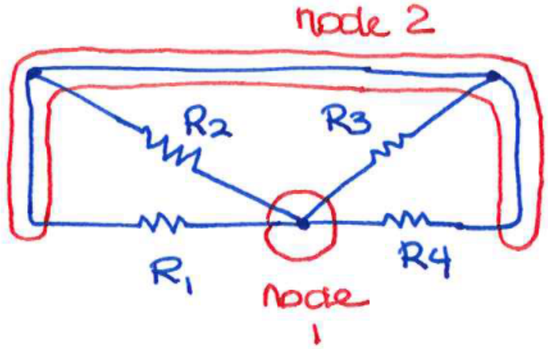
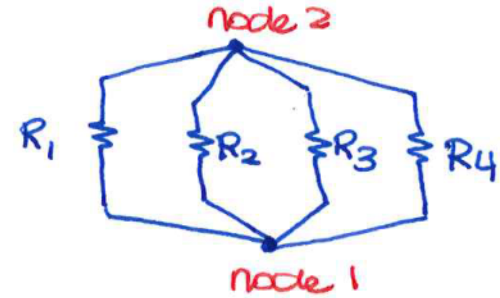
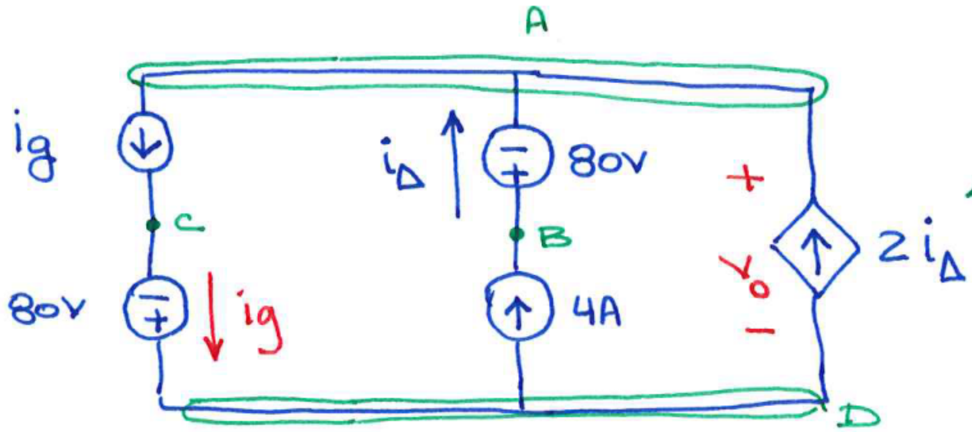
- Watch for parallel connections that may not appear like parallel elements at first glance



$\longleftrightarrow$



Ex: Let $v_o = 100V$. Find the power for each element:



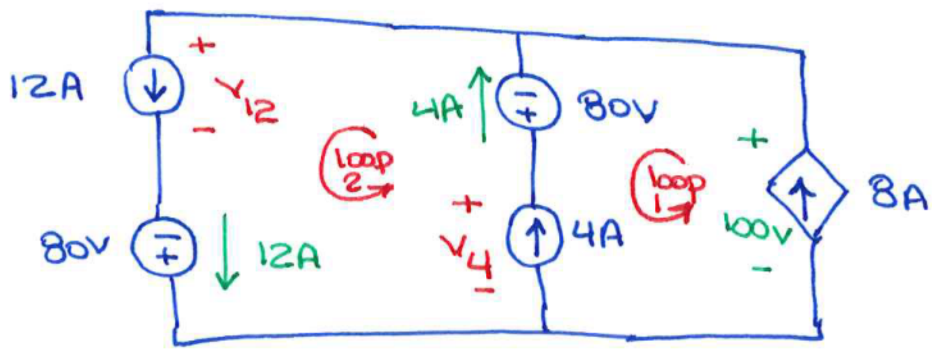
CCCS.
i_Δ: controlling current

Strategy:
- Need all voltages & currents to calculate power
- Use KVL & KCL to find unknown voltages & currents. Assume polarity/direction if unknown.
- Use $p = vi$ or $p = -vi$ based on direction of i & polarity of v for each element.

KCL at node B: $\underset{\text{enter}}{4} - \underset{\text{leave}}{i_\Delta} = 0 \quad \therefore \; i_\Delta = 4\text{ A}$

KCL at node A: $\underset{\text{enter A from CCCS}}{2i_\Delta} + \underset{\text{enter A from 80V src}}{i_\Delta} - \underset{\text{leaves A}}{i_g} = 0 \quad \therefore \; i_g = 12\text{ A}$

We now have all the currents. Two unknown voltages: V_4 & V_{12}. Assumed a polarity for them.



- Choose loop directions arbitrarily.

KVL around loop 1: $-100 - 80 + V_4 = 0$ $\therefore$ $V_4 = 180\ V$

KVL around loop 2: $-V_4 + 80 + V_{12} - 80 = 0$ $\therefore$ $V_{12} = 180\ V$

Element	Power
12 A source	$P = V \cdot i = V_{12} \times 12A = 2160\ W$ ← $P > 0$. Source is absorbing power
left 80V src	$P = -V \cdot i = -(80V)(12A) = -960\ W$
mid 80V src	$P = V \cdot i = (80V)(4A) = 320\ W$
4A src	$P = -V \cdot i = -V_4 \times 4A = -720\ W$
CCCS	$P = -V \cdot i = -(100)(8A) = -800\ W$

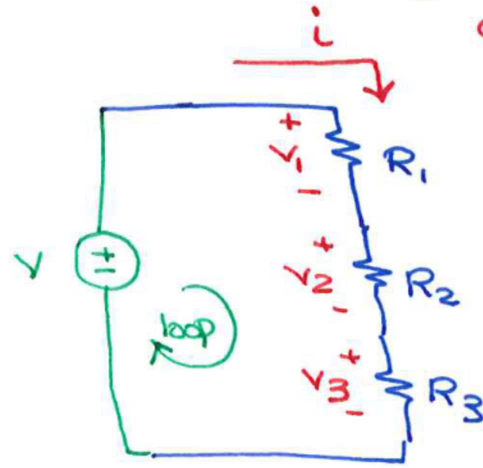
$\sum p = 0$ ✓ YASSSS

conservation of power/energy

Resistive Circuits

- We now have all the tools to begin circuit analysis

Resistors in series



common current for series element

By KVL:

$$-V + V_1 + V_2 + V_3 = 0$$

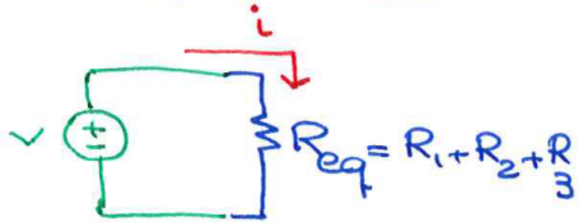
By Ohm's Law:

$$V_1 = i \cdot R_1$$
$$V_2 = i \cdot R_2$$
$$V_3 = i \cdot R_3$$

$$\therefore \quad -V + iR_1 + iR_2 + iR_3 = 0$$

$$\therefore \quad V = i(R_1 + R_2 + R_3) \xrightarrow{\text{looks like}} V = iR$$

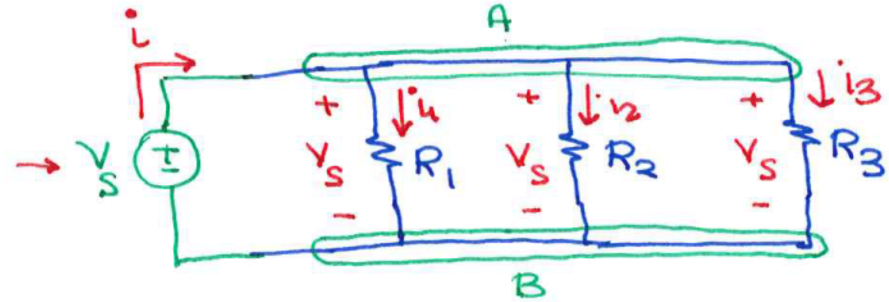
therefore we can replace all resistors in series with an equivalent resistor R_{eq}



$$\boxed{\text{Resistors in series add}}$$

Resistors in Parallel

common voltage for parallel elements →

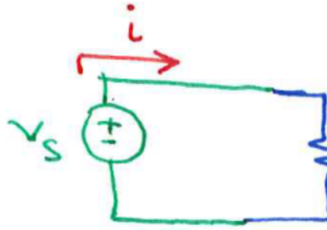


KCL at node A: $\quad i - i_1 - i_2 - i_3 = 0$

From Ohm's Law: $\quad i_1 = \dfrac{V_S}{R_1}, \quad i_2 = \dfrac{V_S}{R_2}, \quad i_3 = \dfrac{V_S}{R_3}$

$$\therefore \quad i - \frac{V_S}{R_1} - \frac{V_S}{R_2} - \frac{V_S}{R_3} = 0 \qquad \therefore \quad i = V_S\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\therefore \quad V_s = i \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}}_{R_{eq}} \quad \xrightarrow{\text{looks like}} \quad V_s = iR$$



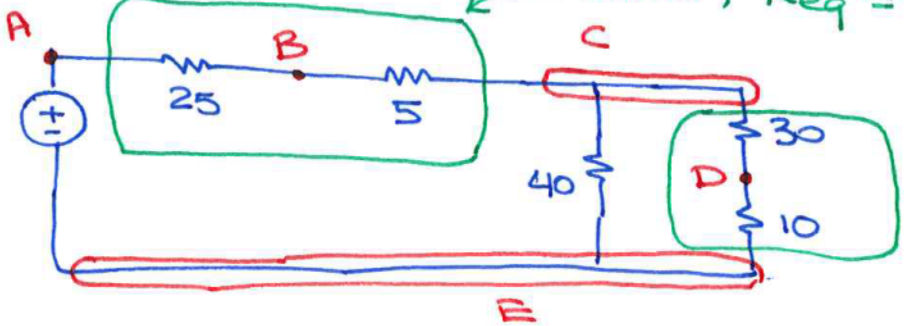
$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

- For 2 resistors in parallel:

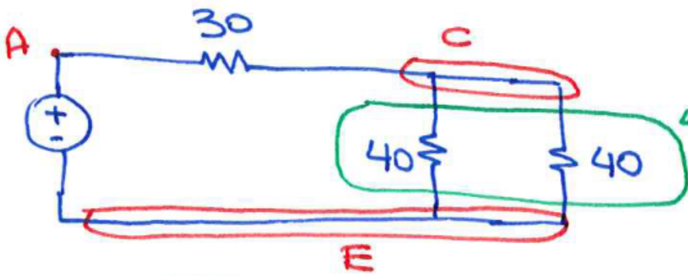
$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 \cdot R_2}{R_1 + R_2} \begin{matrix} \leftarrow \text{product} \\ \leftarrow \text{sum} \end{matrix}$$

Ex: Find a single equivalent resistance for:

series, $R_{eq} = 30\ \Omega$

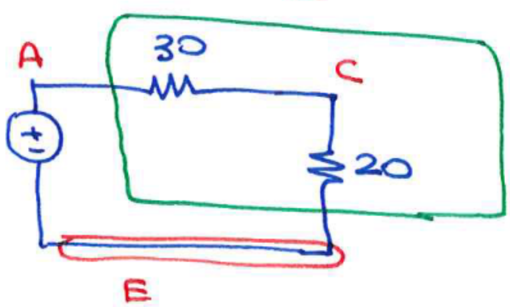


series, $R_{eq} = 40\ \Omega$

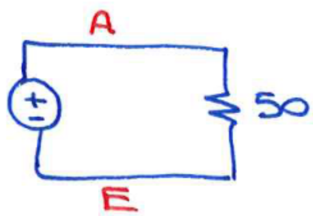


parallel

$$R_{eq} = \frac{40 \times 40}{40 + 40} = 20\ \Omega$$



series, $R_{eq} = 50\ \Omega$

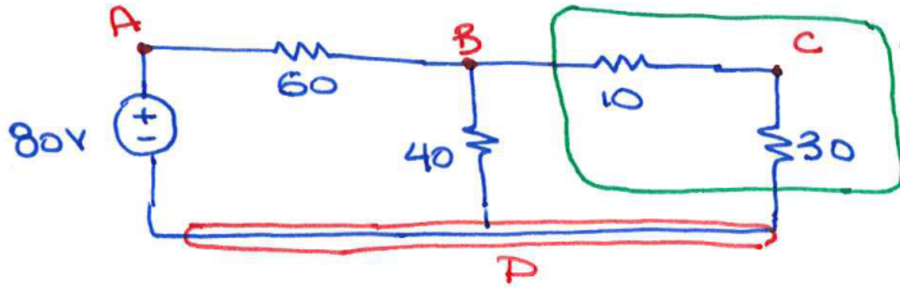


Circuit Analysis using series / parallel equivalents

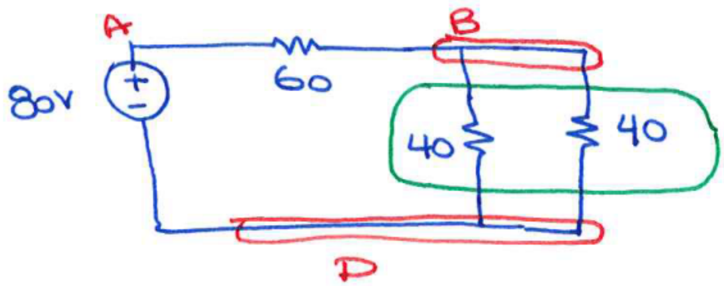
- Circuit analysis : Procedure for determining all voltages & currents in every circuit element

- We may use the above resistor equivalents to analyze a circuit.

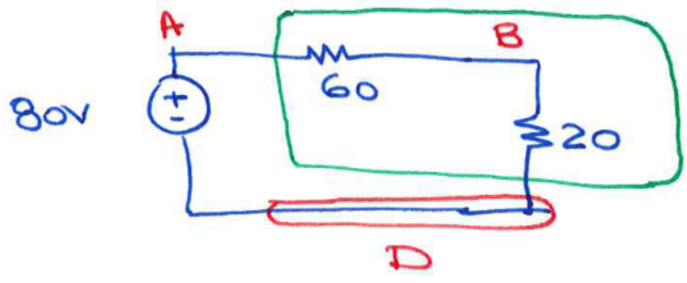
Ex: Find power in each element



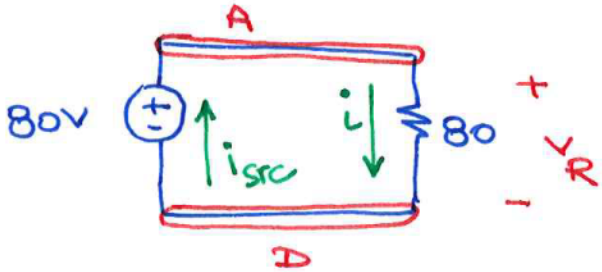
← series
$R_{eq} = 40\,\Omega$



← parallel , $R_{eq} = 20\,\Omega$



← series , $R_{eq} = 80\,\Omega$



$v_R = 80\text{ V}$

How?

i) KVL around loop : $-80 + v_R = 0 \quad \therefore \; v_R = 80\text{ V}$

or

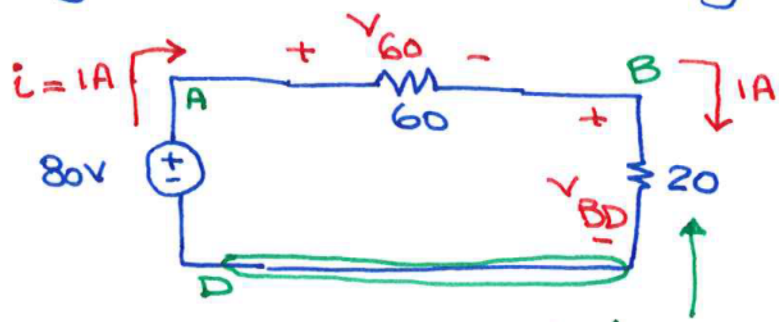
ii) Because of the voltage source, node A is higher than node D by 80V. , i.e. $v_{AD} = 80\text{ V}$

therefore, $v_R = v_{AD} = 80\text{ V}$

Now, from Ohm's Law : $i = \dfrac{V_R}{80\,\Omega} = 1\ A$

To find i_{src}; KCL at node A : $i_{src} - i = 0 \therefore i_{src} = 1\ A$

Key idea : back to the original circuit, step by step:



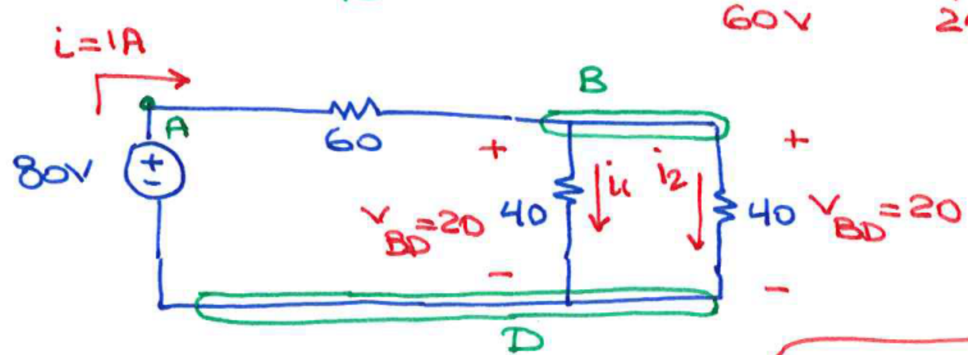
this is a combo of 3 R's.

When replacing Req with original <u>series</u> resistors, these resistors will have the same <u>current</u> as Req

From Ohm's Law: $V_{60} = 60\,\Omega \times 1A = 60\ V$

$V_{BD} = 20\,\Omega \times 1A = 20\ V$

KVL check: $-80 + \underbrace{V_{60}}_{60V} + \underbrace{V_{BD}}_{20V} = 0$ ✓



From Ohm's Law:

$i_1 = \dfrac{V_{BD}}{40\,\Omega} = \dfrac{20V}{40\,\Omega} = 0.5\ A$

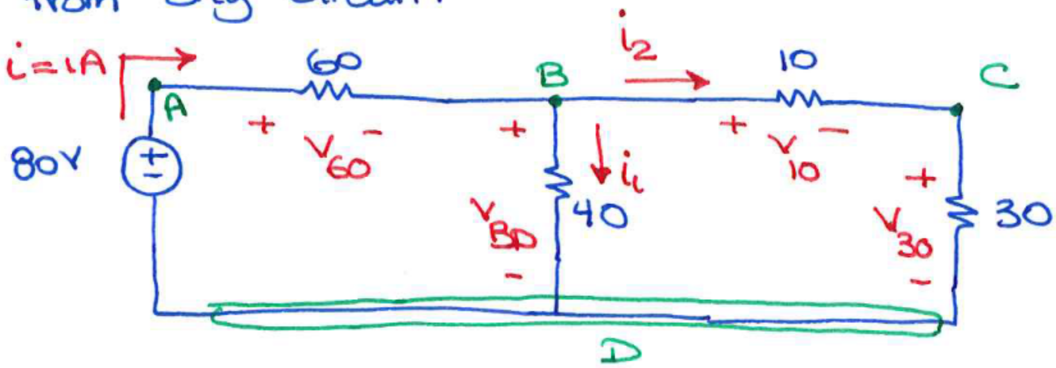
$i_2 = \dfrac{V_{BD}}{40\,\Omega} = 0.5\ A$

When replacing Req with original <u>parallel</u> resistors, they have the same <u>voltage</u>

KCL check at node B: $\underbrace{i}_{1A} - \underbrace{i_1 - i_2}_{0.5A} = 0$ ✓

replace the right 40Ω resistor with series 10Ω & 30Ω resistors

from orig circuit:



From Ohm's Law: $V_{10} = i_2 \times 10 = 5\ V$

$V_{30} = i_2 \times 30 = 15\ V$

KVL check around right loop: $-\underbrace{V_{BD}}_{20} + \underbrace{V_{10}}_{5} + \underbrace{V_{30}}_{15} = 0$ ✓

Now, power:

Element	power	
80V src	$P = -Vi = -(80V)(1A) = -80W$	supplies 80W
60Ω	$P = V_{60} \cdot i = \frac{(V_{60})^2}{60Ω} = i^2(60Ω) = 60\ W$	
40Ω	$P = V_{BD} \cdot i_1 = \frac{V_{BD}^2}{40Ω} = i_1^2(40Ω) = 10\ W$	
10Ω	$P = V_{10} \cdot i_2 = \frac{(V_{10})^2}{10Ω} = i_2^2(10Ω) = 2.5\ W$	
30Ω	$P = V_{30} \cdot i_2 = \frac{(V_{30})^2}{30} = i_2^2 \cdot (30Ω) = 7.5\ W$	

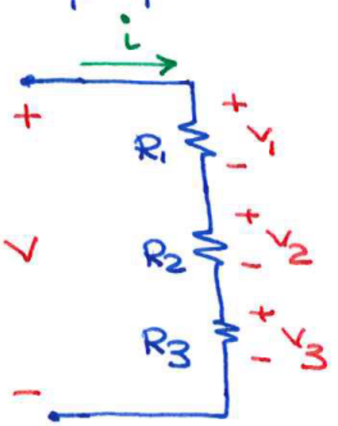
$\sum p = 0$ ✓

energy balance.

- Later, we will use well-established, systematic methods for analysis: Node Voltage & Mesh Current
- Other simple resistive circuit analysis tools: voltage division & current division

Voltage Division :

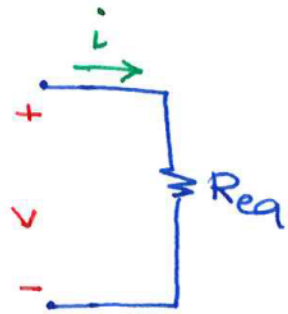
In a series connection of resistors, the total voltage across the series branch of resistors divides among them proportional to their size:



$$R_{eq} = R_1 + R_2 + R_3$$

Ohm's Law on the right circuit:

$$i = \frac{V}{R_{eq}} = \frac{V}{R_1 + R_2 + R_3}$$



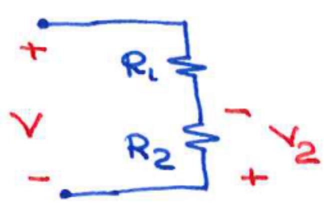
Ohm's Law for the left circuit:

$$V_1 = i \cdot R_1 = \frac{V}{R_1 + R_2 + R_3} \cdot R_1 = \frac{R_1}{R_1 + R_2 + R_3} \cdot V$$

i.e. R_1's portion of total voltage V is the same as its portion of total resistance

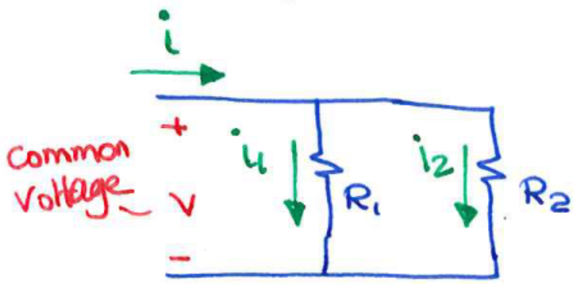
- total voltage $V = V_1 + V_2 + V_3$
- If V_1, V_2, or V_3 have opposite polarity to V, then add a $(-)$ sign, e.g:

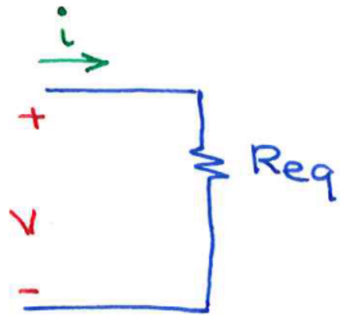
$$V_2 = -\frac{R_2}{R_1 + R_2} \cdot V$$



Current Division

In a parallel connection of resistors, the total current available to the parallel resistors divides among them inversely proportional to their size:





i: total current,
it can only go through parallel resistors R_1 & R_2

$$R_{eq} = \frac{R_1 . R_2}{R_1 + R_2}$$

$$i = i_1 + i_2$$

Ohm's Law on the right circuit : $v = i . R_{eq} = i \dfrac{R_1 . R_2}{R_1 + R_2}$

Ohm's Law for R_1 on the left circuit :

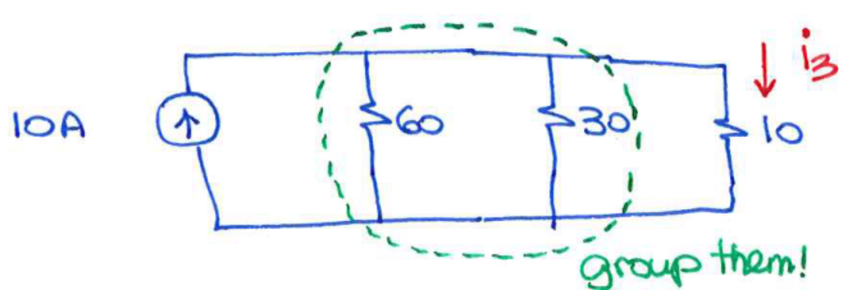
$$i_1 = \frac{v}{R_1} = \frac{i \dfrac{R_1 . R_2}{R_1 + R_2}}{R_1} = \frac{R_2}{R_1 + R_2} . i$$

other branch

i.e. R_1's portion of total current i is the same as R_2's portion of total resistance

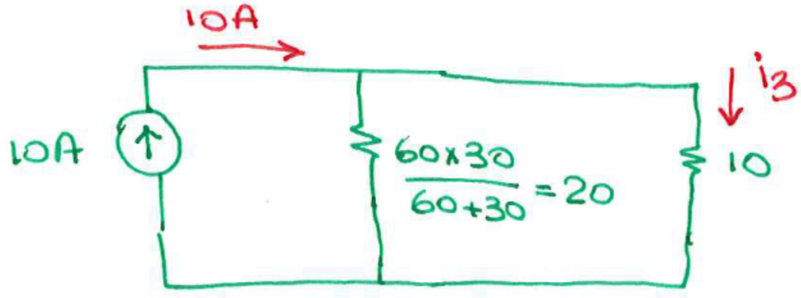
. Not as easy to apply when there are more than 2 resistors in parallel. We can group resistors to apply current division in this case.

Ex: Find i_3



We can't use $i_3 = \cancel{\frac{60+30}{60+30+10} \times 10\text{A}}$

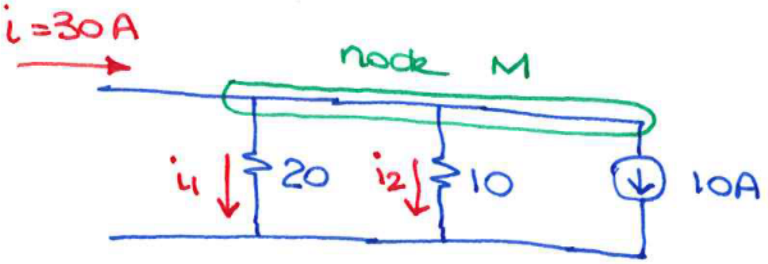
Combine 60Ω & 30Ω in parallel:



Current division gives:

$$i_3 = \frac{20}{10+20} \times 10\text{A} = 6.\bar{6}\ \text{A}$$

Ex 2: How to apply current division here?



To apply current division, we need total current going through parallel resistors : $\underbrace{i_1 + i_2}_{}$

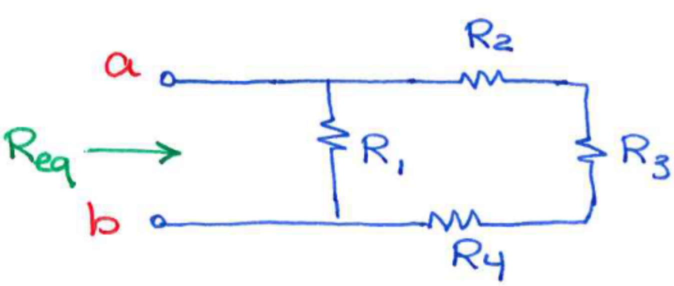
let's call this i_{total}

KCL at node M : $+30 - i_1 - i_2 - 10 = 0$

$$\therefore\ i_{total} = i_1 + i_2 = 20\ \text{A}$$

Now, we can use current div : $i_1 = \frac{10}{10+20} \times i_{total} = 6.\bar{6}\ \text{A}$

. Aside: Finding resistance between 2 nodes/points :



can't say R_1 & R_2 are in series because there could be other circuit elements connected to the left side of a & b.

$$R_{eq} = R_1 \parallel (R_2 + R_3 + R_4)$$