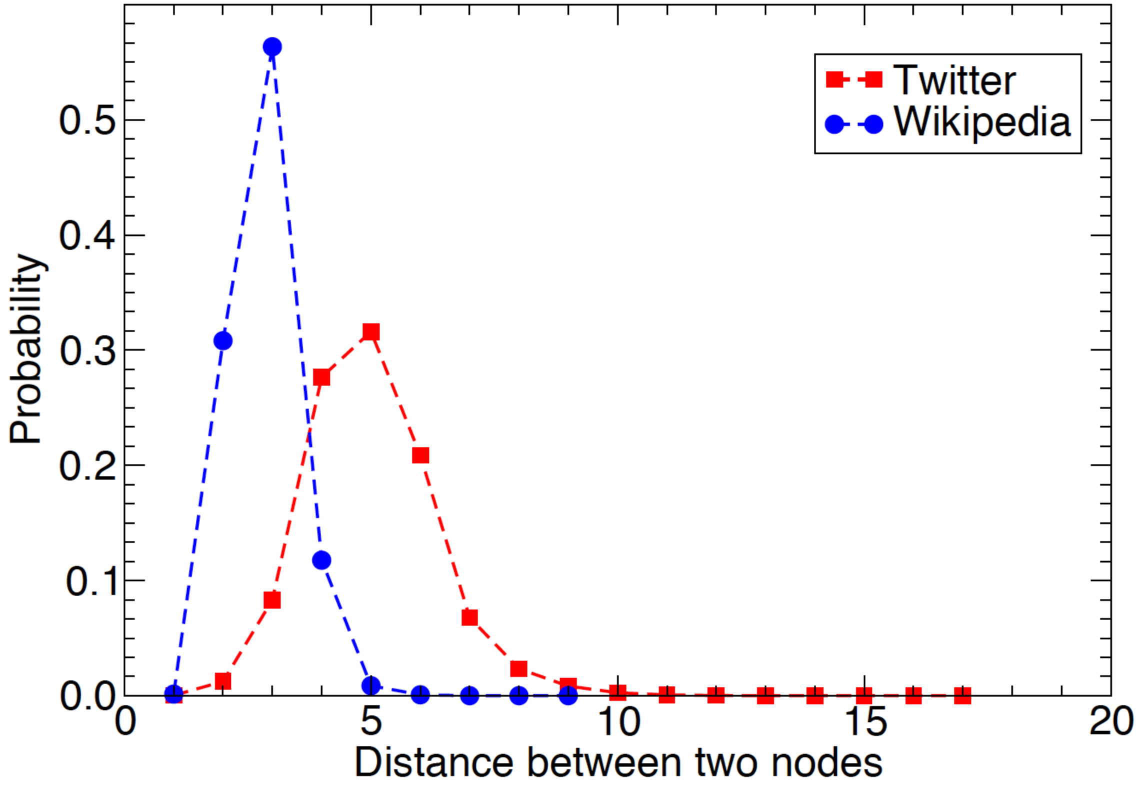
Air travel

- You can travel between almost all pairs of European airports directly or (most of the time) with at most one stop
- All you have to do is **go to a well connected airport**
- This is because there are large degree airports



Cardillo, A et al. (2013). Modeling the multi-layer nature of the European Air Transport Network: Resilience and passengers re-scheduling under random failures. Euro. Phys. J. Special Topics, 215(1), 23-33. [DOI]

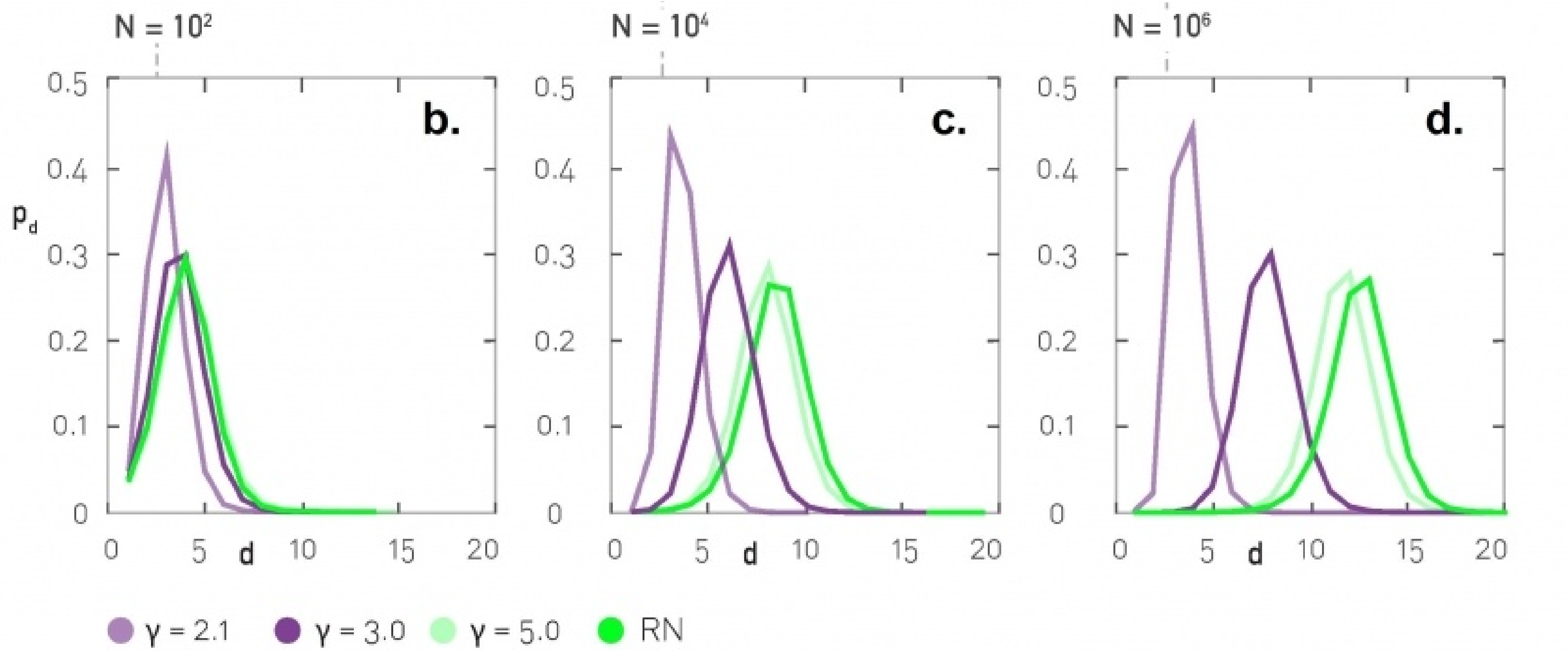
In general, having "hubs" or large degree nodes reduces distances



Source: Menczer, Fortunato, Davis: A First Course on Networks Science. Cambridge, 2020.

Distance distributions: simulation results

Scale-free networks of increasing size, $<k> = 3$



Distance regimes

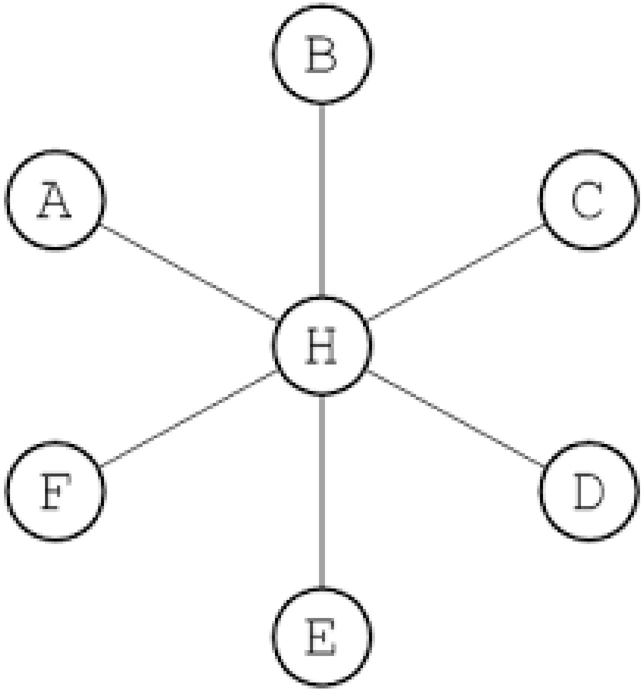
Average distance

- Depends on ɣ and N

$$\langle d \rangle = \begin{cases} \text{const.} & \text{if } \gamma = 2 \\ \log\log \mathrm{N} & \text{if } 2 < \gamma < 3 \\ \log \mathrm{N} / \log\log \mathrm{N} & \text{if } \gamma = 3 \\ \log \mathrm{N} & \text{if } \gamma > 3 \end{cases}$$

Same as in ER graphs

Anomalous regime $\gamma = 2$



Ultra-small world $2 < \gamma < 3$

- Average distance follows log(log(N))
- Example (humans):

$$N \approx 7 \times 10^9$$

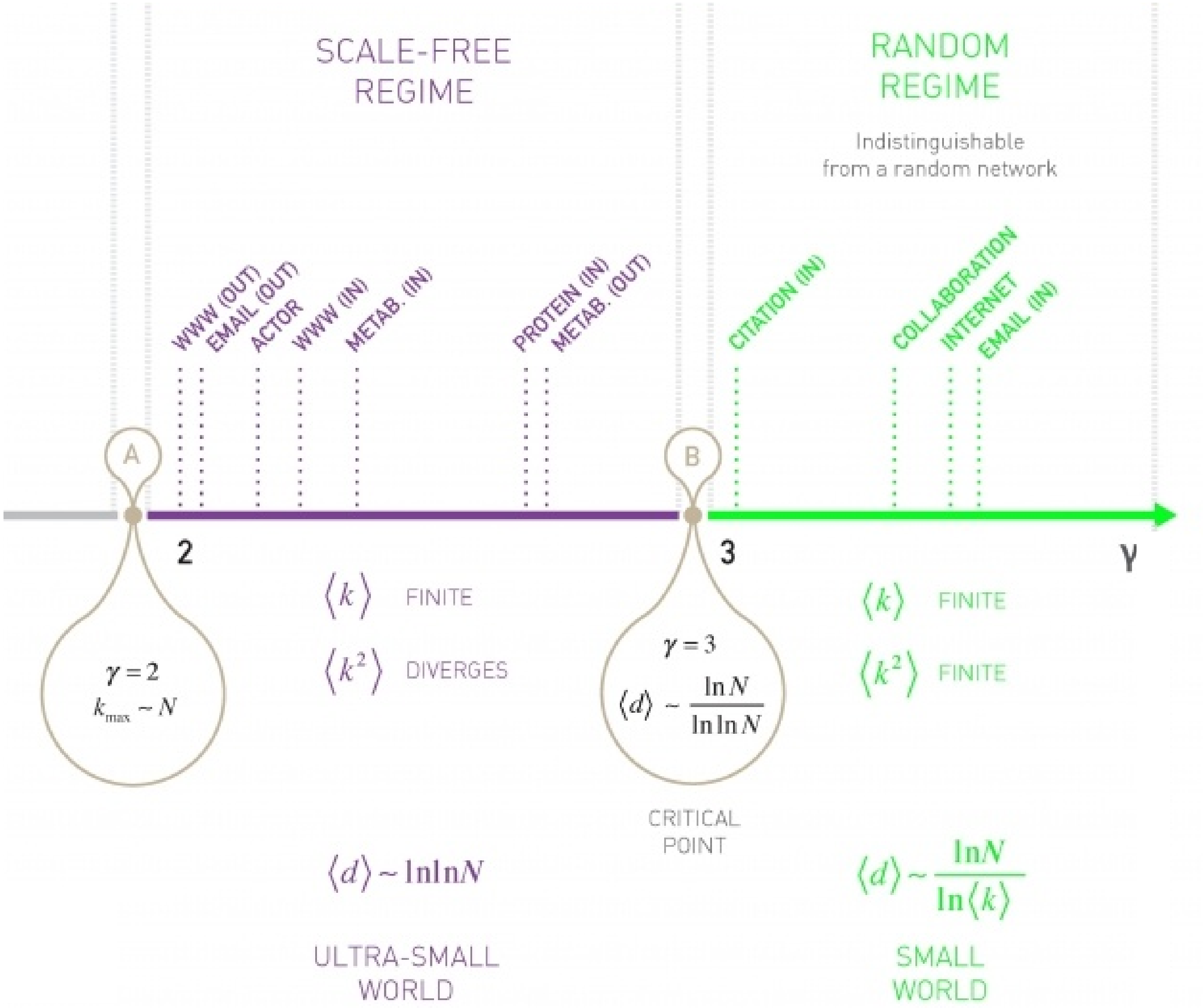
$$\log N \approx 22.66$$

$$\log \log N \approx 3.12$$

Small world $\gamma > 3$

- Average distance follows log(N)
- Similar to ER graphs where it followed log(N)/log($<$k$>$)

The degree distribution exponent plays an important role



When $\gamma > 3$

- In this case it is hard to distinguish this case from an ER graph
- In most real complex networks (but not all)

$$2 < \gamma < 3$$

When $\gamma > 3$

- Remember $$k_{\max} = k_{\min} N^{\frac{1}{\gamma-1}} \qquad N = \left(\frac{k_{\max}}{k_{\min}}\right)^{\gamma-1}$$

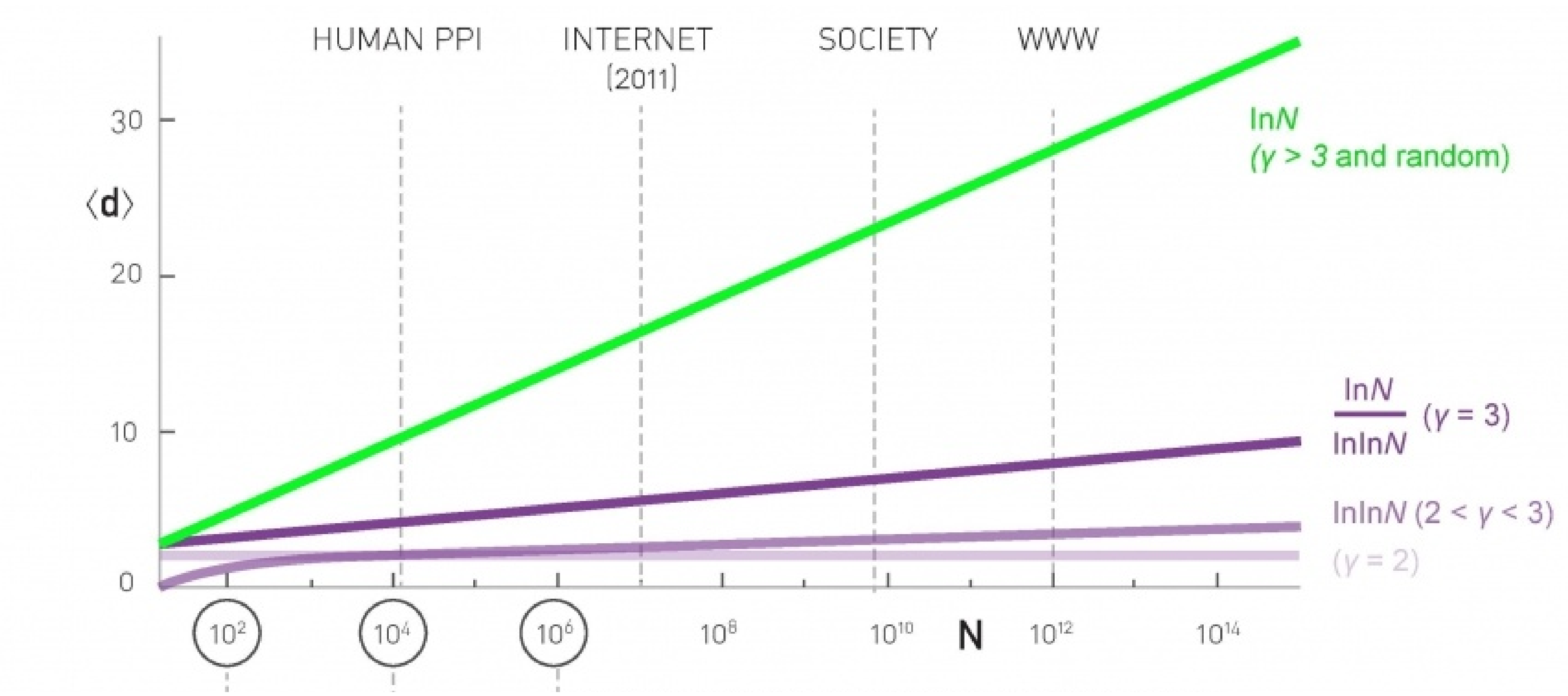
- Observing the scale-free properties requires that

  $k_{max} >> k_{min}$, e.g. $k_{max} = 10\ k_{min}$

- Then if $\gamma = 5, N > 10^8$

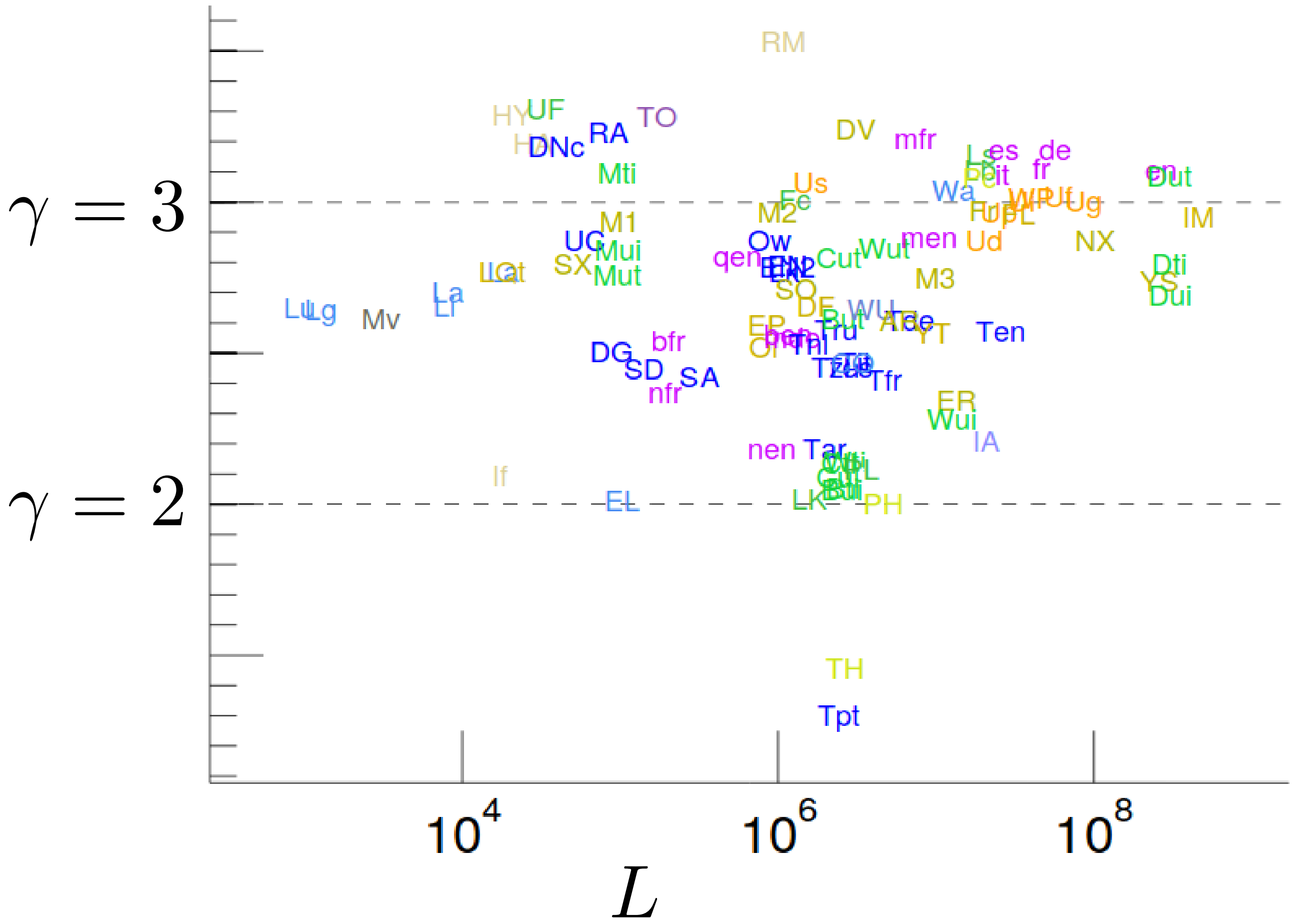
- There are not many such networks for which we have available data

Average distance and N



Examples

http://konect.uni-koblenz.de/statistics/prefatt



EL	Wikipedia elections
LK	Linux kernel mailing list threads
Bui	BibSonomy u-i
Bti	BibSonomy t-i
Cui	CiteULike u-i
If	Infectious
PL	Prosper loans
Cti	CiteULike t-i
Wti	Twitter t-i
nen	Wikinews (en)
Tar	Wikipedia talk, Arabic
Wui	Twitter u-i
ER	Epinions
nfr	Wikinews (fr)
Tfr	Wikipedia talk, French
SD	Slashdot
Tzh	Wikipedia talk, Chinese
Tes	Wikipedia talk, Spanish

Etc.

Summary

Things to remember

- Regimes of distance and connectivity

Practice on your own

- Remember the regimes of a graph given $<k>$

  (It is useful to know this by heart)

- Estimate distance distributions for some graphs