# Closeness

Introduction to Network Science

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Topic 17

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# Sources

- Networks, Crowds, and Markets Ch 3.6B

- Barabási 2016 Section 9.3.2

- P. Boldi and S. Vigna: Axioms for Centrality in Internet Mathematics 2014.

- Esposito and Pesce: Survey of Centrality 2015.

- C. Castillo: Other centrality slides 2016

# Types of centrality measure

- Spectral
  - HITS
  - PageRank

- **Non-spectral**
  - Degree
  - Closeness and harmonic closeness
  - Betweenness

# Is $u$ a well-connected person?

- Degree: $u$ has many connections

- Eigenvector: $u$ is connected to the well-connected

- **Closeness:** $u$ is close to many people

  - Average distance from $u$ is small

- **Betweenness**: many connections pass through $u$

  - Large number of shortest paths pass through $u$

# Closeness

# Closeness

- Distance between two nodes is $d(u,v)$

- **Closeness** is the reciprocal of distances

$$\text{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u,v)}$$

- Some graphs are not connected, in that case $d(u,v)$ can be $\infty$; assuming $1/\infty = 0$ one can define the **harmonic closeness**:
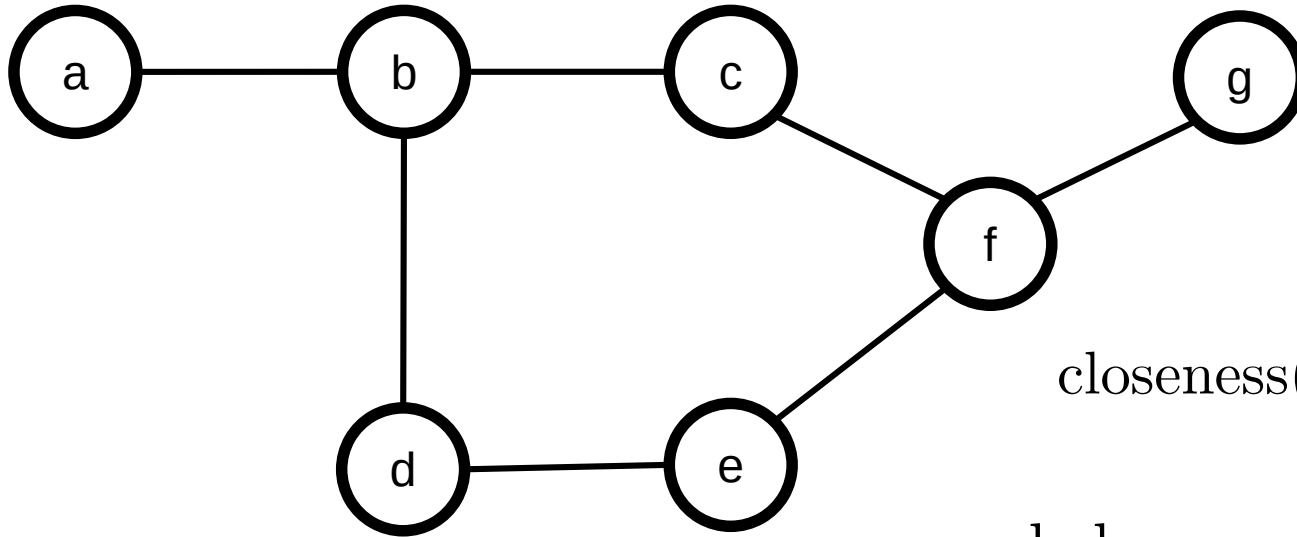
$$\text{hcloseness}(u) = \sum_{v \neq u} \frac{1}{d(u,v)}$$

# Exercise

*Compute closeness and harmonic closeness for all the nodes*

*d(u,v) = 1 if v is a neighbor of u*



$$\text{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u, v)}$$

$$\text{hcloseness}(u) = \sum_{v \in V, v \neq u} \frac{1}{d(u, v)}$$

# Summary

# Things to remember

- Closeness and harmonic closeness definitions
- Try to compute them on your own on a graph