## Models of Influence

### Social Networks Analysis and Graph Algorithms

Prof. Carlos "ChaTo" Castillo — <a href="https://chato.cl/teach">https://chato.cl/teach</a>

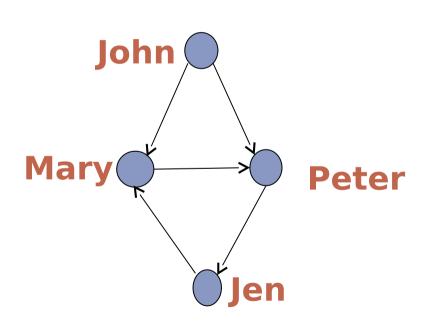


## Sources

- Easley and Kleinberg (2010): Networks, Crowds, and Markets Ch 19
- Carlos Castillo, Wei Chen, Laks V. S. Lakshmanan (2012): Information and Influence Spread in Social Networks, KDD Tutorial.
- Carlos Castillo (2017): Social influence slides

### What are our observables?

**Graph**: users, links/ties



**Log**: user, action, time

User	Action	Time
John	Rates with 5 stars "The Artist"	June 3 <sup>rd</sup>
Peter	Watches "The Artist"	June 5 <sup>th</sup>
Jen	•••	

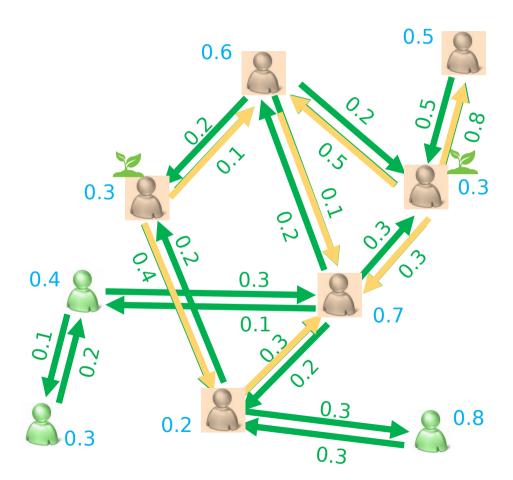
### Two main models

Linear threshold model

Independent cascade model

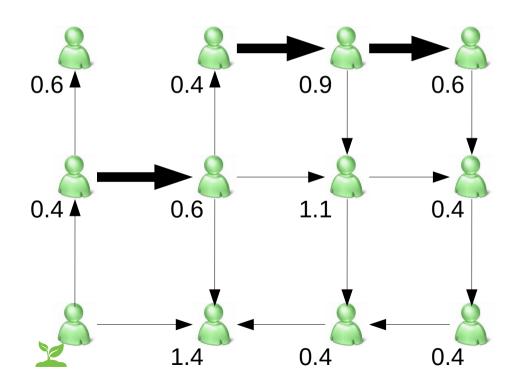
### Linear threshold model

- Nodes have thresholds
- Arcs have weights
- Nodes that receive weighted influence equal or above their threshold become active



### **Exercise**

Add your results to this Google Spreadsheet (Sheet: Linear Threshold)



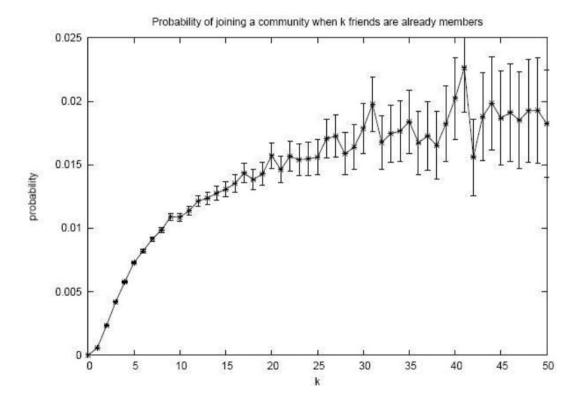
Thick arrows have weight 1.0

Thin arrows have weight 0.5

Execute linear threshold model starting from seed node

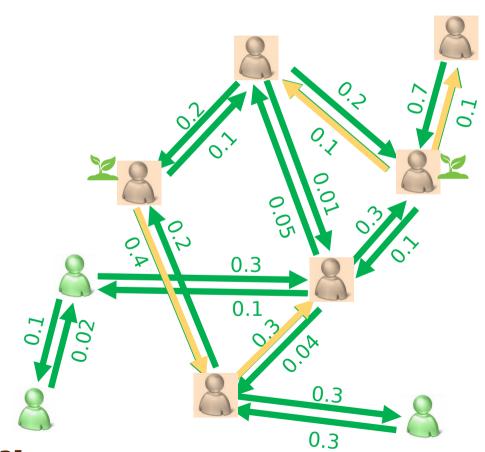
### Linear threshold model

Is the linear threshold model compatible with this observation?



## Independent cascade model

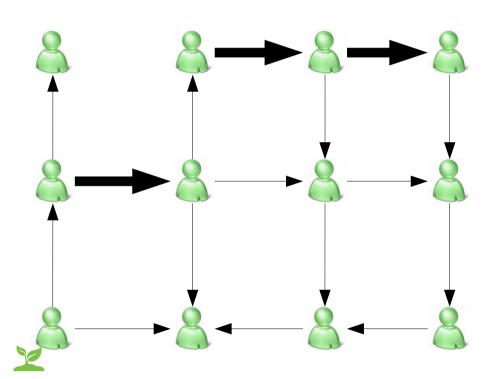
- No thresholds
- Each node, when activating, has one chance of activating each of their neighbors
- Probability of succeeding represented by arc weights



#### **Exercise**

(you need a coin or 1d4)

Add your results to this Google Spreadsheet (Sheet: Independent Cascade)



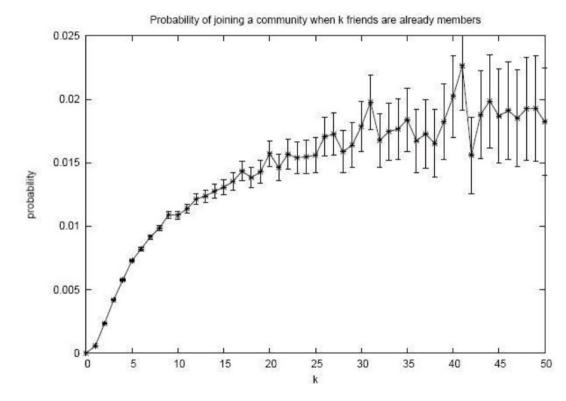
Thick arrows have probability 0.75

Thin arrows have probability 0.5

Execute independent cascade model starting from seed node

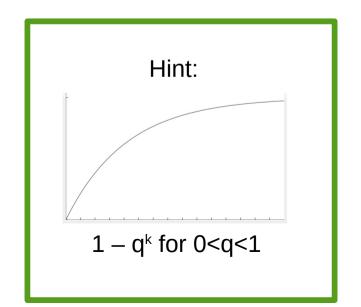
## Independent cascade model

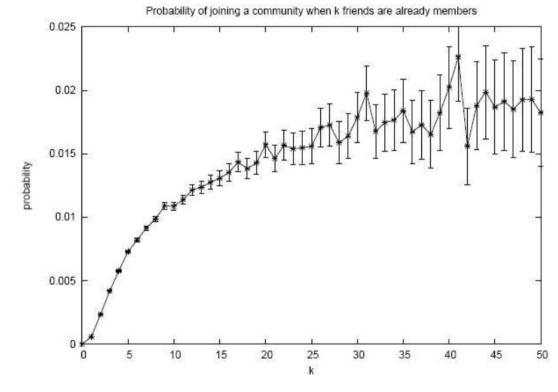
Is the independent cascade model compatible with this observation?



## Independent cascade model

Is the independent cascade model compatible with this observation?





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# What are these models assuming?

(List as many assumptions as you can)

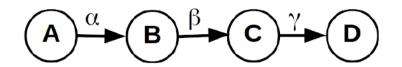
Answer in Nearpod Collaborate https://nearpod.com/student/ Code to be given during class

# Summary

## Things to remember

- Linear threshold model
- Independent cascade model
- Practice executing these models in small graphs by hand
- Practice writing code implementing them

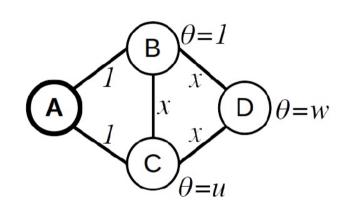
## Practice on your own



- Consider the graph on the top-right, including the infection probabilities indicated in the edges:  $\alpha$ ,  $\beta$  and  $\gamma$ . Let  $X_i$  be the expected number of nodes infected under the Independent Cascade Model for an infection starting at node i, including the node initially infected.
- For instance, if an infection starts from node B, the probability that the number of nodes infected is 2 is P  $(X_B = 2) = \beta \cdot (1 \gamma)$ . This is because for the infected to be 2 we need the infection from B to C to succeed and the infection from C to D to fail.
- Remember that the expectation of a variable X is  $E[X] = x \cdot P(X = x)$ , where the summation is done over the possible values x that the variable can take.
- 1. What is  $E[X_C]$  as a function of  $\gamma$ ?
- 2. What is  $E[X_A]$  as a function of  $\alpha$ ,  $\beta$ ,  $\gamma$ ?

# Practice on your own (cont.)

- Consider this graph and the Linear Threshold model executed on it, starting from seed node A.
- The influence weights are written next to the edges, and the thresholds  $\theta$  are written next to the nodes.
- Indicate what is the range of values of x for node C to be infected, but not node D. Justify briefly your answer.



- 1. \_\_\_\_≤ x< \_\_\_\_\_
- 2. Justification:

# Practice on your own (cont.)

- Consider the graph on the right an the Independent Cascade model executed on it, starting from seed node A.
- ullet The contagion probability of all edges is p
- Indicate what is the probability that at the end of the process:
- 1. Only node A is infected:
- 2. Only nodes A, B are infected:
- 3. Only nodes A, B, C are infected:
- 4. Only nodes A, B, D are infected:

