

# Distances in Scale-Free Networks

## Social Networks Analysis and Graph Algorithms

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- Distance distribution of scale-free networks

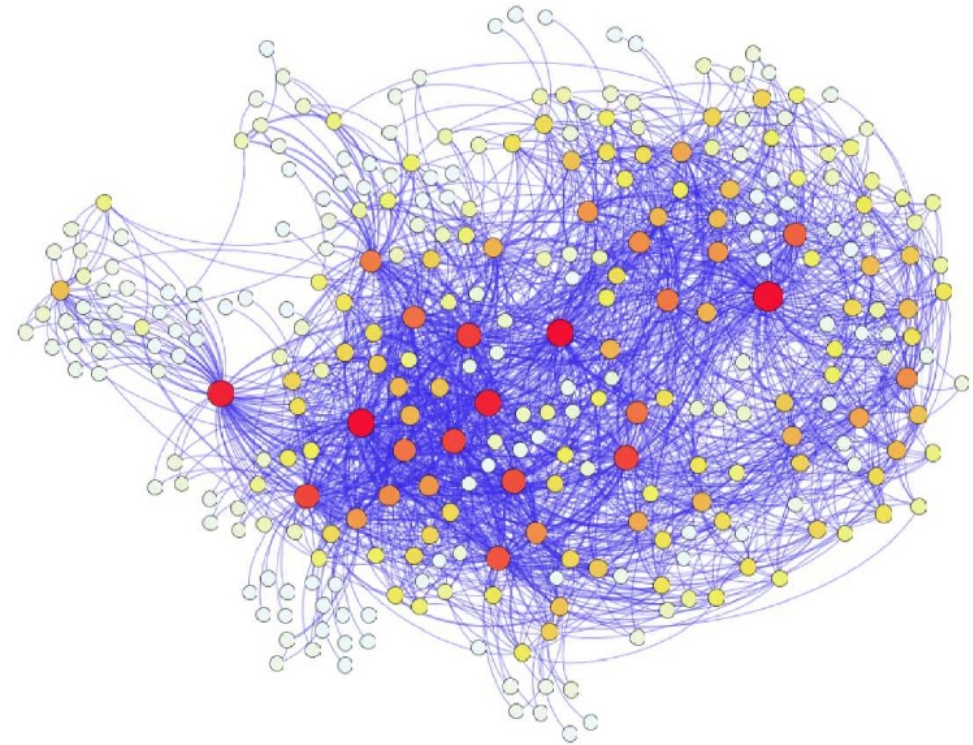
# Sources

- A. L. Barabási (2016). Network Science – Chapter 04
- URLs cited in the footer of specific slides

Consequences of having  
extremely large degree nodes  
*(also known as “large hubs”)*

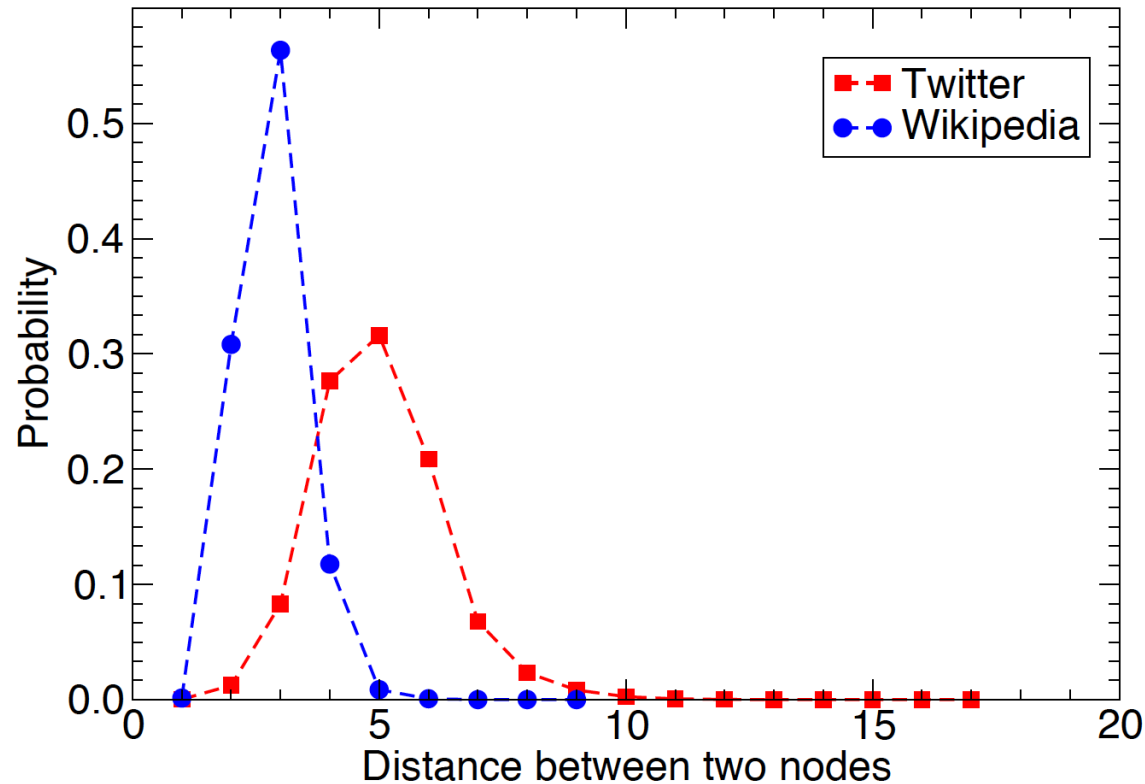
# Air travel

- You can travel between almost all pairs of European airports directly or (most of the time) with at most one stop
- All you have to do is **go to a well connected airport**
- This is because there are large degree airports



Cardillo, A et al. (2013). Modeling the multi-layer nature of the European Air Transport Network: Resilience and passengers re-scheduling under random failures. Euro. Phys. J. Special Topics, 215(1), 23-33. [\[DOI\]](#)

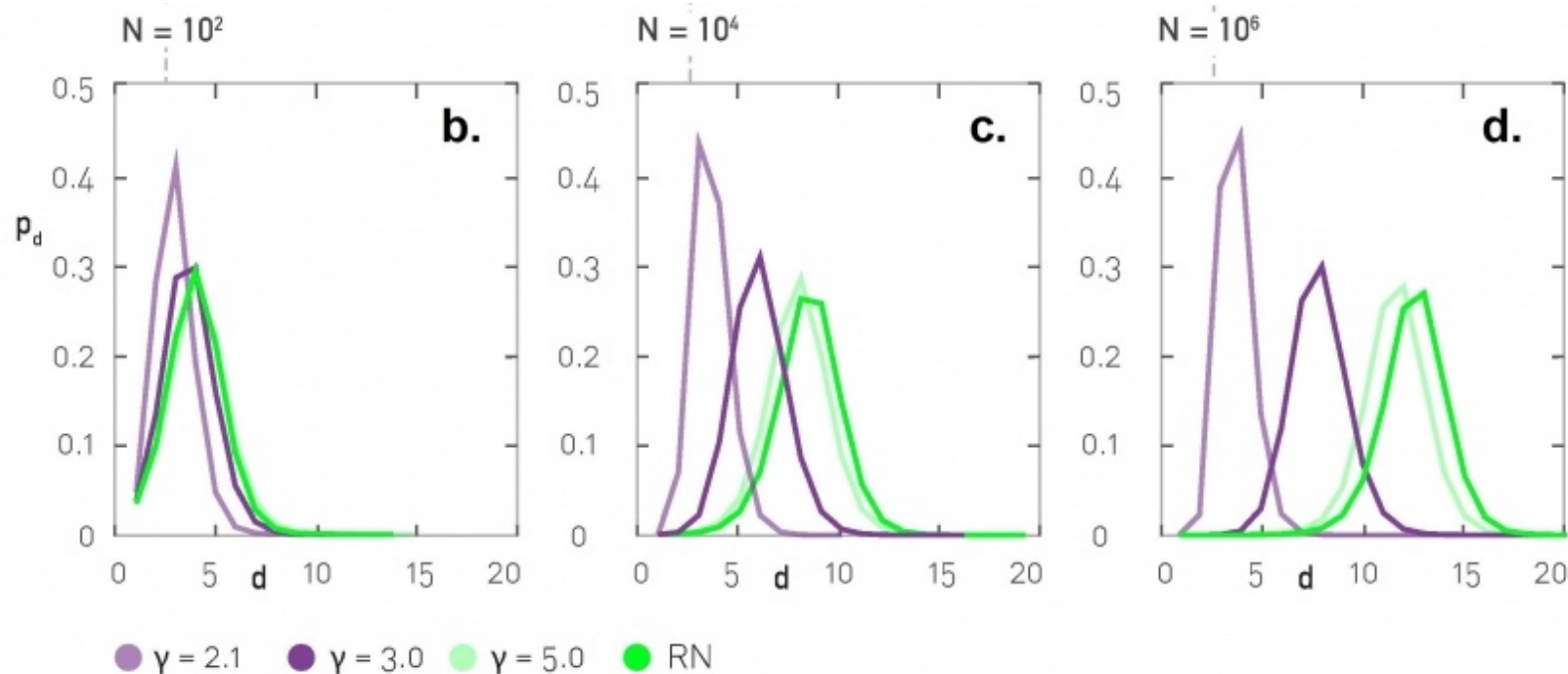
# In general, having “hubs” or large degree nodes reduces distances



# Distance regimes

# Distance distributions: simulation results

Scale-free networks of increasing size,  $\langle k \rangle = 3$





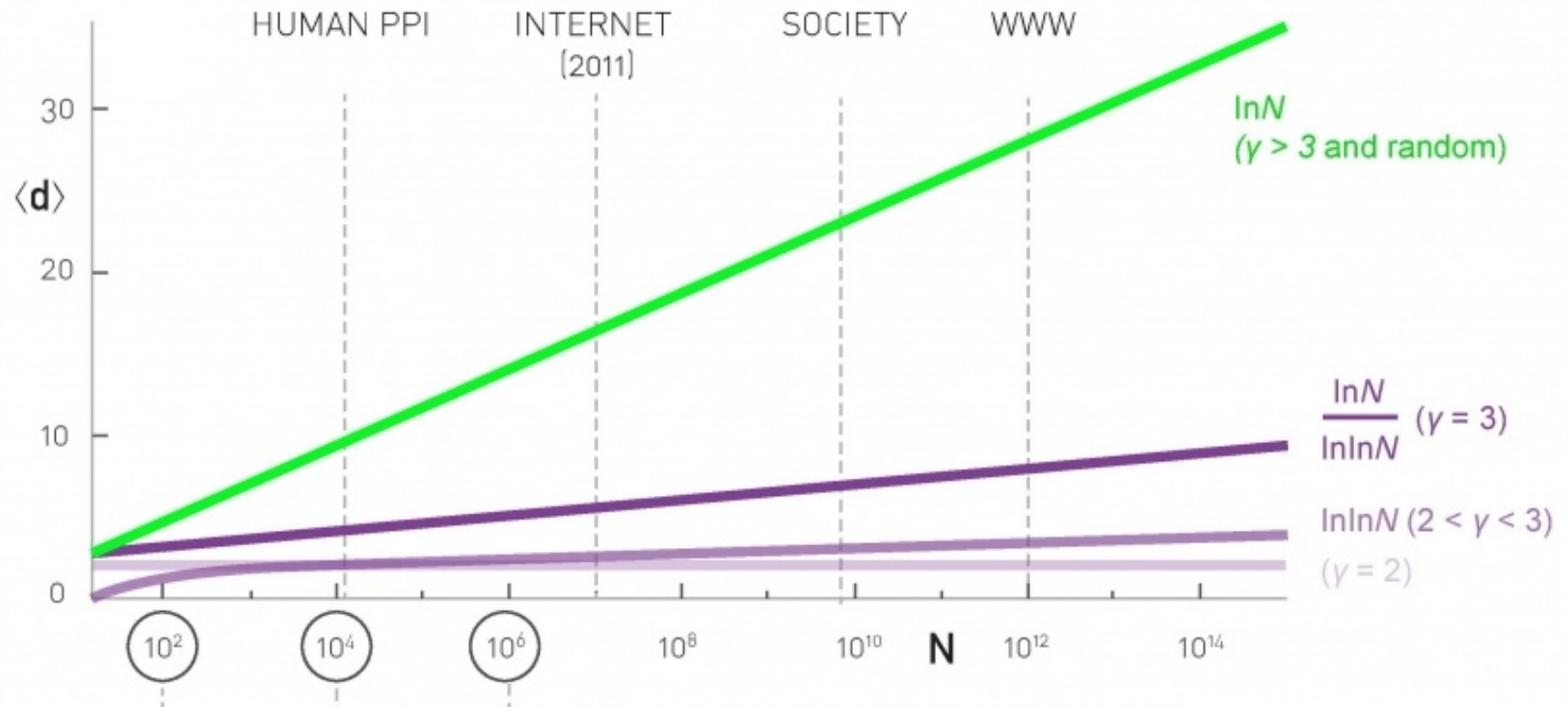
# Average distance

- Depends on  $\gamma$  and  $N$

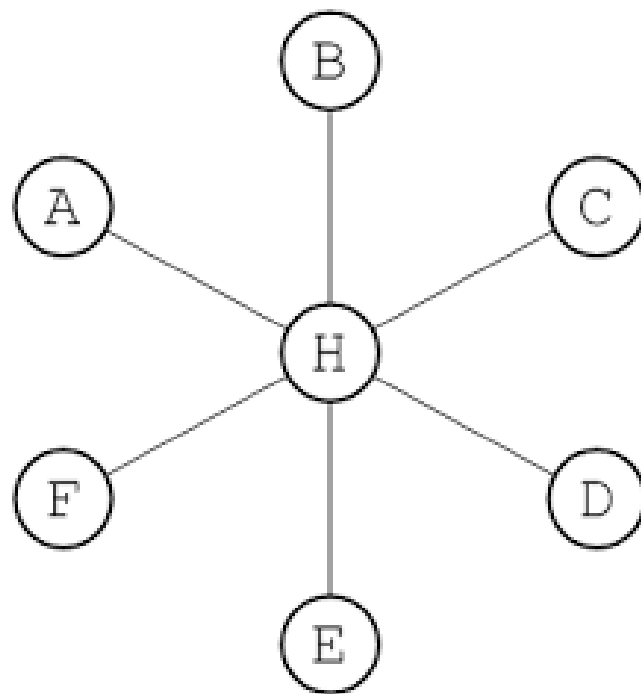
$$\langle d \rangle = \begin{cases} \text{const.} & \text{if } \gamma = 2 \\ \log \log N & \text{if } 2 < \gamma < 3 \\ \log N / \log \log N & \text{if } \gamma = 3 \\ \log N & \text{if } \gamma > 3 \end{cases}$$

← Same as in  
ER graphs

# Average distance and N



# Anomalous regime $\gamma = 2$



# Ultra-small world $2 < \gamma < 3$

- Average distance follows  $\log(\log(N))$
- Example (humans):

$$N \approx 7 \times 10^9$$

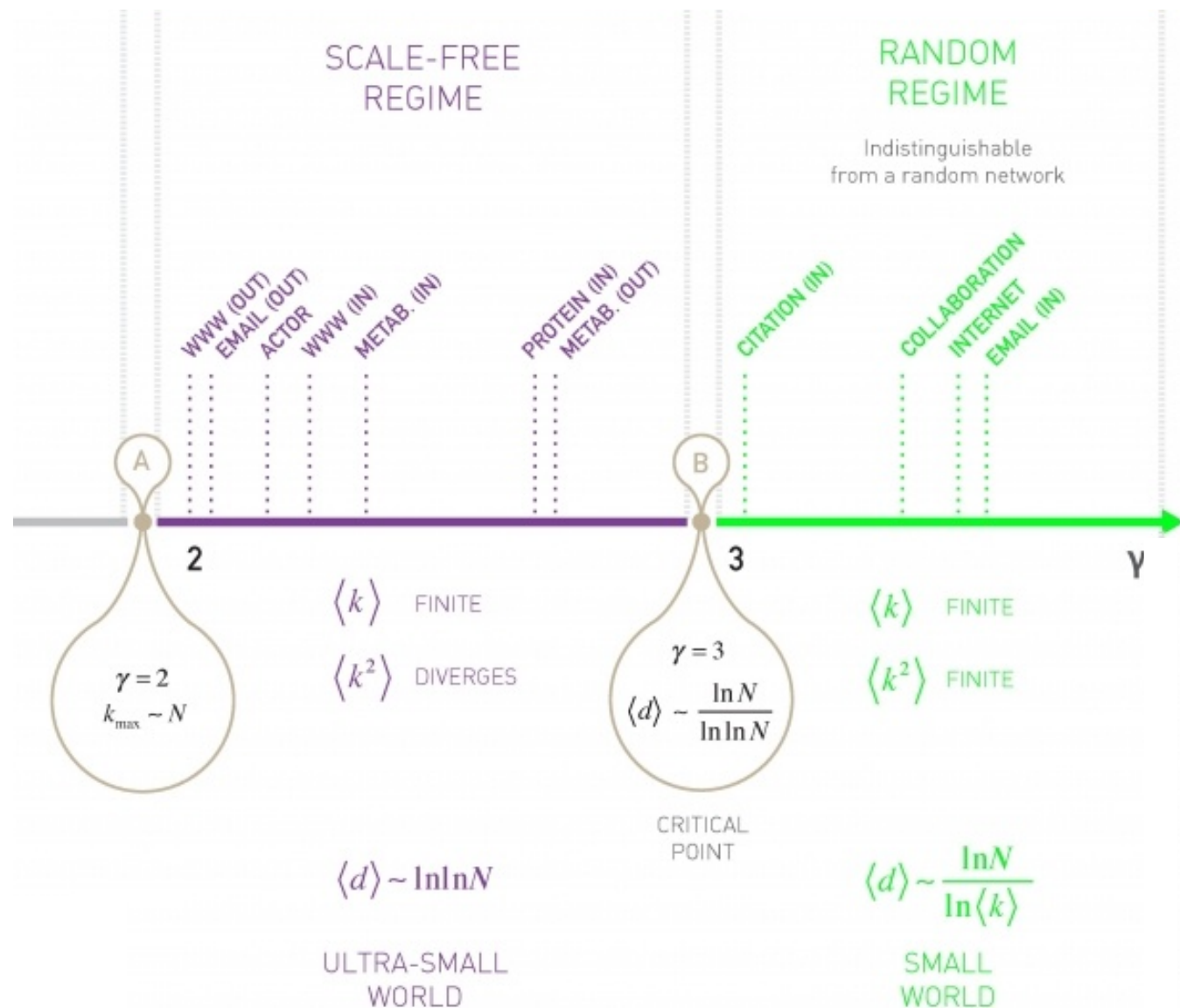
$$\log N \approx 22.66$$

$$\log \log N \approx 3.12$$

# Small world $\gamma > 3$

- Average distance follows  $\log(N)$
- Similar to ER graphs where it followed  $\log(N)/\log(\langle k \rangle)$

The degree distribution exponent plays an important role



# When $\gamma > 3$

- In this case it is hard to distinguish this case from an ER graph
- In most real complex networks (but not all)

$$2 < \gamma < 3$$

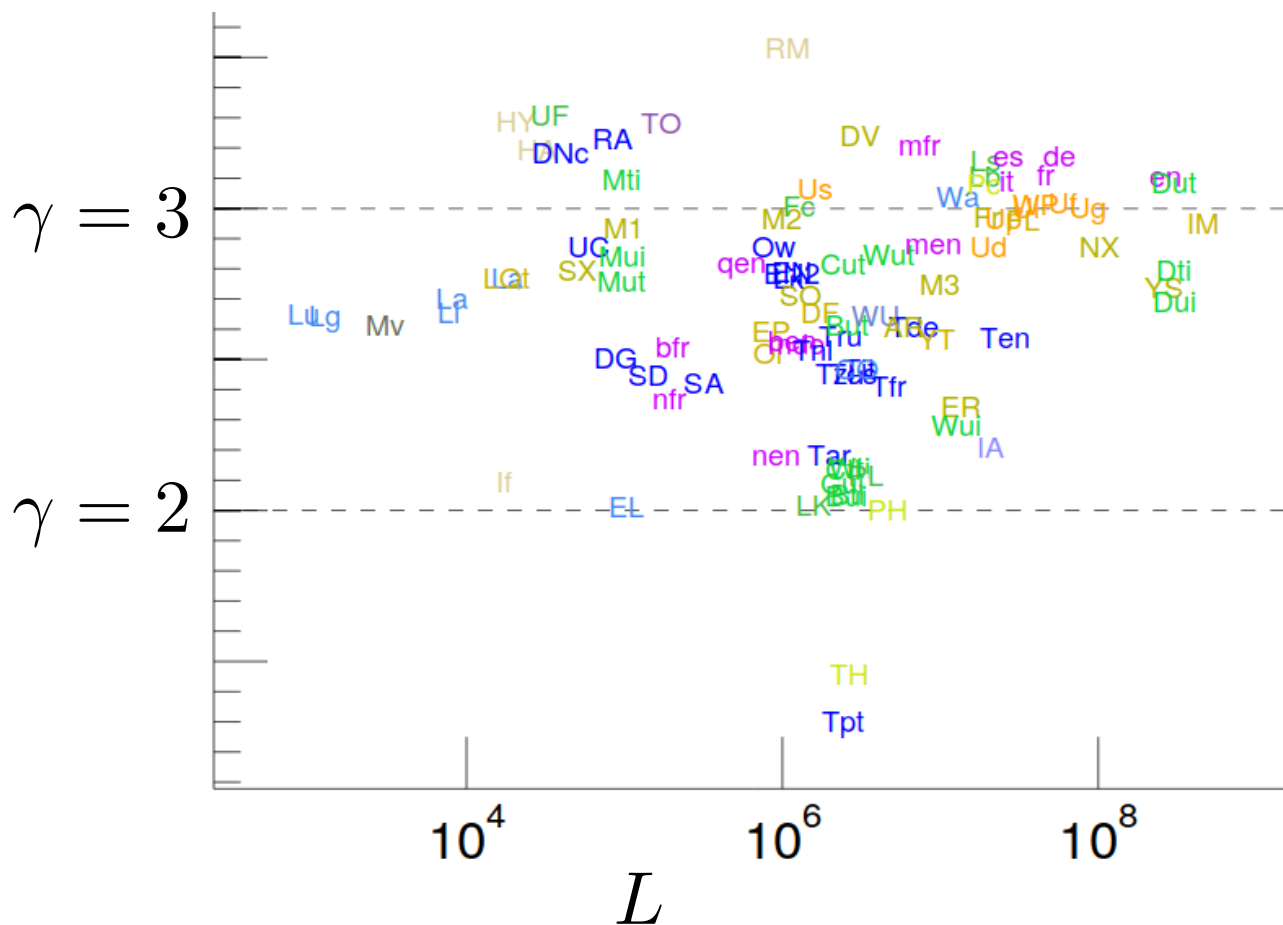
# When $\gamma > 3$

- Remember  $k_{\max} = k_{\min} N^{\frac{1}{\gamma-1}}$   $N = \left( \frac{k_{\max}}{k_{\min}} \right)^{\gamma-1}$
- Observing the scale-free properties requires that  $k_{\max} \gg k_{\min}$ , e.g.  $k_{\max} = 10 k_{\min}$
- Then if  $\gamma = 5$ ,  $N > 10^8$
- There are not many such networks for which we have available data



# Examples

<http://konect.uni-koblenz.de/statistics/prefatt>



<b>EL</b>	Wikipedia elections
<b>LK</b>	Linux kernel mailing list threads
<b>Bul</b>	BibSonomy u-i
<b>Bti</b>	BibSonomy t-i
<b>Cul</b>	CiteULike u-i
<b>If</b>	Infectious
<b>PL</b>	Prosper loans
<b>Cti</b>	CiteULike t-i
<b>Wti</b>	Twitter t-i
<b>nen</b>	Wikinews (en)
<b>Tar</b>	Wikipedia talk, Arabic
<b>Wul</b>	Twitter u-i
<b>ER</b>	Epinions
<b>nfr</b>	Wikinews (fr)
<b>Tfr</b>	Wikipedia talk, French
<b>SD</b>	Slashdot
<b>Tzh</b>	Wikipedia talk, Chinese
<b>Tes</b>	Wikipedia talk, Spanish

Etc.

# Summary

# Things to remember

- Regimes of distance and connectivity

# Practice on your own

- Remember the regimes of a graph given  $\langle k \rangle$   
(It is useful to know this by heart)
- Estimate distance distributions for some graphs