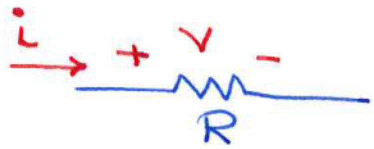
Resistors

- $\underbrace{\text{Voltage}}_{\text{voltage across the resistor}}$ & $\underbrace{\text{Current}}_{\text{current through the resistor}}$ are related by Ohm's Law:

i) if the current through the resistor is shown in the direction of voltage drop:



looks like an energy-absorbing element

then use $\boxed{v = iR}$

ii) If the current is shown in the direction of voltage rise

then use $\boxed{V = -iR}$

- Conductance : From Ohm's Law, $i = \underbrace{\frac{1}{R}}_{\text{conductance, } G} \cdot v$

unit: Siemens (Ω^{-1}), once called mho (℧)

Power & Energy

- In electric circuits, power is the product of voltage & current

$$p = vi$$

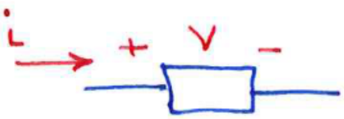
but isn't power the rate of energy transfer?

$$p = \underbrace{\frac{dw}{\cancel{dq}}}_{v} \cdot \underbrace{\frac{\cancel{dq}}{dt}}_{i} = \frac{dw}{dt} \qquad [J/s]$$

p: power in Watts [W]

. If $\underbrace{\text{current is labelled in direction of voltage drop}}$:

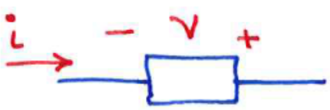
this is called the passive reference direction



which implies the element is absorbing power,

then we must use $\boxed{p = vi}$

. If current is labelled in direction of voltage rise:



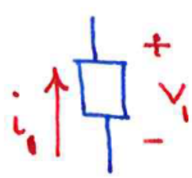
then we must use $\boxed{p = -vi}$

. In both cases (if using $p = vi$ or $p = -vi$),

if $p > 0$, then element is absorbing power

if $p < 0$, then element is supplying power.

Ex: Find power for:



a) $v_1 = 12$ V, $i_1 = 10$ A

Supplies 120 W

$$p = -v_1 i_1 = -(12\,V)(10\,A) = -120\ W$$

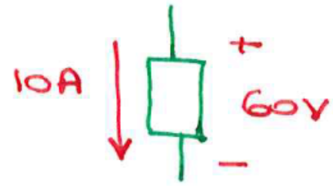
current in dir of voltage rise

b) $v_1 = 60\text{V}$, $i_1 = -10\text{A}$

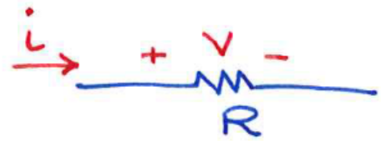
$$p = -v_1 i_1 = -(60\text{V})(-10\text{A}) = 600\text{W}$$ ← absorbs 600W

alternatively, we can change direction of current to match the <u>actual</u> direction

$$p = vi = (60\text{V})(10\text{A}) = 600\text{W}$$



Power in resistors



Power : $p = vi$

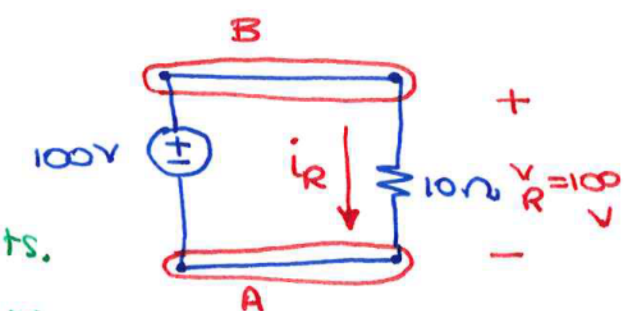
Ohm's Law : $v = iR$

$\therefore$ $\boxed{p = i^2 \cdot R}$ or $\boxed{p = \dfrac{v^2}{R}}$

P is always positive for resistors. They will always absorb power.

Ex: Consider the circuit.

Find power for both elements.



Need to find unknown voltages & currents.

For now, let's logic our way through this.

Later, we will develop systematic ways of finding voltage & current.

<u>step 1</u>: Find voltage across resistor. It is connected to the same 2 points as the 100V source. (B is higher in potential than A by 100V.

step 2: choose an arbitrary direction for i_R

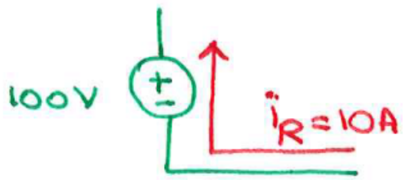
step 3: From Ohm's Law: $i_R = \frac{V_R}{10\Omega} = \frac{100V}{10\Omega} = 10A$

For the resistor, $P = V_R \cdot i_R = (100V)(10A) = 1000\,W$

absorbs 1000 W

can also use $P = (i_R)^2 \cdot R = \frac{(V_R)^2}{R}$

step 4: Current i_R will continue to flow through the source:



$P_{source} = - V \cdot i_R = -(100V)(10A) = -1000\,W$

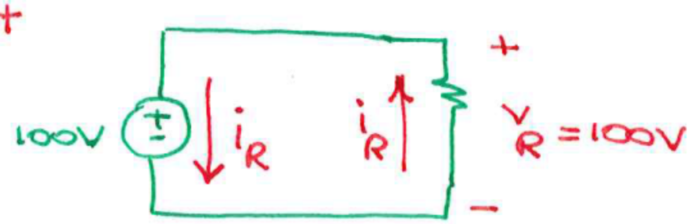
current sees (−) terminal first

supplies 1000 W

. What if we chose i_R to be in the opposite direction in Step 2?

current sees (−) first

Ohm's Law: $V_R = - i_R \cdot R$



$\therefore\ i_R = \frac{-V_R}{R} = \frac{-100V}{10\Omega} = -10\,A$

same <u>actual</u> current as before

For the resistor, $P = - V_R \cdot i_R = -(100V)(-10A) = 1000\,W$

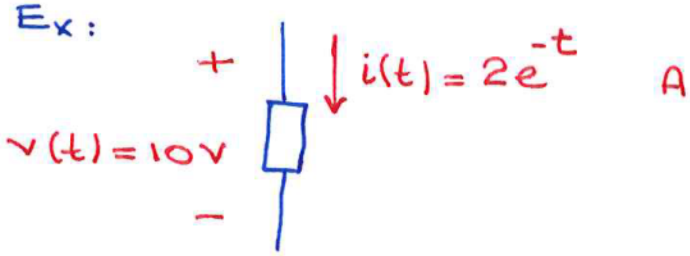
for the source, $P_{source} = (100V) \cdot i_R = (100V)(-10A) = -1000\,W$

Energy

: we have $p = \frac{dw}{dt}$ therefore $w = \int_{t_1}^{t_2} p(t)\,dt$

- Power companies measure energy to determine monthly bill i.e. usage of power over time

Ex:

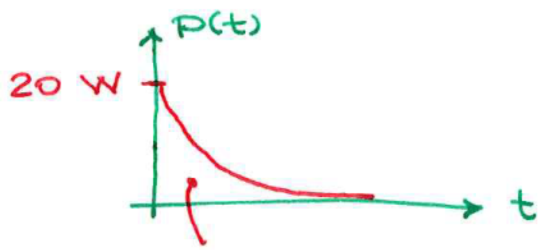


a) Compute power

b) Compute energy from $t=0 \rightarrow \infty$

a) Current in dir of voltage drop

$\therefore \quad p(t) = v(t).\,i(t) = 10 \times 2e^{-t} = 20e^{-t}$ W



energy from $t=0 \rightarrow \infty$ is the area under the curve

b) $w = \int_0^\infty 20e^{-t}\,dt = -20e^{-t}\Big|_0^\infty = 0 - (-20) = 20$ J

$w > 0 \quad \therefore$ absorbs energy

Ex2: Assume energy cost is \$ 0.12 per $\underbrace{\text{KiloWatt-hour (kWh)}}$

Electrical bill for 30 days: \$ 60.^{00}

Power is constant over this time period.

$1\,W = 1\,J/s$

$\therefore 1\,J = 1\,W.s$

$= \frac{1}{1000}\,kW.\,\frac{1}{3600}\,h$

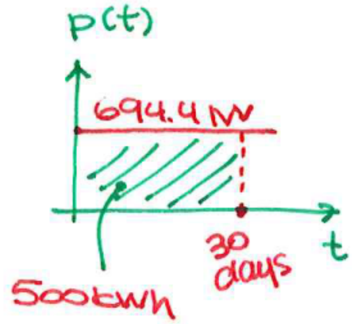
$\therefore 1\,kWh = 3600 \times 1000\,J$

a) Find power in Watts

b) Voltage = 120V. Find current.

a) Total energy consumed in 30 days is:

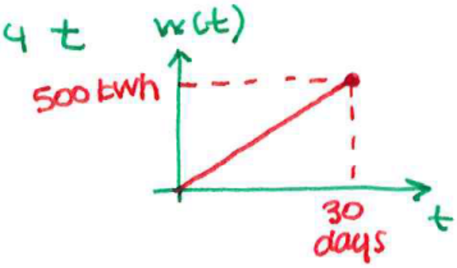
$$W_{30\,days} = \frac{\$\,60.^{00}}{\$\,0.12\,/kWh} = 500 \quad kWh$$



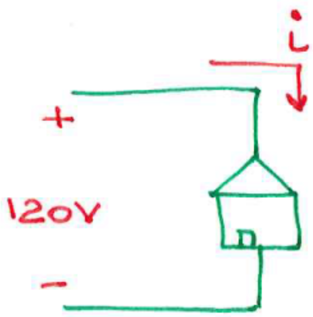
$$p = \frac{W_{30\,day}}{t} = \frac{500\ kWh}{(30 \times 24)\ h} = 694.4\ W$$

let's also find $w(t)$:

$$w(t) = \int_0^t p(t)\,dt = \int_0^t 694.4\,dt = 694.4\,t$$



b)



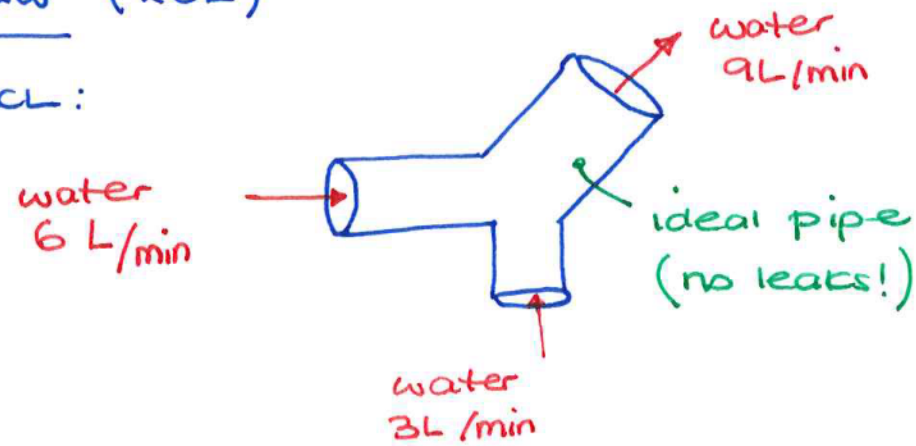
$$p = vi \quad \therefore i = \frac{P}{V} = \frac{694.4\ W}{120\ \ V} = 5.8\ A$$

Kirchoff's Laws

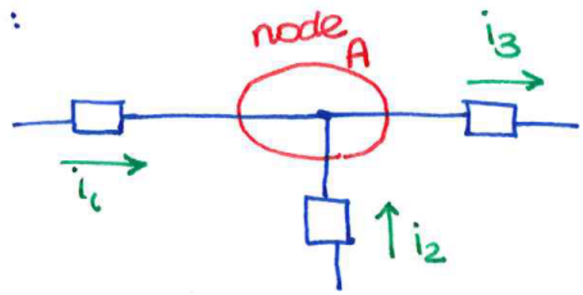
- So far, we have seen:
  - fundamental electrical quantities v, i (and p, w)
  - circuit elements with their own v & i relationship
- Kirchoff's Laws define how v & i distribute in a circuit.

Kirchoff's Current Law (KCL)

- Fluid analogy of KCL:



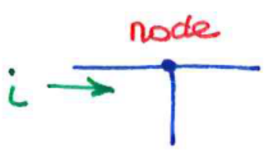
- Consider a node in a circuit:



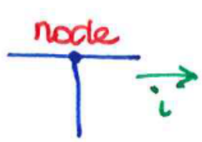
node: joining of 2 or more circuit elements

KCL states: $\boxed{\text{Algebraic sum of all currents at a node must be zero.}}$

- Choose a consistent way to distinguish incoming & outgoing currents at a node:



current entering the node adds.



current leaving the node subtracts

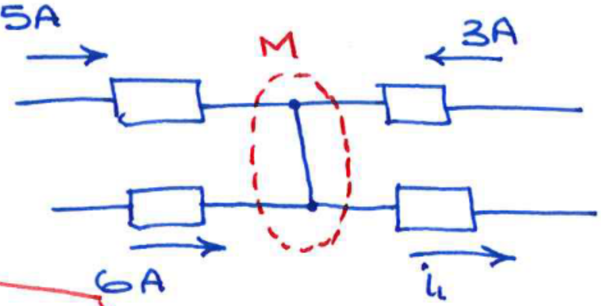
Later, in the Node Voltage Method, we will use the opposite convention!

- Sum of currents at node A in the circuit above:

$$\underbrace{i_1 + i_2}_{\text{enter A}} - \underbrace{i_3}_{\text{leave A}} = 0$$

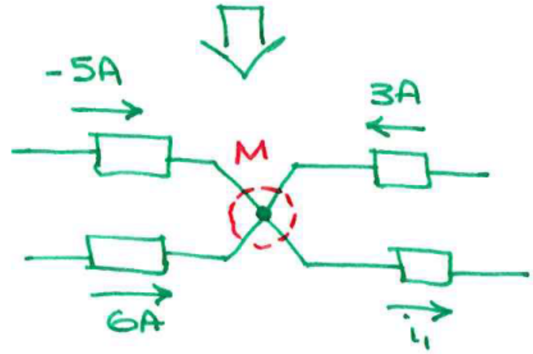
so $\quad i_1 + i_2 = i_3$

Ex: Determine i_1



Note that M is a single node.

Connection points connected directly to each other by conductors constitute one node

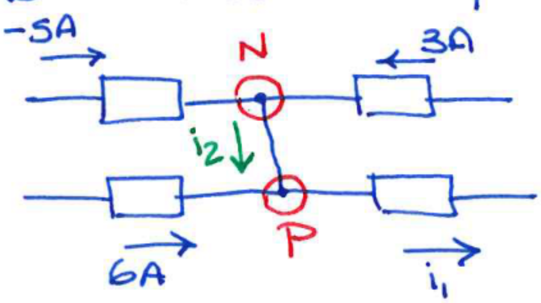


KCL at node M:

$$\underbrace{-5+3+6}_{\text{enter M}} \underbrace{-\,i_1}_{\text{leaves M}} = 0$$

$$\therefore\ i_1 = 4\ \text{A}$$

- Alternatively, we can write KCL for each connection point inside Node M:



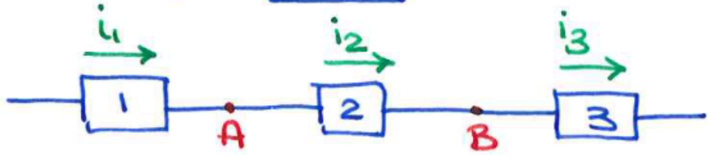
- Chose an arbitrary direction for i_2
- KCL at connection point N:

$$\underbrace{-5+3}_{\text{enter N}} \underbrace{-\,i_2}_{\text{leave N}} = 0 \qquad \therefore\ i_2 = -2\ \text{A}$$

- KCL at connection point P:

$$\underbrace{\overset{-2}{i_2} + 6}_{\text{enter P}} \underbrace{-\,i_1}_{\text{leaves P}} = 0 \qquad \therefore\ i_1 = 4\ \text{A}$$

. Let's consider <u>series</u> connection of elements



Two elements are in series if they are <u>connected at one node only</u> and there is <u>no other element connected to that node</u>.

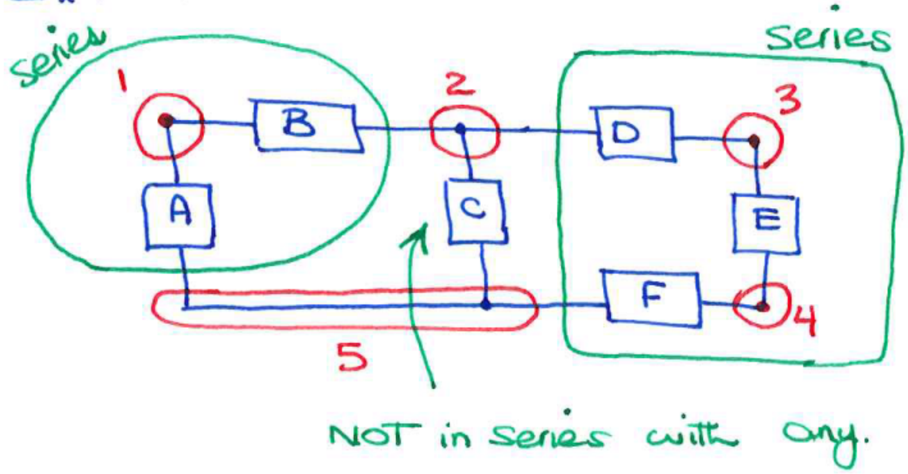
KCL at node A: $i_1 - i_2 = 0$ $\quad \therefore\ i_1 = i_2$

KCL at node B: $i_2 - i_3 = 0$ $\quad \therefore\ i_2 = i_3$

i.e. $\boxed{\text{Elements in series have the same current}}$ * $\quad i_1 = i_2 = i_3$

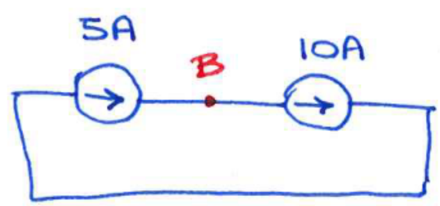
* if the currents are labelled with the same direction

Ex: What's in series here?



Examine all nodes & identify the ones at which <u>only 2 elements</u> are joined

. Circuits that violate KCL:

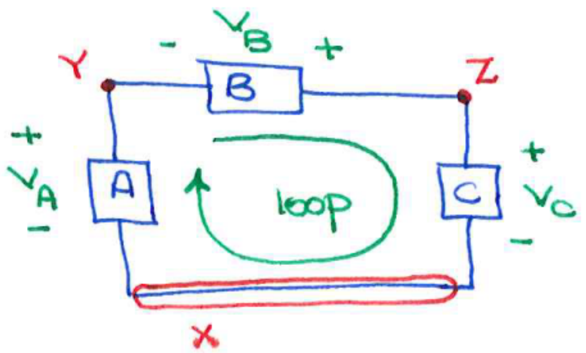


KCL at node B:

$5 - 10 \neq 0$

Kirchoff's Voltage Law (KVL)

- Derived from conservation of energy.
- Consider a loop in a circuit:



loop: a closed path starting at a node & finishing back at the same node

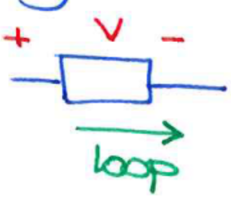
loop: $Y \to Z \to X \to Y$

KVL states: $\boxed{\text{Algebraic sum of all voltages around a loop must be zero.}}$

- Loop direction is chosen arbitrarily! It has no relation to current direction

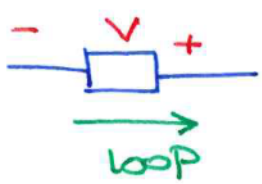
- When summing voltages around a loop, by convention:

1) if



then add V

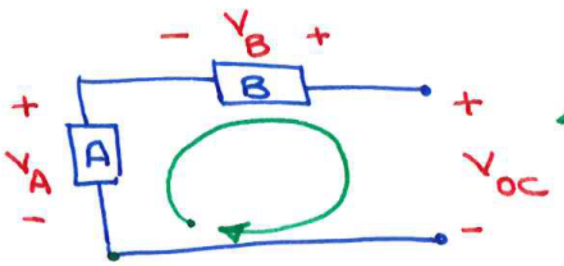
2) if



then subtract V

- KVL around the example circuit gives: $-V_B + V_C - V_A = 0$

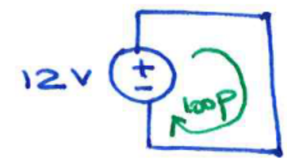
- Loops can contain open circuits. For example,



important to account for potential difference across the open circuit

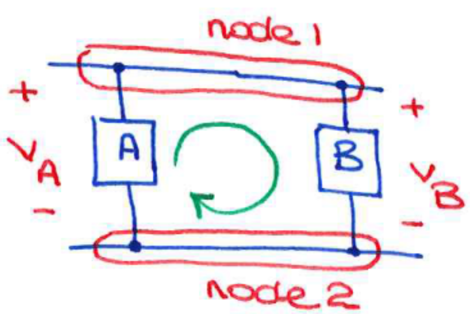
KVL: $-V_A - V_B + V_{oc} = 0$

- Circuits prohibited by KVL:



$-12 \neq 0$

- Consider <u>parallel</u> elements:



Two elements are parallel if their terminals are <u>directly</u> connected to each other

in other words,

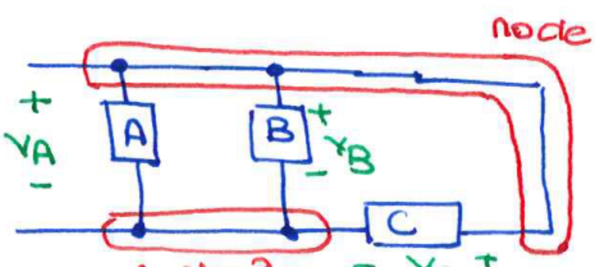
they are connected to the same 2 nodes

KVL around the loop: $-V_A + V_B = 0$ $\therefore V_A = V_B$

$\boxed{\text{. Elements in parallel have the same voltage}}$ *

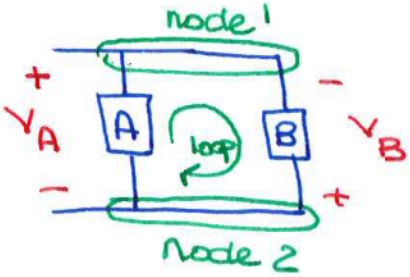
* if the polarities are labelled consistently.

- We can have more than 2 elements in parallel:



$V_A = V_B = V_C$

. What if the polarities were not labelled consistently?



$v_A = -v_B$

can confirm with KVL:

$-v_A - v_B = 0$

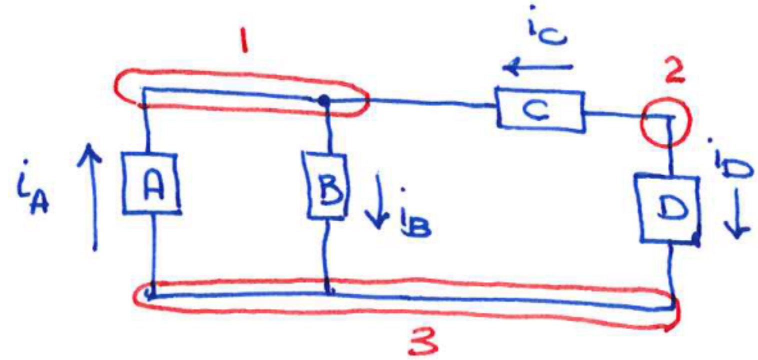
$\therefore\ v_A = -v_B$

Ex:

a) What's in series, parallel?

b) i_c in terms of i_D

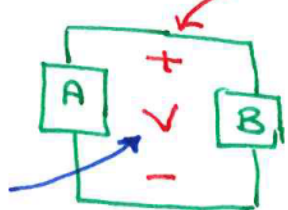
c) $i_A = 3A$, $i_c = 1A$. Find i_B, i_D.



a) series: C & D. Connected at one node only, Nothing else connected there.

parallel: A & B. Connected to the same 2 nodes (1 & 3)

common voltage



no circuit element separating terminals of A & B

b) C & D are in series $\therefore$ must have the same current

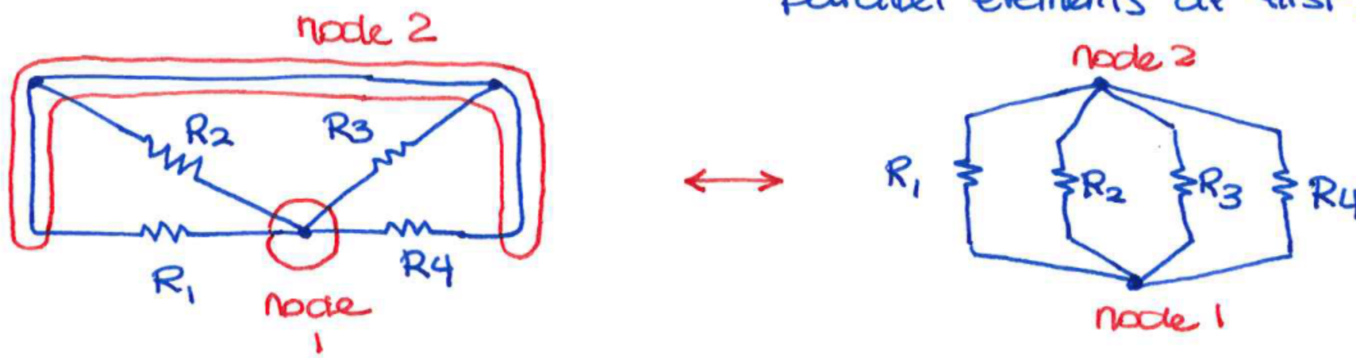
paying attention to directions: $i_c = -i_D$

or, from KCL at node 2: $-i_c - i_D = 0$ $\quad \therefore\ i_c = -i_D$

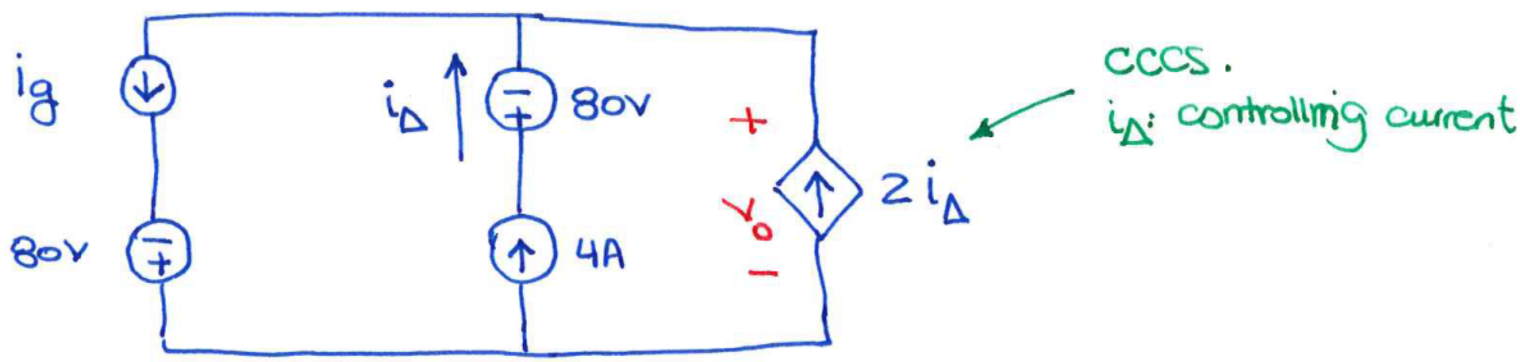
c) $i_D = -i_c = -1\ A$

KCL at node 1: $\underbrace{i_A}_{3A} - i_B + \underbrace{i_c}_{1A} = 0$ $\quad \therefore\ i_B = 4\ A$

. Watch for parallel connections that may not appear like parallel elements at first glance



Ex: Let $v_o = 100V$. Find the power for each element:



Strategy: . Need all voltages & currents to calculate power
. Use KVL & KCL to find unknown voltages & currents. Assume polarity / direction if unknown.