

K-Core Decomposition

Social Networks Analysis and Graph Algorithms

Prof. Carlos “ChaTo” Castillo — <https://chato.cl/teach>



Universitat
Pompeu Fabra
Barcelona

Sources

- Barabási 2016 Chapter 9
- [Networks, Crowds, and Markets](#) Ch 3
- C. Castillo (2017) [Dense Sub-Graphs](#)
- Tutorial by A. Beutel, L. Akoglu, C. Faloutsos [[Link](#)]
- Frieze, Gionis, Tsourakakis: “Algorithmic techniques for modeling and mining large graphs (AMAZING)” [[Tutorial](#)]
- A survey of algorithms for dense sub-graph discovery [[link](#)]

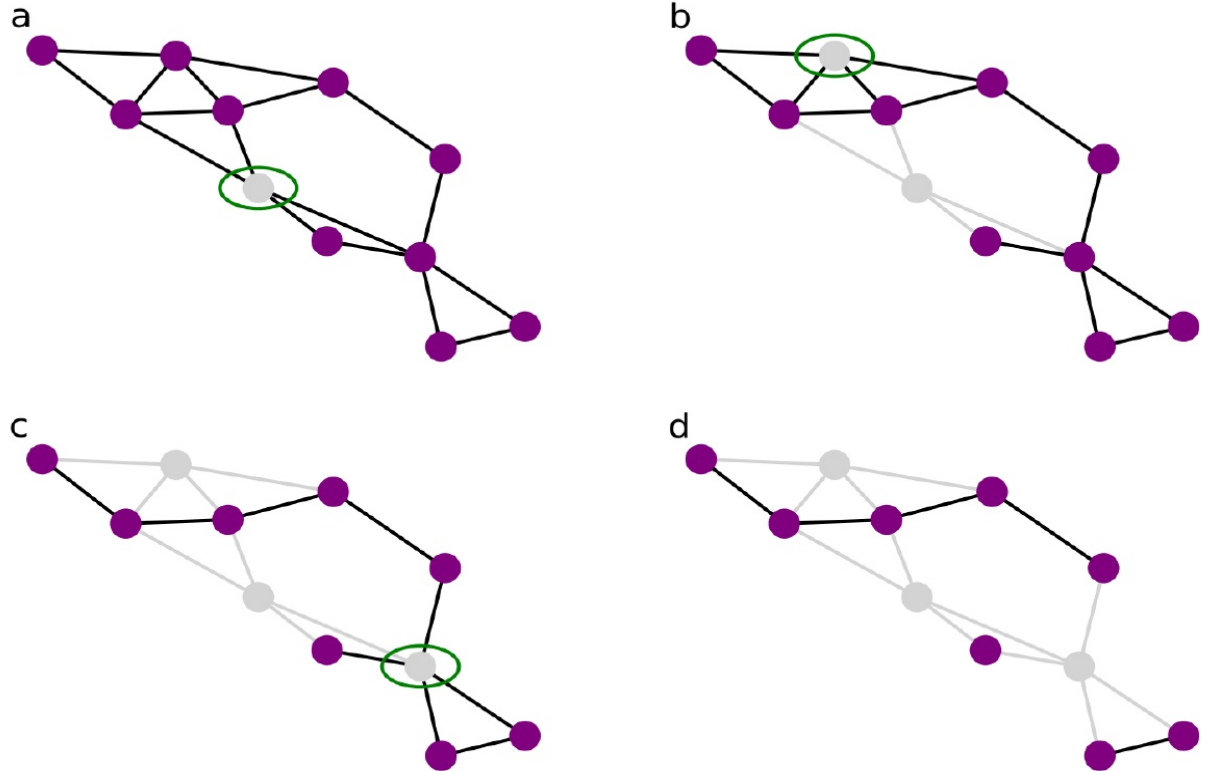
Robustness

Robustness to failure

- A system is **robust** if the failure of some of its components does not affect its function
- Aircrafts, for instance, have:
 - Separate primary and back-up flight instruments (airspeed, altimeter, ...);
 - Multiple independent hydraulic systems
 - Primary and alternate/emergency landing gear;
 - Multiple sources of power and priorities for energizing components

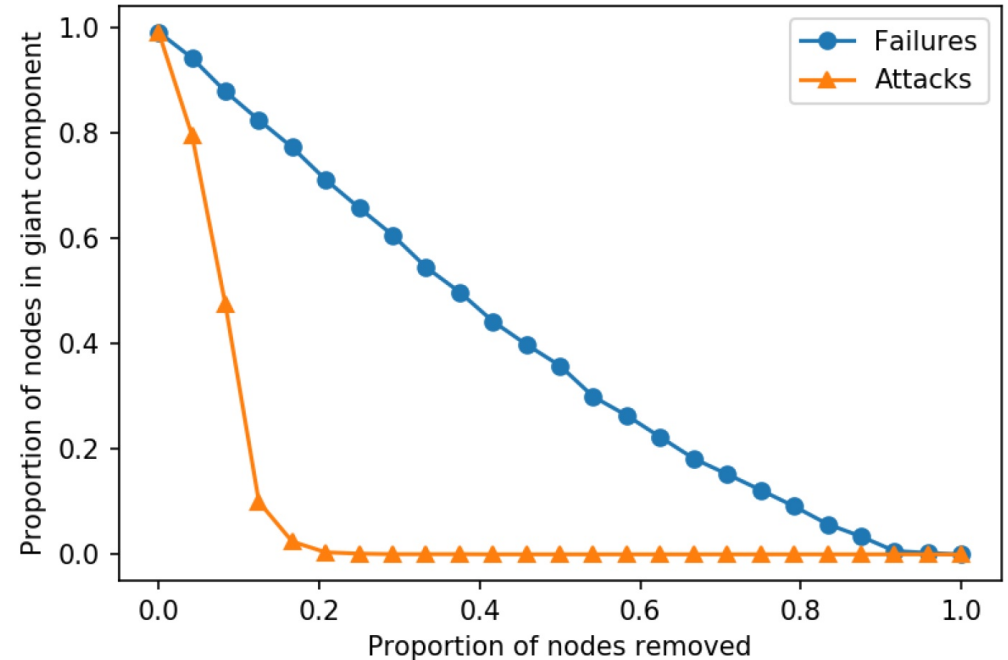
Robustness in networks

The targeted
removal of 4 nodes
disconnects this
graph into three
components



Destroying the giant connected component

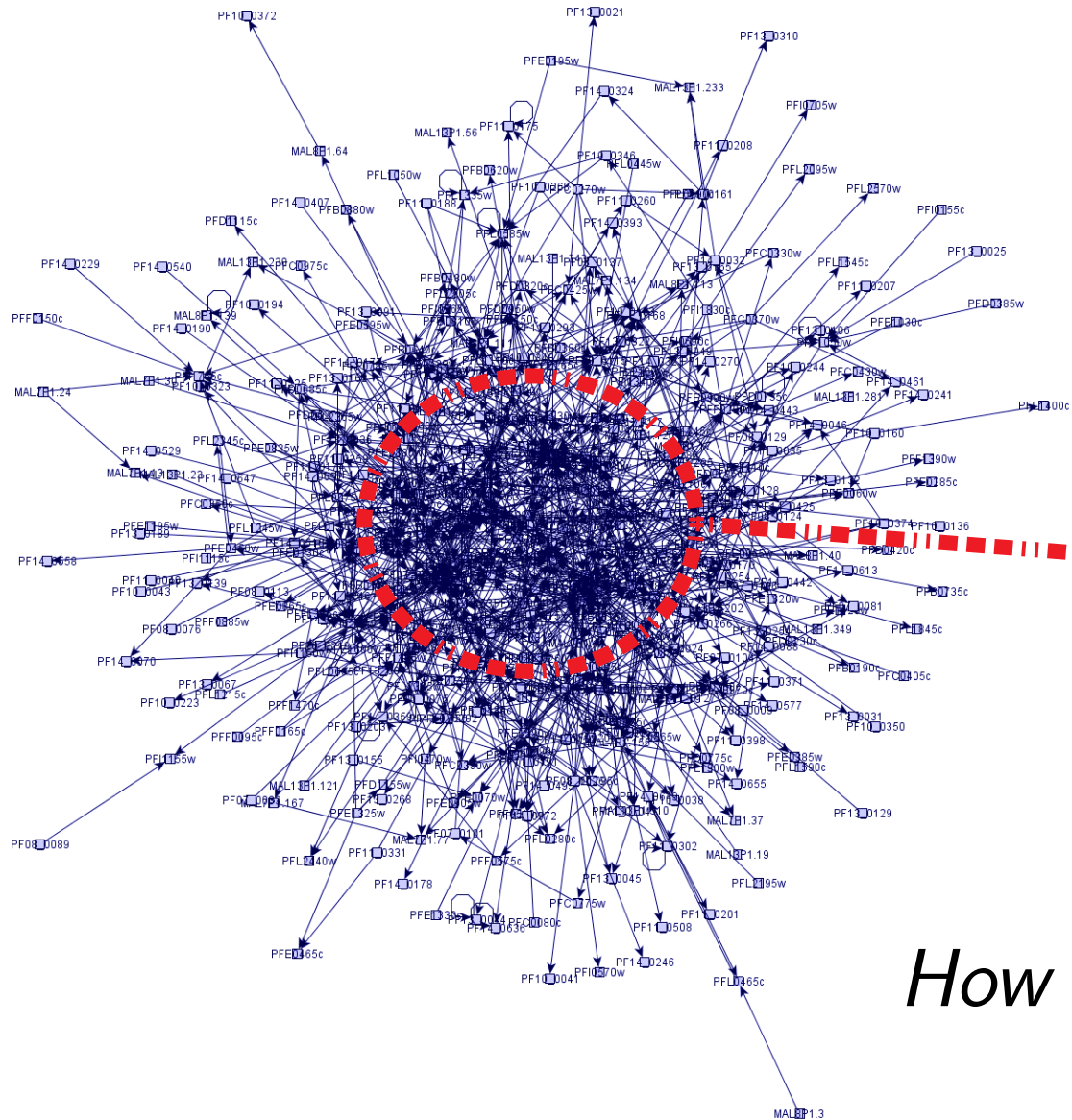
The size of the giant component in this network of flights is reduced slowly by random removals (“failures”), but reduced quickly by removing high-degree edges (“attacks”)



Many networks look
like “hairballs”

They have a **core**
and a **periphery**

How can we find the core?

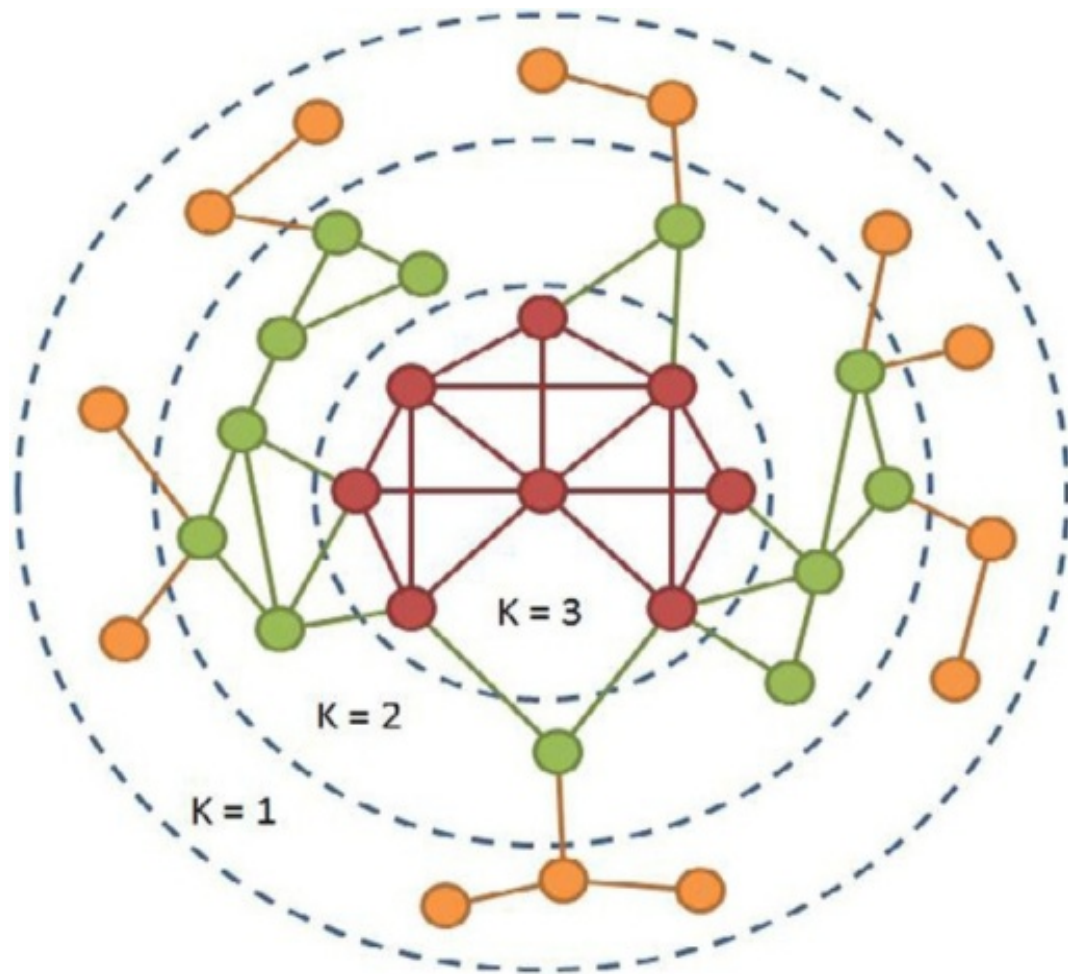


k-core decomposition
is a method to decompose
a graph into *layers*

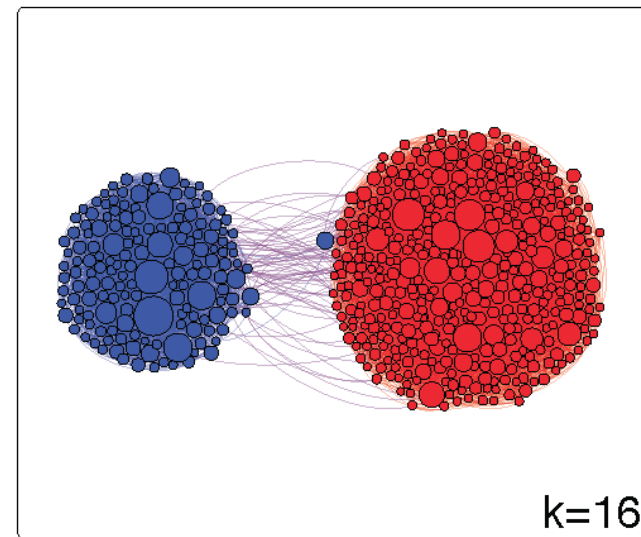
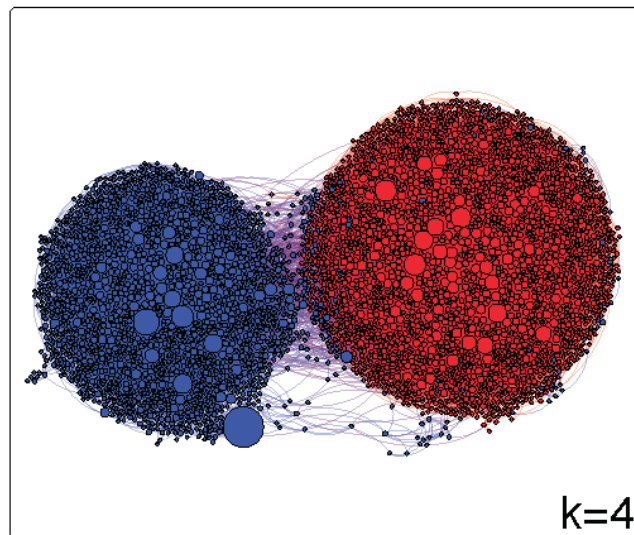
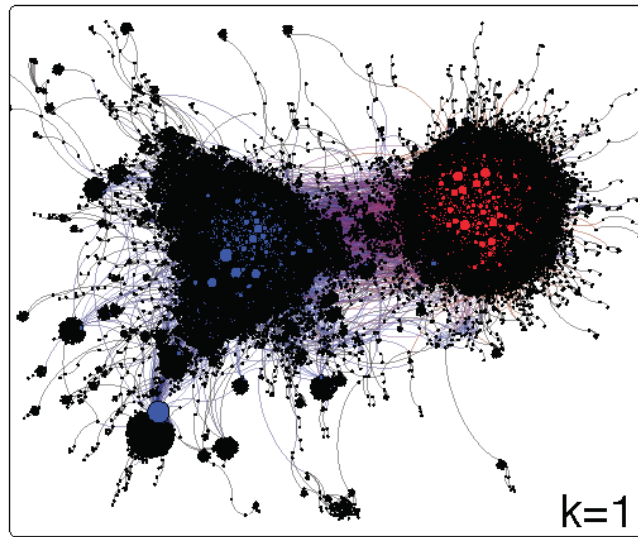
k-core decomposition

- Remove all nodes having degree 1
 - Those are in the 1-core
- Remove all nodes having degree 2 *in the remaining graph*
 - Those nodes are in the 2-core
- Remove all nodes having degree 3 *in the remaining graph*
 - Those nodes are in the 3-core
- Etc.

Example 1

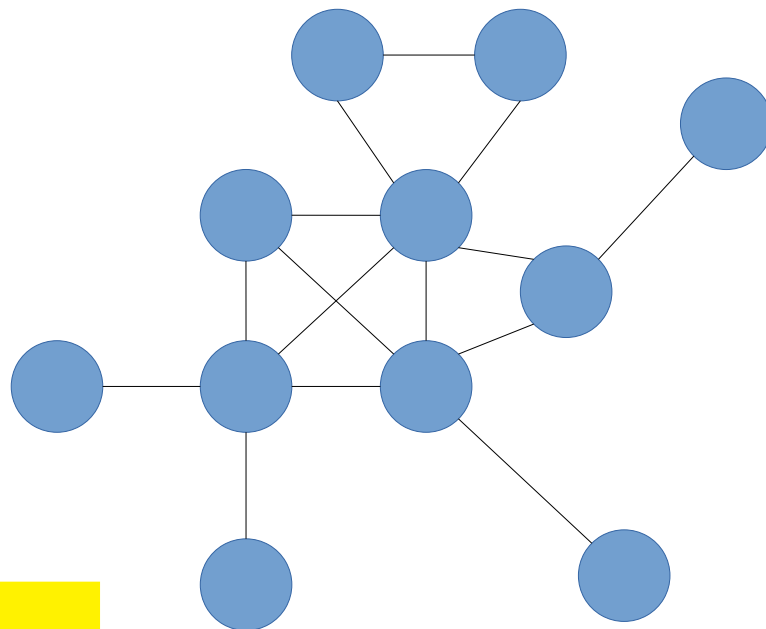


Example 2



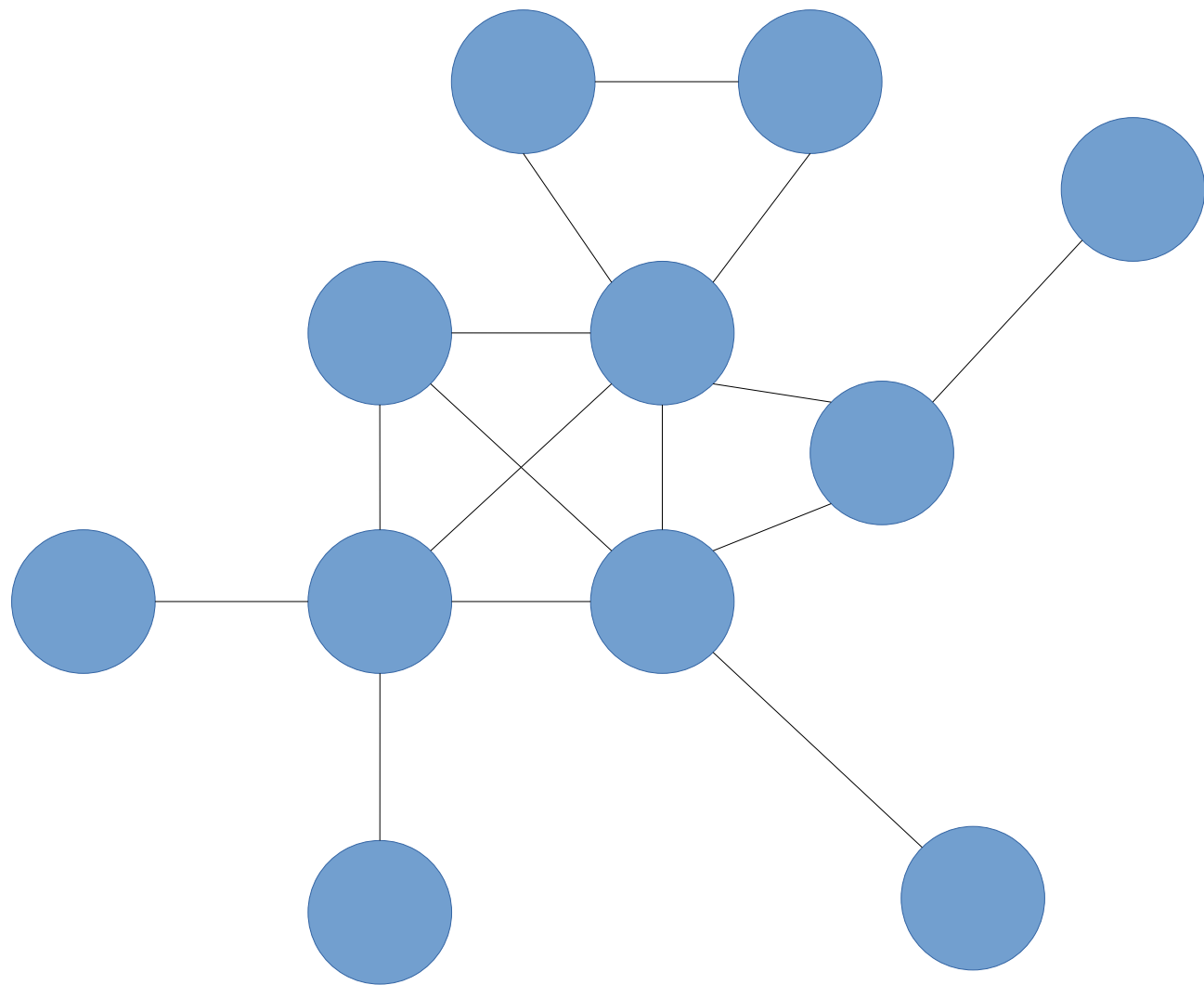
Exercise

For each node in the graph, indicate the max k-core to which it belongs



Draw in Nearpod Draw-it
<https://nearpod.com/student/>
Code to be given during class

<http://www.cpt.univ-mrs.fr/~barrat/NHM.pdf>



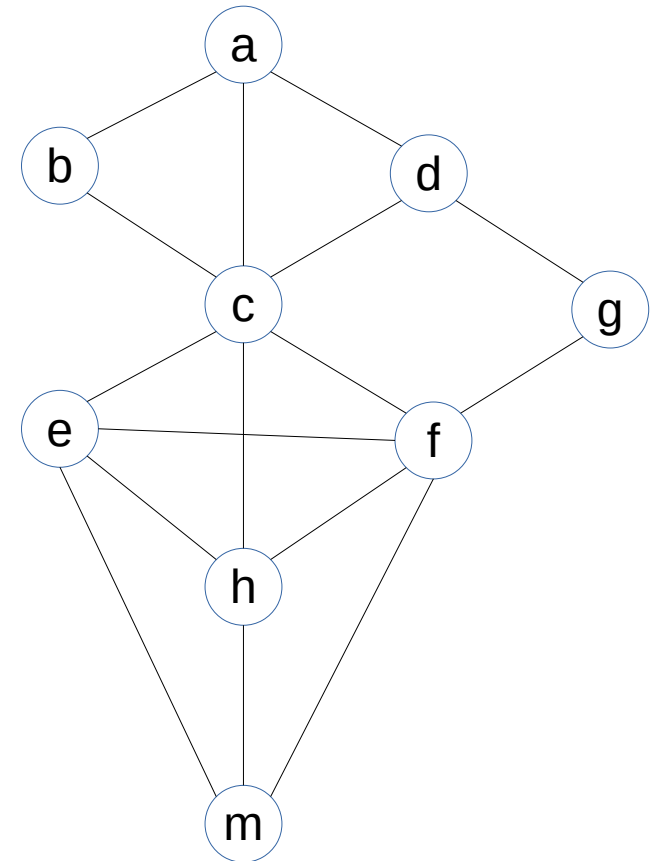
Summary

Things to remember

- What does it mean for a network to be robust?
- What is the k -core decomposition
- How to compute it on a graph

Practice on your own

Find the 3-core of this graph



Solution by Vivekanand Khyade (start at 01:23)
<https://youtu.be/8sNZ5d8eNC8?t=83>