

# Closeness

## Social Networks Analysis and Graph Algorithms

Prof. Carlos “ChaTo” Castillo — <https://chato.cl/teach>



Universitat  
Pompeu Fabra  
*Barcelona*

# Sources

- D. Easley and J. Kleinberg (2010). Networks, Crowds, and Markets – [Section 3.6B](#)
- P. Boldi and S. Vigna (2014). [Axioms for Centrality](#) in *Internet Mathematics*.
- Esposito and Pesce (2015): [Survey of Centrality](#).
- F. Menczer, S. Fortunato, C. A. Davis (2020). A First Course in Network Science – Chapter 02

# Types of centrality measure

- **Non-spectral**
  - Degree
  - Closeness and harmonic closeness
  - Betweenness
- Spectral
  - HITS
  - PageRank

# Is $u$ a well-connected person?

- Degree:  $u$  has many connections
- **Closeness:**  $u$  is close to many people
  - Average distance from  $u$  is small
- **Betweenness:** many connections pass through  $u$ 
  - Large number of shortest paths pass through  $u$
- PageRank:  $u$  is connected to the well-connected

# Closeness

# Closeness

- Distance between two nodes is  $d(u, v)$
- **Closeness** is the reciprocal of the sum of distances

$$\text{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u, v)}$$

- Some graphs are not connected, in that case  $d(u, v)$  can be  $\infty$ ; assuming  $1/\infty = 0$  one can define the **harmonic closeness**:

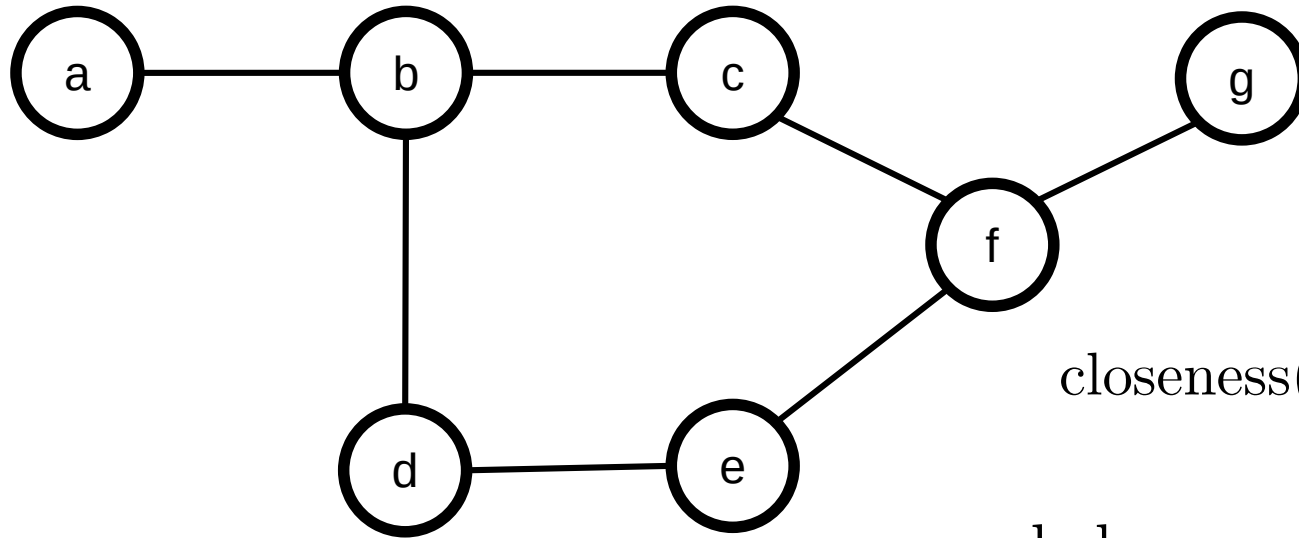
$$\text{hcloseness}(u) = \sum_{v \neq u} \frac{1}{d(u, v)}$$

# Exercise

Answer in  
[Google Spreadsheet](#)

Compute closeness and harmonic closeness for all the nodes

$d(u,v) = 1$  if  $v$  is a neighbor of  $u$



$$\text{closeness}(u) = \frac{1}{\sum_{v \in V, v \neq u} d(u, v)}$$

$$\text{hcloseness}(u) = \sum_{v \in V, v \neq u} \frac{1}{d(u, v)}$$

# Summary



# Things to remember

- Closeness and harmonic closeness definitions
- Try to compute them on your own on a graph
- Practice drawing examples of graphs in which a chosen node has high degree but low closeness, or viceversa