

K-Core Decomposition

Social Networks Analysis and Graph Algorithms

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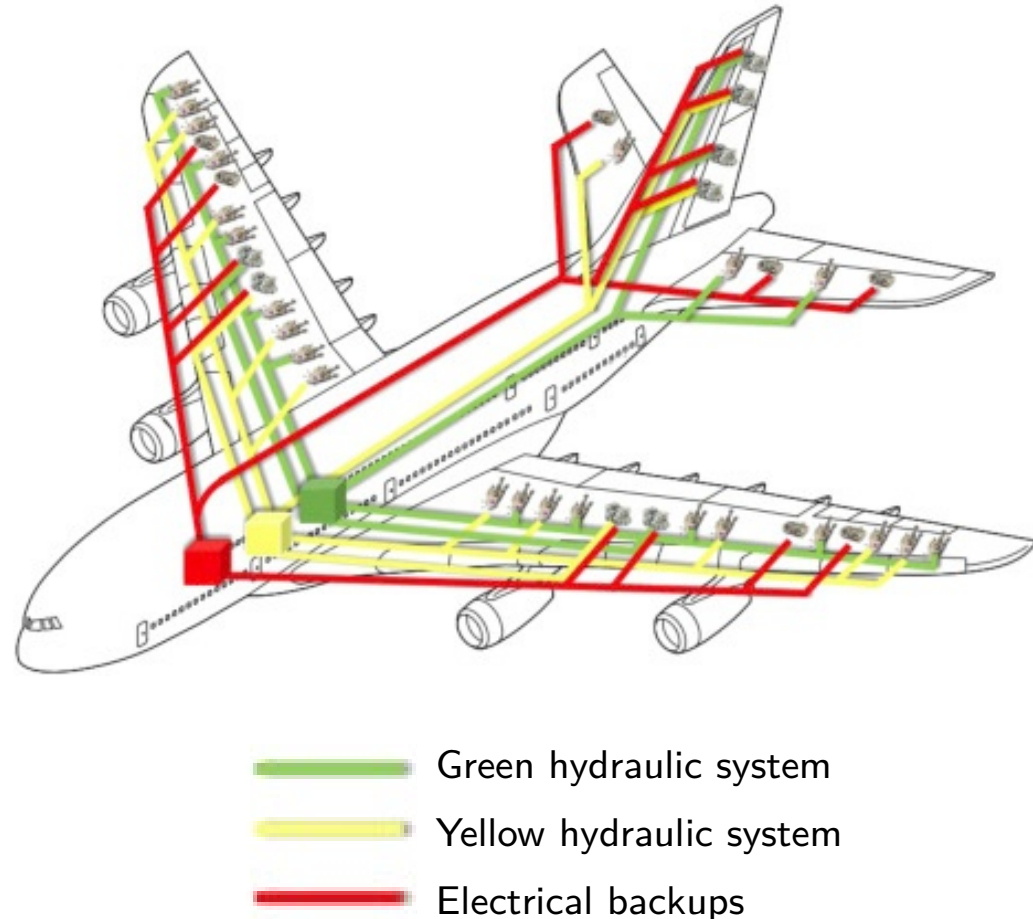
Sources

- A. L. Barabási (2016). Network Science – [Chapter 09](#)
- A. Beutel, L. Akoglu, C. Faloutsos (2015). [Tutorial at KDD](#)
- A. Frieze, A. Gionis, C. Tsourakakis (2013). “Algorithmic techniques for modeling and mining large graphs (AMAZING)” [Tutorial at KDD](#)
- V. E. Lee, N. Ruan, R. Jin, C. Aggarwal (2010). A survey of algorithms for dense sub-graph discovery. [Chapter 10](#) of “Managing and Mining Graph Data”
- URLs cited in the footer of slides

Robustness

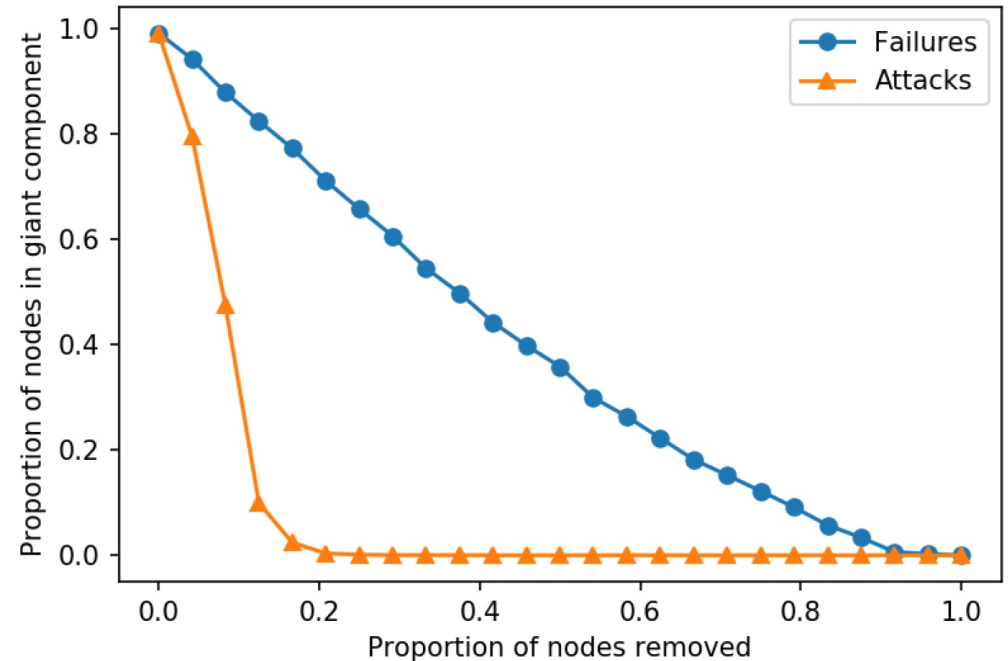
Robustness to failure

- A system is **robust** if the failure of some of its components does not affect its function
- Aircrafts, for instance, have:
 - Separate primary and back-up flight instruments (airspeed, altimeter, ...);
 - 3+ independent hydraulic systems
 - Primary and emergency landing gear;
 - Multiple sources of power



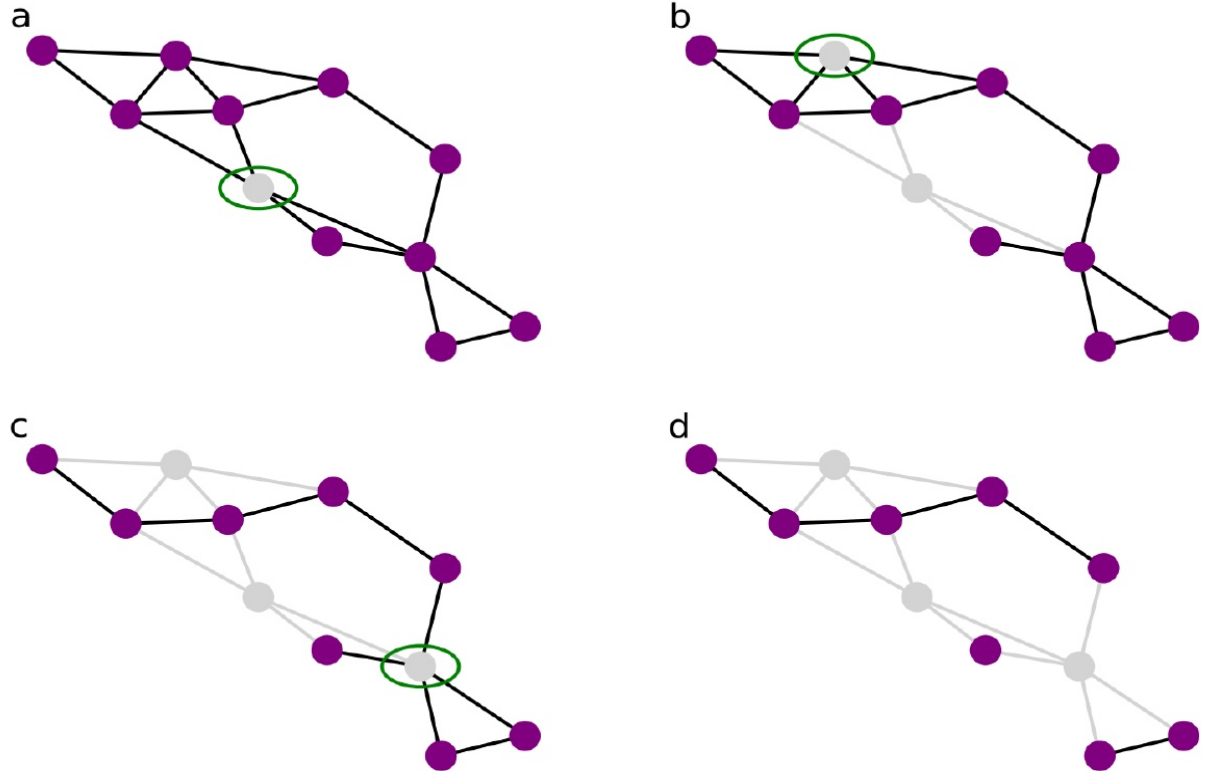
Robust networks maintain connectedness

The size of the giant component in this network of flights is reduced slowly by random removals (“failures”), but reduced quickly by removing high-degree edges (“attacks”)



Targeted removal of nodes

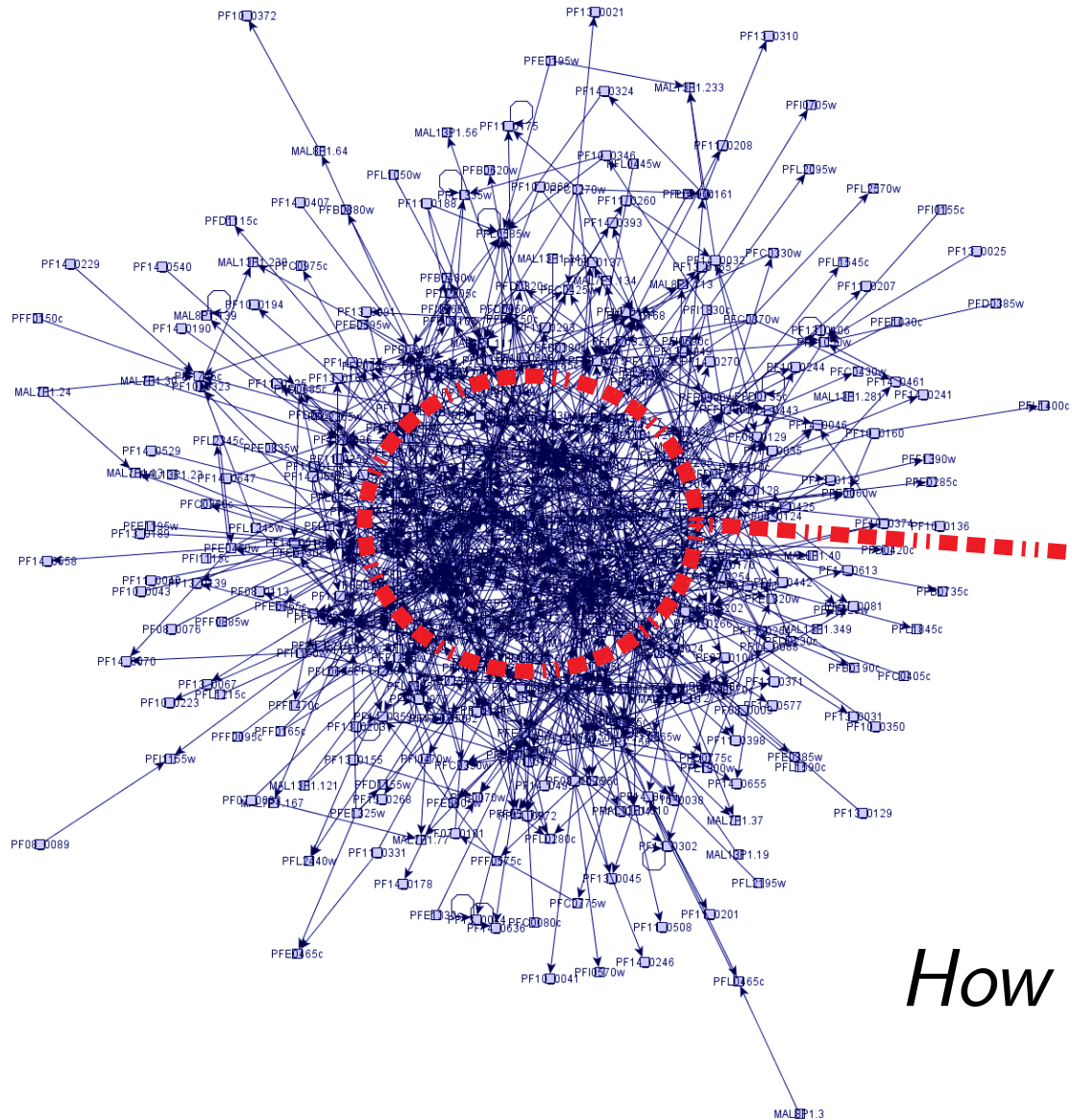
The targeted
removal of 4 nodes
disconnects this
graph into three
components



Many networks look
like “hairballs”

They have a **core**
and a **periphery**

How can we find the core?

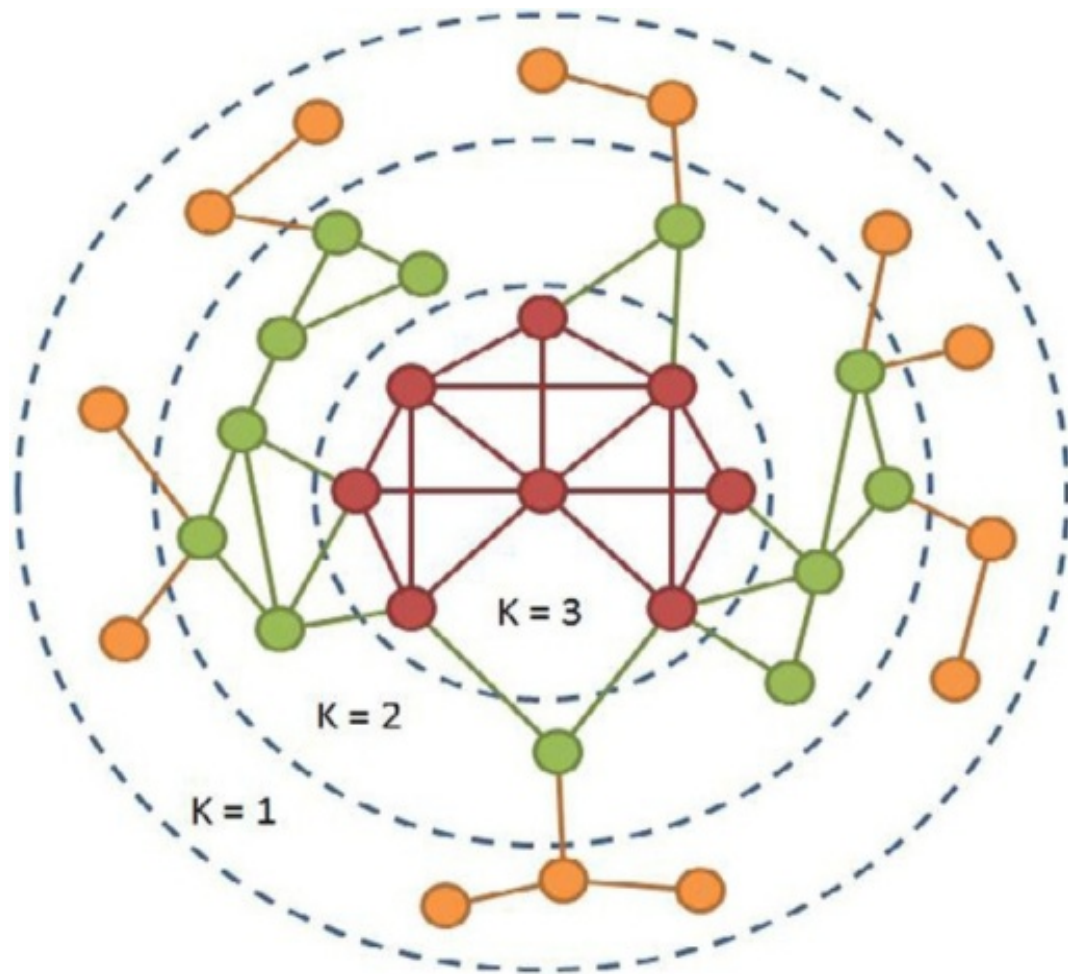


k-core decomposition
is a method to decompose
a graph into *layers*

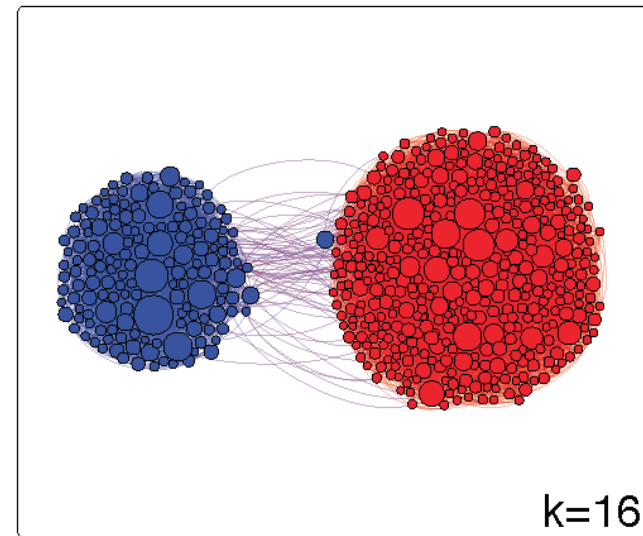
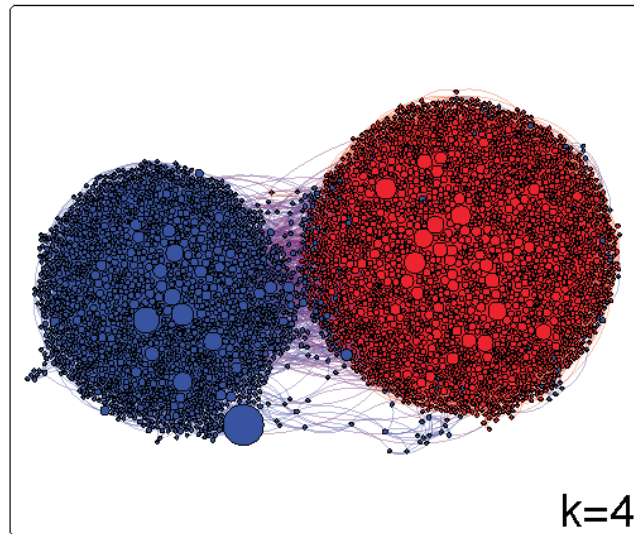
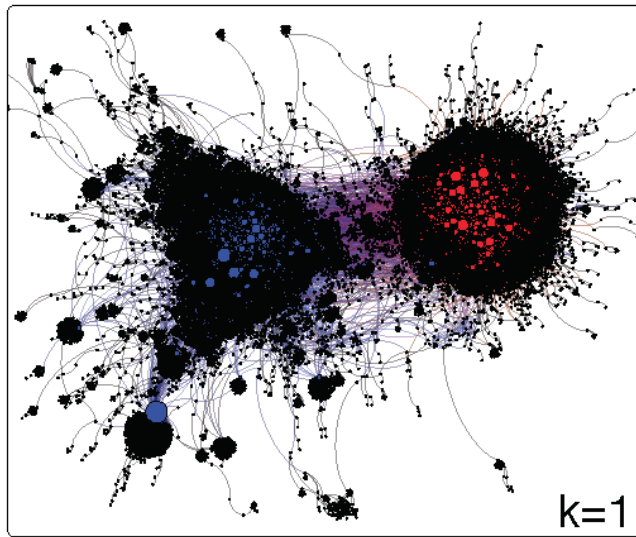
k-core decomposition

- Remove all nodes having degree 1
 - Those are in the 1-core
- Remove all nodes having degree 2 *in the remaining graph*
 - Those nodes are in the 2-core
- Remove all nodes having degree 3 *in the remaining graph*
 - Those nodes are in the 3-core
- Etc.

Example 1

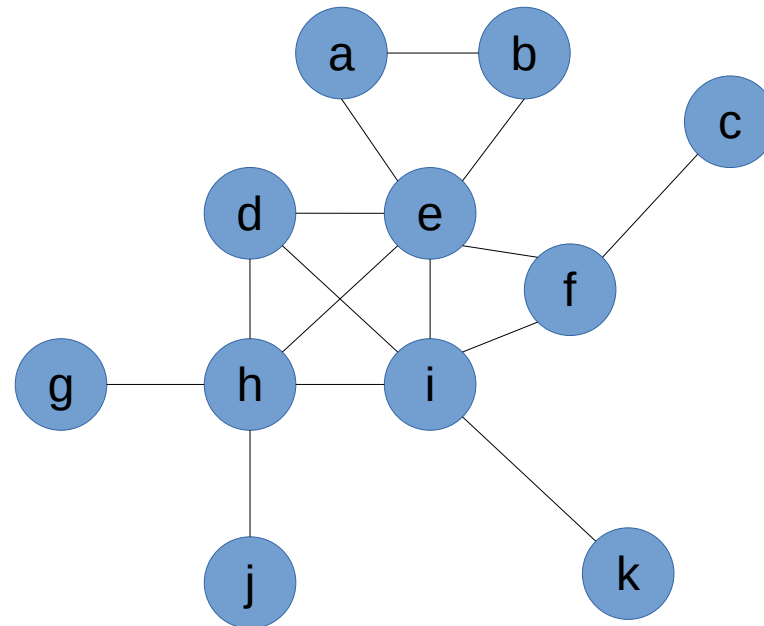


Example 2



Exercise

For each node in the graph, indicate the max k-core to which it belongs



<http://www.cpt.univ-mrs.fr/~barrat/NHM.pdf>

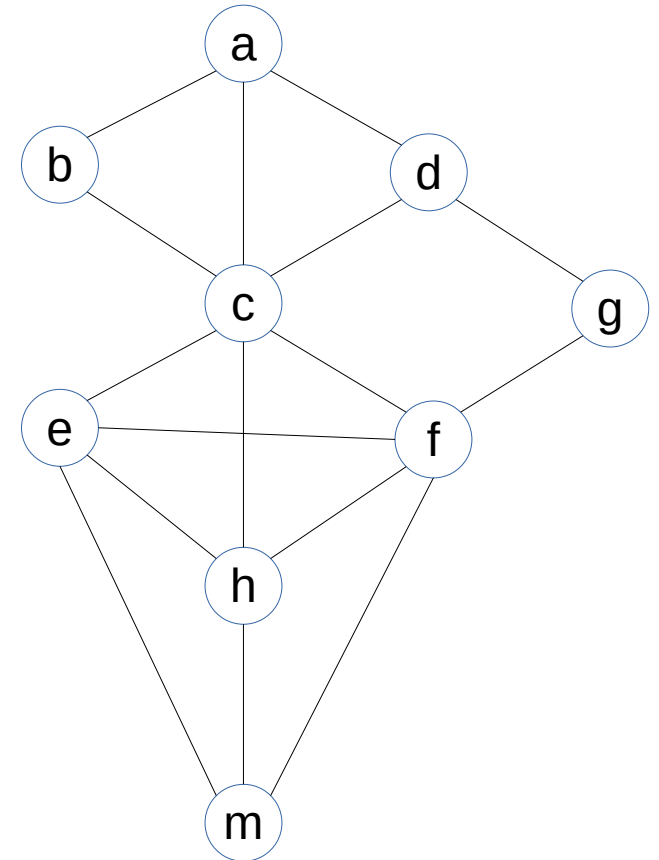
Summary

Things to remember

- What does it mean for a network to be robust?
- What is the k -core decomposition
- How to compute it on a graph

Practice on your own

Find the 3-core of this graph



Solution by Vivekanand Khyade (start at 01:23)
<https://youtu.be/8sNZ5d8eNC8?t=83>