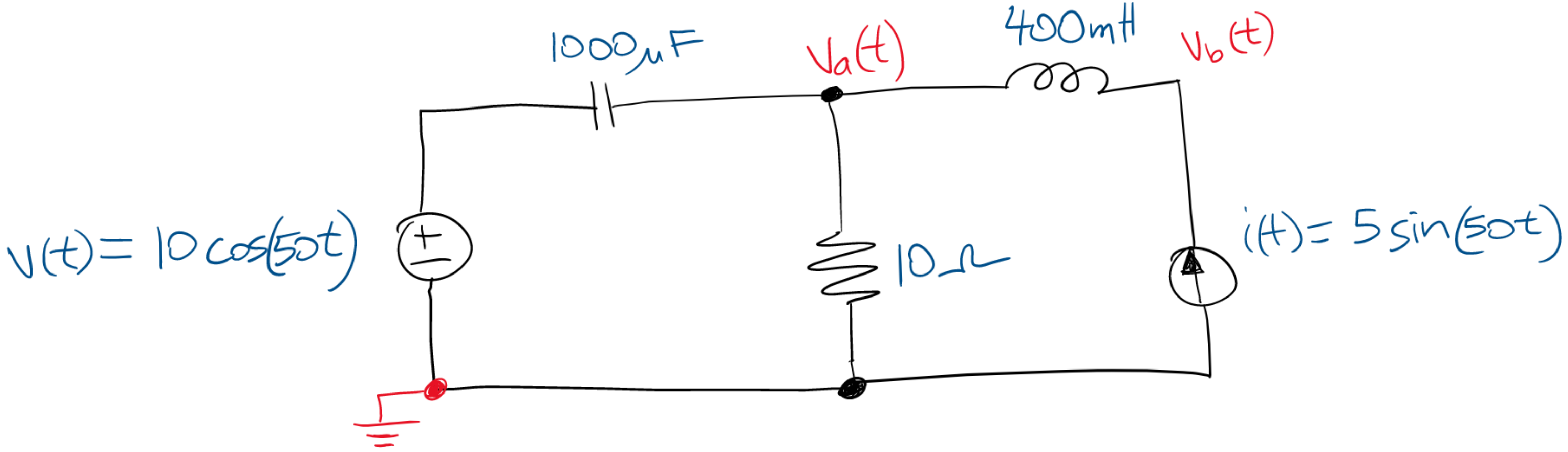
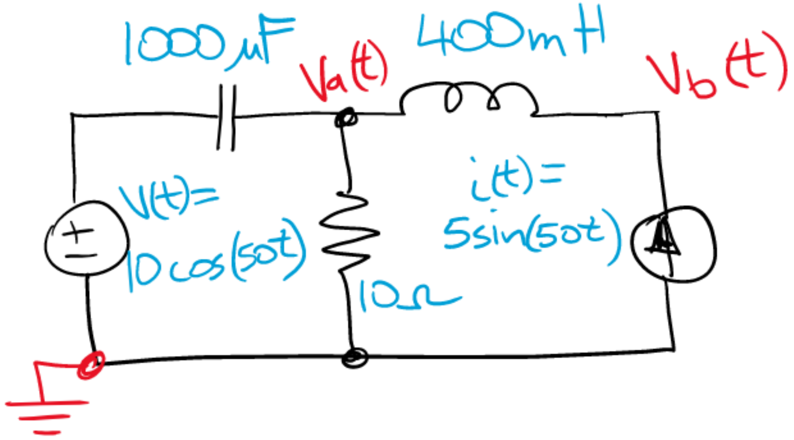
Example: Find the node voltages $V_a(t)$ and $V_b(t)$.



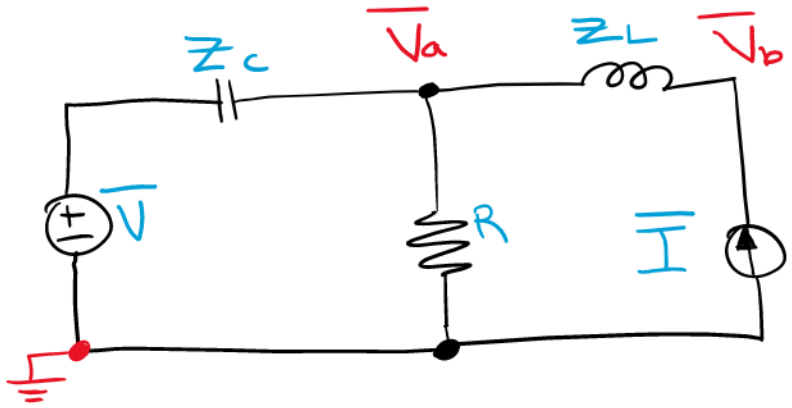
Example: Find the node voltages $V_a(t)$ and $V_b(t)$.



Let's write the node voltage equations symbolically. It's exactly the same format as for DC analysis.

Node A:

$$\frac{\overline{V_a} - \overline{V}}{Z_c} + \frac{\overline{V_a}}{R} + \frac{\overline{V_a} - \overline{V_b}}{Z_L} = 0 \quad \text{(eqn1)}$$



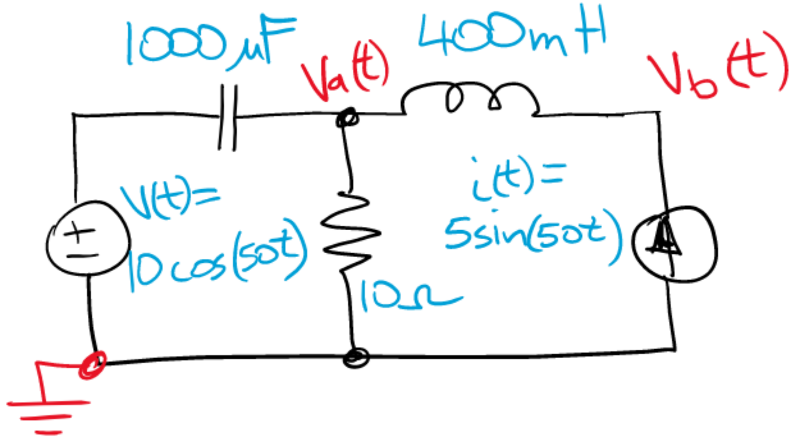
Node B:

$$\frac{\overline{V_b} - \overline{V_a}}{Z_L} - \overline{I} = 0 \quad \text{(eqn2)}$$

The basic AC circuit analysis is the same as for DC circuit analysis.

* What is $-\overline{I}$? Is it different than $\overline{I}$?

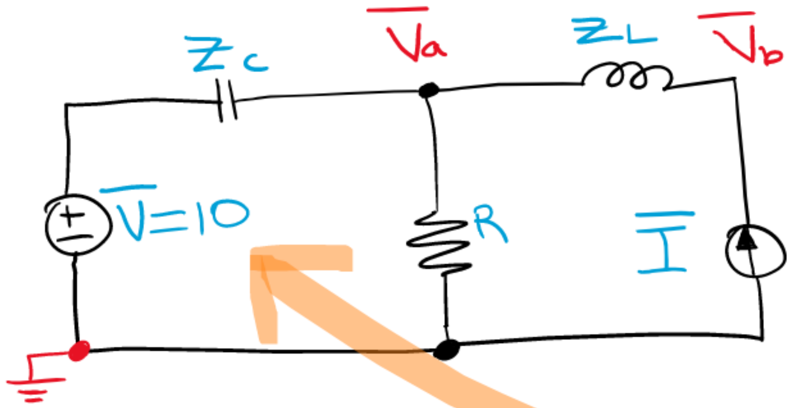
Example: Find the node voltages $V_a(t)$ and $V_b(t)$.



$$\frac{\bar{V}_a-\bar{V}}{Z_c}+\frac{\bar{V}_a}{R}+\frac{\bar{V}_a-\bar{V}_b}{Z_L}=0 \quad \text{(eqn1)}$$

$$\frac{\bar{V}_b-\bar{V}_a}{Z_L}-\bar{I}=0 \quad \text{(eqn2)}$$

Let's develop the expressions for the phasors and impedances.



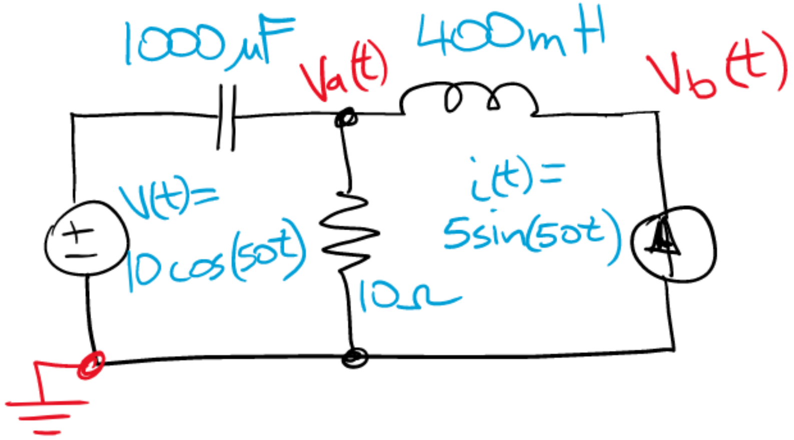
$\bar{V}$: $v(t) = 10\cos(50t)$

$v(t) = 10\cos(50t + 0°)$

ω disappears

$\bar{V} = 10\angle 0° = 10$

Example: Find the node voltages $V_a(t)$ and $V_b(t)$.



$$\frac{\overline{V_a}-\overline{V}}{Z_c}+\frac{\overline{V_a}}{R}+\frac{\overline{V_a}-\overline{V_b}}{Z_L}=0 \quad \text{(eqn1)}$$

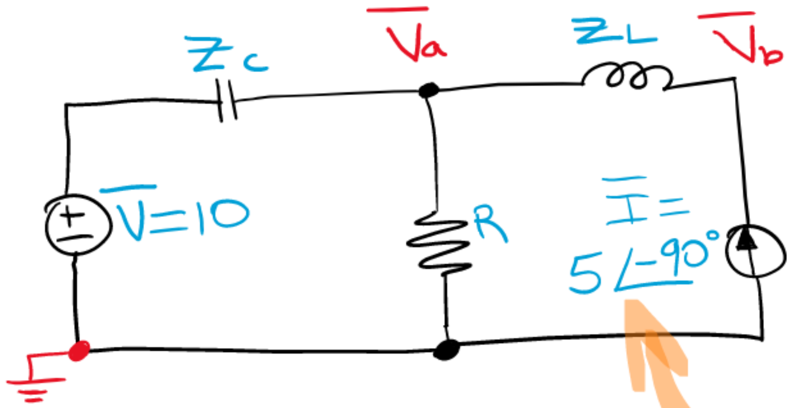
$$\frac{\overline{V_b}-\overline{V_a}}{Z_L}-\overline{I}=0 \quad \text{(eqn2)}$$

Let's develop the expressions for the phasors and impedances.

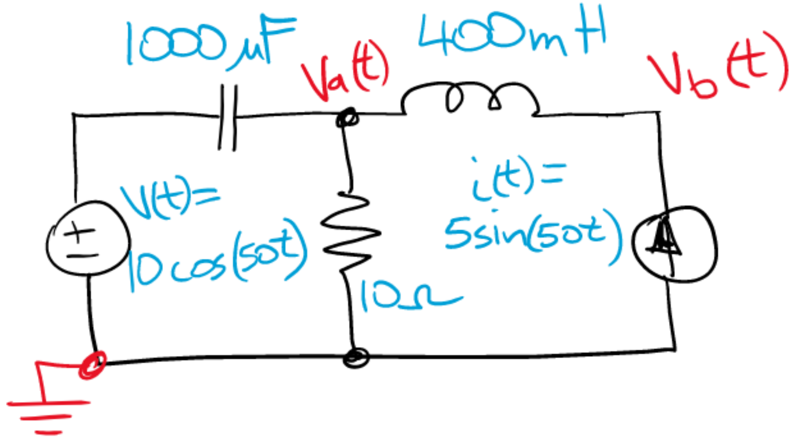
$\overline{I}$: $i(t)=5\sin(50t) \rightarrow$ must convert to cosine form

$$i(t)=5\cos(50t-90^\circ)$$

$$\overline{I}=5\angle-90^\circ$$



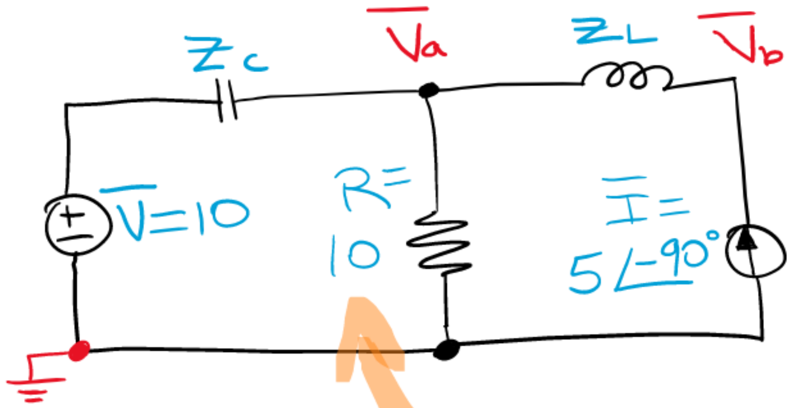
Example: Find the node voltages $V_a(t)$ and $V_b(t)$.



$$\frac{\bar{V}_a - \bar{V}}{Z_c} + \frac{\bar{V}_a}{R} + \frac{\bar{V}_a - \bar{V}_b}{Z_L} = 0 \quad \text{(eqn1)}$$

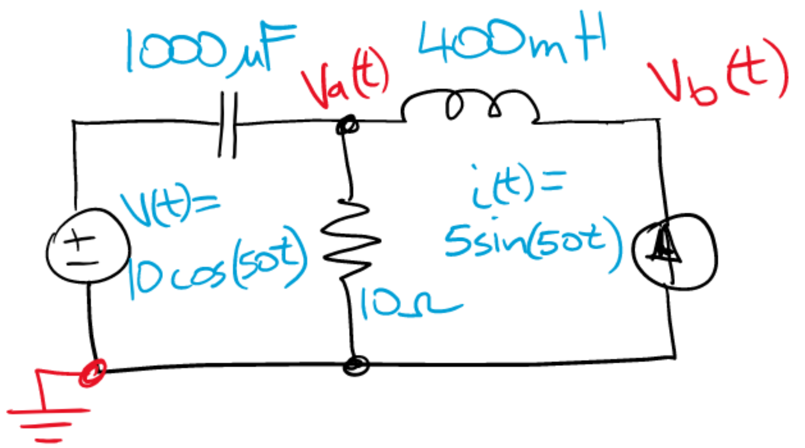
$$\frac{\bar{V}_b - \bar{V}_a}{Z_L} - \bar{I} = 0 \quad \text{(eqn2)}$$

Let's develop the expressions for the phasors and impedances.



R : $R = 10\,\Omega$

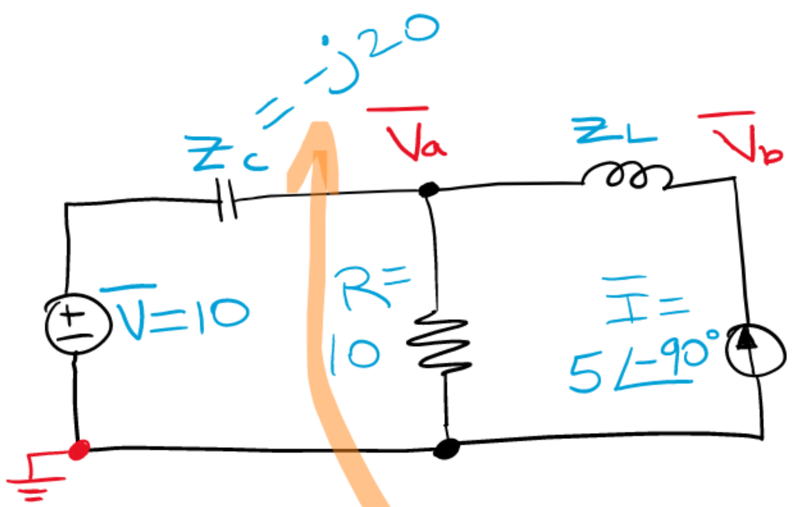
Example: Find the node voltages $V_a(t)$ and $V_b(t)$.



$$\frac{\overline{V_a}-\overline{V}}{Z_C}+\frac{\overline{V_a}}{R}+\frac{\overline{V_a}-\overline{V_b}}{Z_L}=0 \quad \text{(eqn1)}$$

$$\frac{\overline{V_b}-\overline{V_a}}{Z_L}-\overline{I}=0 \quad \text{(eqn2)}$$

Let's develop the expressions for the phasors and impedances.



Z_C : $Z_C = \frac{1}{j\omega C}$

$C = 1000\,\mu F$

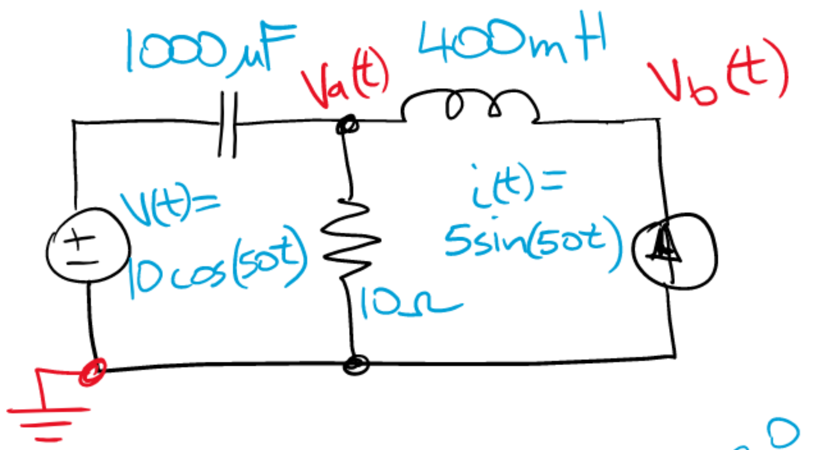
$\omega = 50 \frac{\text{radian}}{\text{second}}$

from $\cos(50t)$ and $\sin(50t)$

$$Z_C = \frac{1}{j \times 50 \times 1000 \times 10^{-6}} = -j20\,\Omega$$

recall $\frac{1}{j} = -j$

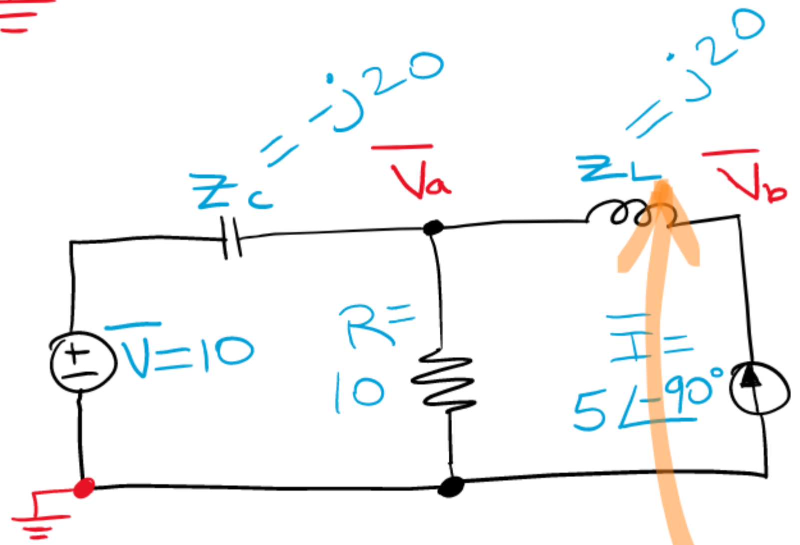
Example: Find the node voltages $V_a(t)$ and $V_b(t)$.



$$\frac{\overline{V_a}-\overline{V}}{Z_c}+\frac{\overline{V_a}}{R}+\frac{\overline{V_a}-\overline{V_b}}{Z_L}=0 \quad \text{(eqn1)}$$

$$\frac{\overline{V_b}-\overline{V_a}}{Z_L}-\overline{I}=0 \quad \text{(eqn2)}$$

Let's develop the expressions for the phasors and impedances.



Z_L :

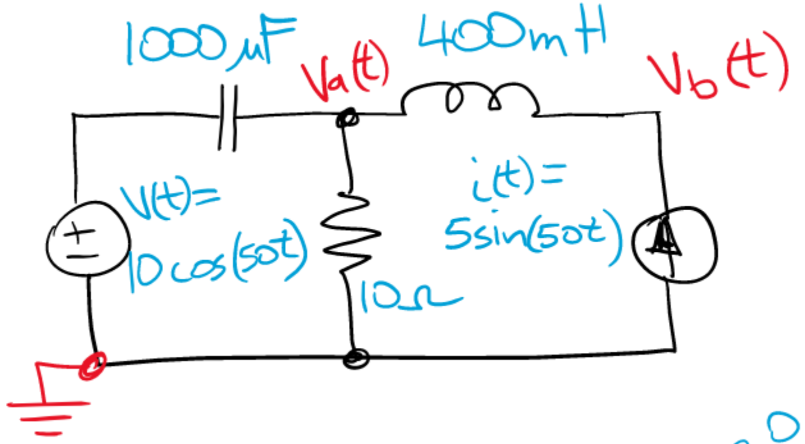
$$Z_L = j\omega L$$

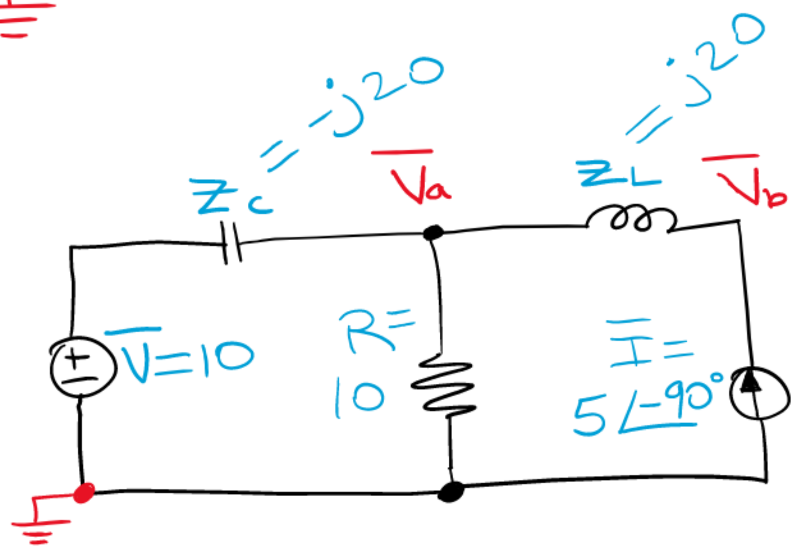
$$\omega = 50 \ \frac{\text{radian}}{\text{second}}$$

$$L = 400 \ mH$$

$$Z_L = j \times 50 \times 400 \times 10^{-3} = j20\,\Omega$$

Example: Find the node voltages $V_a(t)$ and $V_b(t)$.





$$\frac{\overline{V_a} - \overline{V}}{Z_C} + \frac{\overline{V_a}}{R} + \frac{\overline{V_a} - \overline{V_b}}{Z_L} = 0 \quad \text{(eqn1)}$$

$$\frac{\overline{V_b} - \overline{V_a}}{Z_L} - \overline{I} = 0 \quad \text{(eqn2)}$$

Plug in values into eqn 1

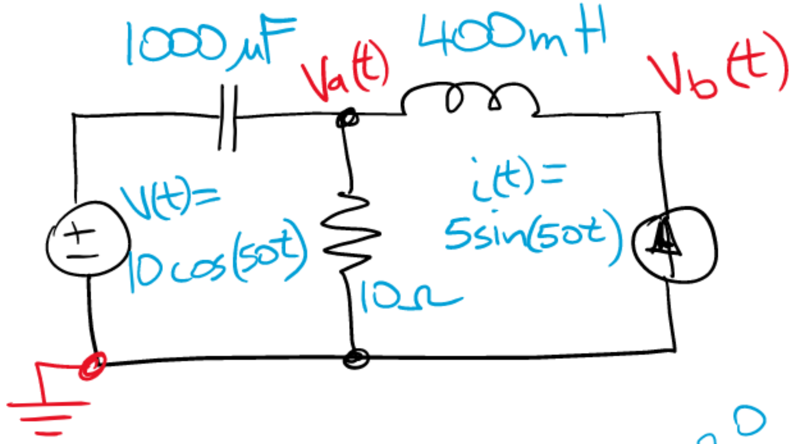
$$\frac{\overline{V_a} - 10}{-j20} + \frac{\overline{V_a}}{10} + \frac{\overline{V_a} - \overline{V_b}}{j20} = 0$$

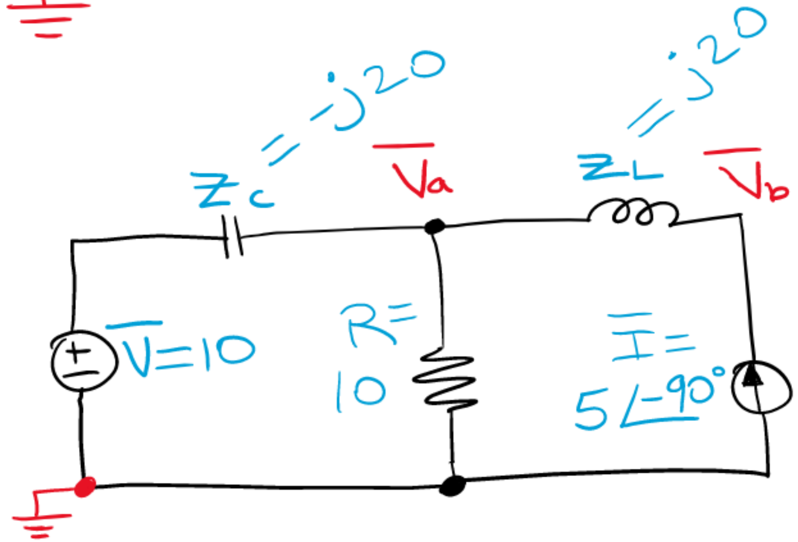
$$(\times j20) \quad -(\overline{V_a} - 10) + j2\overline{V_a} + \overline{V_a} - \overline{V_b} = 0$$

Simplify

$$j2\overline{V_a} - \overline{V_b} = -10 \quad \text{(eqn 3)}$$

Example: Find the node voltages $V_a(t)$ and $V_b(t)$.





$$\frac{\bar{V}_a-\bar{V}}{Z_c}+\frac{\bar{V}_a}{R}+\frac{\bar{V}_a-\bar{V}_b}{Z_L}=0 \quad \text{(eqn1)}$$

$$\frac{\bar{V}_b-\bar{V}_a}{Z_L}-\bar{I}=0 \quad \text{(eqn2)}$$

$$j2\bar{V}_a-\bar{V}_b=-10 \quad \text{(eqn 3)}$$

Plug in values into eqn 2

$$\frac{\bar{V}_b-\bar{V}_a}{j20}-5\angle{-90^\circ}=0$$

mixed rectangular and polar form

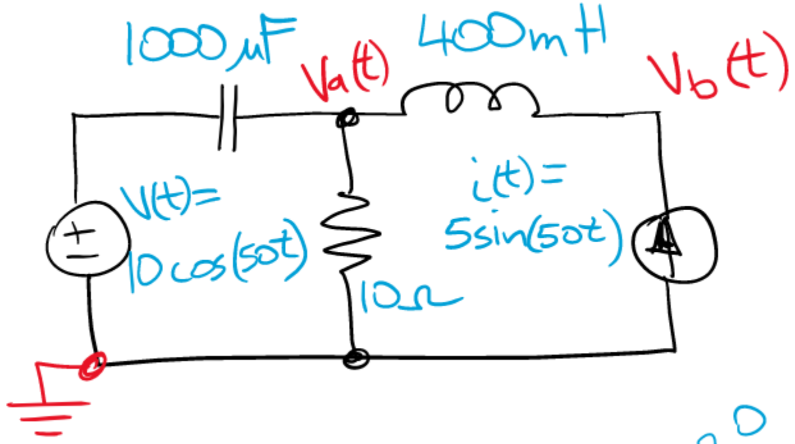
$$5\angle{-90^\circ}=0-j5=\bar{I}$$

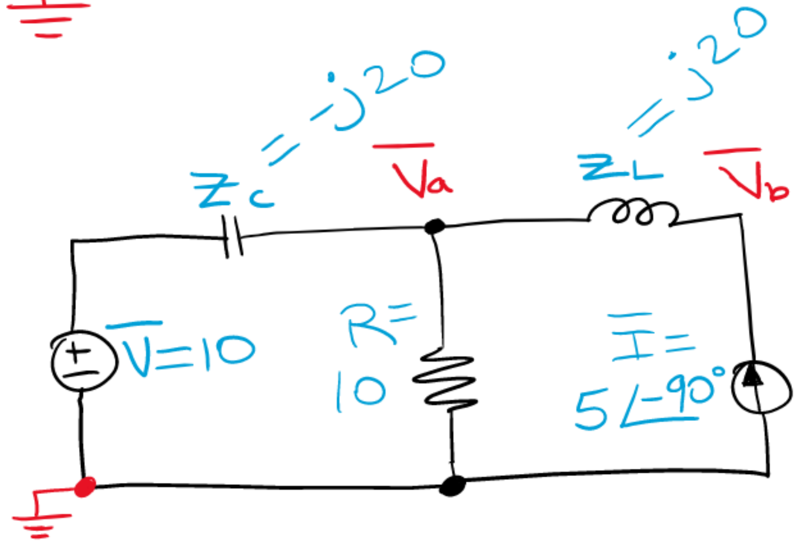
Then $$\frac{\bar{V}_b-\bar{V}_a}{j20}-(-j5)=0$$

$$(\times j20) \quad \bar{V}_b-\bar{V}_a-100=0$$

$$\bar{V}_b-\bar{V}_a=100 \quad \text{(eqn4)}$$

Example: Find the node voltages $V_a(t)$ and $V_b(t)$.





$j2\overline{V}_a - \overline{V}_b = -10$ (eqn 3) $\overline{V}_b - \overline{V}_a = 100$ (eqn 4)

Add eqn 3 and eqn 4

$$
\begin{array}{rl}
 & j2\overline{V}_a - \overline{V}_b = -10 \\
+ & -\overline{V}_a + \overline{V}_b = 100 \\
\hline
 & -\overline{V}_a + j2\overline{V}_a + 0 = 90
\end{array}
$$

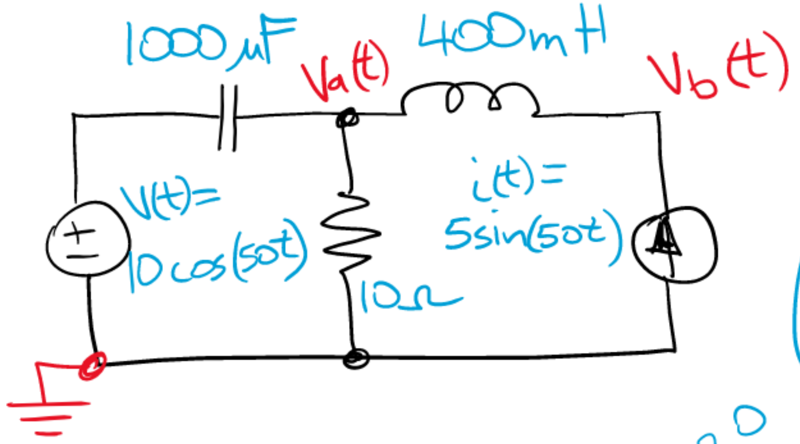
(factor $\overline{V}_a$) $\overline{V}_a(-1+j2) = 90 \Rightarrow \overline{V}_a = \dfrac{90}{-1+j2}$

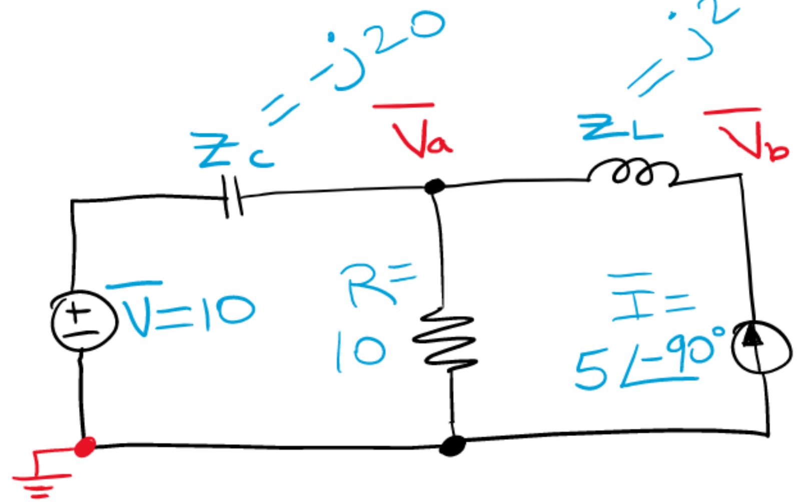
need to convert to standard form: $\overline{X} = Re[\overline{X}] + Im[\overline{X}]$

(multiply by complex conjugate) $\overline{V}_a = \dfrac{90}{-1+j2} \times \dfrac{-1-j2}{-1-j2} = \dfrac{-90-j180}{1+4} = \dfrac{-90-j180}{5}$

$$\overline{V}_a = -18 - j36$$

Example: Find the node voltages $v_a(t)$ and $v_b(t)$.





$$j2\bar{V}_a - \bar{V}_b = -10 \quad \text{(eqn 3)}$$

$$\bar{V}_b - \bar{V}_a = 100 \quad \text{(eqn 4)}$$

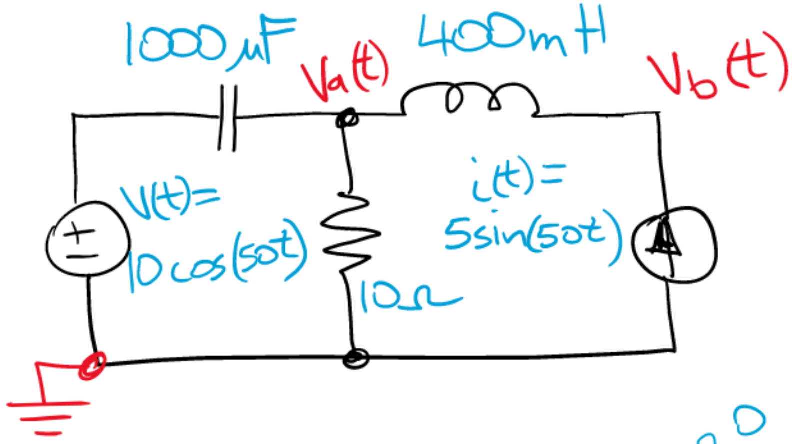
$$\bar{V}_a = -18 - j36$$

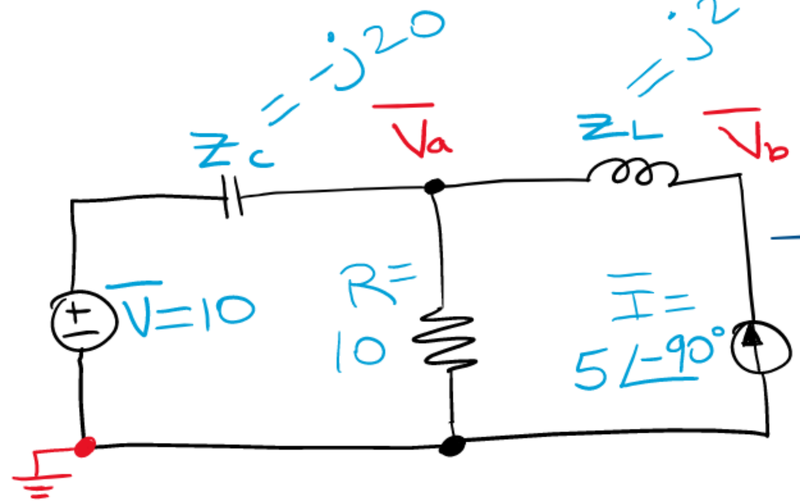
(Sub $\bar{V}_a$ into eqn 4)

$$\bar{V}_b - (-18 - j36) = 100$$

$$\bar{V}_b = 82 - j36$$

Example: Find the node voltages $V_a(t)$ and $V_b(t)$.

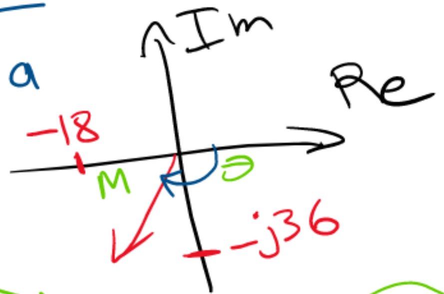




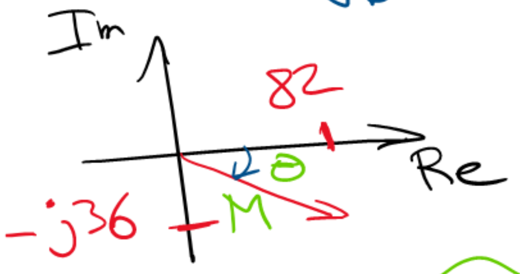
$\overline{V_a} = -18 - j36$ $\qquad$ $\overline{V_b} = 82 - j36$

To find the time domain (ie $V_a(t), V_b(t)$) representation, we must convert the rectangular representation to polar representation.

$\overline{V_a}$



$\overline{V_b}$



$$\overline{V_a} = \underbrace{\sqrt{18^2 + 36^2}}_{M} \angle \underbrace{-180^\circ + \tan^{-1}\left(\frac{36}{18}\right)}_{\theta}$$

$$\overline{V_a} = 40.25 \angle -116.6^\circ$$

$$V_a(t) = 40.25 \cos(\underbrace{50}_{\omega}t - 116.6^\circ)$$

$$\overline{V_b} = \underbrace{\sqrt{36^2 + 82^2}}_{M} \angle \underbrace{\tan^{-1}\left(\frac{-36}{82}\right)}_{\theta}$$

$$\overline{V_b} = 89.55 \angle -23.7^\circ$$

$$V_b(t) = 89.55 \cos(\underbrace{50}_{\omega}t - 23.7^\circ)$$

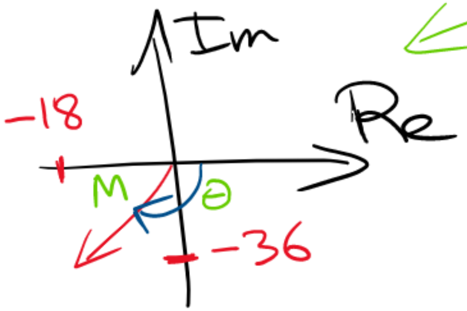
Now, convert from Polar form to time domain

A couple ways to solve for $V_a(t)$

$$\bar{V}_a = \frac{90}{-1+j2}$$

$$\bar{V}_a = \frac{90}{-1+j2} \times \frac{-1-j2}{-1-j2} = \frac{-90-j180}{1+4}$$

$$\bar{V}_a = \frac{-90-j180}{5} = -18-j36$$



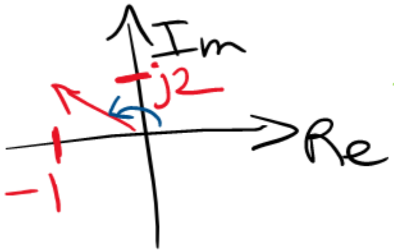
$$\bar{V}_a = \sqrt{(-18)^2+(-36)^2}\ \angle -180+\tan^{-1}\left(\frac{36}{18}\right)$$

$$\bar{V}_a = 40.25\ \angle -116.6^\circ$$

$$V_a(t) = 40.25\cos(50t-116.6^\circ)$$

same

$$\bar{V}_a = \frac{90}{-1+j2}$$



$$-1+j2 \Rightarrow \sqrt{(-1)^2+(2)^2}\ \angle 180^\circ - \tan^{-1}\left(\frac{2}{1}\right)$$

$$= \sqrt{5}\ \angle 116.6^\circ$$

Then

$$\bar{V}_a = \frac{90}{-1+j2} = \frac{90}{\sqrt{5}\ \angle 116.6^\circ}$$

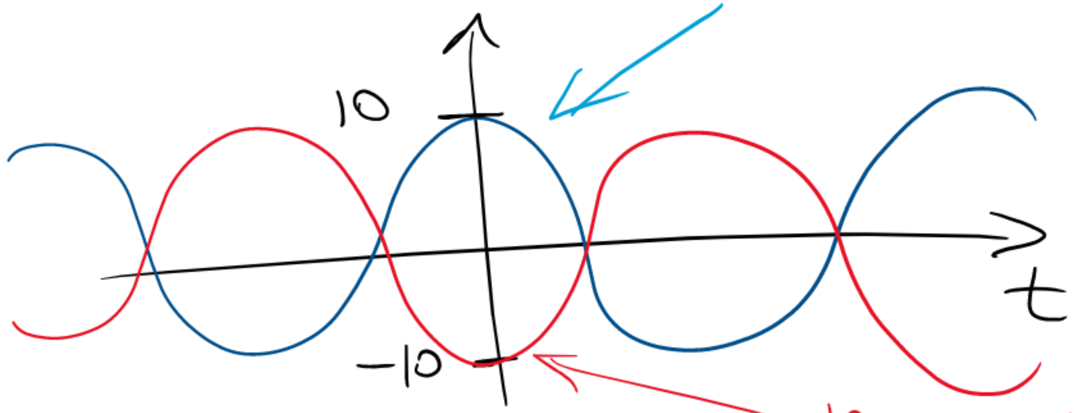
$$\bar{V}_a = 40.25\ \angle -116.6^\circ$$

$$V_a(t) = 40.25\cos(50t-116.6^\circ)$$

What is the difference between $\bar{I}$ and $-\bar{I}$ or $\bar{V}$ and $-\bar{V}$? Don't AC waveforms go positive and negative anyway?

Let $\bar{V} = 10\angle 0^\circ$

then $v(t) = 10\cos(wt)$



This implies $-\bar{V} = -10\angle 0^\circ$

then $-v(t) = -10\cos(wt)$

we can also say:

$$-v(t) = -10\cos(wt) = 10\cos(wt+180^\circ) = 10\cos(wt-180^\circ)$$

$$-\bar{V} = -10\angle 0^\circ = 10\angle 180^\circ = 10\angle -180^\circ$$