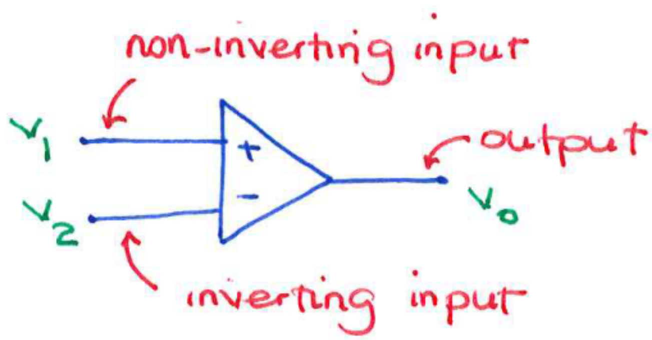
Operational Amplifier

- An operational amplifier (op amp) is a complex electronic circuit element that implements a VCVS.
- Many engineering applications:
  - video / audio amplifier
  - Telecommunication
  - Instrumentation (precise measurement devices)
- Circuit symbol:

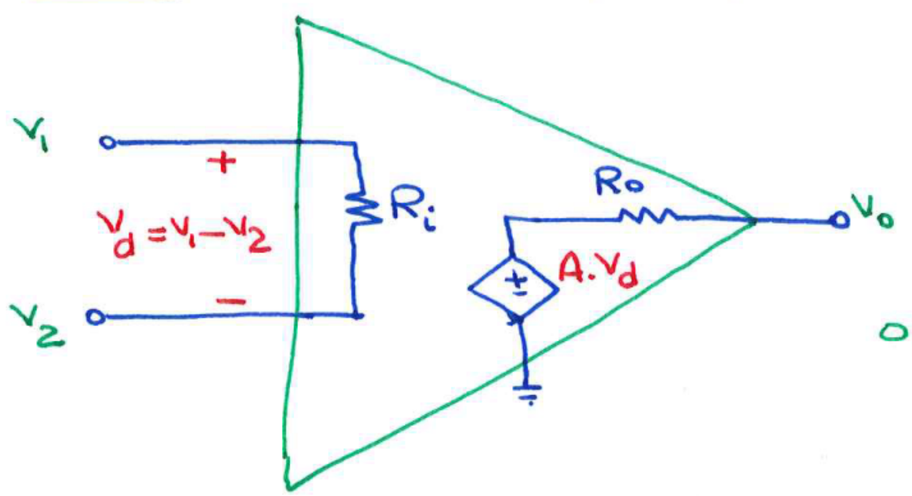


- The op amp amplifies the difference between input voltages v_d (where $v_d = v_1 - v_2$) :

$$v_o = A \cdot v_d$$

big number!

- The model for a real op amp:

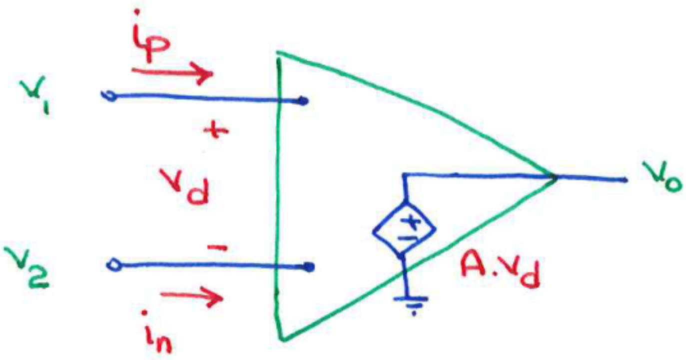


input resistance $R_i \simeq 1M\Omega$

output resistance $R_o \simeq 50\Omega$

open loop gain $A \simeq 10^5$

- The model for an ideal op amp:



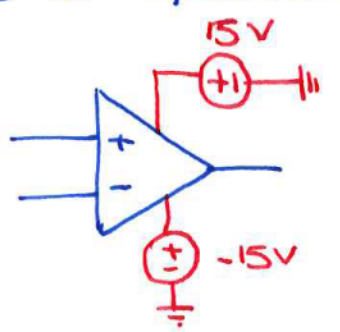
- $R_i = \infty$

  i.e. open circuit seen from the inputs

  $\therefore \quad i_n = i_p = 0$

- $R_o = 0\ \Omega$ (zero output resistance)
- $A = \infty$ (infinite open loop gain)

- Same behaviour for AC or DC. For now, we will use DC only.
- Since op amp is an electronic circuit, it requires an external source to operate:



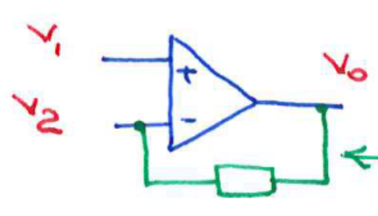
when we draw op amp circuits, we seldom draw external power sources. We simply assume they're there!

The Summing Point Constraints ← KEY!

1) The Virtual Open circuit : For an ideal op amp, $R_i = \infty$ (open circuit)

   therefore, $\boxed{i_n = i_p = 0}$

2) the virtual short circuit : For ideal op amp circuits with negative feedback, the 2 input terminals are at the same voltage: $\boxed{v_1 = v_2}$

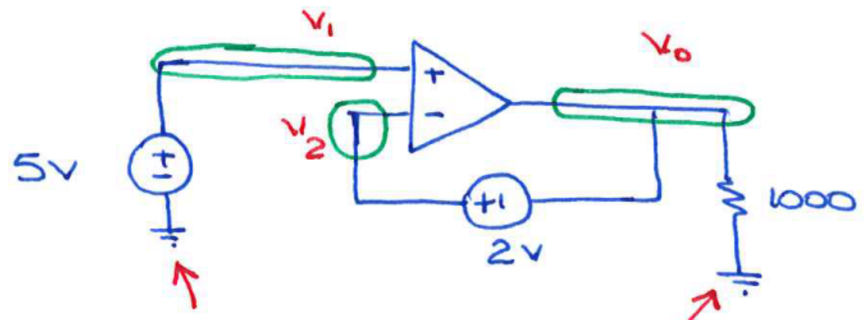


← negative feedback (output connected back to − input)

Example: Summing Point Constraints

1) Find V_o

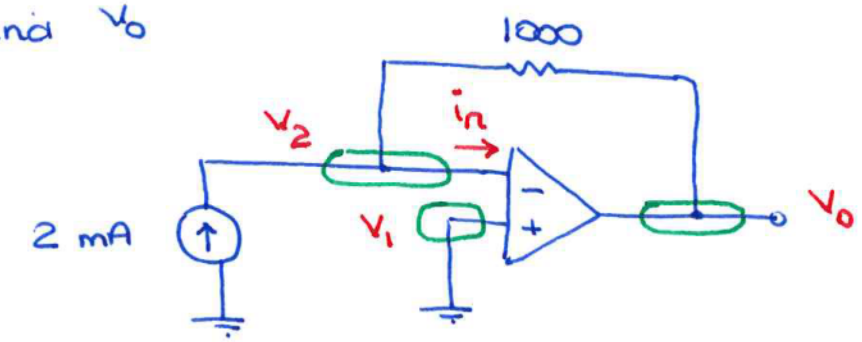
① $V_1 = 5\ V$ (node voltage is 5 V)



② $V_2 = 5\ V$ from virtual short ($V_1 = V_2$)

③ $V_2 - V_o = 2\ V$ $\quad \therefore \quad V_o = 5 - 2 = 3\ V$

2) Find V_o



Node 1: + input of op amp

Node 2: − input of op amp

① $V_1 = 0$ connected directly to ref node

② $V_2 = V_1$ from virtual short $\therefore V_2 = 0\ V$

③ Node eq at 2:

$$-2\ mA + \frac{\overset{0}{V_2} - V_o}{1000} + \overbrace{i_n}^{0\ \text{(from virtual open)}} = 0$$

$\therefore V_o = -2\ V$

voltage of output node w.r.t ref

· Easiest way to analyze amplifier circuits:

Tip 1: All the interesting stuff happens at the op amp input terminals. Start there! Take advantage of summing point constraints.

$$v_1 = v_2$$
$$i_n = i_p = 0$$

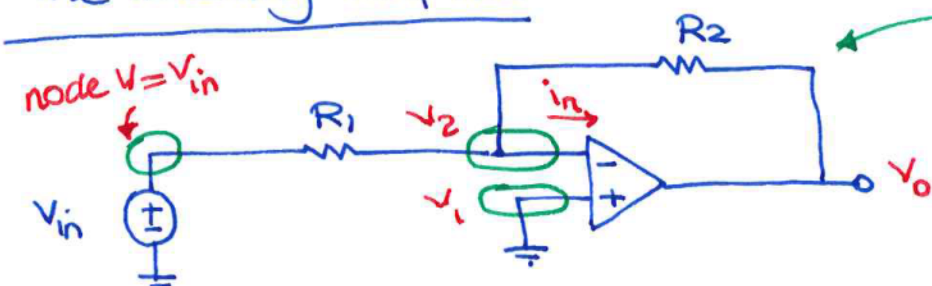
Tip 2: Node equations at the input terminals tend to simplify the job

Tip 3: Seldom need to write node eq at output node when solving the circuit.

· Many interesting & useful circuits can be made with op amps. Some of the basic configurations:

The Inverting Amplifier

negative feedback



$v_1 = 0$ directly connected to ref node

$v_2 = v_1$ from virtual short $\therefore$ $v_2 = 0$

Node eq at 2: $$\frac{\overset{0}{v_2} - v_{in}}{R_1} + \frac{\overset{0}{v_2} - v_o}{R_2} + \underset{0}{\cancel{i_n}} = 0$$ (virtual open)

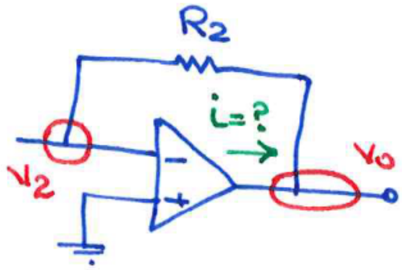
$$\therefore -\frac{v_{in}}{R_1} = \frac{v_o}{R_2} \qquad \therefore \boxed{v_o = \frac{-R_2}{R_1} \cdot v_{in}}$$

. We often use closed loop gain A_v to define performance of an amplifier circuit

$$\boxed{A_v = \frac{V_o}{V_{in}}}$$

. For inverting amplifier circuit, $A_v = \dfrac{V_o}{V_{in}} = \dfrac{-R_2}{R_1}$

. Why didn't we write node eq at output node?

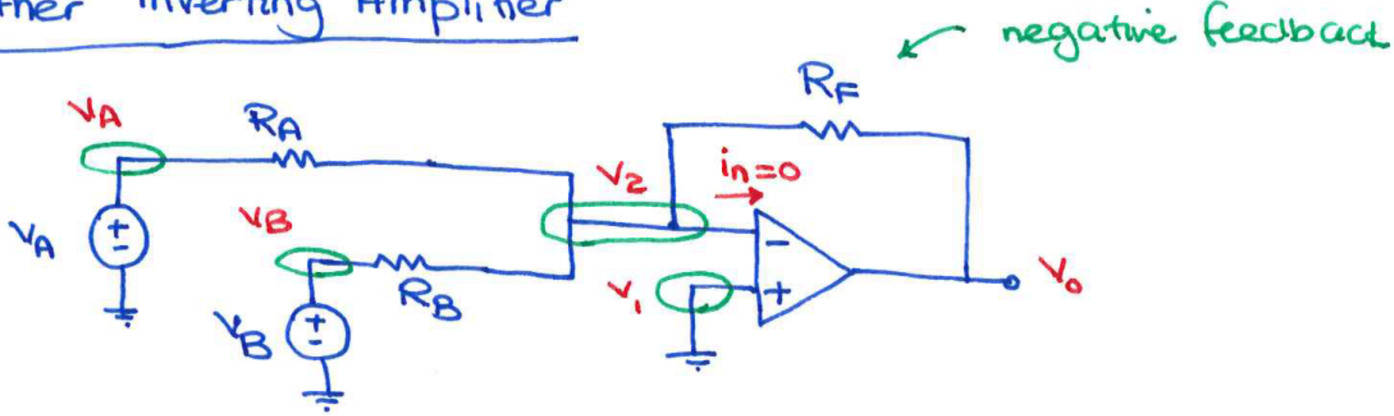


$$\frac{V_o - V_2}{R_2} - i = 0$$

an extra unknown.

. If we need to find i, we can use this equation after we've found all the node voltages.

Another Inverting Amplifier



$$\underbrace{v_2 = \overbrace{v_1 \quad = \quad 0}^{\text{direct to GND}}}_{\text{virtual short}}$$

Node eq at 2: $$\frac{\overset{0}{v_2} - v_A}{R_A} + \frac{\overset{0}{v_2} - v_B}{R_B} + \frac{\overset{0}{v_2} - v_o}{R_F} + \overset{0}{\cancel{i_n}} = 0 \quad \text{(virtual open)}$$

$$\therefore \quad \frac{v_o}{R_F} = \frac{-v_A}{R_A} - \frac{v_B}{R_B}$$

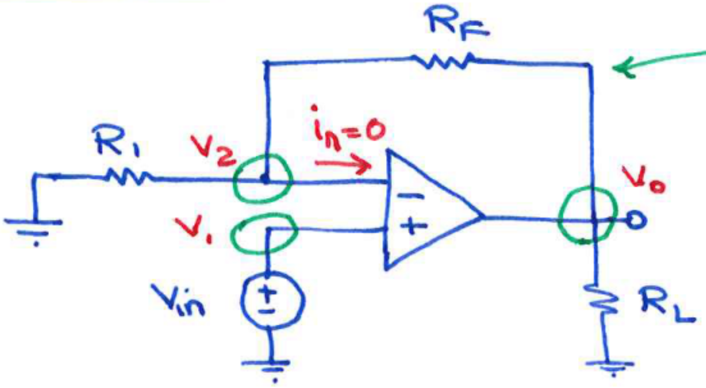
$$\therefore \quad v_o = \frac{-R_F}{R_A} \cdot v_A + \frac{-R_F}{R_B} \cdot v_B$$

. If we choose $R_A = R_B = R$, then $v_o = \frac{-R_F}{R}(v_A + v_B)$

This would be an inverting summing amplifier

e.g. part of an audio mixing circuit

The non-inverting amplifier



negative feedback

let's add a load to the output

This is similar to the inverting amplifier except V_{in} is moved to the + terminal

$$\underbrace{V_2 = V_1}_{\text{virtual short}} \overbrace{= V_{in}}^{\text{voltage source } V_{in} \text{ between node 1 \& ref}}$$

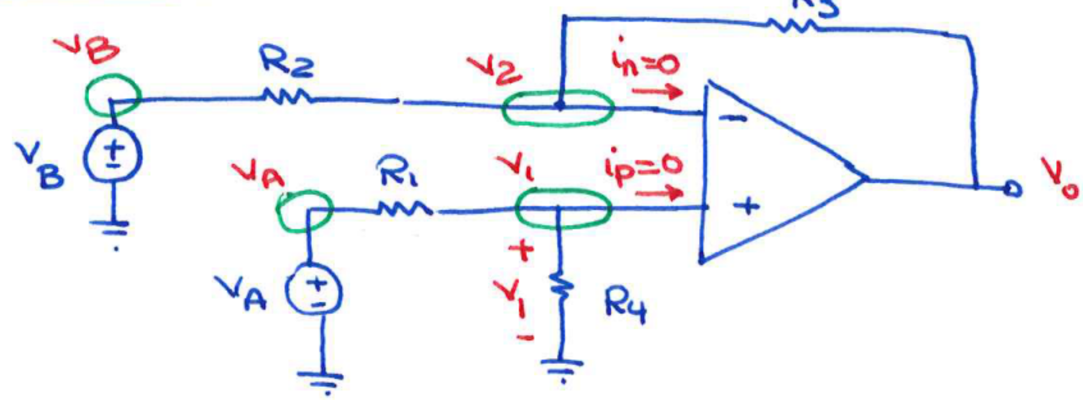
Node eq at 2: $$\frac{\overset{V_{in}}{V_2} - 0}{R_1} + \frac{\overset{V_{in}}{V_2} - V_o}{R_F} + \overset{0 \text{ (virtual open)}}{\cancel{i_n}} = 0$$

$$\therefore \frac{V_{in}}{R_1} + \frac{V_{in} - V_o}{R_F} = 0 \qquad \therefore V_o = \left(1 + \frac{R_F}{R_1}\right) \cdot V_{in}$$

- Closed loop gain $A_v = \frac{V_o}{V_{in}} = 1 + \frac{R_F}{R_1}$ ← note: positive gain

- Note that R_L has no effect on closed loop gain of this (or any other) amplifier circuit

The Differential Amplifier Circuit

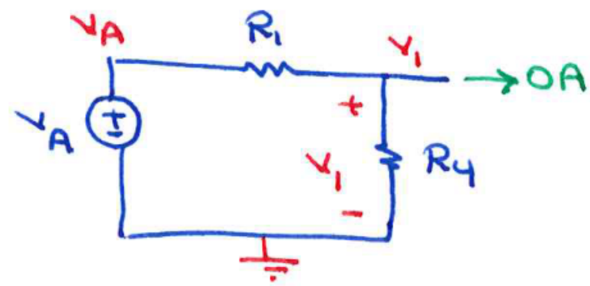


Node eq at 1: $\dfrac{v_1 - v_A}{R_1} + \dfrac{v_1 - 0}{R_4} + \cancel{i_p}^{\,0} = 0$

$\therefore \; v_1 = v_A \left(\dfrac{R_4}{R_1 + R_4} \right)$ (i)

Note that this is a voltage division equation

How?



Since $i_p = 0$, R_1 & R_4 share the same current

i.e. look like series resistors $\therefore \; v_1 = \dfrac{R_4}{R_1 + R_4} \cdot v_A$

Node eq at 2: $\dfrac{v_2 - v_B}{R_2} + \dfrac{v_2 - v_0}{R_3} + \cancel{i_n}^{\,0} = 0$ (ii)

$v_1 = v_2$ therefore, we can plug in (i) into (ii) to get:

$$v_0 = \left(\frac{R_2 + R_3}{R_2} \right) \left(\frac{R_4}{R_1 + R_4} \right) \cdot v_A - \left(\frac{R_3}{R_2} \right) \cdot v_B$$

If we choose $\dfrac{R_3}{R_2} = \dfrac{R_4}{R_1}$, then it can be shown that:

$$v_0 = \frac{R_3}{R_2} \left(v_A - v_B \right)$$

← amplifies the difference between input voltages v_A & v_B

3. **[22 marks.]** The op-amps in Fig. P3 are ideal and are operating in their linear regions.

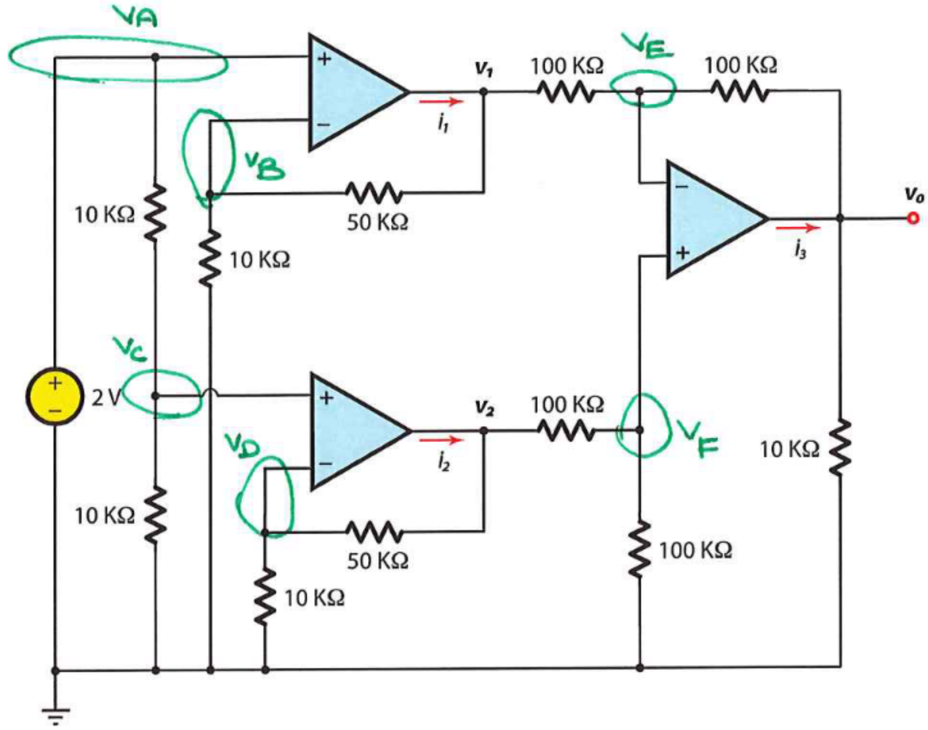


Fig. P3. Find v_1, v_2, v_0, then i_1, i_2, i_3.

(a) [6] Determine node voltages v_1 and v_2.

(b) [8] Determine the output voltage v_0.

(c) [8] Determine the op-amp output currents i_1, i_2, and i_3.

Ex: 2013 Final exam, Q3

a) $V_A = 2$ V (2V source between node A & ref node)

$\underline{V_B = V_A} = 2$ V

virtual short

virtual open

Node eq at B: $\dfrac{\overset{2}{V_B} - V_1}{50K} + \dfrac{\overset{2}{V_B}}{10K} + 0 = 0$

x 50K: $2 - V_1 + 10 = 0$ $\therefore \boxed{V_1 = 12\text{ V}}$

virtual open

Node eq at C: $\dfrac{V_C - \overset{2V}{V_A}}{10K} + \dfrac{V_C - 0}{10K} + 0 = 0$

$\therefore V_C = 1$ V

$V_D = V_C = 1$ V from virtual short

Node eq at D: $\dfrac{\overset{1}{V_D} - 0}{10K} + \dfrac{\overset{1}{V_D} - V_2}{50K} + 0 = 0$

$\therefore \boxed{V_2 = 6\text{ V}}$

b) Node eq at F: $\dfrac{V_F - \overset{6}{V_2}}{100K} + \dfrac{V_F - 0}{100K} + 0 = 0$

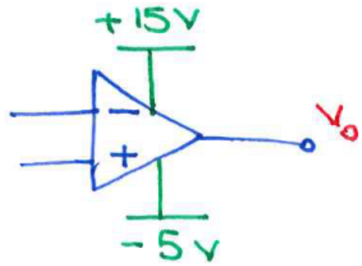
$\therefore V_F = 3$ V

$V_E = V_F = 3$ V from virtual short

Node eq at E: $\dfrac{\overset{3}{V_E} - \overset{12}{V_1}}{100K} + \dfrac{\overset{3}{V_E} - V_o}{100K} + 0 = 0$

$\therefore \boxed{V_o = -6\text{V}}$

Modification: what if op amp 3 looked like this:



the output of this op amp must remain in the range:

$-5 \leqslant V_o \leqslant +15$. In this case, op amp 3 is "saturated"

(not in linear region) $\therefore$ $V_o = -5$ V

c) KCL at node 1: $$-i_1 + \frac{\overset{12}{V_1} - \overset{3}{V_E}}{100k} + \frac{\overset{12}{V_1} - \overset{2}{V_B}}{50k} = 0$$

$$\therefore i_1 = 0.29 \text{ mA}$$

KCL at node 2: $$-i_2 + \frac{\overset{6}{V_2} - \overset{3}{V_F}}{100k} + \frac{\overset{6}{V_2} - \overset{1}{V_D}}{50k} = 0$$

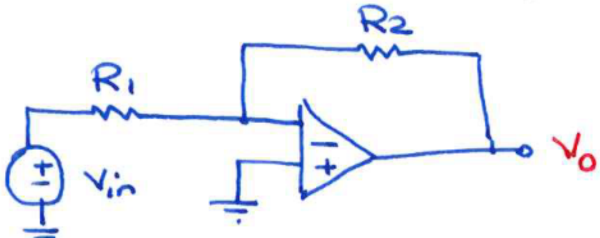
$$\therefore i_2 = 0.13 \text{ mA}$$

KCL at V_o node: $$-i_3 + \frac{\overset{-6}{V_o} - \overset{3}{V_E}}{100k} + \frac{\overset{-6}{V_o} - 0}{10k} = 0$$

$$\therefore i_3 = -0.69 \text{ mA}$$

Another application of op amps: comparator

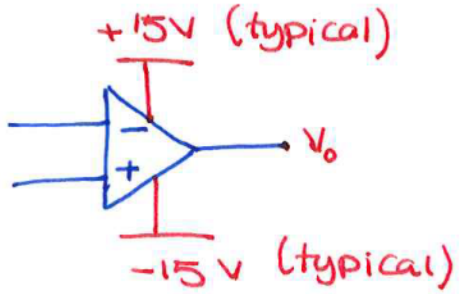
- Consider an inverting amplifier:



we showed that $V_o = \frac{-R_2}{R_1} \cdot V_{in}$

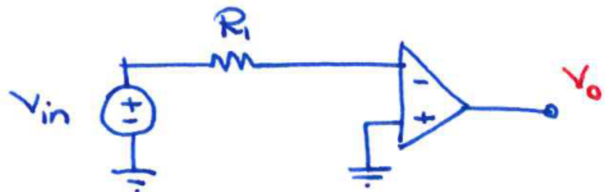
Resistor R2 is a critical component here. It provides negative feedback to allow this circuit to operate. (allows us to use $V_1 = V_2$)

- Also, recall that op amp itself requires an external power source to operate:



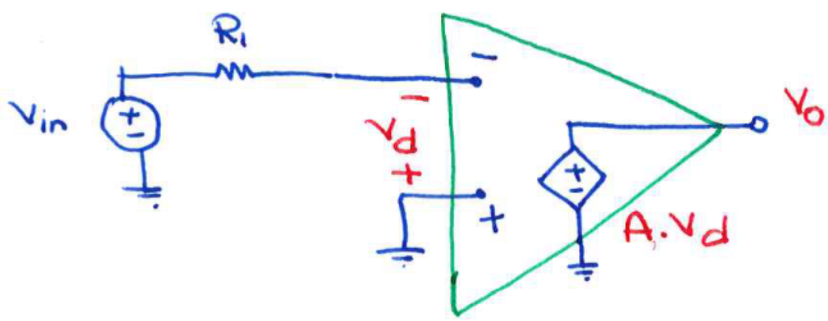
With external power sources as shown, V_o is allowed to be in the range $-15 \leq V_o \leq 15$

- Now, suppose R2 is removed:

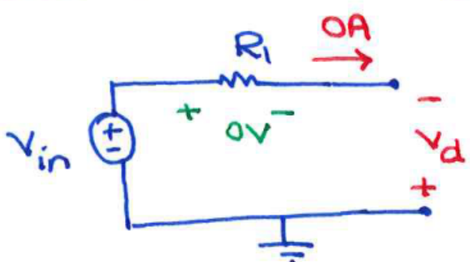


this means that we are operating the op amp <u>open loop</u>.

Using the ideal op amp model:



KVL : Sum of voltages around the loop connected to the input:



$$-V_{in} - V_d = 0 \quad \therefore \quad V_d = -V_{in}$$

i.e. cannot use virtual short ($V_1 = V_2$ or $V_d = 0$) in open loop operation output voltage v_o is given by:

open loop gain. Big number! (∞ for ideal op amp)

$$\underbrace{v_o = A \cdot v_d}_{} = -A \cdot v_{in}$$

VCVS between output node & ref node

$$\begin{cases} \text{if } v_{in} < 0, & v_o = +[\text{big number}] \\ \text{if } v_{in} > 0, & v_o = -[\text{big number}] \end{cases}$$

here, [big number] is limited by external power sources:

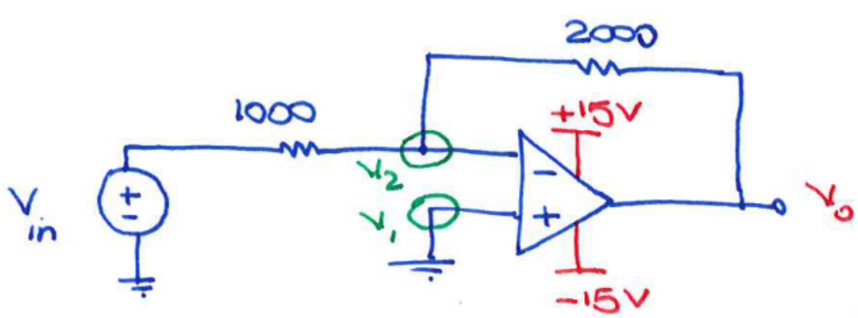
$$\therefore \begin{cases} \text{if } v_{in} < 0, & v_o = +15\text{ V} \\ v_{in} > 0, & v_o = -15\text{ V} \end{cases}$$

operates only at extremes.

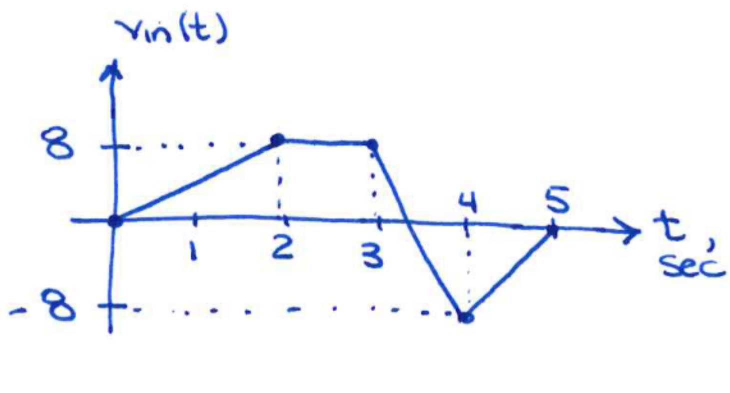
- This behaviour makes op amps useful as <u>voltage comparators</u>. (See Lab 3).

<u>Op amp example with voltage signals</u>

- Consider the following inverting amplifier:



$v_{in}(t)$ is given as:



Draw $v_o(t)$.

First, write V_o in terms of V_{in}:

$$\underbrace{V_1 = \overbrace{V_2 = 0}^{\text{direct to GND}}}_{\text{virtual short}}$$

Node eq at 2 : $\dfrac{\cancel{V_2}^{\,0} - V_{in}}{1000} + \dfrac{\cancel{V_2}^{\,0} - V_o}{2000} = 0 \quad \therefore V_o = -2V_{in}$