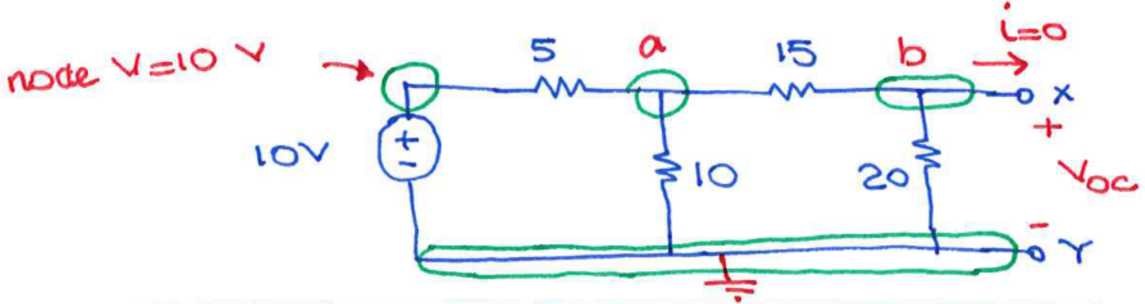
Ex: Find thevenin Equivalent of the following circuit as seen from X & Y :



Determining a thevenin Equivalent is two separate analysis problems: Find V_{oc} & find i_{sc}

1) Find V_{oc} using method of your choice.

V_{oc} is the voltage across the 20Ω resistor in this circuit.

Let's try Nodal. V_{oc} is the node voltage at b (terminal X is part of node b, terminal Y is part of ref node)

$V_{oc} = V_{XY} = V_X - V_Y = V_b - 0 = V_b$

KCL at a : $\frac{V_a - 10}{5} + \frac{V_a}{10} + \frac{V_a - V_b}{15} = 0$

$\therefore \; 11V_a - 2V_b = 60$ (i)

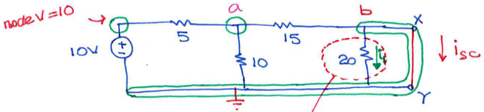
KCL at b : $\frac{V_b - V_a}{15} + \frac{V_b}{20} + 0 = 0$

$\therefore \; -4V_a + 7V_b = 0$ (ii)

Solve (i) & (ii). Only interested in V_b. $V_b = 3.48$ V

$V_t = V_{oc} = V_{XY} = V_b = 3.48$ V

2) Now, find i_{sc} using any method. Let's try Nodal again



"Shorted out". It can be removed from our analysis

why? 1) $i_t = 0$ Since potential difference across the resistor = $V_b - V_b = 0$ V. It is connected to the same node on both sides

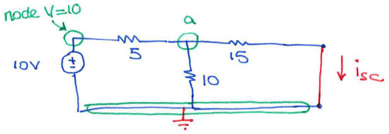
or 2) It is parallel with a short circuit between X & Y (R=0)

$R_{eq} = \frac{0 \times 20}{0 + 20} = 0$

• what happened to V_b? Node b now directly connected to ref node (is part of ref node) $\therefore V_b = 0$

important: SC changes the circuit. Must treat circuit as a whole new analysis problem

redraw:



KCL at a:
$$\frac{V_a - 10}{5} + \frac{V_a}{10} + \underbrace{\frac{V_a}{15}}_{} = 0$$

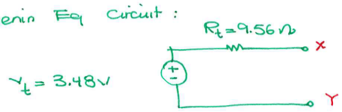
Note: this is i_{sc}. Current from node a to ref node through 15 Ω resistor

$$\therefore V_a = 5.45 \text{ V}$$

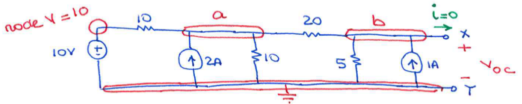
$$i_{sc} = \frac{V_a - 0}{15} = 0.364 \text{ A}$$

Finally, $R_t = \dfrac{V_{oc}}{i_{sc}} = \dfrac{3.48\text{V}}{0.364\text{ A}} = 9.56\ \Omega$

Thevenin Eq Circuit:



Ex 2: Find thevenin Equivalent of:



1) Find V_{oc}: Let's use node voltage.

$$V_{oc} = V_{XY} = V_b - 0 = V_b$$

KCL at node a: $$\frac{V_a - 10}{10} - 2 + \frac{V_a}{10} + \frac{V_a - V_b}{20} = 0$$

$$\therefore\ 5V_a - V_b = 60 \qquad (i)$$

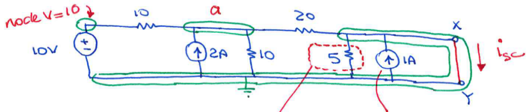
KCL at node b: $$\frac{V_b - V_a}{20} + \frac{V_b}{5} - 1 = 0$$

$$\therefore\ -V_a + 5V_b = 20 \qquad (ii)$$

Solve (i) and (ii). Only interested in V_b. $V_b = 6.\bar{6}$ V

$$V_t = V_{oc} = V_b = 6.6 \text{ V}$$

2) Now, find i_{sc}



resistance is futile!
5Ω resistor is shorted out

1A source remains.
• Current sources don't get shorted out

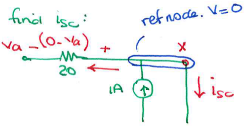
Let's try node voltage again to find V_a:

KCL at a:

$$\frac{V_a - 10}{10} - 2 + \frac{V_a}{10} + \frac{V_a}{20} = 0$$

Solve this to get $V_a = 12\text{ V}$

Use KCL at a connection point to find i_{sc}:

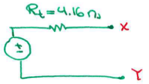


$$\frac{0 - \overset{12V}{V_a}}{20} - 1 + i_{sc} = 0$$

$$\therefore\ i_{sc} = 1.6\text{ A}$$

. Finally, $R_t = \dfrac{V_{oc}}{i_{sc}} = \dfrac{6.6\text{ V}}{1.6\text{ A}} = 4.16\ \Omega$

. Thevenin Equivalent circuit :



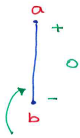
$v_t = 6.\bar{6}\text{ V}$

Finding R_t Directly

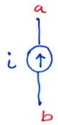
. If a circuit has no dependent sources, then we may use an alternative method to find R_t by zeroing the sources:

. How to zero sources ?

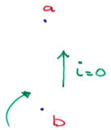
if we let $V = 0$,
the voltage source
becomes a short circuit



effective resistance $= 0\,\Omega$



If we let $i = 0$,
the current source becomes an open circuit

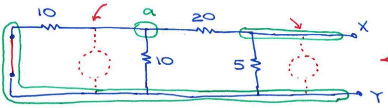


effective resistance $= \infty$

- Now, find resistance as seen by the terminals by combining series & parallel resistors

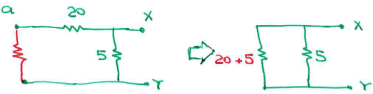
- Repeat previous example. Find R_t by zeroing sources:

two current sources are now OC



voltage source is now SC

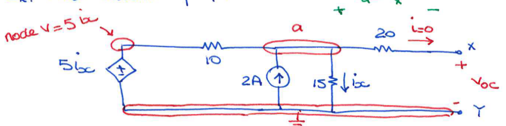
Three nodes: a, X, and Y



$$R_{eq} = R_t = 5 \parallel 25 = \frac{5 \times 25}{5 + 25} = 4.16\ \Omega$$

- Reminder: this method <u>cannot</u> be used if a circuit has dependent sources. Must determine i_{sc}, $R_t = \frac{V_{oc}}{i_{sc}}$ for those circuits.

Ex: Find thevenin Eq of:



Let's find V_{oc} using node voltage

With terminals X & Y open, no current flows through 20Ω resistor ∴ Voltage across 20Ω resistor $= 0$

$$\underbrace{V_a - V_x}_{} = 0 \qquad \therefore \; V_a = V_x$$

$$V_a = V_x = V_{XY} = V_{oc} = V_t$$

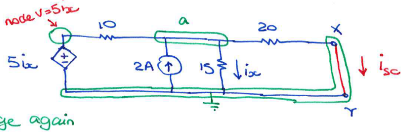
↑ Y is part of ref node

- Node eq at a:

$$\frac{V_a - 5i_x}{10} - 2 + \underbrace{\frac{V_a}{15}}_{\text{this is also } i_x} + 0 = 0$$

Solve to get $V_a = V_{oc} = V_t = 15\text{ V}$

- Now, find i_{sc}



- Nothing is shorted out

Let's use node voltage again

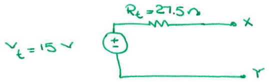
Node eq at a:

$$\frac{V_a - 5i_x}{10} - 2 + \underbrace{\frac{V_a}{15}}_{i_x} + \underbrace{\frac{V_a - 0}{20}}_{\text{this is } i_{sc}} = 0$$

Solve for V_a to get: $V_a = 10.91\text{ V}$ ← Note that V_a is different from the OC condition

then $i_{sc} = \dfrac{V_a - 0}{20} = 0.545\text{ A}$

Finally, $R_t = \dfrac{V_{oc}}{i_{sc}} = \dfrac{15\text{ V}}{0.545\text{ A}} = 27.5\ \Omega$



Summary of Thevenin Equivalents

1) Find V_t. Have to find V_{oc}

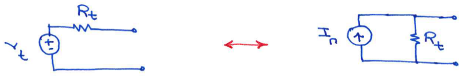
2) Find R_t:
   - Find i_{sc}. $R_t = \dfrac{V_{oc}}{i_{sc}}$

   OR

   - if no dep. sources, turn off indep sources, find R_{eq} by combining series / parallel resistors

Norton Equivalent

- Provides an alternative form to Thevenin Equivalent:



Thevenin ↔ Norton

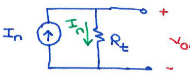
$V_t = V_{oc}$

$R_t = \dfrac{V_{oc}}{i_{sc}}$

$I_n = i_{sc}$

$= \dfrac{V_t}{R_t}$

- Check to see if Thevenin & Norton have the same OC & SC characteristics:

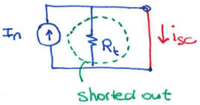


V_{oc} is the voltage across R_t.

From Ohm's Law:

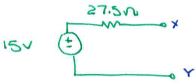
$V_{oc} = I_n \cdot R_t = i_{sc} \cdot R_t = \frac{V_t}{\cancel{R_t}} \cdot \cancel{R_t}$

$= V_t$ ← same as Thevenin Eq Circuit

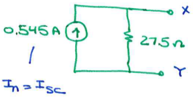


$i_{sc} = I_n = \frac{V_t}{R_t}$ ← same as i_{sc} for Thevenin Eq Circuit

- Last example, re-visited:

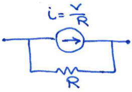


↔



- Thevenin & Norton are related by <u>Source Transformation</u>:

↔

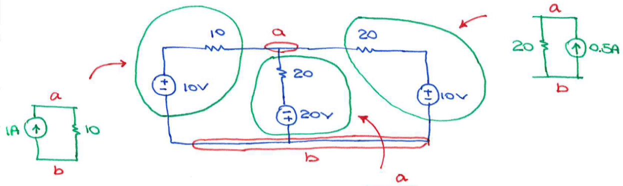


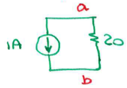
voltage source in series with resistor

current source in parallel with resistor

- Source transformation can be applied to any circuit. i.e. It is not exclusive to Thevenin / Norton

Source Transformation as a handy simplification

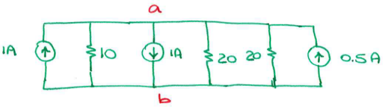




Note: Direction of current source towards + terminal of voltage source

we now have:

6 parallel elements!

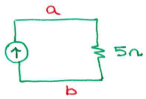


$$R_{eq} = 20\Omega \,//\, 20\Omega \,//\, 10\Omega = 5\Omega$$

- we can combine current sources in parallel (paying attention to directions)

∴ equivalent circuit is :

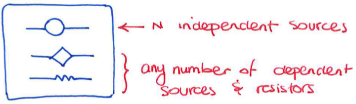
$1-1+0.5 = 0.5A$



- We can also combine voltage sources in series :

Principle of superposition in circuit analysis

- Given a circuit



an important (and often required method in AC analysis) is superposition. We first present it in DC circuits.

Method:
1) Let only one independent source be active
2) Zero all other N-1 independent sources
3) Determine the response (voltage or current) at desired location in the circuit. Let this be m'

Repeat this one independent source at a time to find m'', m''', ...

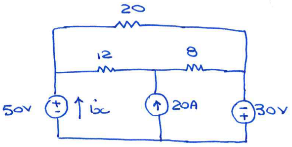
- Keep dependent sources ON at all times

Superposition: The total response (voltage or current) m is the sum of N individual responses $m' + m'' + \dots + m^N$

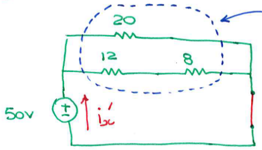
- Ex: Find i_x by superposition

We have 3 independent sources

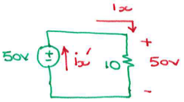
Activate one source at a time, zero the other two.



i) 50 V source by itself

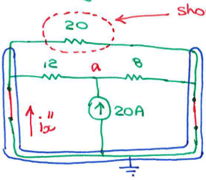


$(8+12) \parallel 20 = 10\,\Omega$



$$i_x' = \frac{50\,V}{10} = 5\,A$$

ii) 20 A source by itself



shorted out! Connected to the same node on both ends.

Find i_x'' using method of your choice.

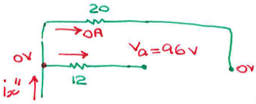
Let's try nodal

Node eq at a: $\dfrac{V_a - 0}{12} + \dfrac{V_a - 0}{8} - 20 = 0 \quad \therefore V_a = 96\,V$

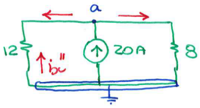
To find i_x'', use KCL:

$$-i_x'' + \frac{0-96}{12} + 0 = 0$$

$$\therefore i_x'' = -8\,A$$



• Aside: this circuit can be re-drawn as:



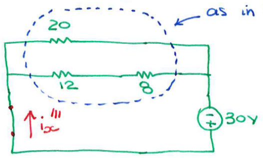
we can find i_x'' using current division:

$$i_x'' = -\frac{8}{8+12} \times 20A = -8A$$

direction of i_x'' w.r.t 20A source

iii) 30V source by itself

as in i), $R_{eq} = 10\Omega$



$$i_x''' = \frac{30V}{10\Omega} = 3A$$

Finally, by superposition:

$$i_x = i_x' + i_x'' + i_x'''$$
$$= 5 - 8 + 3$$
$$= 0 \text{ A}$$