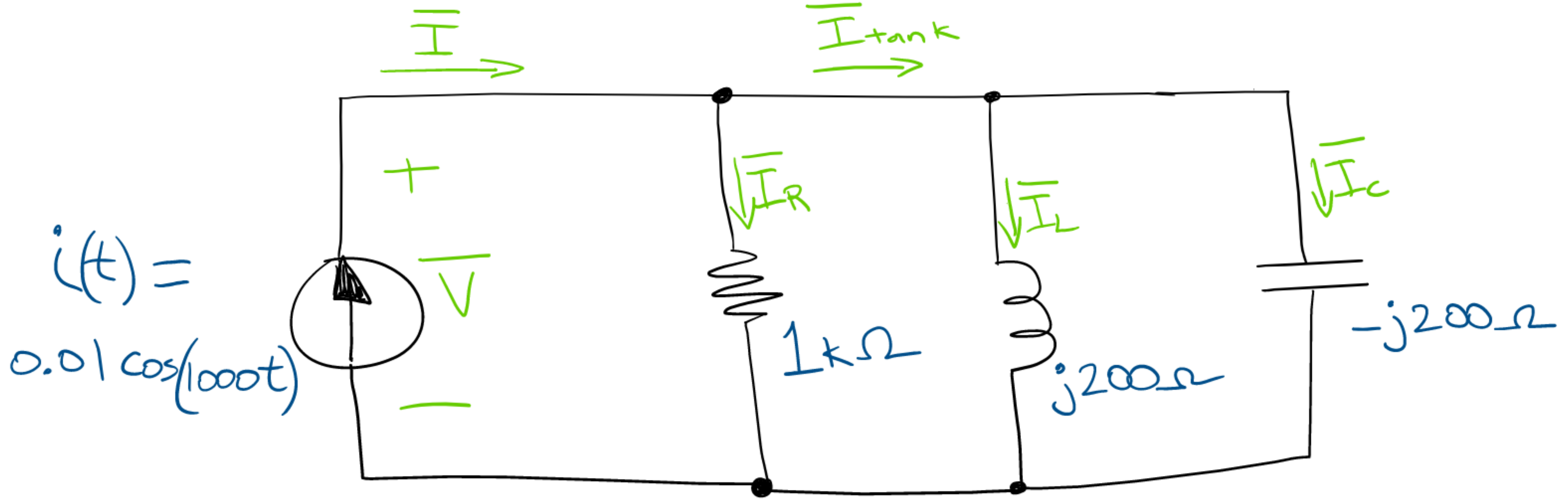
Example

Find the phasor voltage $\overline{V}$ and the phasor currents $\overline{I}$, $\overline{I}_R$, $\overline{I}_L$, $\overline{I}_c$, and $\overline{I}_{tank}$.



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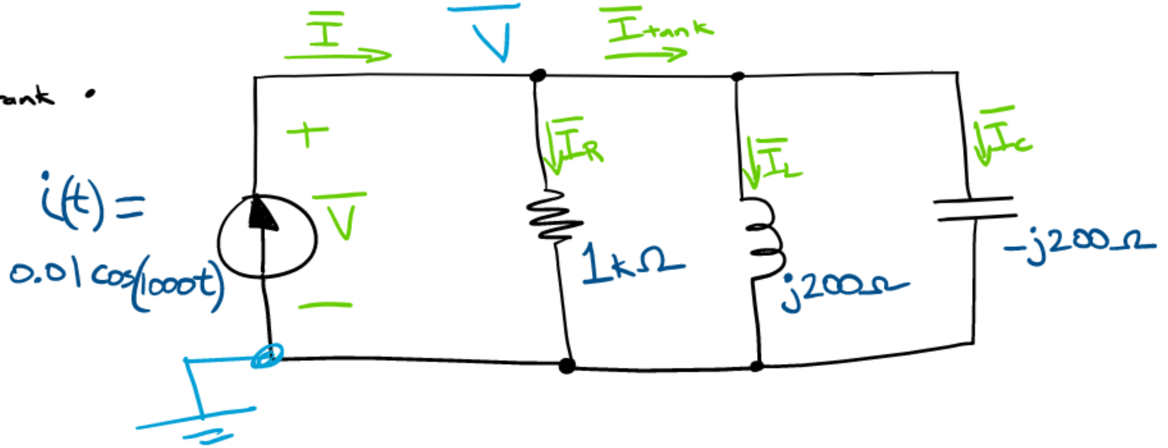

Solution:

Note: we can observe that

1. The impedances for L and C are given directly ($j200\Omega$ and $-j200\Omega$), so no need to use $\omega = 1000$.
2. For node voltage analysis, we can just use $\bar{V}$ to denote the node voltage.

Conceptual question:

Does the polarity of $\bar{V}$ matter?

See previous video to understand the difference between $\bar{V}$ and $-\bar{V}$

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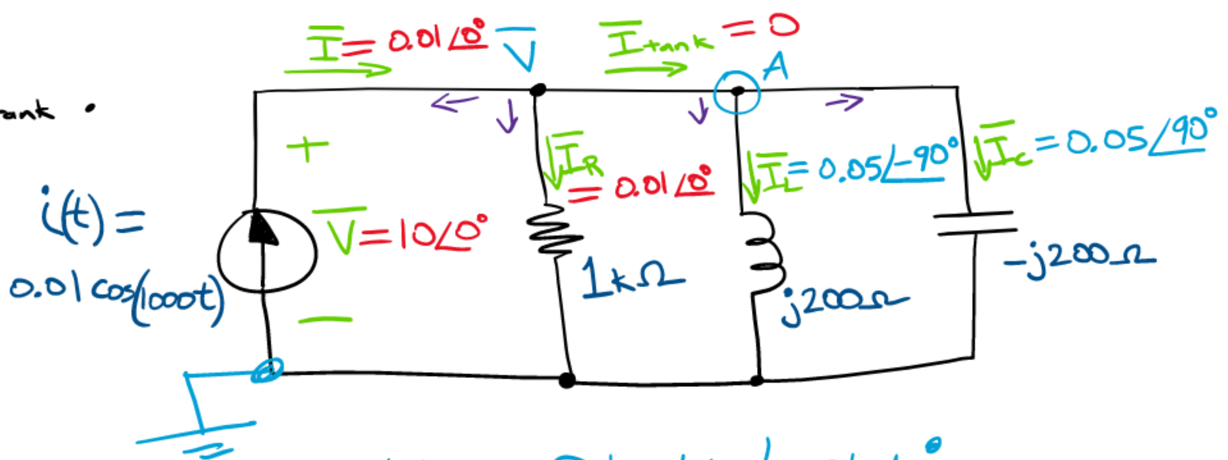


Convert $i(t)$ to a phasor:

$$i(t) = 0.01\cos(1000t)$$

$$\overline{I} = 0.01\angle 0^\circ = 0.01$$

Use node voltage analysis to find $\overline{V}$:

$$-0.01 + \frac{\overline{V}}{1000} + \frac{\overline{V}}{j200} + \frac{\overline{V}}{-j200} = 0$$

$$1000 \times \frac{\overline{V}}{-j200} = 1000 \times \frac{j\overline{V}}{200}$$

$$(\times 1000) \quad -10 + \overline{V} \underbrace{- j5\overline{V} + j5\overline{V}}_{=0} = 0$$

$$\overline{V} = 10 = 10\angle 0^\circ$$

Use Ohm's Law:

$$\overline{I}_R = \frac{\overline{V}}{R} = \frac{10V}{1000\Omega} = 0.01A = 0.01\angle 0^\circ$$

$$\overline{I}_L = \frac{\overline{V}}{Z_L} = \frac{10V}{j200\Omega} = -j0.05A = 0.05\angle -90^\circ$$

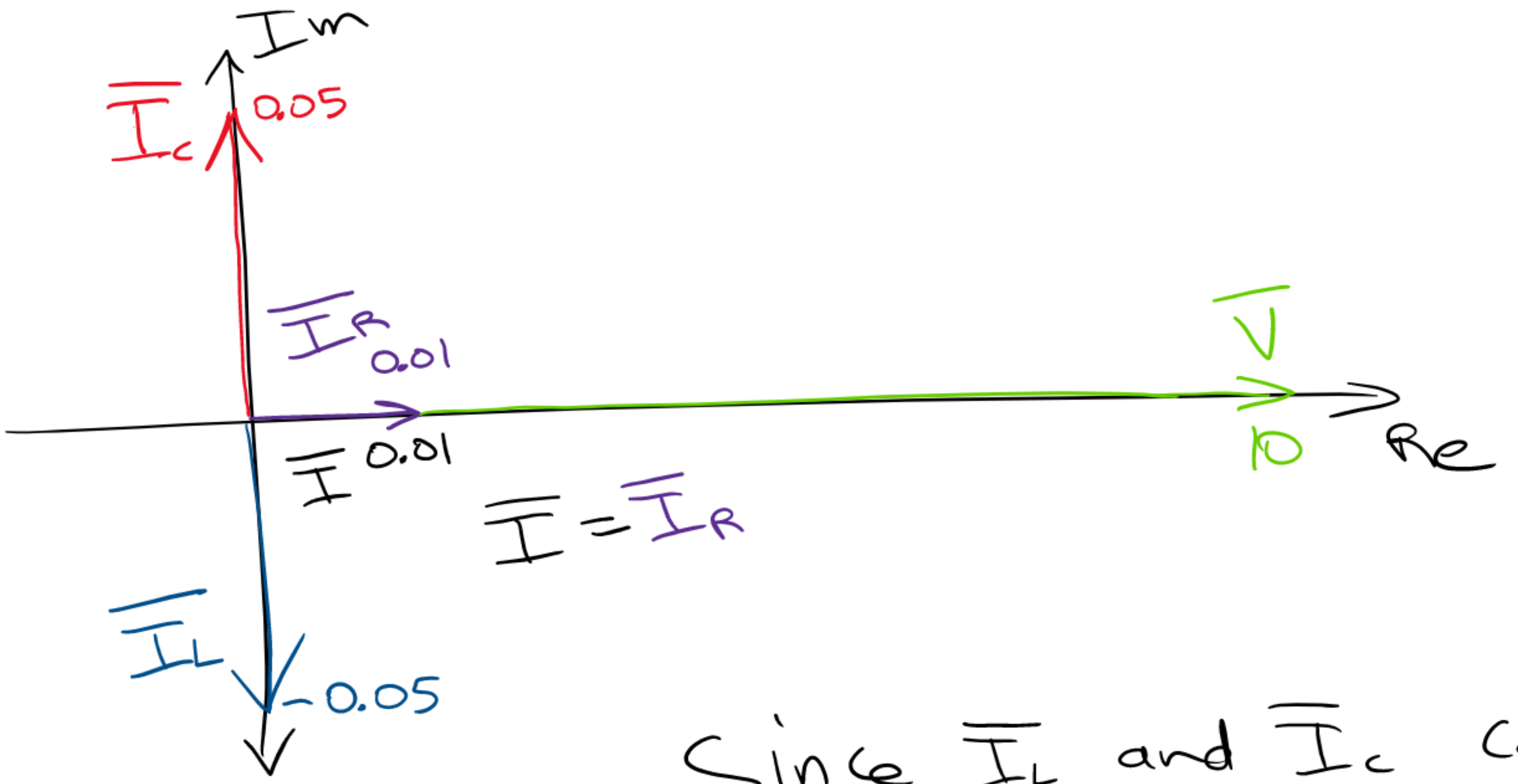
$$\overline{I}_c = \frac{\overline{V}}{Z_c} = \frac{10V}{-j200\Omega} = j0.05A = 0.05\angle 90^\circ$$

Use KCL at the connection point A:

$$\overline{I}_{tank} = \overline{I}_L + \overline{I}_c = -j0.05A + j0.05A$$

$$\overline{I}_{tank} = 0\,A$$

Let's draw the current and voltage phasors:



$\overline{I} = \overline{I}_R$

$\overline{V} = 10\angle 0^\circ$

$\overline{I} = 0.01\angle 0^\circ$

$\overline{I}_R = 0.01\angle 0^\circ$

$\overline{I}_L = 0.05\angle -90^\circ$

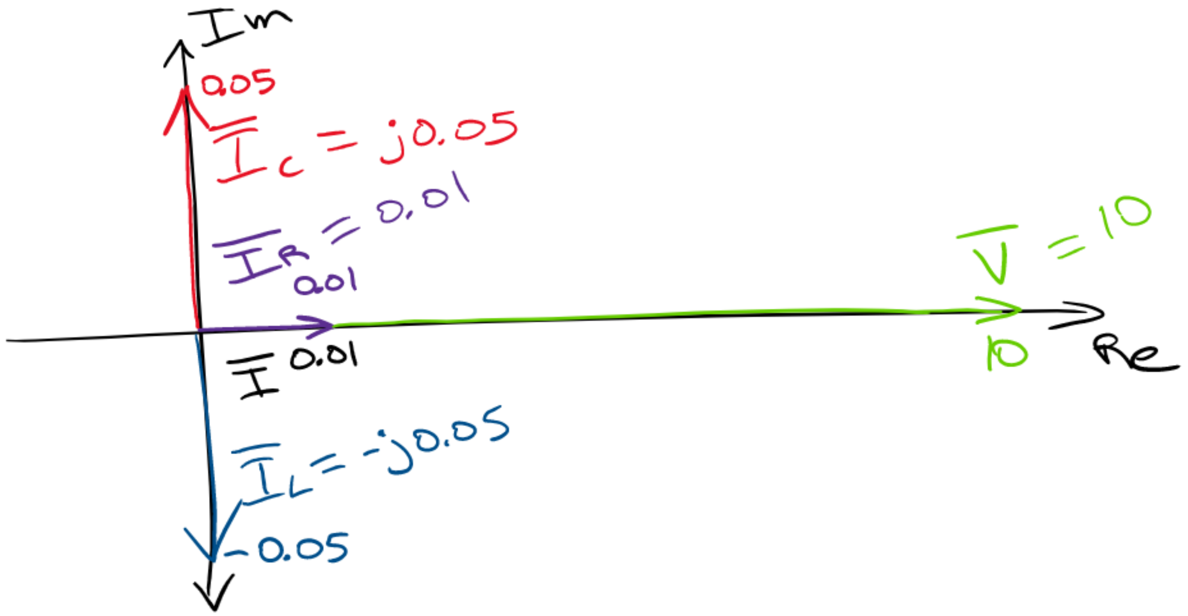
$\overline{I}_c = 0.05\angle 90^\circ$

$\overline{I}_{tank} = 0$

Since $\overline{I}_L$ and $\overline{I}_c$ cancel each other out, $\overline{I}_{tank}$ is 0.

$$\overline{I}_{tank} = \overline{I}_L + \overline{I}_c = -j200 + j200$$

$$\overline{I}_{tank} = 0$$



$\overline{V} = 10\angle 0^\circ$

$\overline{I} = 0.01\angle 0^\circ$

$\overline{I}_R = 0.01\angle 0^\circ$

$\overline{I}_L = 0.05\angle -90^\circ$

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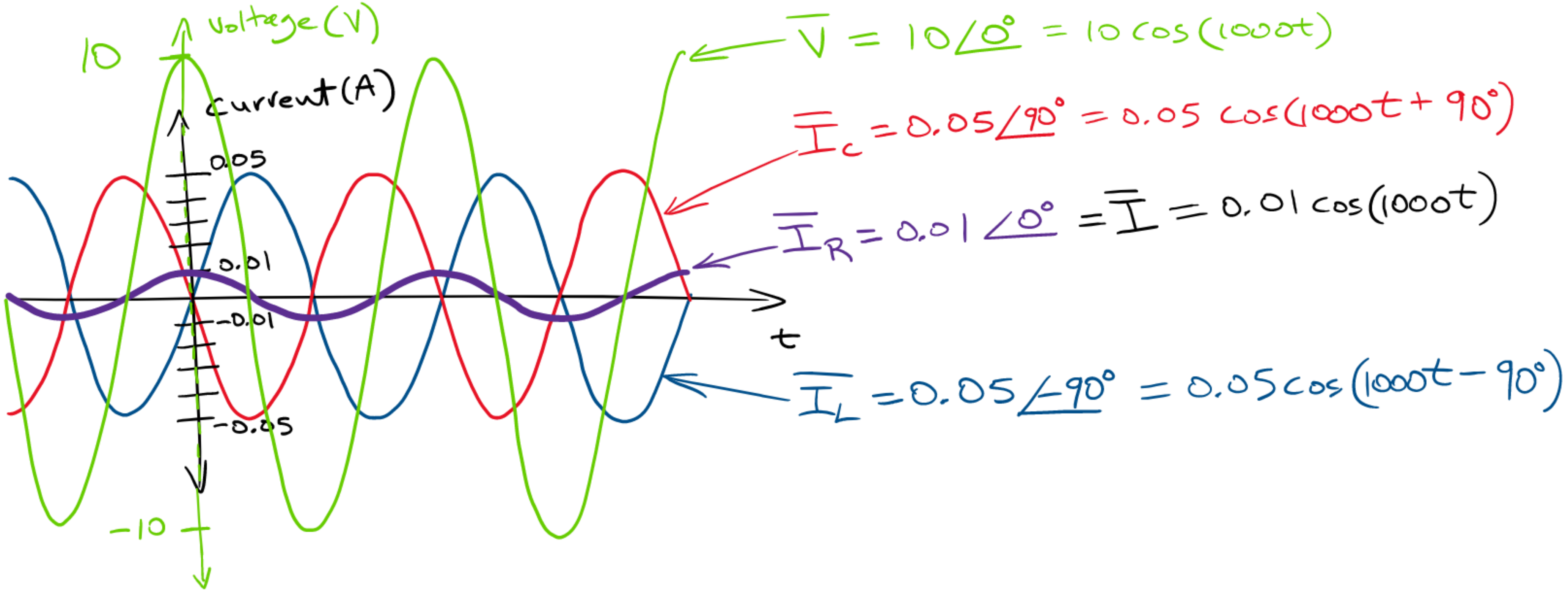
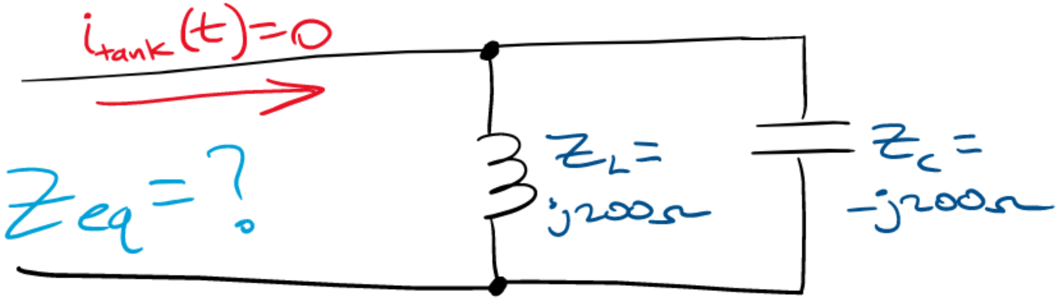
$\overline{I}_{tank} = 0$



Let's consider only the inductor and capacitor in this circuit



$Z_{eq} = ?$

How can we have 0 A current going into the L-C circuit?

$$Z_{eq} = \frac{j200 \times (-j200)}{j200 + (-j200)} = \frac{40\,000}{0} = \pm\infty$$

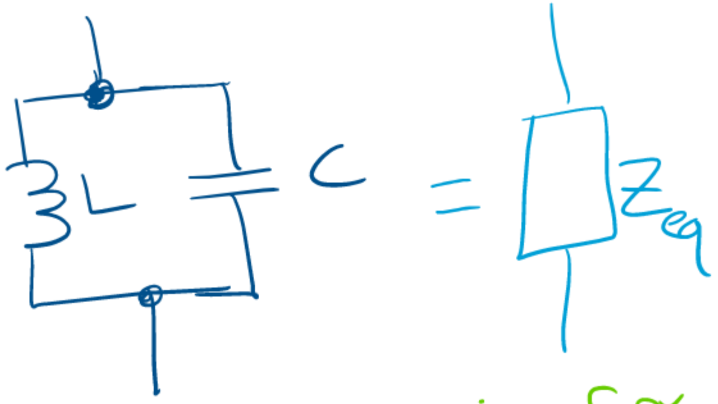
So, the parallel combination of the L-C circuit is $\pm\infty\,\Omega \Rightarrow$ appears as open-circuit.

But, this is <u>only</u> at the frequency <u>$\omega = 1000$.</u>

This is a special case called <u>parallel resonance</u>.

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Where does $\omega_0 = \frac{1}{\sqrt{LC}}$ come from?



For an L-C circuit, the Z_L is equal in magnitude to Z_C

or $Z_L = -Z_C$

condition for

$Z_{eq} = \frac{Z_L Z_C}{Z_L + Z_C}$

denominator goes to zero.

$Z_L = j\omega L$, $Z_C = \frac{1}{j\omega C}$

at resonance frequency, ω_0, $Z_L = -Z_C$

Then,

$$j\omega_0 L = -\frac{1}{j\omega_0 C}$$

$(\times \omega_0)$ $\omega_0^2 jL = -\frac{1}{jC}$

$(\times -j)$ $\omega_0 = \frac{1}{\sqrt{LC}}$ $\longrightarrow$ $Z_{eq} = \pm\infty\ \Omega$ at ω_0 (open-circuit)

A series L-C is also interesting

at $\omega_0 = \frac{1}{\sqrt{LC}}$ $\longrightarrow$ Series resonance

This L-C circuit appears as a <u>Short-circuit</u>

E.g. $Z_L = j200\,\Omega$, $Z_C = -j200\,\Omega$

$$Z_{eq} = Z_L + Z_C = j200 - j200 = 0\,\Omega$$