Epidemics

Social Networks Analysis and Graph Algorithms

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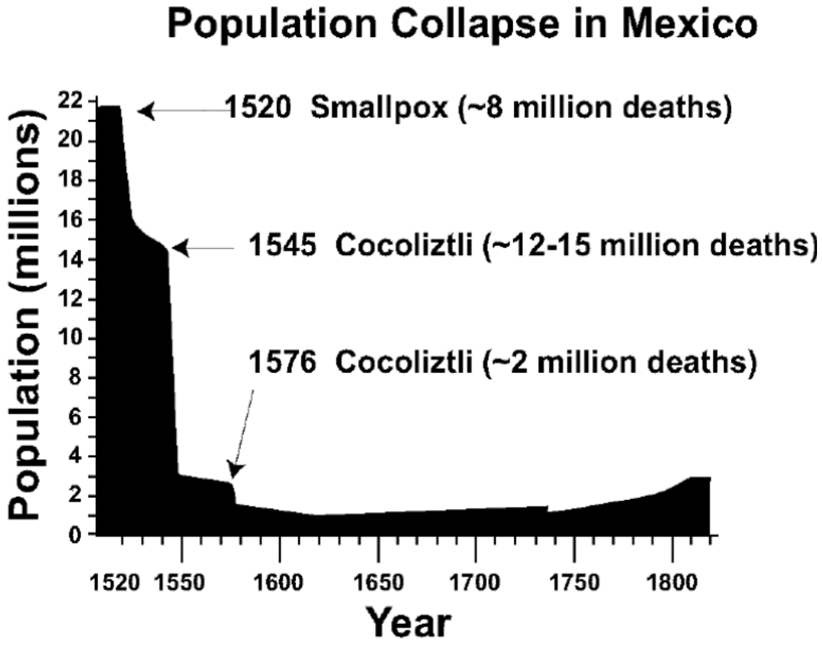
Sources

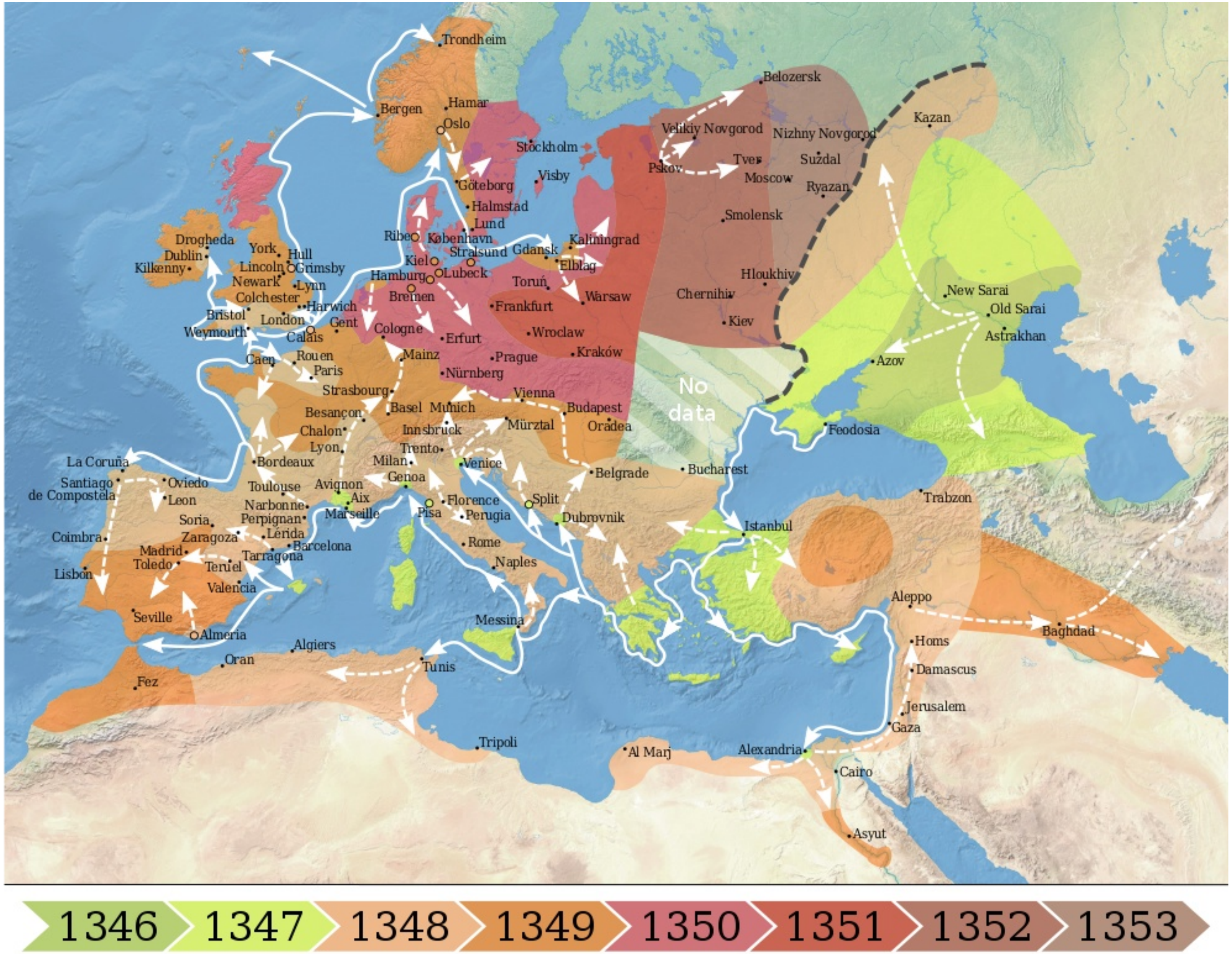
- A. L. Barabási (2016). Network Science – Chapter 10
- D. Easley and J. Kleinberg (2010). Networks, Crowds, and Markets — Chapter 21
- URLs cited in the footer of slides

Examples: human epidemics

- Influenza, measles, STDs
- The "Black Death" [next slide]
- Smallpox and other diseases brought by Europeans to America since early 1500s



https://en.wikipedia.org/wiki/Cocoliztli_Epidemic_of_1545-1548



https://commons.wikimedia.org/wiki/File:1346-1353_spread_of_the_Black_Death_in_Europe_map.svg

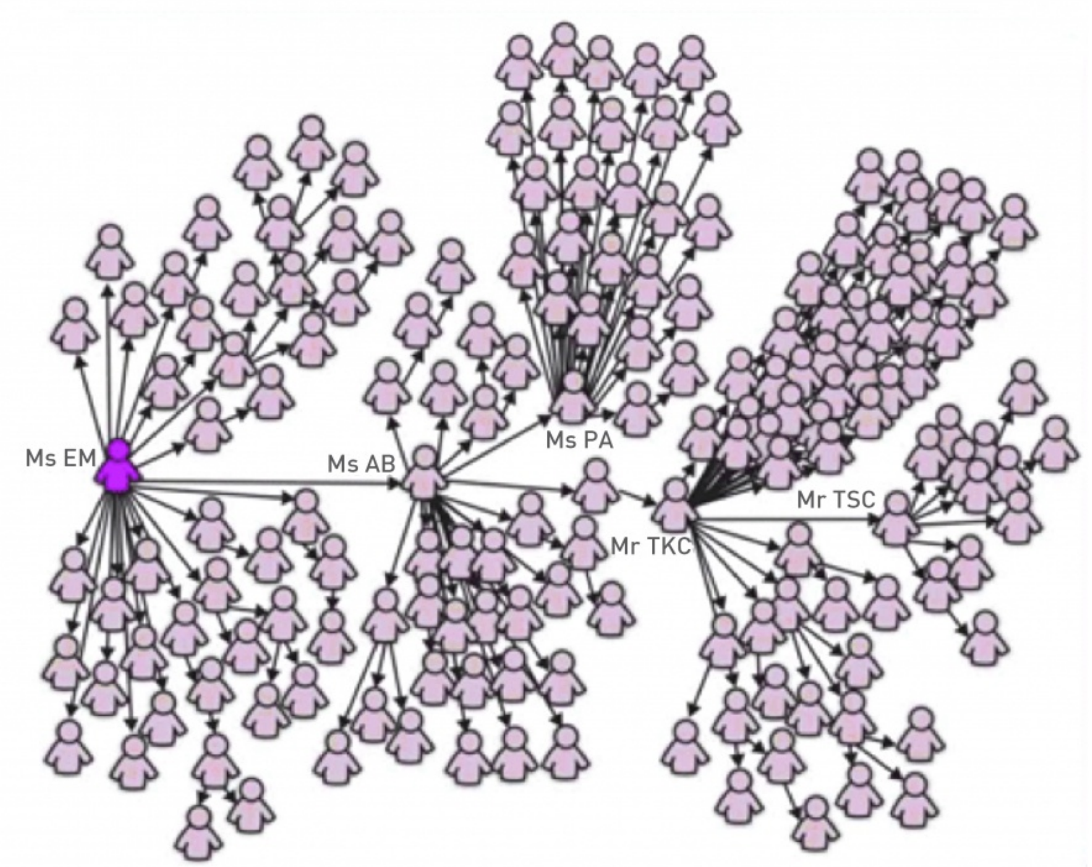
The “Black Death” (Bubonic plague)

1300s

Killed 30%-60% of the total population of Europe

SARS Outbreak (2003)

- February 21st: Chinese doctor who have been several treating “atypical pneumonia” cases check-ins into hotel in Hong Kong
  - Hospitalized on Feb 22nd
  - Died on March 4th
- March 1st: “Ms. E. M.” returns to Singapore after visiting Hong Kong
  - Graph depicts 144 out of the first 206 SARS patients in Singapore
  - Ms. E. M. lived, various of her family members died

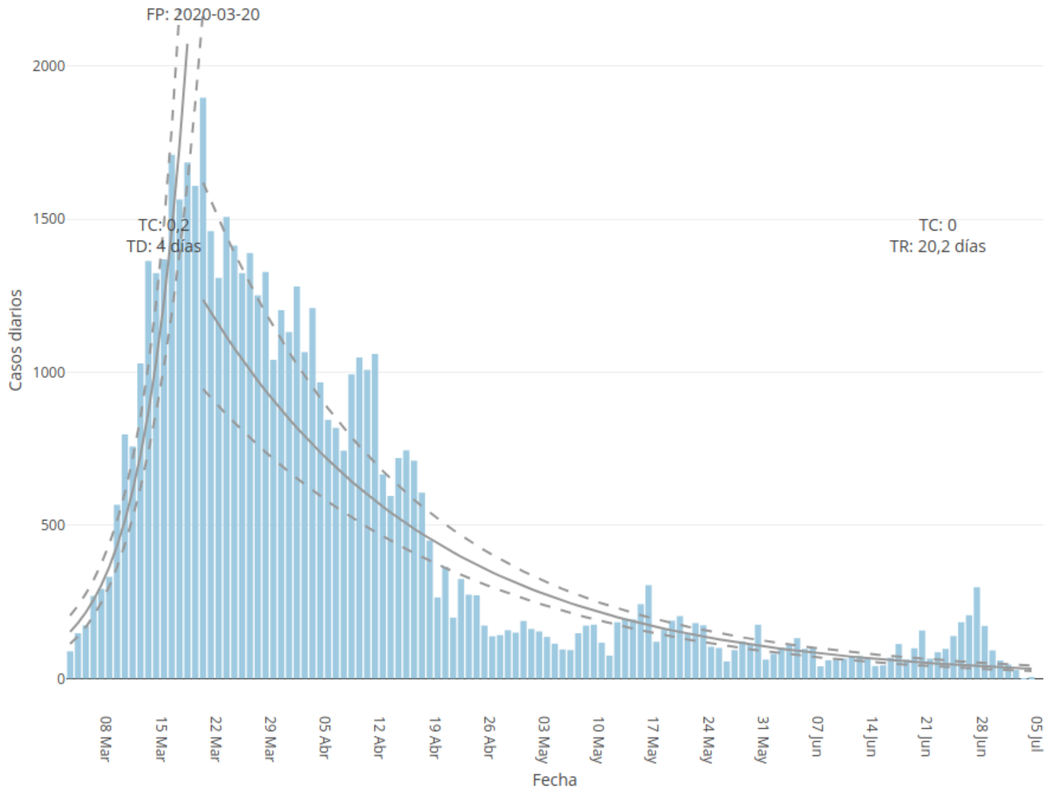


https://en.wikipedia.org/wiki/Timeline_of_the_SARS_outbreak

COVID-19

Why this curve?

How can one make this kind of forecast?



COVID-19 cases in Catalonia as of July 2020 - https://cnecovid.isciii.es/covid19/#ccaa

Diffusion of ideas vs diseases

- Adopting a new idea, behavior, fashion, product, taste, may also spread from person to person: "social contagion"
- There is a certain agency of the receiver
- In diffusion of diseases, we assume **there is no agency: each contagion is random**

Simple model: branching process

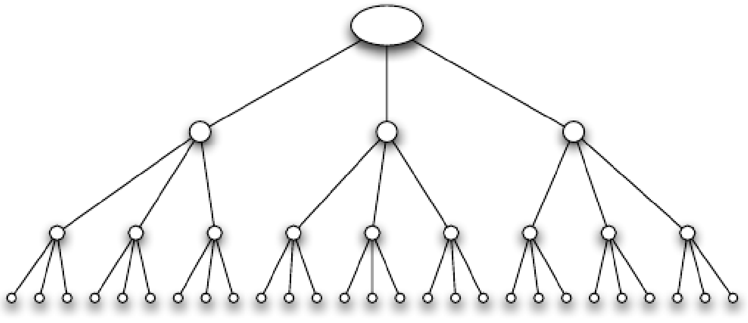
Modeling epidemics

- There are many factors:
  - Contagiousness
  - Length of infectious period,
  - Severity
  - ...
- Structure of contacts in a population

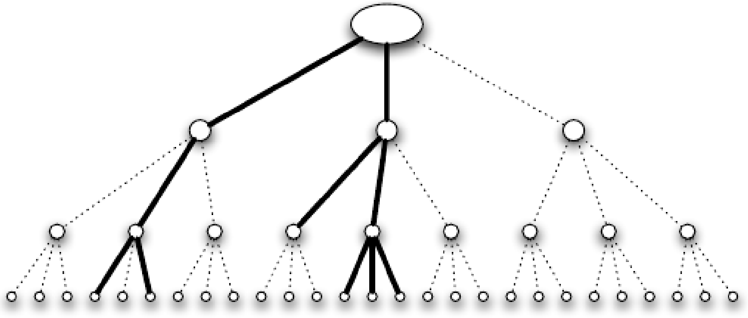
Simple model: branching process

- Each person interacts with other k people
- Each interaction ends in infection with probability β

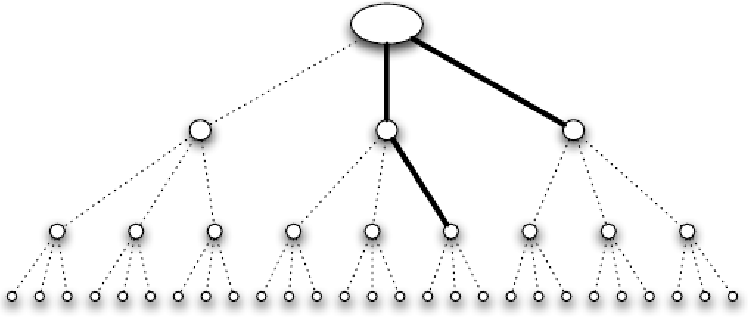
Example: k=3



(a) *The contact network for a branching process*



(b) *With high contagion probability, the infection spreads widely*



(c) *With low contagion probability, the infection is likely to die out quickly*

Transmission rate or "Basic reproductive number" R_0

- Each person interacts with other k people
- Each interaction ends in infection with probability β

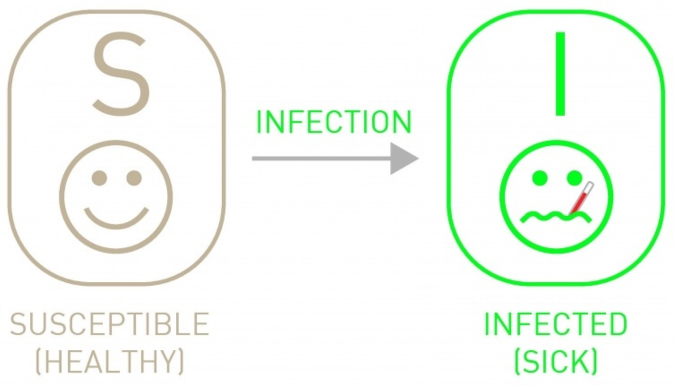
- What is the expected number of cases caused by a single individual, R_0?
- What do you think happens if $R_0 < 1$?
- What do you think happens if $R_0 > 1$?

Changing $R_0 = \beta k$

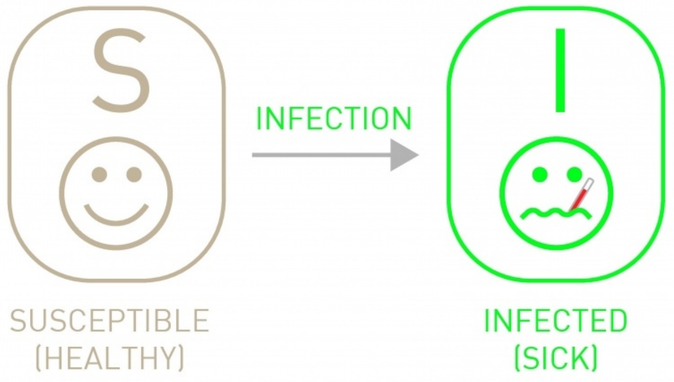
Disease	Transmission	R_0
Measles	Airborne	12-18
Pertussis	Airborne droplet	12-17
Diptheria	Saliva	6-7
Smallpox	Social contact	5-7
Polio	Fecal-oral route	5-7
Rubella	Airborne droplet	5-7
Mumps	Airborne droplet	4-7
HIV/AIDS	Sexual contact	2-5
SARS	Airborne droplet	2-5
Influenza (1918 strain)	Airborne droplet	2-3

- Sanitary practices (reduce β)
- Quarantine (reduces k)

The SI model



The SI model



- Susceptible:
  - The node can catch the disease
- Infected:
  - The node has the disease and can spread it
  - It will stay sick forever

Notation

- Number of susceptible $S(t)$
  - Fraction of susceptible $s(t) = S(t) \ / \ N$
- Number of infected $I(t)$
  - Fraction of infected $i(t) = I(t) \ / \ N$
- $s(t) + i(t) = 1$

How many susceptible neighbors a node has?

$$\langle k \rangle \frac{S(t)}{N} = \langle k \rangle \, s(t)$$

How many new infections are produced?

(for every infected, iterate through its susceptible neighbors, infect with probability β)

$$i(t) \, \underbrace{\langle k \rangle \, s(t)}\beta$$

infected   susceptible neighbors   infection probability

Prove that $i(t) = \dfrac{i_0 e^{\beta\langle k\rangle t}}{1 - i_0 + i_0 e^{\beta\langle k\rangle t}}$

Begin from: $\dfrac{di(t)}{dt} = i(t)\left\langle k\right\rangle(1 - i(t))\beta$

First, place all terms with $i(t)$ on the left side

Second, use $\dfrac{1}{x\cdot(1-x)} = \dfrac{1}{x} + \dfrac{1}{1-x}$

Third, integrate from $t=0$ to t and denote by $i_0 = i(t=0)$

$$\int \frac{1}{x}dx = \log x + C \qquad \int \frac{1}{1-x}dx = -\log(1-x) + C$$

Behavior in the limit $t \to \infty$

- What is the limit of when $t \to \infty$?

$$i(t) = \frac{i_0 e^{\beta \langle k \rangle t}}{1 - i_0 + i_0 e^{\beta \langle k \rangle t}}$$

- Hint: similar to

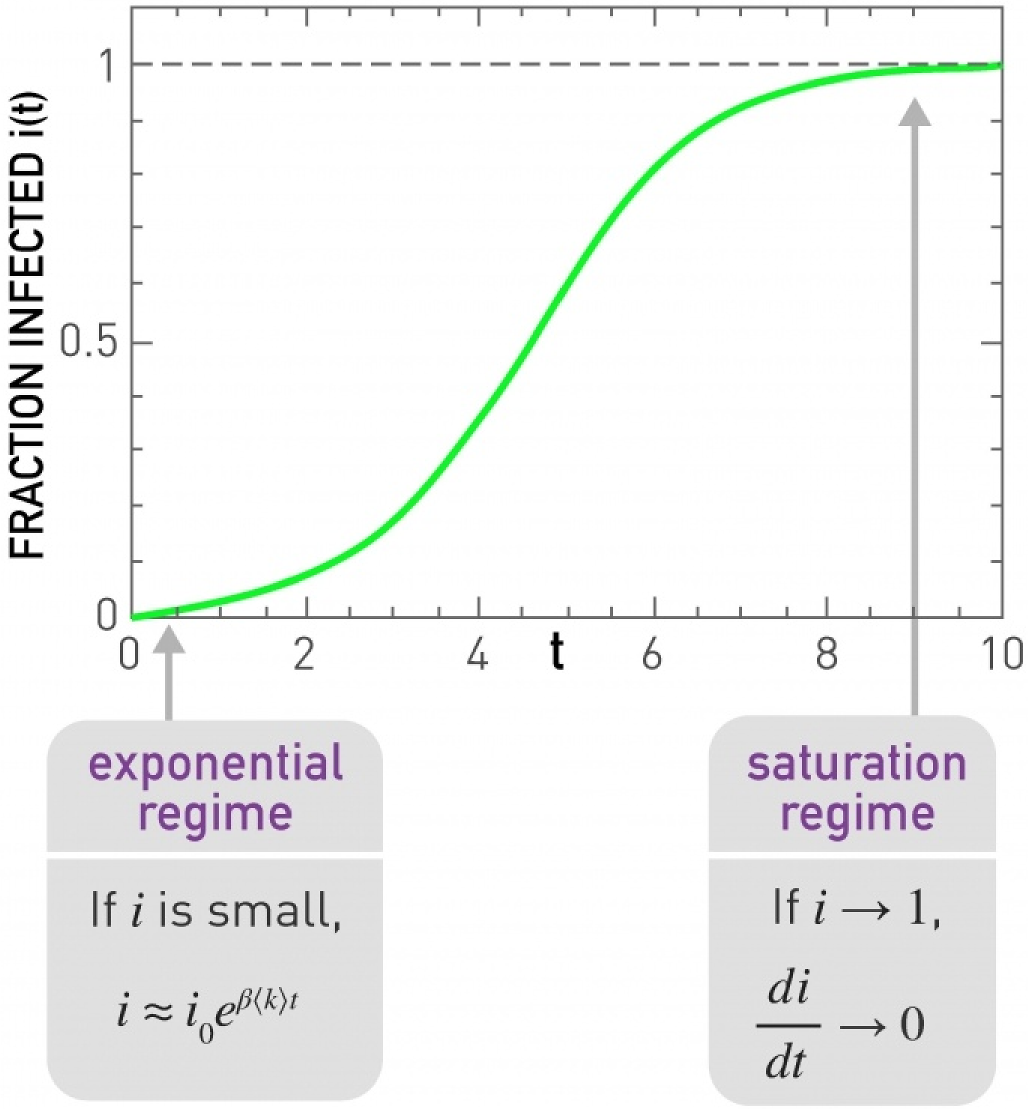
$$f(t) = \frac{e^t}{1 + e^t}$$

Infected as a function of time (SI)

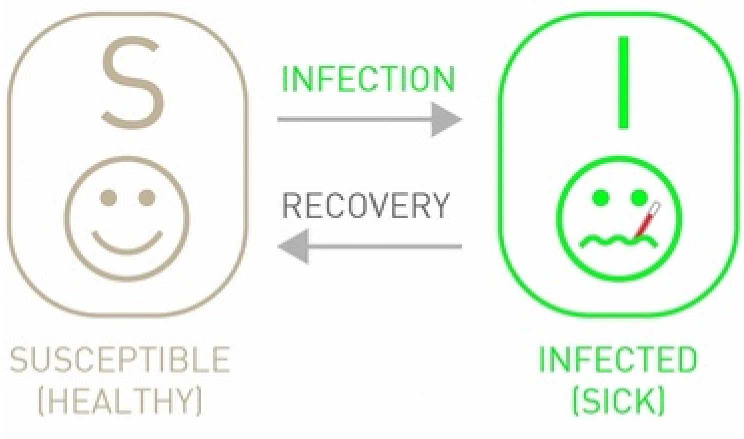
$$i(t) = \frac{i_0 e^{\beta \langle k \rangle t}}{1 - i_0 + i_0 e^{\beta \langle k \rangle t}}$$

Characteristic time
(to infect 1/e ≃ 36% of people):

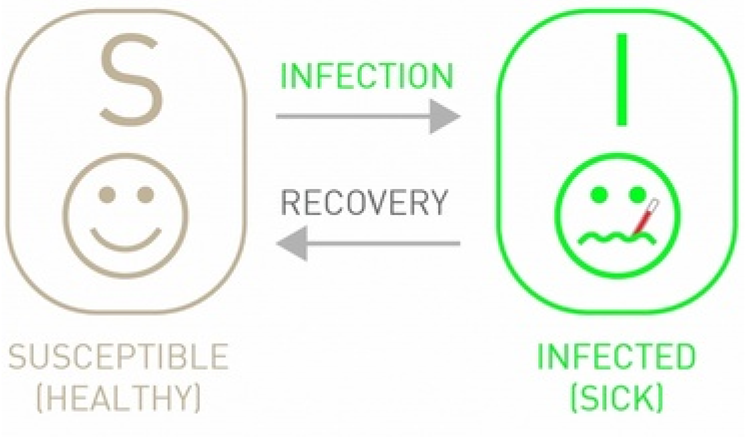
$$\tau = \frac{1}{\beta \langle k \rangle}$$



The SIS model



The SIS model



- Susceptible:
  - The node can catch the disease
- Infected:
  - The node has the disease and can spread it
  - After some time, it recovers … but it becomes susceptible again

Infection dynamics

$$\frac{di(t)}{dt} = \beta \langle k \rangle i(t)(1 - i(t)) - \mu i(t)$$

- μ is the recovery rate, i.e., the probability of becoming susceptible again in an unit of time

$$i(t) = \left(1 - \frac{\mu}{\beta \langle k \rangle}\right) \frac{Ce^{(\beta\langle k \rangle - \mu)t}}{1 + Ce^{(\beta\langle k \rangle - \mu)t}}$$

- C is a constant that depends on i_0

Behavior in the limit $t \to \infty$

- What is the limit of when $t \to \infty$?

$$i(t) = \left(1 - \frac{\mu}{\beta \langle k \rangle}\right) \frac{Ce^{(\beta\langle k\rangle - \mu)t}}{1 + Ce^{(\beta\langle k\rangle - \mu)t}}$$

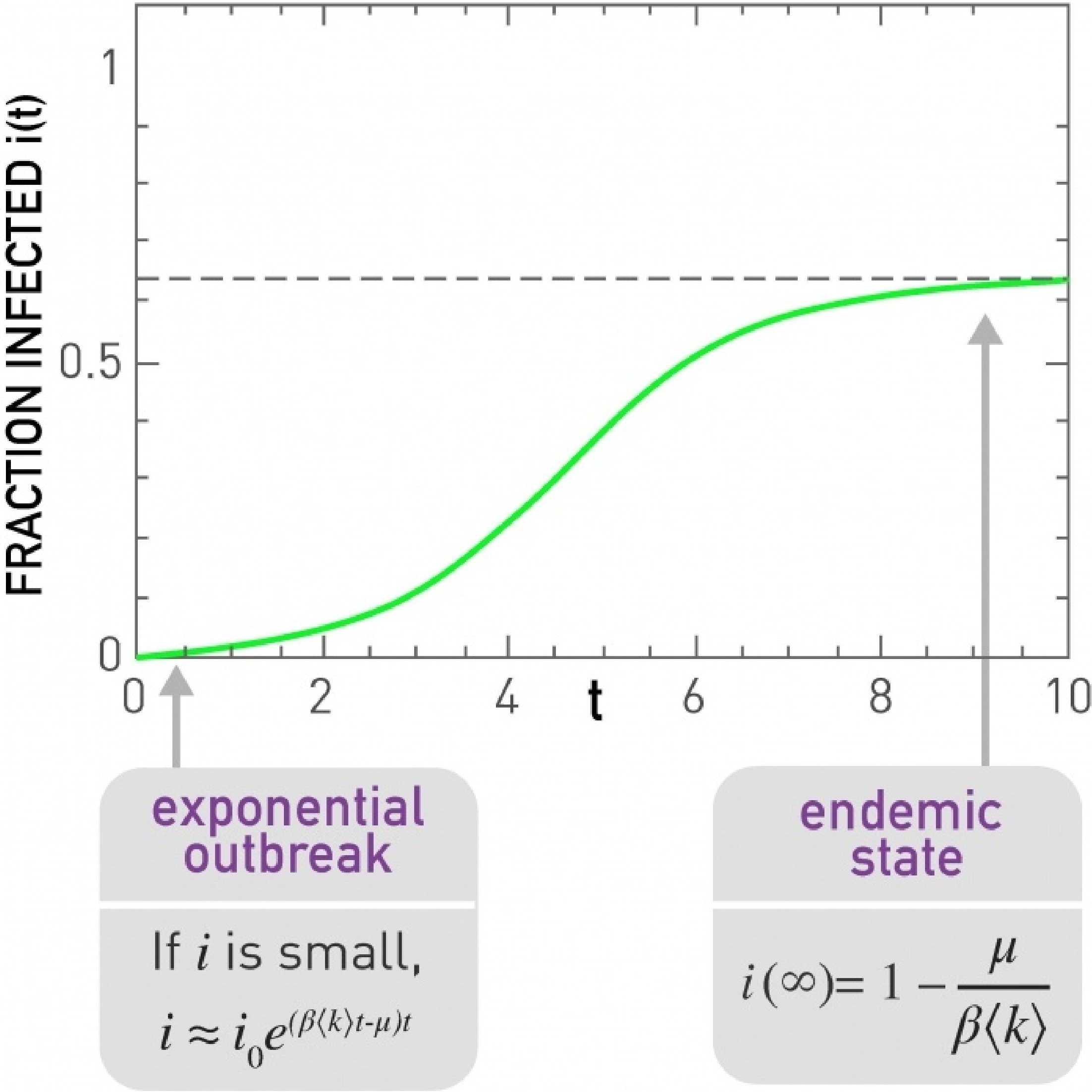
- Hint: similar to

$$f(t) = \alpha \frac{e^t}{1 + e^t}$$

Infected as a function of time (SIS)

$$i(t) = \left(1 - \frac{\mu}{\beta\langle k\rangle}\right)\frac{Ce^{(\beta\langle k\rangle-\mu)t}}{1+Ce^{(\beta\langle k\rangle-\mu)t}}$$

This is in the case $\mu < \beta\langle k\rangle$

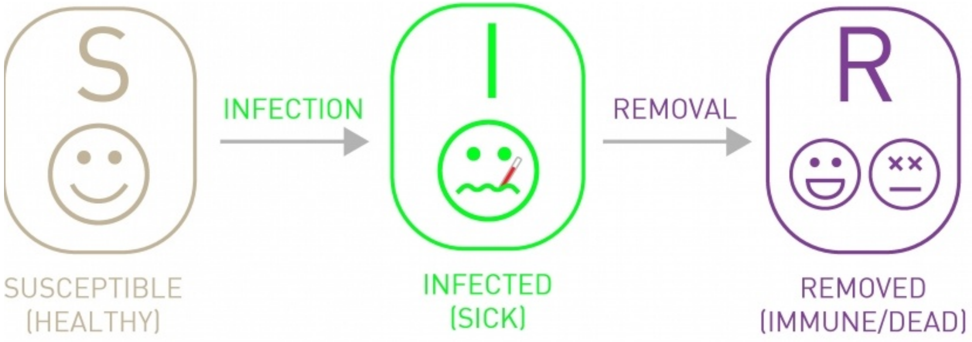


What happens if $\mu > \beta \langle k \rangle$?

- Remember: $$\frac{di(t)}{dt} = \beta \langle k \rangle i(t)(1 - i(t)) - \mu i(t)$$

The SIR model

The SIR model



- Susceptible:
  - The node can catch the disease
- Infected:
  - The node has the disease and can spread it
- Removed:
  - The node no longer has the disease, and cannot catch it or propagate it again (could be dead, could be immune)

Infection dynamics in SIR

$$\frac{di(t)}{dt} = \beta \langle k \rangle i(t)(1 - r(t) - i(t)) - \mu i(t)$$
$$\frac{dr(t)}{dt} = \mu i(t)$$
$$\frac{ds(t)}{dt} = -\frac{di(t)}{dt} - \frac{dr(t)}{dt} = -\beta \langle k \rangle i(t)(1 - r(t) - i(t))$$

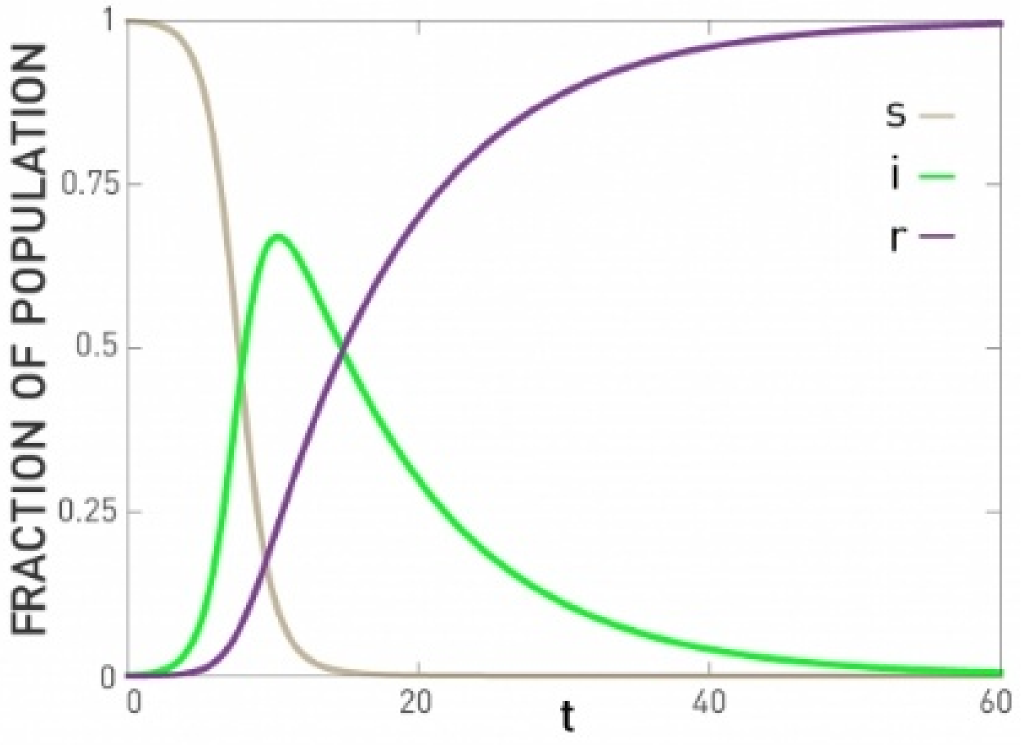
- No closed form solution

Infection dynamics (SIR)

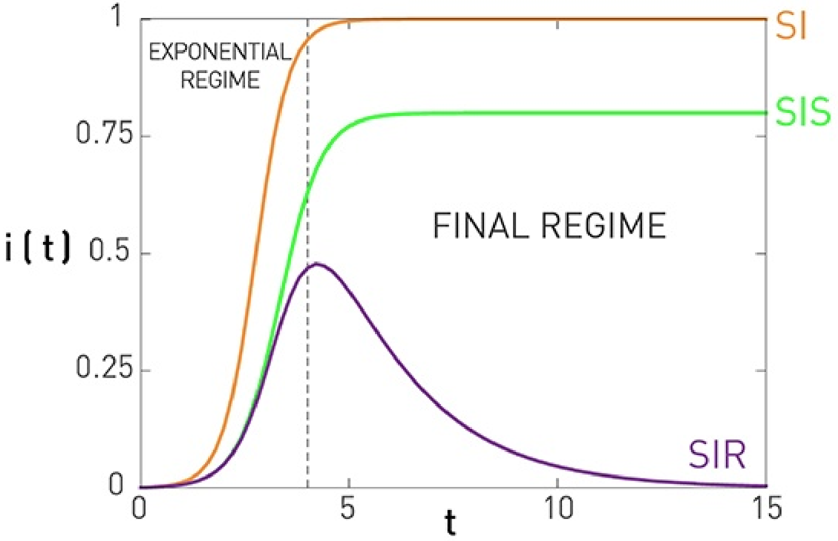
$$\frac{di(t)}{dt} = \beta \langle k \rangle i(t)(1 - r(t) - i(t)) - \mu i(t)$$

$$\frac{dr(t)}{dt} = \mu i(t)$$

$$\frac{ds(t)}{dt} = -\beta \langle k \rangle i(t)(1 - r(t) - i(t))$$



Comparison of $i(t)$



	SI	SIS	SIR
Exponential Regime: Number of infected individ-uals grows exponentially	$i = \frac{i_0 e^{\beta\langle k\rangle t}}{1 - i_0 + i_0 e^{\beta\langle k\rangle t}}$	$i = \left(1 - \frac{\mu}{\beta\langle k\rangle}\right)\frac{Ce^{(\beta\langle k\rangle - \mu)t}}{1 + Ce^{(\beta\langle k\rangle - \mu)t}}$	No closed solution
Final Regime: Saturation at $t \rightarrow = \infty$	$i(\infty) = 1$	$i(\infty) = 1 - \frac{\mu}{\beta\langle k\rangle}$	$i(\infty) = 0$
Epidemic Threshold: Disease does not always spread	No threshold	$R_0 = 1$	$R_0 = 1$

Things to remember

- SI, SIS, SIR models
- Which are the states in each process and which are the possible transitions
- Equations for number of nodes in each state
- Regimes under different parameters
- Practice executing by hand and write code if it helps you remember better each process

Practice on your own

Under the **SIS** model, $$i(t) = \left(1 - \frac{\mu}{\beta \langle k \rangle}\right) \frac{Ce^{(\beta\langle k\rangle - \mu)t}}{1 + Ce^{(\beta\langle k\rangle - \mu)t}}$$

1. When $\mu < \beta \langle k \rangle$ what is the limit of $i(t)$?
2. How is this state called?
3. What happens when $\mu > \beta \langle k \rangle$?
4. What conditions lead to large values of $i(t)$?

Practice on your own (cont.)

- In the **SIRS** epidemic model, there are three possible states for a node: susceptible, infected, and recovered. Susceptible nodes can become infected, infected nodes can become recovered, and *recovered nodes can become susceptible again*.
- During one unit of time, with probability β an infected node can infect one of its contacts, with probability μ, an infected node can recover, and with probability σ, a recovered node can become susceptible again.
- Let s(t) be the fraction of susceptible nodes, i(t) be the fraction of infected nodes, r(t) the fraction of recovered nodes, and $<$k$>$ the average degree of the graph. Write the equations, simplifying them appropriately, for:

$$1. \frac{di(t)}{dt} \quad 2. \frac{dr(t)}{dt} \quad 3. \frac{ds(t)}{dt}$$

4. Is $\sigma > \mu$ sufficient to say that the recovered will tend to zero in the long run?