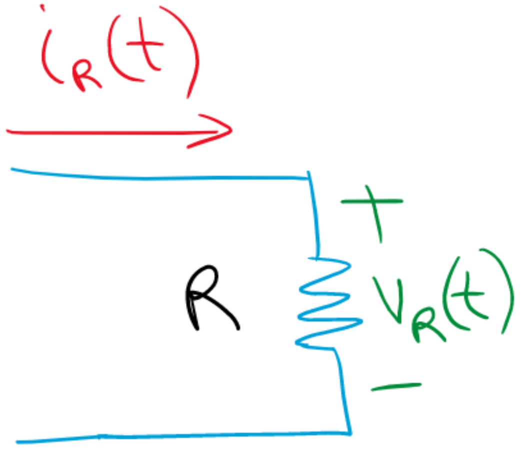
Introduction to Complex Impedance

Impedance is similar to resistance, in that it relates the voltage and current relationship of circuit elements like resistors, capacitors, and inductors.

However, impedance is more general as it is affected by frequency and phase relationships of the circuit elements.



Resistors

The impedance of resistors is easiest to understand, given in the following relationship

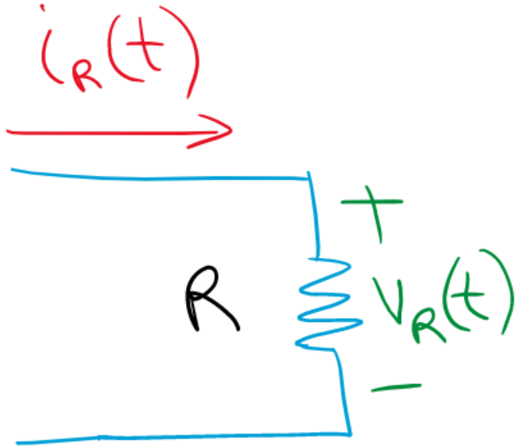
$$\boxed{\overline{V}_R = R\,\overline{I}_R}$$

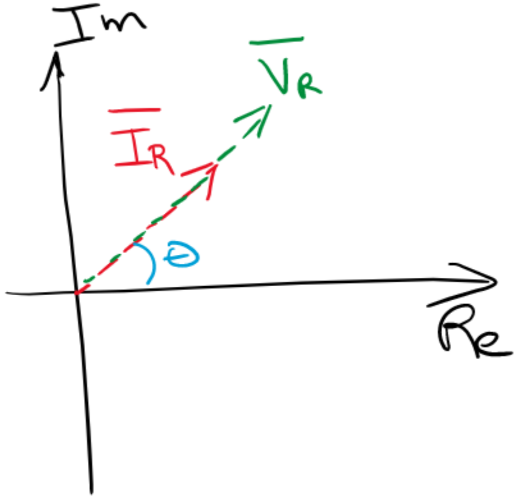
where R is real-valued and is in units Ω.

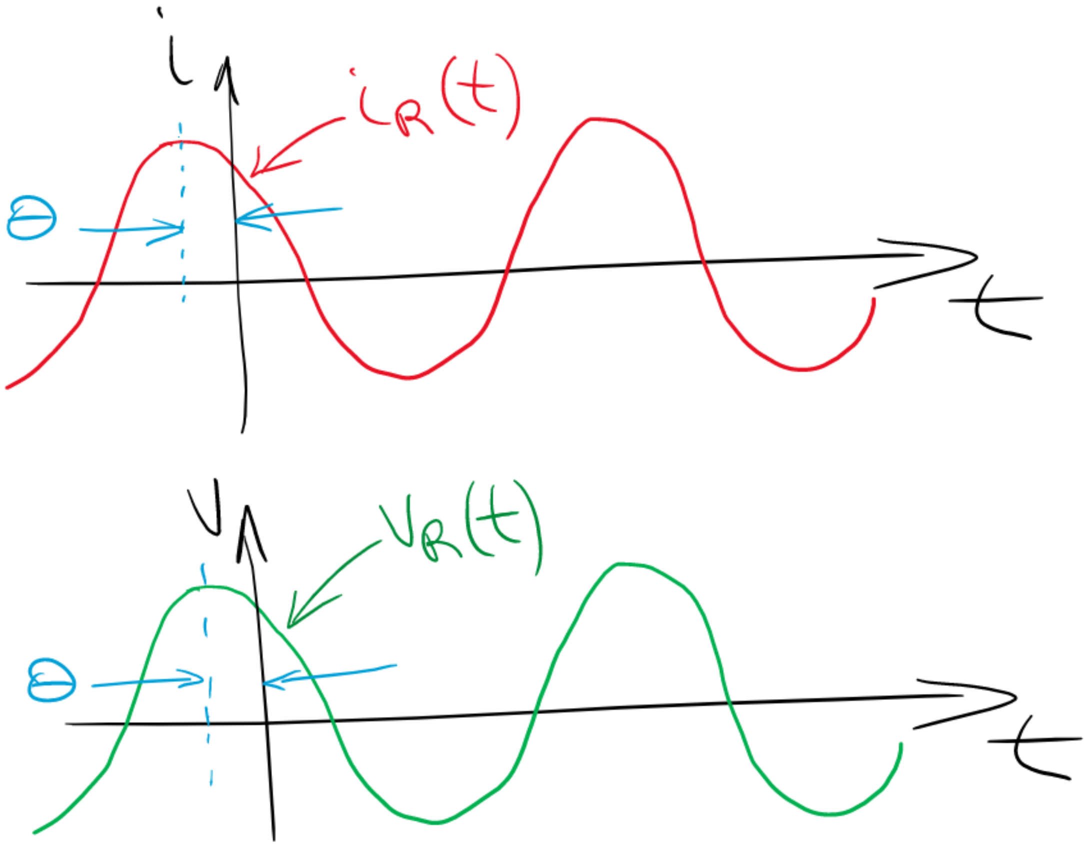
what does this mean?

Resistors

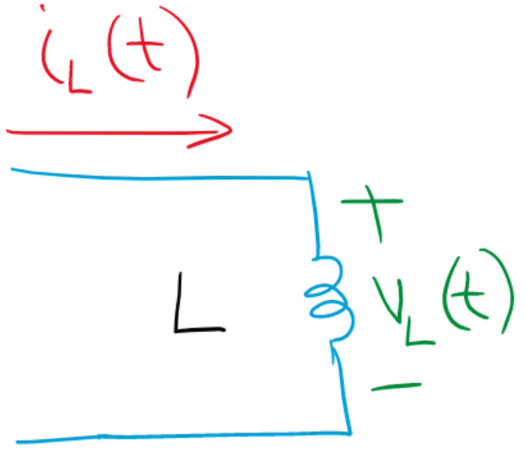
$$\overline{V}_R = R\,\overline{I}_R$$







Because R is real-valued, there is no phase-shift between the voltage and current waveforms.



Inductors

Inductors have a more complicated impedance than resistors, given by:

$$\boxed{\overline{V}_L = Z_L \overline{I}_L}$$

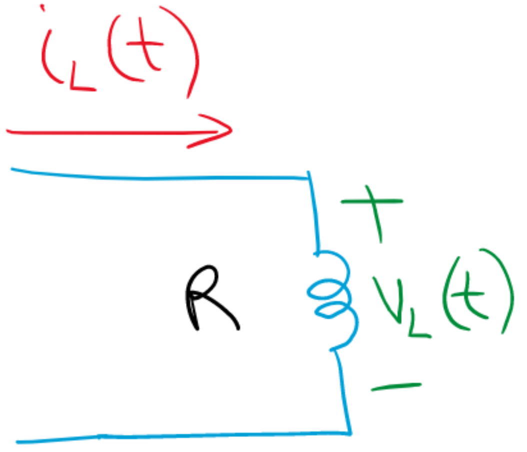
where

$$\boxed{Z_L = j\omega L = \omega L \angle 90^\circ}$$

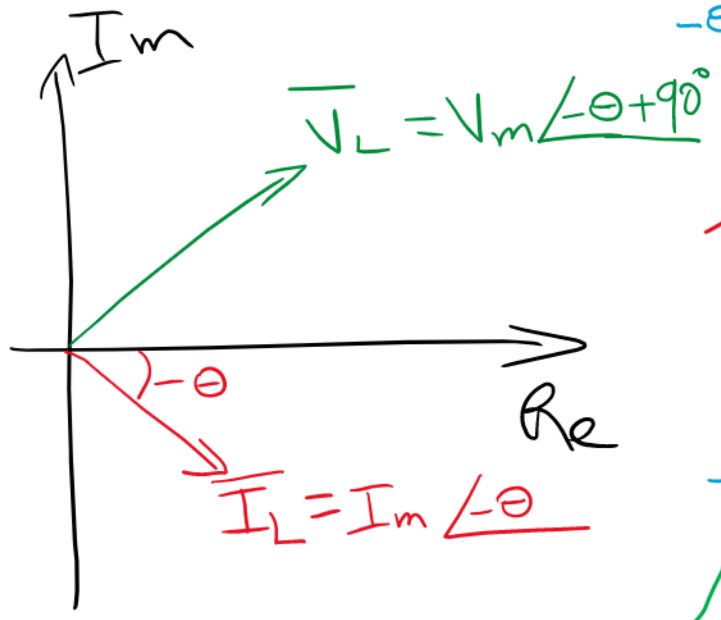
and Z_L is in units Ω.

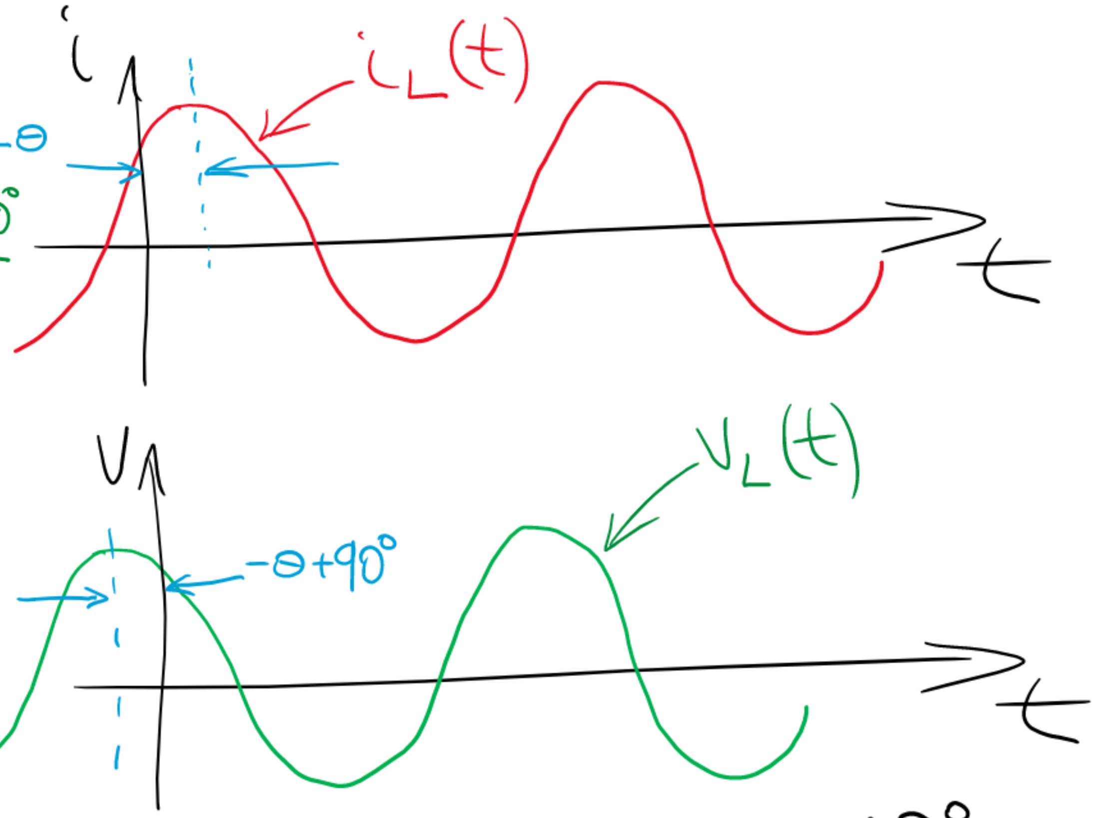
What does this mean?

Inductors



$$\overline{V}_L = Z_L \overline{I}_L = j\omega L \overline{I}_L = \omega L \overline{I}_L \angle 90^\circ$$





Because Z_L has positive imaginary value, this represents a phase-shift of $+90^\circ$. For the inductor, the voltage waveform will be shifted 90° ahead of the current waveform.

* Voltage leads current in an inductor by 90°.

Inductors

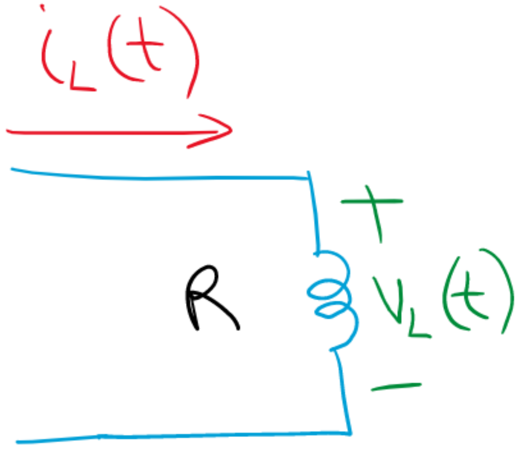
$$\overline{V}_L = Z_L \overline{I}_L = j\omega L \overline{I}_L = \omega L \overline{I}_L \angle 90^\circ$$

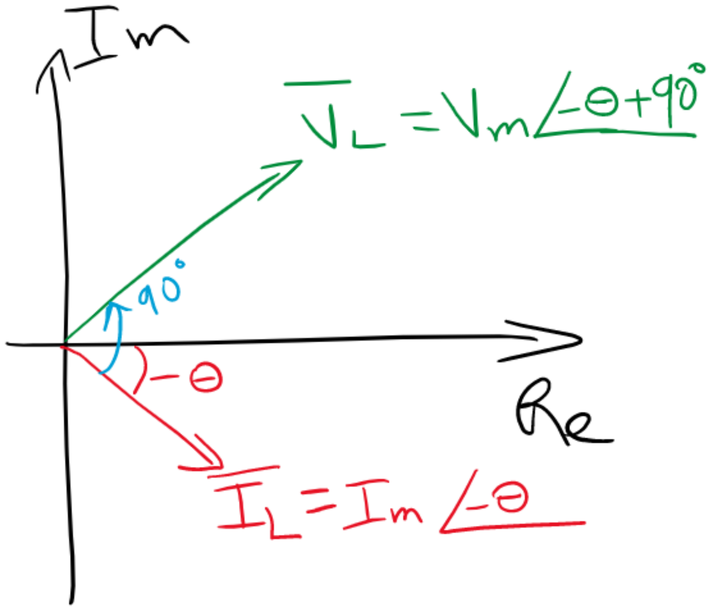
other formulations:

$$\overline{I}_L = \frac{\overline{V}_L}{Z_L} = \frac{\overline{V}_L}{j\omega L} = \frac{\overline{V}_L}{\omega L} \angle -90^\circ$$

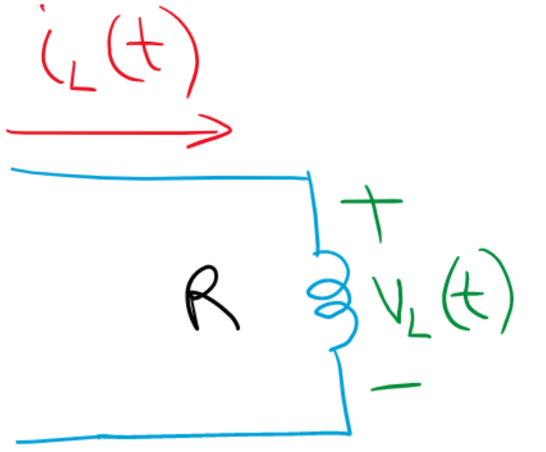
or

$$Z_L = \frac{\overline{V}_L}{\overline{I}_L}$$

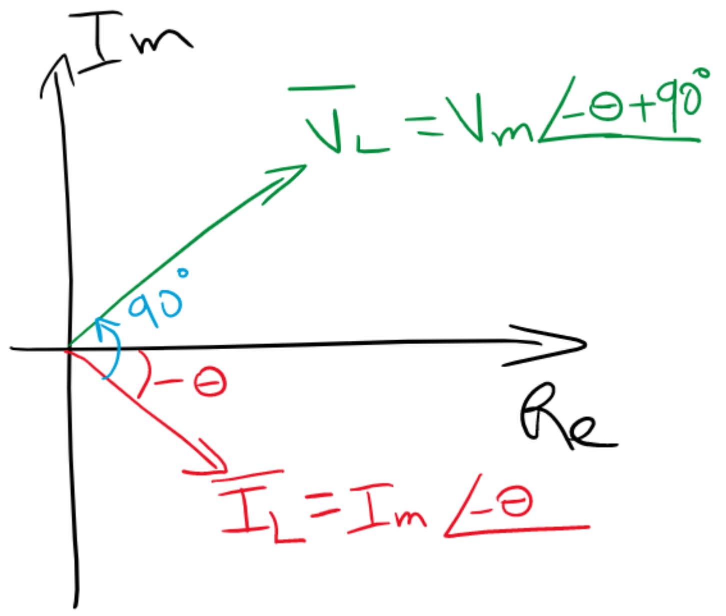




Inductors



$$\overline{V}_L = Z_L \overline{I}_L = j\omega L \overline{I}_L = \omega L \overline{I}_L \angle 90^\circ$$



A memory aid

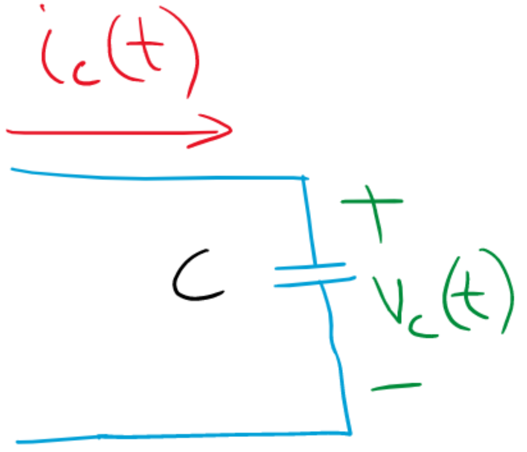
Voltage (Electromotive Force) → E

Inductor → L

Current → I

E L I

Voltage <u>leads</u> Current in an Inductor



Capacitors

Capacitors have a more complicated impedance than resistors, given by:

$$\boxed{\overline{V}_c = Z_c \overline{I}_c}$$

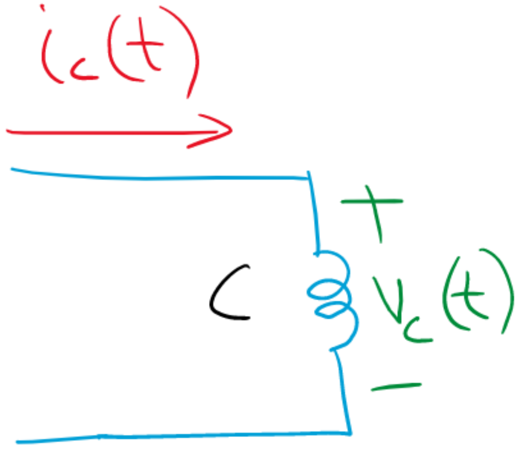
where

$$\boxed{Z_c = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{\omega C}\angle -90^\circ}$$

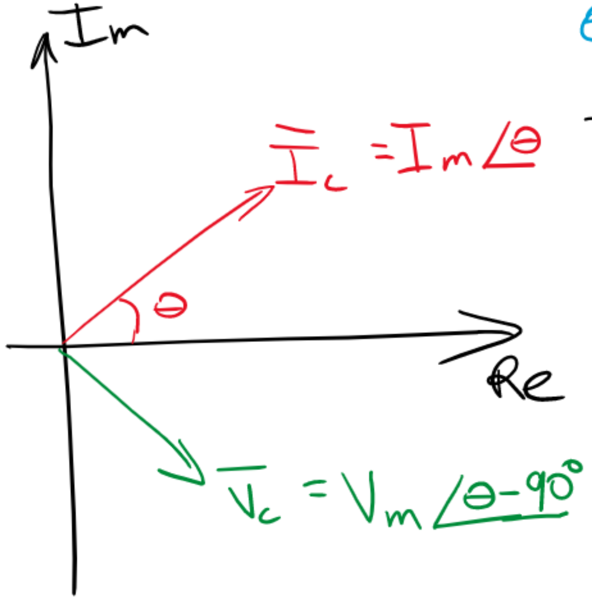
and Z_c is in units Ω.

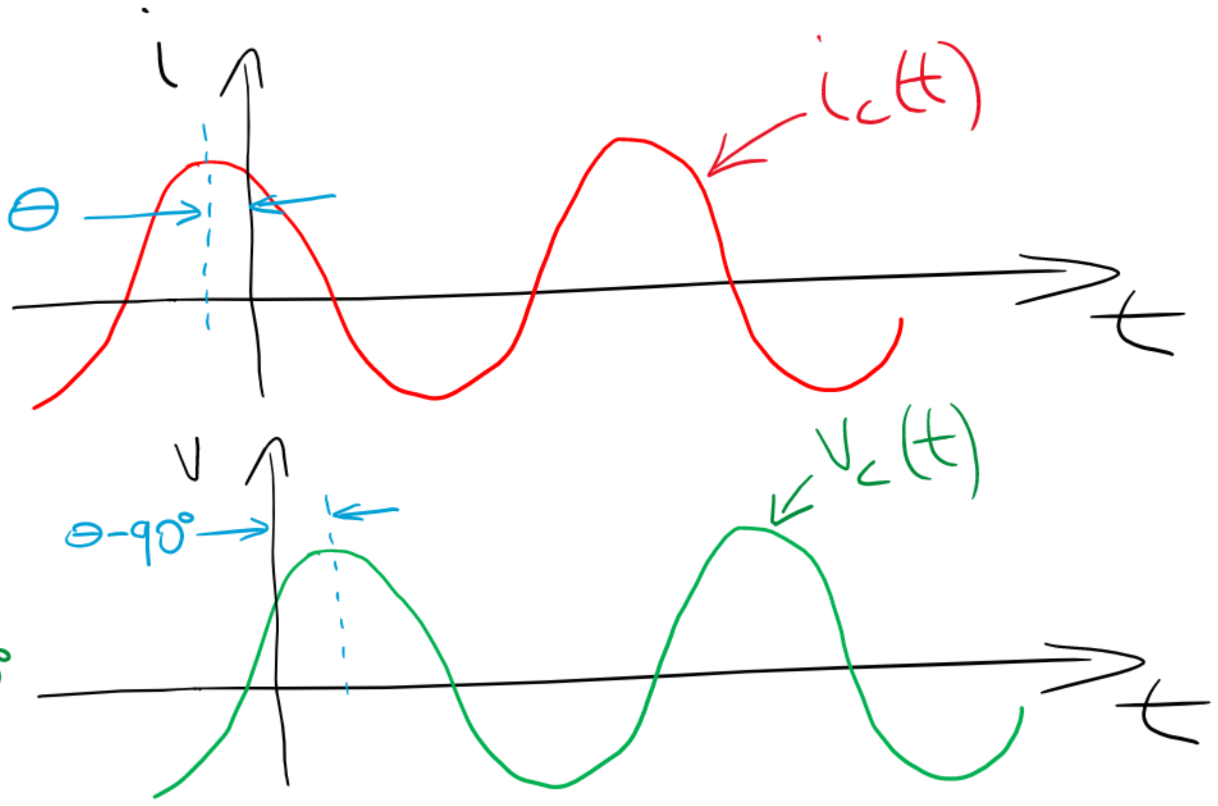
What does this mean?

Capacitors



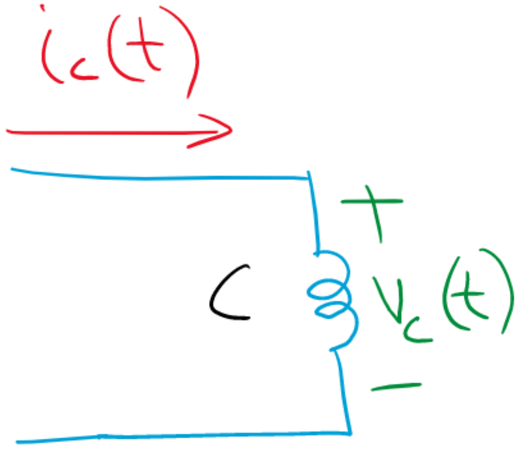
$$\overline{V_c} = Z_c \overline{I_c} = \frac{1}{j\omega C}\overline{I_c} = \frac{1}{\omega C}\overline{I_c}\angle -90^\circ$$





Because Z_c has negative imaginary value, this represents a phase-shift of -90°. For the capacitor, the current waveform will be shifted 90° ahead of the voltage waveform.

* Current leads voltage in a capacitor by 90°.



Capacitors

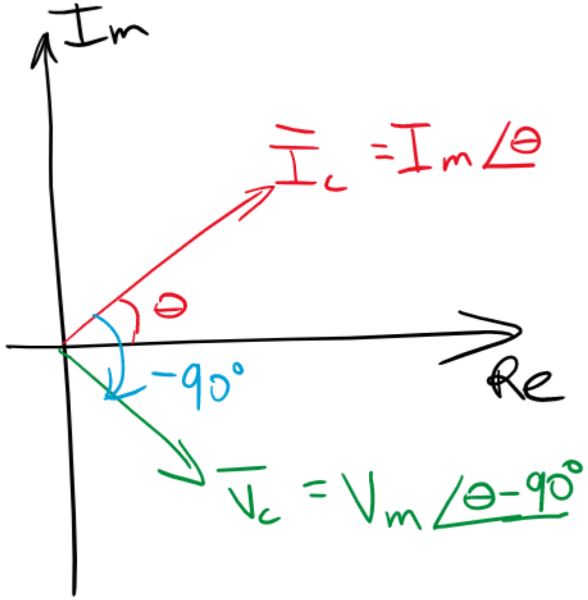
$$\overline{V}_c = Z_c \overline{I}_c = \frac{1}{j\omega C}\overline{I}_c = \frac{1}{\omega C}\overline{I}_c \angle{-90^\circ}$$

Other formulations

$$\overline{I}_c = \frac{\overline{V}_c}{Z_c} = \frac{\overline{V}_c}{\frac{1}{j\omega C}} = j\omega C\,\overline{V}_c = \omega C\,\overline{V}_c \angle 90^\circ$$

or

$$Z_c = \frac{\overline{V}_c}{\overline{I}_c}$$



Memory aid

I C E

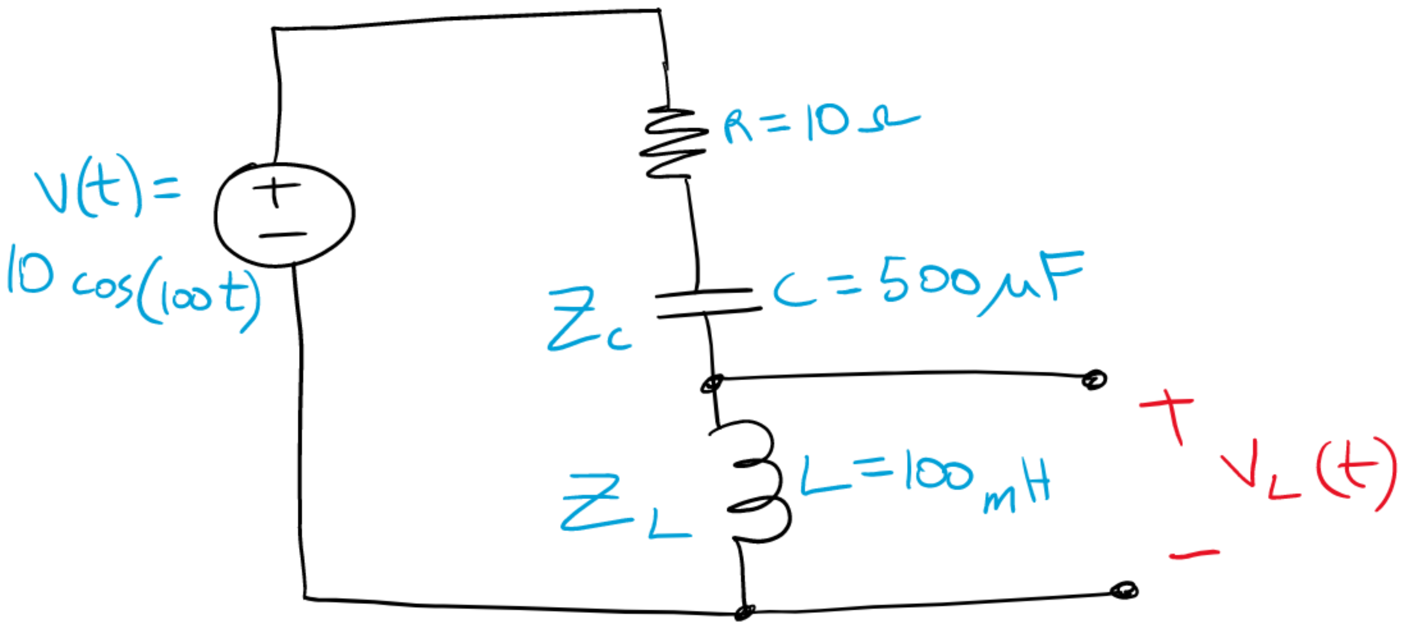
Current

Capacitor

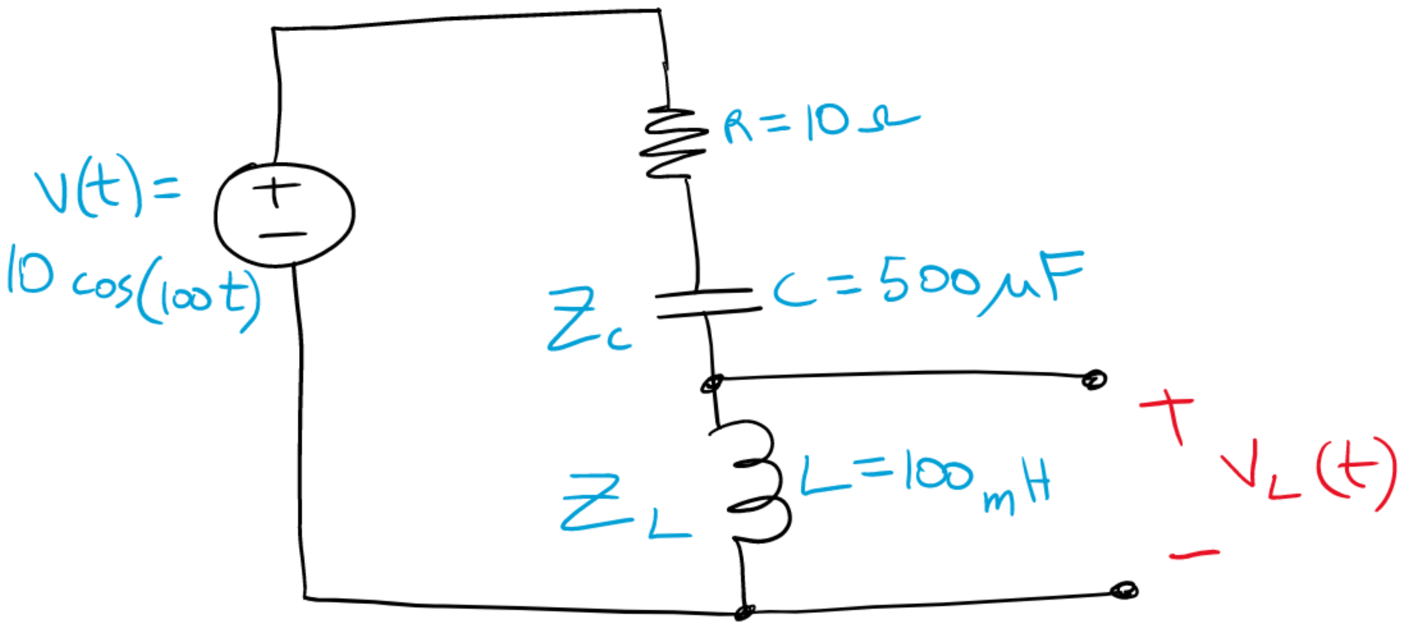
Voltage

Current <u>leads</u> voltage in a capacitor.

<u>Example</u> Find $v_L(t)$ given $v(t) = 10\cos(100t)$.



Example Find $v_L(t)$ given $v(t) = 10\cos(100t)$.



$$R = 10\,\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \times 100 \times 500 \times 10^{-6}} = -j20\,\Omega$$

$$Z_L = j\omega L = j \times 100 \times 100 \times 10^{-3} = j10\,\Omega$$

$$\overline{V} = 10\angle 0^\circ = 10\,V$$

Solution

Recognize this is a voltage divider.

$$\overline{V}_L = \frac{Z_L}{R + Z_C + Z_L} \times \overline{V}$$

Plug in values

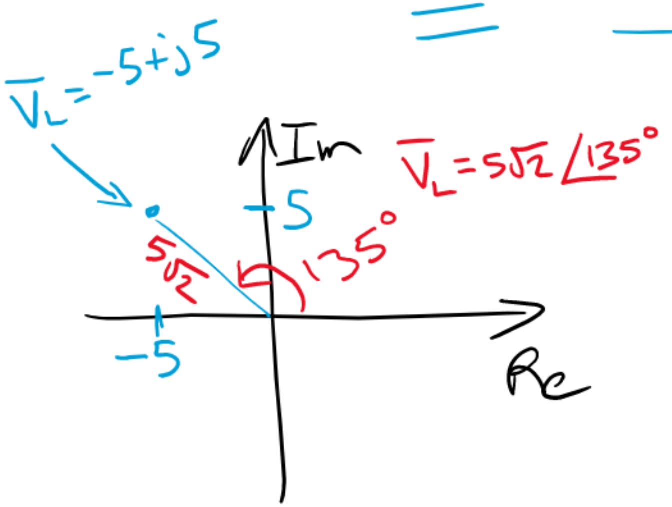
$$\overline{V}_L = \frac{j10}{10 + (-j20) + j10} \times 10$$

$$\overline{V}_L = \frac{j100}{10 - j10}$$

$$\bar{V}_L = \frac{j100}{10-j10}$$

$$= \frac{j100}{10-j10} \times \frac{10+j10}{10+j10} = \frac{j1000-1000}{100+100}$$

$$= \frac{-1000+j1000}{200} = -5+j5$$



Convert $\bar{V}_L$ to polar form so we can easily write the time domain expression for $V_L(t)$.

$$\bar{V}_L = \sqrt{5^2+5^2}\ \angle\left(180° - \tan^{-1}\left(\frac{5}{5}\right)\right)$$

$$\bar{V}_L = 5\sqrt{2}\angle 135°$$

Finally,

$$V_L(t) = 5\sqrt{2}\cos(100t + 135°)$$