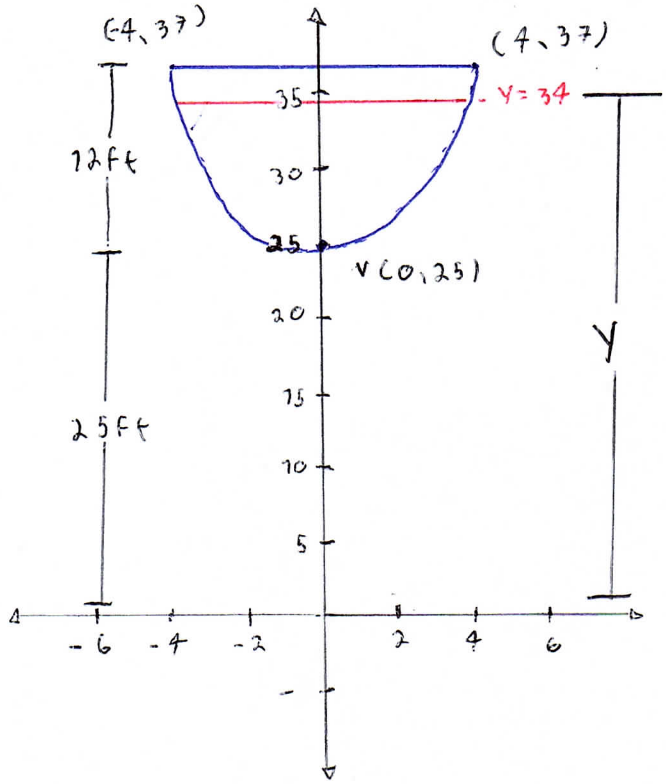
1. Se construye una torre y sobre ella se ubica un depósito de longitud 25 pie, ubicado de forma tal que se está en posición horizontal, para contener agua. El deposito está vacío, posee secciones transversales en forma de un segmento parabólico, cuyo borde superior mide 8 pie y tiene una altura de 12 pie. Para llenarlo se bombea agua desde una fuente ubicada 25 pies por debajo del deposito.

a. plantee la integral para obtener el trabajo requerido al llenar el depósito de agua hasta 3/4 de su altura.

longitud = 25 ft alto = 12 ft



* planteando volumen

$V = 25 \cdot 2x \cdot \Delta y$

* encontrando x

- primero la ec. de la parabola vertical → $(x-h)^2 = 4p(y-k)$

$(x-0)^2 = 4p(y-25)$

$x^2 = 4p(y-25)$;

- calculamos "p" sustituyendo el punto $(4, 37)$

$4^2 = 4p(37-25)$

$16 = 4p(12)$

$16 = 48p \to p = \frac{16}{48} \to \boxed{p = \frac{1}{3}}$

- sustituimos "p" en la ec.

$x^2 = 4(\frac{1}{3})(y-25)$

$x^2 = \frac{4}{3}(y-25)$

$x^2 = \frac{4}{3}y - \frac{100}{3}$

- ahora obtenemos "x"

$\sqrt{x^2} = \sqrt{4/3\, y - 100/3}$

$\boxed{x = \sqrt{\frac{4}{3}y - \frac{100}{3}}}$

volumen

$V = 50\sqrt{\frac{4}{3}y - \frac{100}{3}}\ \Delta y$

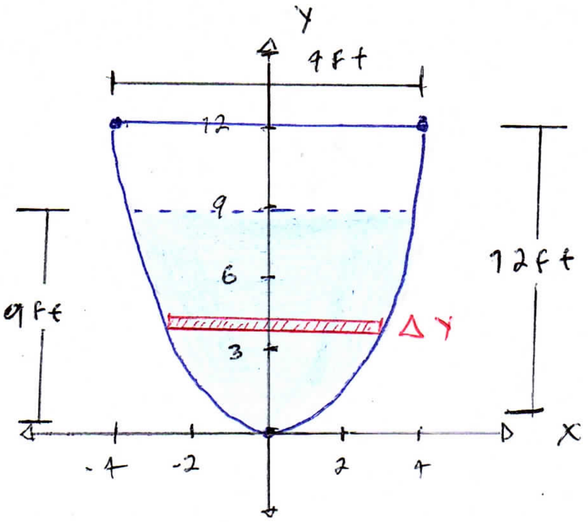
$D = y$

* integral

$W = \int_{25}^{34} \gamma\, 50\sqrt{\frac{4}{3}y - \frac{100}{3}}\,(y)\,dy$

$\boxed{W = 50\,(62.4)\int_{25}^{34} y\sqrt{\frac{4}{3}y - \frac{100}{3}}\,dy}$

b. Después de llenar hasta 3/4 de su altura, calcule la fuerza ejercida por el agua sobre la sección transversal.



profundidad $= h(y) = 9 - y$

Longitud $= 2x$

* encontrando "x" a partir de la ec. canónica, utilizando $p = \frac{1}{3}$ del anterior ejercicio y $V(0,0)$

$$(x-h)^2 = 4p(y-k)$$

$$(x-0)^2 = 4p(y-0)$$

$$x^2 = 4\left(\tfrac{1}{3}\right)y$$

$$x^2 = \frac{4}{3}y$$

$$x = \sqrt{\frac{4}{3}y}$$

$$L(y) = 2\sqrt{\frac{4}{3}y}$$

* planteamos la integral

$$F = \rho\int_0^b h(y)\,L(y)\,dy$$

$$F = \rho\int_0^9 (9-y)\left(2\sqrt{4/3\,y}\right)dy$$

$$F = 2\rho\int_0^9 (9-y)\sqrt{\tfrac{4}{3}}\sqrt{y}\,dy$$

$$F = 2\sqrt{\tfrac{4}{3}}\,\rho\int_0^9 (9-y)\sqrt{y}\,dy$$

$$F = 2\sqrt{\tfrac{4}{3}}\,\rho\int_0^9 \left(9y^{1/2} - y^{3/2}\right)dy$$

$$F = 2\sqrt{4/3}\,\rho\left[9\cdot\tfrac{2}{3}y^{3/2} - \tfrac{2}{5}y^{5/2}\right]_0^9 \rightarrow 2\sqrt{4/3}\,\rho\left[6y^{3/2} - \tfrac{2}{5}y^{5/2}\right]_0^9$$

$$F = 2\sqrt{4/3}\,\rho\left\{\left[6(9)^{3/2} - \tfrac{2}{5}(9)^{5/2}\right] - \left[6(0)^{3/2} - \tfrac{2}{5}(0)^{5/2}\right]\right\}$$

$$F = 2\sqrt{4/3}\,\rho\left\{\left[6(27) - \tfrac{2}{5}(243)\right] - \left[6(0) - \tfrac{2}{5}(0)\right]\right\}$$

$$F = 2\sqrt{4/3}\,\rho\left[\left(162 - \tfrac{486}{5}\right) - (0-0)\right]$$

$$F = 2\sqrt{4/3}\,\rho\left[\tfrac{324}{5} - 0\right] \rightarrow 2\sqrt{4/3}\,\rho\left(\tfrac{324}{5}\right)$$

$$F = 2\sqrt{4/3}\,(62.4)\left(\tfrac{324}{5}\right) = 9338.17$$

$$\boxed{R/\quad F = \rho\int_0^9 (9-y)\left(2\sqrt{\tfrac{4}{3}y}\right)dy = 9338.17\ lb}$$

2. Calcular masa M_x, M_y y la coordenada del centro de masa para una lamina de densidad constante "ρ" limitada por $y = \frac{1}{2}$, $y = \ln(3x)$, $x = 4$. Trazar el dibujo y ubicar gráficamente el centro de masa.

$Y = \ln(3x) \equiv x = \frac{e^Y}{3}$

* buscamos interceptos con las rectas

①

$\frac{1}{2} = \ln(3x)$

$3x = e^{1/2}$

$x = \frac{e^{1/2}}{3} \to x = 0.55$

$f(0.55) = 0.5 \quad (0.55, 0.5)$

②

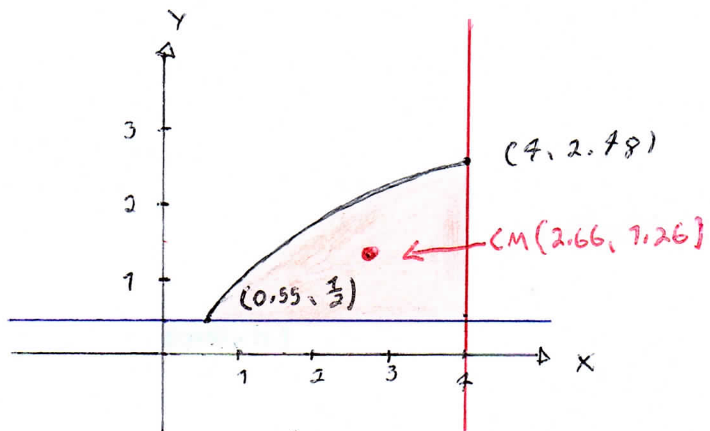
$Y = \ln(3x), \; x = 4$

$y = \ln(3(4))$

$Y = \ln(12)$

$Y = 2.48$

$f(2.48) = 4 \quad (4, 2.48)$



* integraremos en terminos de "y" para más facilidad

$m = \rho \int_a^b [f(y) - g(y)]\, dy$

$m = \rho \int_{1/2}^{2.48} \left[4 - \frac{e^Y}{3}\right] dy$

$m = 4\rho \int_{1/2}^{2.48} dy - \frac{1}{3}\rho \int_{1/2}^{2.48} e^y\, dy$

$m = 4\rho [y]_{1/2}^{2.48} - \frac{1}{3}\rho [e^y]_{1/2}^{2.48}$

$m = 4\rho [2.48 - \frac{1}{2}] - \frac{1}{3}\rho [e^{2.48} - e^{1/2}]$

$m = 4\rho (\frac{99}{50}) - \frac{1}{3}\rho (10.29)$

$m = \frac{198}{25}\rho - 3.43\rho \to \rho(\frac{198}{25} - 3.43)$

$\boxed{m = 4.49\,\rho}$

* Calculamos Mx

$$Mx = \rho \int_a^b y\,[f(y) - g(y)]\,dy$$

$$Mx = \rho \int_{1/2}^{2.48} y\left[4 - \frac{e^y}{3}\right] dy \rightarrow \rho \int_{1/2}^{2.48} \left(4y - y\frac{e^y}{3}\right) dy$$

$$Mx = 4\rho \int_{1/2}^{2.48} y\,dy - \frac{1}{3}\rho \int_{1/2}^{2.48} y e^y\,dy$$

$$Mx = 4\rho\left[\frac{1}{2}y^2\right]_{1/2}^{2.48} - \frac{1}{3}\rho \underbrace{\int_{1/2}^{2.48} y e\,dy}$$

$u = y \quad du = dy$

$v = e^y \quad dv = e^y dy$

$$\int y e^y\,dy$$

$$= u \cdot v - \int v\,du$$

$$= ye^y - \int e^y\,dy$$

$$\boxed{\int ye\,dy = ye^y - e^y + C}$$

$$Mx = 4\rho\left[\frac{y^2}{2}\right]_{1/2}^{2.48} - \frac{1}{3}\rho\left[ye^y - e^y\right]_{1/2}^{2.48}$$

$$Mx = 4\rho\left[\frac{(2.48)^2}{2} - \frac{(1/2)^2}{2}\right] - \frac{1}{3}\rho\left\{\left[(2.48)e^{2.48} - e^{2.48}\right] - \left[(\tfrac{1}{2})e^{1/2} - e^{1/2}\right]\right\}$$

$$Mx = 4\rho\left[\frac{6.15}{2} - \frac{(1/4)}{2}\right] - \frac{1}{3}\rho\left\{\left[29.61 - e^{2.48}\right] - \left[0.824 - e^{1/2}\right]\right\}$$

$$Mx = 4\rho\left[3.075 - \frac{1}{8}\right] - \frac{1}{3}\rho\left[17.67 - (-0.825)\right]$$

$$Mx = 4\rho\left[\frac{59}{20}\right] - \frac{1}{3}\rho\left[17.67 + 0.825\right]$$

$$Mx = 4\rho\left(\frac{59}{20}\right) - \frac{1}{3}\rho\left[18.495\right]$$

$$Mx = 11.8\rho - 6.165\rho \rightarrow \rho(11.8 - 6.165)$$

$$Mx = 5.64\rho$$

$$\boxed{Mx = 5.64\rho}$$

* Calculamos M_y

$$M_y = \rho \int_a^b \frac{[f(y) + g(y)]}{2} [f(y) - g(y)]\, dy$$

$$M_y = \frac{1}{2} \rho \int_a^b [f(y)^2 - g(y)^2]\, dy$$

$$M_y = \frac{1}{2} \rho \int_{1/2}^{2.48} \left[4^2 - \left(\frac{e^y}{3}\right)^2\right] dy$$

$$M_y = \frac{1}{2} \rho \int_{1/2}^{2.48} \left[16 - \frac{e^{2y}}{9}\right] dy$$

$$u = 2y \qquad du = 2dy \qquad \frac{du}{2} = dy$$

$$M_y = \frac{16}{2} \rho \int_{1/2}^{2.48} dy - \frac{1}{2}\left(\frac{1}{9}\right) \rho \int_{1/2}^{2.48} e^{2y}\, dy$$

$$M_y = 8 \rho \int_{1/2}^{2.48} dy - \frac{1}{18} \rho \int_{1/2}^{2.48} e^u \left(\frac{du}{2}\right)$$

$$M_y = 8 \rho \int_{1/2}^{2.48} dy - \frac{1}{36} \rho \int_{1/2}^{2.48} e^u\, du$$

$$M_y = 8\rho [y]_{1/2}^{2.48} - \frac{1}{36}\rho [e^u]_{1/2}^{2.48} \rightarrow 8\rho [y]_{1/2}^{2.48} - \frac{1}{36}\rho [e^{2y}]_{1/2}^{2.48}$$

$$M_y = 8\rho [2.48 - 1/2] - \frac{1}{36}\rho [e^{2(2.48)} - e^{2(1/2)}]$$

$$M_y = 8\rho \left(\frac{99}{50}\right) - \frac{1}{36}\rho [e^{4.96} - e^1] \rightarrow \frac{396}{25}\rho - \frac{1}{36}\rho (139.88)$$

$$M_y = \frac{396}{25}\rho - 3.89\rho \rightarrow \rho\left(\frac{396}{25} - 3.89\right)$$

$$\boxed{M_y = 11.95\rho}$$

* encontrando el centro de masa

$$\bar{x} = \frac{M_y}{m} = \frac{11.95\rho}{4.49\rho} \qquad \boxed{\bar{x} = 2.66}$$

$$\bar{y} = \frac{M_x}{m} = \frac{5.64\rho}{4.49\rho} \qquad \boxed{\bar{y} = 1.26}$$

$$\boxed{R/\ CM(2.66,\ 1.26)}$$