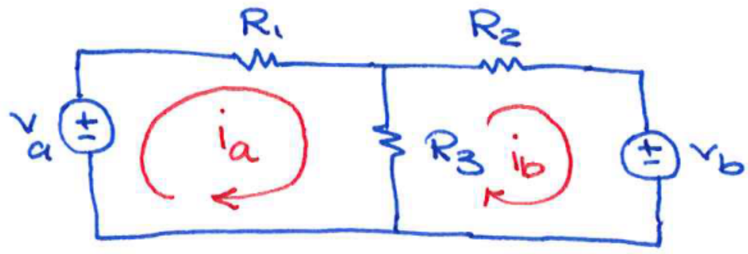
The Mesh Current Method

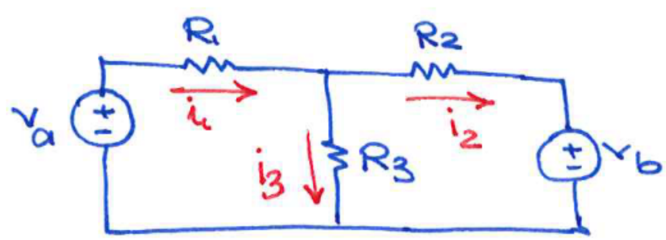
- Another useful, systematic method for circuit analysis



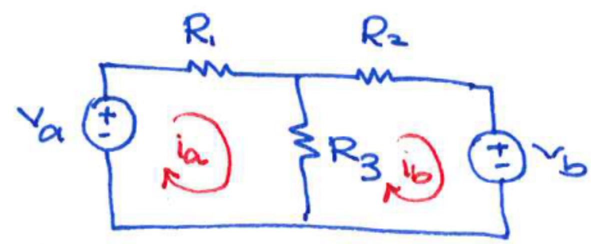
i_a & i_b : mesh current

- imagined currents circulating in a closed loop or mesh.

- Mesh currents are different from branch currents:
  - We use branch currents to write KCL
  - Branch currents can be measured with an ammeter
  - Branch current is used in Ohm's Law.



branch currents



mesh currents

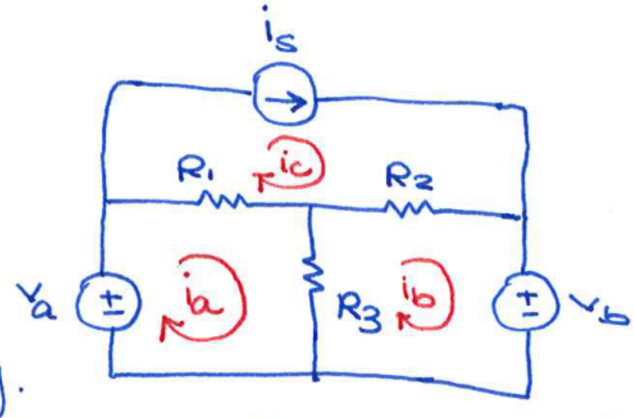
$$i_1 = i_2 + i_3$$

$$i_1 = i_a$$
$$i_2 = i_b$$
$$i_3 = i_a - i_b$$

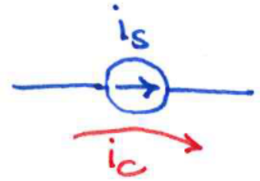
- 3 main steps:

1) Identify meshes & any known mesh currents
2) Write a mesh equation (KVL) for each mesh; Develop a system of equations. Only unknowns should be mesh currents.
3) Solve for mesh currents

Step 1 : Identify Meshes



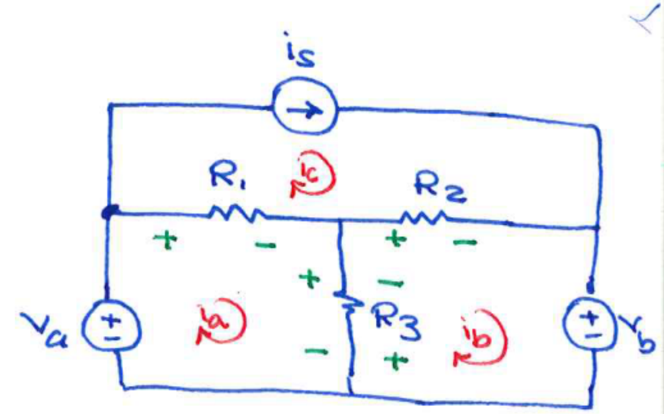
- Imagine a circuit as a window pane; Assign a mesh current to each pane.
- Assumed mesh directions arbitrarily.
- Usually, mesh current method is easier (more systematic) if all meshes have the same direction
- Note the current source in the upper mesh:



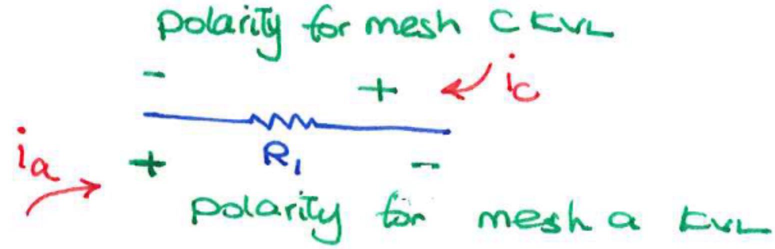
i_c labelled with the same direction as i_s : $i_c = i_s$

i.e. i_c is known immediately

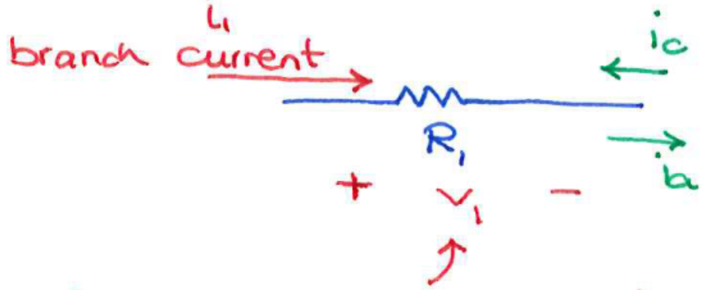
Step 2 : Form mesh equations



- Labelling the circuit:
  Include a polarity on each resistor inside each mesh.
  Assume voltage drop in direction of mesh current
- Consider R_1 :



- When writing KVL for <u>mesh a</u>: we need to determine <u>branch</u> currents in terms of mesh currents to find voltages across resistors:



voltage across R_1 due to branch current i_1

chose i_1 in the same direction as i_a $\therefore$ $i_1 = i_a - i_c$

From Ohm's Law: $V_1 = i_1 . R_1 = (i_a - i_c) . R_1$

• Now, sum voltages around mesh a (clockwise direction; same as i_a)

$$-v_a + (i_a - i_c) R_1 + (i_a - i_b) R_3 = 0 \qquad (i)$$

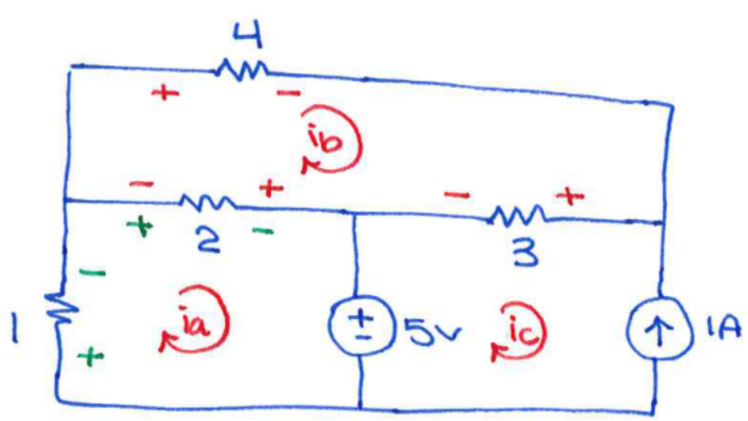
saw (−) terminal first

. Similarly, for mesh b:

$$(i_b - i_a) R_3 + (i_b - i_c) R_2 + v_b = 0 \qquad (ii)$$

Step 3: Solve 2 equations (i) & (ii) to find 2 unknowns i_a & i_b

Ex: Find power in 3Ω resistor using mesh current



We know that $i_c = -1A$

Two unknowns i_a & i_b

. Avoid mesh eq at c since we don't know the voltage across the 1A source

• Once all mesh currents are calculated, we can write KVL around mesh c to find voltage across 1A source

KVL around mesh a :

$$\underbrace{i_a \times 1}_{\text{only one mesh current through in resistor}} + 2\overbrace{(i_a - i_b)}^{\text{2 mesh currents in opposite direction}} + 5 = 0$$

$$\therefore \quad 3\, i_a - 2\, i_b = -5 \qquad \text{(i)}$$

KVL around mesh b: (use polarities labelled in red)

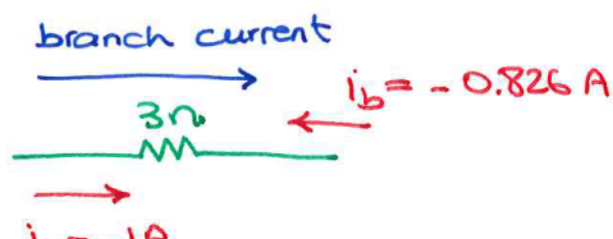
$$4\, i_b + 3\,(i_b - \underbrace{i_c}_{-1}) + 2\,(i_b - i_a) = 0$$

$$\therefore \quad -2\, i_a + 9\, i_b = -3 \qquad \text{(ii)}$$

. Solving (i) and (ii) gives $i_a = -2.217$ A , $i_b = -0.826$ A

. To find power in 3Ω resistor :

- assumed direction for branch current

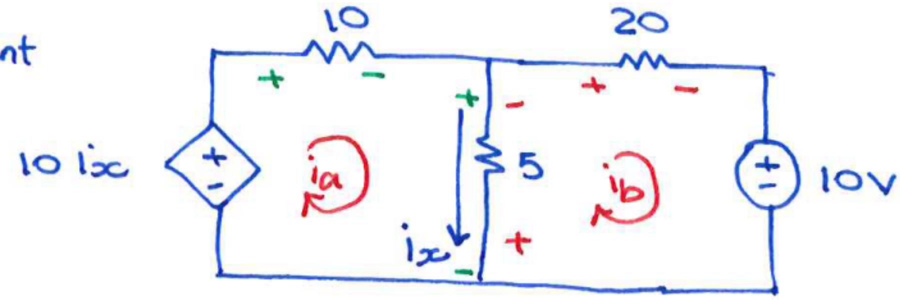


$$\text{branch current} = i_c - i_b$$
$$= -1 - (-0.826)$$
$$= -0.174 \text{ A}$$

$$P = i^2 R = (-0.174)^2 \times 3 = 0.09 \text{ W}$$

Ex (Dep. Source). Solve for mesh currents

Note: i_x is a <u>branch</u> current



• KVL around mesh a:

$$-10 i_x + 10 i_a + \underbrace{5(i_a - i_b)}_{\text{ignored } i_x \text{ when writing this}} = 0$$

need to write i_x in terms of i_a & i_b $\therefore$ $i_x = i_a - i_b$

$$\therefore \quad -10(i_a - i_b) + 10 i_a + 5(i_a - i_b) = 0$$

$$\therefore \quad i_a = -i_b \qquad (i)$$

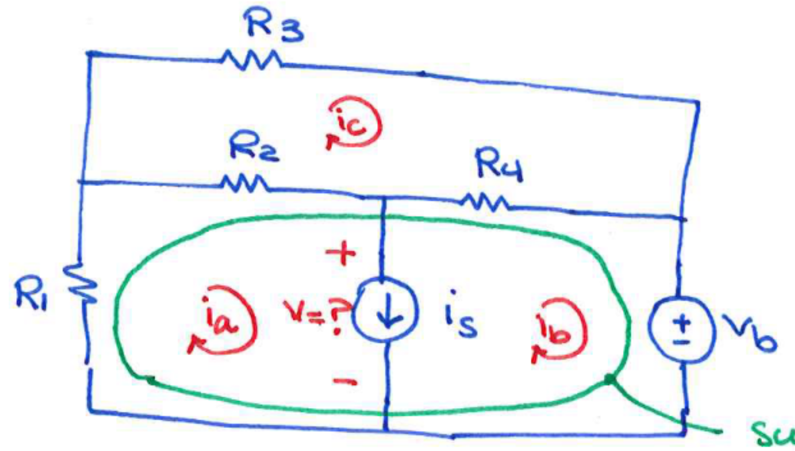
• KVL around mesh b:

$$5(i_b - i_a) + 20 i_b + 10 = 0$$

$$\therefore \quad -5 i_a + 25 i_b = -10 \qquad (ii)$$

• Solving (i) and (ii) gives: $i_a = \frac{1}{3}$ A , $i_b = \frac{-1}{3}$ A

Special case: Circuits with current sources shared by 2 meshes.



supermesh i_a & i_b

- KVL for mesh c: no problem!
- when writing KVL for mesh a & b, we have an unknown voltage across the current source!

Mesh a: $R_1 i_a + R_2(i_a - i_c) + V = 0$

extra unknown!

Mesh b: $R_4(i_b - i_c) + V_b - V = 0$

adding these 2 eliminates V:

$$R_1 i_a + R_2(i_a - i_c) + R_4(i_b - i_c) + V_b = 0 \quad (i)$$

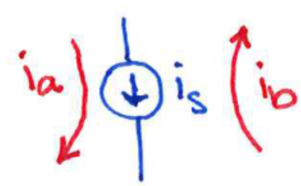
We can arrive at (i) directly with a supermesh.

i) KVL around super mesh:

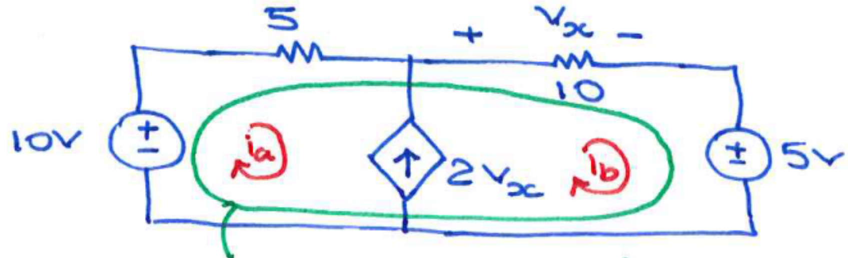
$$\underbrace{R_1 i_a + R_2(i_a - i_c)}_{\text{mesh a side}} + \underbrace{R_4(i_b - i_c) + V_b = 0}_{\text{mesh b side}}$$

ii) From inside the supermesh:

$i_s = i_a - i_b$ Dependence Equation



Ex : Find mesh currents



Current source (VCCS)

Shared by 2 meshes

$\therefore$ need a super mesh

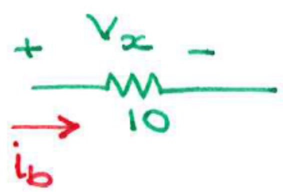
supermesh i_a & i_b

. KVL around super mesh : $-10 + 5i_a + 10i_b + 5 = 0$

$\therefore \ 5i_a + 10i_b = 5 \qquad (i)$

. Dependence Eq : $i_b - i_a = 2V_x$

Need to write V_x in terms of mesh current(s):



From Ohm's Law : $V_x = 10 . i_b$

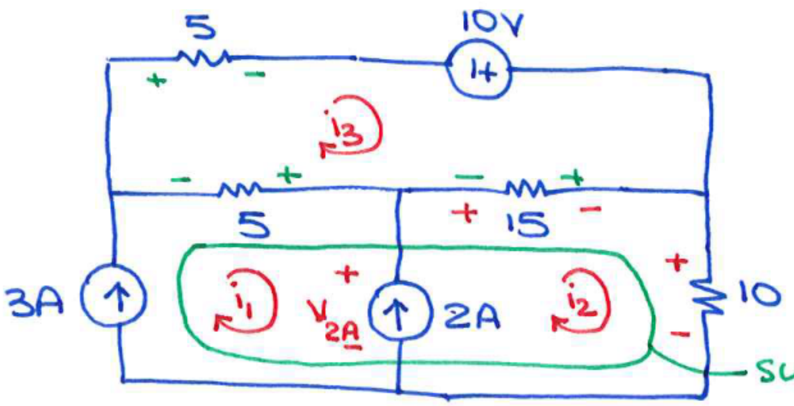
$\therefore \ i_b - i_a = 2(10 i_b) \quad \therefore \ i_a = -19 \ i_b \qquad (ii)$

. Solve (i) and (ii) to get $i_a = 1.12$ A , $i_b = -0.058$ A

Ex: 2012 MT

a) Find Mesh Currents

b) Find power in 2A current source



a) $i_1 = 3A$

2A source shared by 2 meshes $\therefore$ supermesh!

- Avoid KVL for this supermesh since voltage across 3A source is unknown.
- Dependence Eq from super mesh: $i_2 - i_1 = 2 \quad \therefore i_2 = 5\,A$
- KVL around mesh 3: $5i_3 - 10 + 15(i_3 - \underbrace{i_2}_{5}) + 5(i_3 - \underbrace{i_1}_{3}) = 0$

$$\therefore i_3 = 4\,A$$

b) Need V_{2A} to find power. Assumed a polarity for V_{2A}.

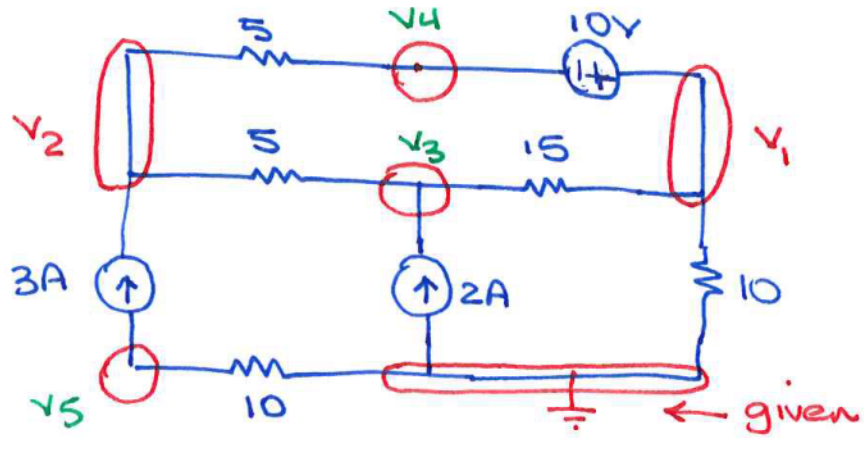
Let's try KVL around mesh 2:

$$-V_{2A} + 15(\underbrace{i_2}_{5} - \underbrace{i_3}_{4}) + 10\,\underbrace{i_2}_{5} = 0 \qquad \therefore V_{2A} = 65\,V$$

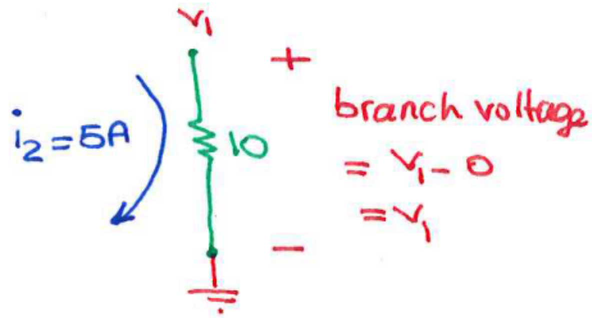
$$P_{2A} = -(V_{2A}).(2A) = -65 \times 2 = -130\ W$$

supplies 130 W

Ex: Using the mesh currents in prev ex, find node voltages V_1 & V_2.

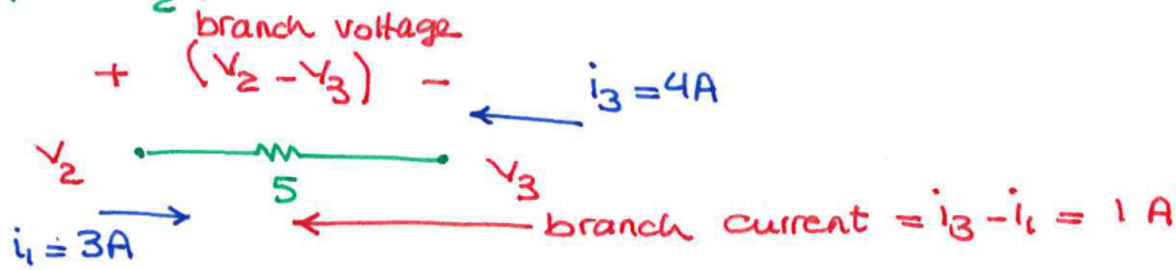


Let's use the 10Ω resistor to find V_1:



from Ohm's Law : $V_1 = 5 \times 10 = 50$ V

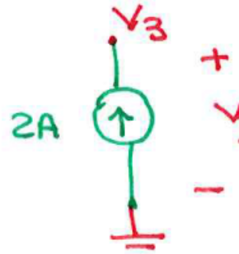
• To find V_2, let's use the 5Ω resistor in the middle:



Ohm's Law: branch voltage across R = – branch current thru R × R

$\therefore \quad V_2 - V_3 = -1 \times 5$
$= -5$

but we know the voltage across 2A source:



+

$V_{2A} = V_3 - 0 = 65 \quad \therefore \quad V_3 = 65$ V

–

Finally, $V_2 - V_3 = -5 \quad \therefore \quad V_2 = 60$ V

Choosing between node voltage & mesh current methods

- Pick the method with fewer unknowns in a given circuit
- Node voltage: Look for nodes with voltage sources attached. May eliminate unknowns by good choice of ref node.
- Mesh current: Look for meshes where current is fixed in value due to current sources.

Summary of circuit analysis methods

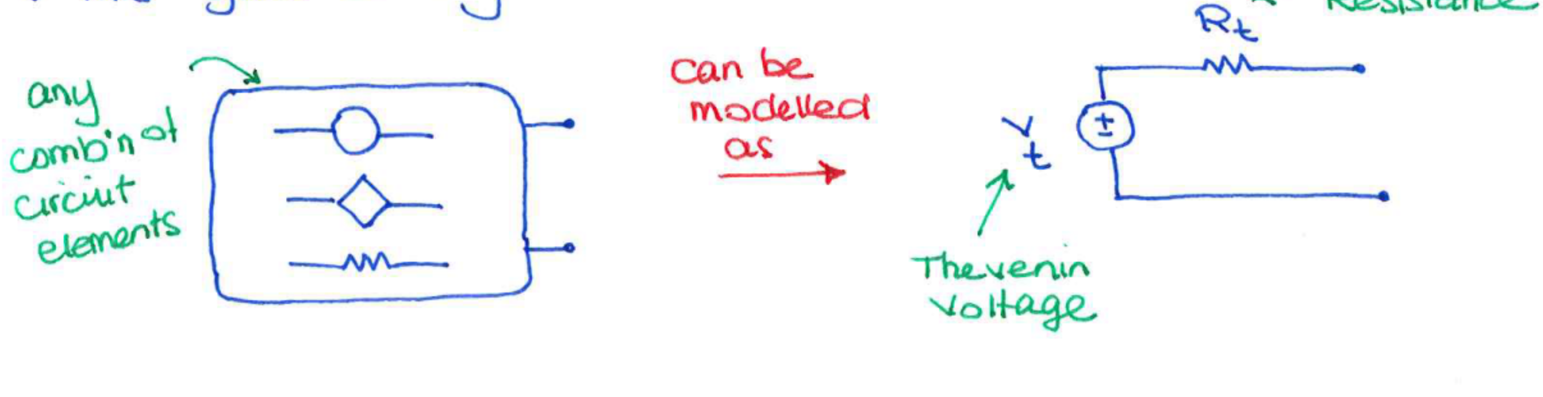
- So far:
  1) Circuit Simplification, KVL, KCL, Ohm's Law
  2) Node Voltage
  3) Mesh Current

Coming next:
  4) Thevenin Equivalent
  5) Superposition

Thevenin Equivalent Circuits

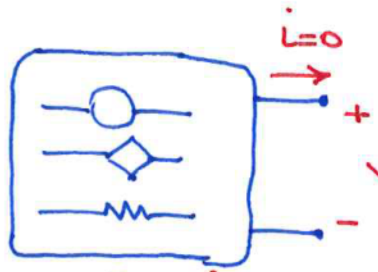
- Thevenin theorem states: An electric circuit containing voltage sources, current sources, and resistors with two terminals is electronically equivalent to a network with <u>one</u> voltage source and <u>one</u> resistor.

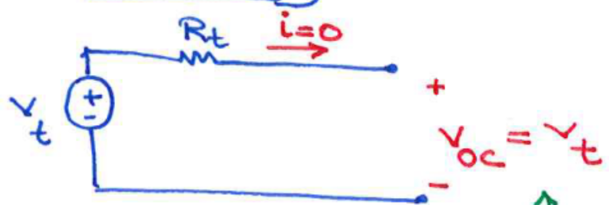
- This gives a way to model complex circuits:



- The voltage-current characteristics are identical at the two terminals. v_t & R_t are found by considering operating extremes: open circuit, short circuit

Open circuit



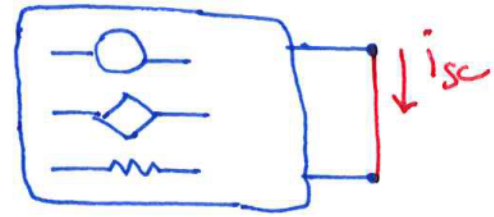


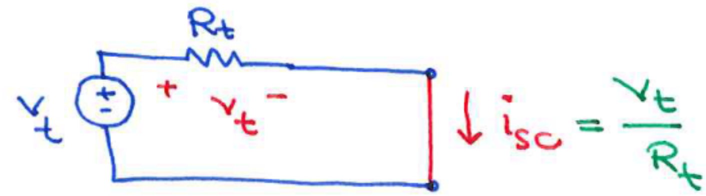
why?

KVL around loop; voltage across $R = 0$

therefore $\boxed{v_t = v_{oc}}$

Short circuit



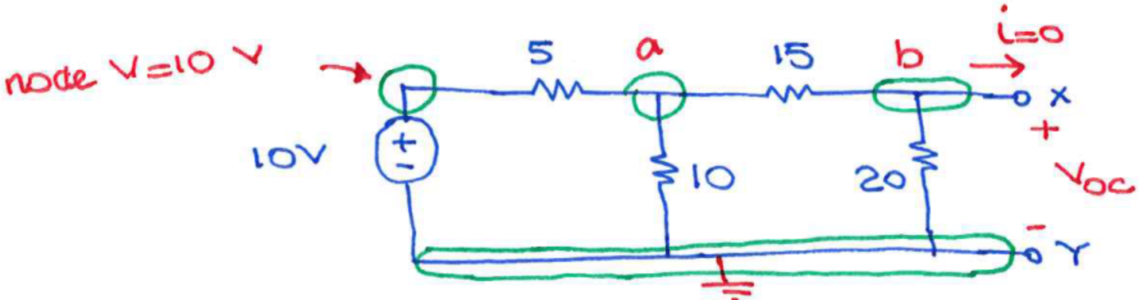


$$i_{sc} = \frac{v_t}{R_t} = \frac{v_{oc}}{R_t}$$

$$\boxed{R_t = \frac{v_{oc}}{i_{sc}}}$$

- Later, a short cut for finding R_t for circuits <u>without</u> dep sources

Ex: Find thevenin Equivalent of the following circuit as seen from X & Y:



Determining a thevenin Equivalent is two <u>separate</u> analysis problems: Find v_{oc} & find i_{sc}