

```
In [31]: from sympy import * # Librería para operaciones simbólicas  
from ubogsla18p import * # Librería personal
```

Tipos de datos

Escalares ¶

```
In [32]: # asignando una constante entera  
a=3+5  
imprimir('a=',a)
```

Out[32]: $a = 8$

```
In [33]: # asignando una constante racional  
a=Rational(1,2)+Rational(1,3)  
imprimir('a=',a)
```

Out[33]: $a = \frac{5}{6}$

```
In [34]: # asignando una constante de punto flotante  
a=2*3.14  
imprimir('a=',a)
```

Out[34]: $a = 6.28$

```
In [35]: # asignando variables  
c=symbols('c')  
b=3*c+5*c  
imprimir('b=',b)
```

Out[35]: $b = 8c$

```
In [36]: d=b+1/c  
imprimir('d=',d)
```

Out[36]: $d = 8c + \frac{1}{c}$

```
In [37]: # simplificando fraccionarios  
e=cancel(d)  
imprimir(e)
```

Out[37]: $\frac{1}{c}(8c^2 + 1)$

Vectores

```
In [38]: # asignando vectores
v=Matrix([2, 3, 5])
imprimir('v=',v,',   v[0]=' ,v[0],', v[1]=' ,v[1],', v[2]=' ,v[2])
```

```
Out[38]: 
$$v = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, v[0] = 2, v[1] = 3, v[2] = 5$$

```

Matrices

```
In [39]: # asignando matrices de constantes
A=Matrix([[2, 3], [5,7]])
imprimir('A=',A,',   A[0,0]=' ,A[0,0],', A[0,1]=' ,A[0,1],', A[1,0]=' ,A[1,0],', A[1
```

```
Out[39]: 
$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, A[0,0] = 2, A[0,1] = 3, A[1,0] = 5, A[1,1] = 7$$

```

```
In [40]: print('El tamaño de A es ',A.shape)
```

```
El tamaño de A es  (2, 2)
```

```
In [41]: print('sus columnas son ',A.cols,' y sus renglones ', A.rows )
```

```
sus columnas son  2  y sus renglones  2
```

```
In [42]: # asignando matrices de variables
A=Matrix(symbols('a:4:3')).reshape(4,3)
imprimir('A=',A)
```

```
Out[42]: 
$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix}$$

```

Matriz identidad

```
In [43]: imprimir('I_2=',eye(2),r', \ I_3=',eye(3))
```

```
Out[43]: 
$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```

Matriz de ceros

```
In [44]: imprimir(r'\theta_{2 \times 3}=',zeros(2,3),r', \ \theta_{2 \times 2}=',zeros(2,2))
```

```
Out[44]:  $0_{2 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, 0_{2 \times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 
```

Matrices iguales

Dos matrices son iguales si tienen el mismo tamaño y los mismos elementos.

```
In [45]: zeros(2,3) == zeros(2,2)
```

```
Out[45]: False
```

```
In [46]: 0*eye(2) == zeros(2,2)
```

```
Out[46]: True
```

Escalar, vector, matriz [video \(cap02a.mp4\)](#) [html \(cap02a\cap00present.html\)](#)

Operaciones entre matrices

[Nakos, Sec 3.1 y 2.1]

Escalar por matriz

[video \(cap02b.mp4\)](#) [html \(cap02b\cap00present.html\)](#)

```
In [47]: D=c*A
imprimir('D=',c,A,'=',D)
```

```
Out[47]:  $D = c \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} a_{00}c & a_{01}c & a_{02}c \\ a_{10}c & a_{11}c & a_{12}c \\ a_{20}c & a_{21}c & a_{22}c \\ a_{30}c & a_{31}c & a_{32}c \end{bmatrix}$ 
```

```
In [48]: B=(1/c)*A
imprimir('B=',1/c,A,'=',B)
```

```
Out[48]:  $B = \frac{1}{c} \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} \frac{a_{00}}{c} & \frac{a_{01}}{c} & \frac{a_{02}}{c} \\ \frac{a_{10}}{c} & \frac{a_{11}}{c} & \frac{a_{12}}{c} \\ \frac{a_{20}}{c} & \frac{a_{21}}{c} & \frac{a_{22}}{c} \\ \frac{a_{30}}{c} & \frac{a_{31}}{c} & \frac{a_{32}}{c} \end{bmatrix}$ 
```

Suma de matrices

[video \(cap02c.mp4\)](#) [html \(cap02c\cap00present.html\)](#)

```
In [49]: C=A+B
imprimir('C=',A,'+',B,'=',C)
```

$$\text{Out[49]: } C = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} + \begin{bmatrix} \frac{a_{00}}{c} & \frac{a_{01}}{c} & \frac{a_{02}}{c} \\ \frac{a_{10}}{c} & \frac{a_{11}}{c} & \frac{a_{12}}{c} \\ \frac{a_{20}}{c} & \frac{a_{21}}{c} & \frac{a_{22}}{c} \\ \frac{a_{30}}{c} & \frac{a_{31}}{c} & \frac{a_{32}}{c} \end{bmatrix} = \begin{bmatrix} a_{00} + \frac{a_{00}}{c} & a_{01} + \frac{a_{01}}{c} & a_{02} + \frac{a_{02}}{c} \\ a_{10} + \frac{a_{10}}{c} & a_{11} + \frac{a_{11}}{c} & a_{12} + \frac{a_{12}}{c} \\ a_{20} + \frac{a_{20}}{c} & a_{21} + \frac{a_{21}}{c} & a_{22} + \frac{a_{22}}{c} \\ a_{30} + \frac{a_{30}}{c} & a_{31} + \frac{a_{31}}{c} & a_{32} + \frac{a_{32}}{c} \end{bmatrix}$$

```
In [50]: C1=C.applyfunc(cancel)
imprimir('C=',C,'=',C1)
```

$$\text{Out[50]: } C = \begin{bmatrix} a_{00} + \frac{a_{00}}{c} & a_{01} + \frac{a_{01}}{c} & a_{02} + \frac{a_{02}}{c} \\ a_{10} + \frac{a_{10}}{c} & a_{11} + \frac{a_{11}}{c} & a_{12} + \frac{a_{12}}{c} \\ a_{20} + \frac{a_{20}}{c} & a_{21} + \frac{a_{21}}{c} & a_{22} + \frac{a_{22}}{c} \\ a_{30} + \frac{a_{30}}{c} & a_{31} + \frac{a_{31}}{c} & a_{32} + \frac{a_{32}}{c} \end{bmatrix} = \begin{bmatrix} \frac{1}{c}(a_{00}c + a_{00}) & \frac{1}{c}(a_{01}c + a_{01}) & \frac{1}{c}(a_{02}c + a_{02}) \\ \frac{1}{c}(a_{10}c + a_{10}) & \frac{1}{c}(a_{11}c + a_{11}) & \frac{1}{c}(a_{12}c + a_{12}) \\ \frac{1}{c}(a_{20}c + a_{20}) & \frac{1}{c}(a_{21}c + a_{21}) & \frac{1}{c}(a_{22}c + a_{22}) \\ \frac{1}{c}(a_{30}c + a_{30}) & \frac{1}{c}(a_{31}c + a_{31}) & \frac{1}{c}(a_{32}c + a_{32}) \end{bmatrix}$$

Matriz opuesta

[video \(cap02d.mp4\)](#) [html \(cap02d\cap00present.html\)](#)

```
In [51]: imprimir('-A=-',A,'=',-A)
```

$$\text{Out[51]: } -A = - \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} -a_{00} & -a_{01} & -a_{02} \\ -a_{10} & -a_{11} & -a_{12} \\ -a_{20} & -a_{21} & -a_{22} \\ -a_{30} & -a_{31} & -a_{32} \end{bmatrix}$$

Resta de matrices

[video \(cap02e.mp4\)](#) [html \(cap02e\cap00present.html\)](#)

```
In [52]: D=A-B
imprimir('C=',A,'-',B,'=',D,D.applyfunc(cancel))
```

Out[52]:

$$C = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} - \begin{bmatrix} \frac{a_{00}}{c} & \frac{a_{01}}{c} & \frac{a_{02}}{c} \\ \frac{a_{10}}{c} & \frac{a_{11}}{c} & \frac{a_{12}}{c} \\ \frac{a_{20}}{c} & \frac{a_{21}}{c} & \frac{a_{22}}{c} \\ \frac{a_{30}}{c} & \frac{a_{31}}{c} & \frac{a_{32}}{c} \end{bmatrix} = \begin{bmatrix} a_{00} - \frac{a_{00}}{c} & a_{01} - \frac{a_{01}}{c} & a_{02} - \frac{a_{02}}{c} \\ a_{10} - \frac{a_{10}}{c} & a_{11} - \frac{a_{11}}{c} & a_{12} - \frac{a_{12}}{c} \\ a_{20} - \frac{a_{20}}{c} & a_{21} - \frac{a_{21}}{c} & a_{22} - \frac{a_{22}}{c} \\ a_{30} - \frac{a_{30}}{c} & a_{31} - \frac{a_{31}}{c} & a_{32} - \frac{a_{32}}{c} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{c}(a_{00}c - a_{00}) & \frac{1}{c}(a_{01}c - a_{01}) & \frac{1}{c}(a_{02}c - a_{02}) \\ \frac{1}{c}(a_{10}c - a_{10}) & \frac{1}{c}(a_{11}c - a_{11}) & \frac{1}{c}(a_{12}c - a_{12}) \\ \frac{1}{c}(a_{20}c - a_{20}) & \frac{1}{c}(a_{21}c - a_{21}) & \frac{1}{c}(a_{22}c - a_{22}) \\ \frac{1}{c}(a_{30}c - a_{30}) & \frac{1}{c}(a_{31}c - a_{31}) & \frac{1}{c}(a_{32}c - a_{32}) \end{bmatrix}$$

Combinación lineal y matriz por vector

[video \(cap02f.mp4\)](#) [html \(cap02f\cap00present.html\)](#)

```
In [53]: b=Matrix(symbols('b0:3'))
imprimir('b=',b)
```

Out[53]:

$$b = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

```
In [54]: u=Matrix(symbols('u0:4'))
v=Matrix(symbols('v0:4'))
w=Matrix(symbols('w0:4'))
imprimir('u=',u,', v=',v,', w=',w)
```

Out[54]:

$$u = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix}, v = \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}, w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

```
In [55]: y=b[0]*u+b[1]*v+b[2]*w
imprimir('y=b_0 u + b_1 v + b_2 w= ',y)
```

Out[55]:

$$y = b_0 u + b_1 v + b_2 w = \begin{bmatrix} b_0 u_0 + b_1 v_0 + b_2 w_0 \\ b_0 u_1 + b_1 v_1 + b_2 w_1 \\ b_0 u_2 + b_1 v_2 + b_2 w_2 \\ b_0 u_3 + b_1 v_3 + b_2 w_3 \end{bmatrix}$$

```
In [56]: A=juntar(u,v,w)
         imprimir('A=',A)
```

Out[56]:

$$A = \begin{bmatrix} u_0 & v_0 & w_0 \\ u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix}$$

```
In [57]: y=A*b
         imprimir('y=',A,b,'=',y)
```

Out[57]:

$$y = \begin{bmatrix} u_0 & v_0 & w_0 \\ u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_0 u_0 + b_1 v_0 + b_2 w_0 \\ b_0 u_1 + b_1 v_1 + b_2 w_1 \\ b_0 u_2 + b_1 v_2 + b_2 w_2 \\ b_0 u_3 + b_1 v_3 + b_2 w_3 \end{bmatrix}$$

Multiplicación matricial o matriz por vectores

[html \(cap02g\cap00present.html\)](html\cap02g\cap00present.html)

```
In [58]: A=Matrix(symbols('a:4:3')).reshape(4,3)
         imprimir('A=',A)
```

Out[58]:

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix}$$

```
In [59]: B=Matrix(symbols('b:3:2')).reshape(3,2)
         imprimir('B=',B)
```

Out[59]:

$$B = \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \\ b_{20} & b_{21} \end{bmatrix}$$

```
In [60]: C=A*B
         imprimir('C=',A,B,'=',C)
```

Out[60]:

$$C = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \\ b_{20} & b_{21} \end{bmatrix} = \begin{bmatrix} a_{00}b_{00} + a_{01}b_{10} + a_{02}b_{20} & a_{00}b_{01} + a_{01}b_{11} + a_{02}b_{21} \\ a_{10}b_{00} + a_{11}b_{10} + a_{12}b_{20} & a_{10}b_{01} + a_{11}b_{11} + a_{12}b_{21} \\ a_{20}b_{00} + a_{21}b_{10} + a_{22}b_{20} & a_{20}b_{01} + a_{21}b_{11} + a_{22}b_{21} \\ a_{30}b_{00} + a_{31}b_{10} + a_{32}b_{20} & a_{30}b_{01} + a_{31}b_{11} + a_{32}b_{21} \end{bmatrix}$$

Propiedades

Dadas las matrices A, B, C de tamaños adecuados y los escalares a, b, c :

	<i>Reales</i>	<i>Matrices</i>
<i>Suma conmutativa</i>	$a + b = b + a$	$A + B = B + A$
<i>Mult. conmutativa</i>	$ab = ba$	$a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ pero $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} a \neq$ $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$
<i>Suma asociativa</i>	$(a + b) + c = a + (b + c)$	$(A + B) + C = A + (B + C)$
<i>Mult. asociativa</i>	$(ab)c = a(bc)$	$(ab)C = a(bC)$ $(aB)C = a(BC)$ $(AB)C = A(BC)$
<i>Modulativa</i>	$a + 0 = a$	$A + 0_{m \times n} = A$
<i>Identidad</i>	$1a = a$	$1A = A$ $IA = A$ $AI = A$
<i>Nulidad</i>	$0a = 0$	$0A_{m \times n} = 0_{m \times n}$ $0_{m \times k}A_{k \times n} = 0_{m \times n}$ $A_{m \times k}0_{k \times n} = 0_{m \times n}$
<i>Opuesto</i>	$a + (-a) = 0$	$A + (-A) = 0_{m \times n}$
<i>Distributiva I</i>	$(a + b)c = ac + bc$	$(a + b)C = aC + bC$ $(A + B)C = AC + BC$
<i>Distributiva D</i>	$a(b + c) = ab + ac$	$a(B + C) = aB + aC$ $A(B + C) = AB + AC$