In [31]:
```
from sympy import * # Librería para operaciones simbólicas
from ubogsla18p import * # Librería personal
```

Tipos de datos

Escalares

In [32]:
```
# asignando una constante entera
a=3+5
imprimir('a=',a)
```

Out[32]: $a = 8$

In [33]:
```
# asignando una constante racional
a=Rational(1,2)+Rational(1,3)
imprimir('a=',a)
```

Out[33]: $a = \frac{5}{6}$

In [34]:
```
# asignando una constante de punto flotante
a=2*3.14
imprimir('a=',a)
```

Out[34]: $a = 6.28$

In [35]:
```
# asignando variables
c=symbols('c')
b=3*c+5*c
imprimir('b=',b)
```

Out[35]: $b = 8c$

In [36]:
```
d=b+1/c
imprimir('d=',d)
```

Out[36]: $d = 8c + \frac{1}{c}$

In [37]:
```
# simplificando fraccionarios
e=cancel(d)
imprimir(e)
```

Out[37]: $\frac{1}{c}\left(8c^2 + 1\right)$

Vectores

In [38]:
```
# asignando vectores
v=Matrix([2, 3, 5])
imprimir('v=',v,',   v[0]=',v[0],', v[1]=',v[1],', v[2]=',v[2])
```

Out[38]:

$$v = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, v[0] = 2, v[1] = 3, v[2] = 5$$

Matrices

In [39]:
```
# asignando matrices de constantes
A=Matrix([[2, 3], [5,7]])
imprimir('A=',A,',   A[0,0]=',A[0,0],', A[0,1]=',A[0,1],', A[1,0]=',A[1,0],', A[1
```

Out[39]:

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, A[0,0] = 2, A[0,1] = 3, A[1,0] = 5, A[1,1] = 7$$

In [40]:
```
print('El tamaño de A es ',A.shape)
```

```
El tamaño de A es  (2, 2)
```

In [41]:
```
print('sus columnas son ',A.cols,' y sus renglones ', A.rows )
```

```
sus columnas son  2  y sus renglones  2
```

In [42]:
```
# asignando matrices de variables
A=Matrix(symbols('a:4:3')).reshape(4,3)
imprimir('A=',A)
```

Out[42]:

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix}$$

Matriz identidad

In [43]:
```
imprimir('I_2=',eye(2),r', \ I_3=',eye(3))
```

Out[43]:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matriz de ceros

In [44]:
```
imprimir(r'0_{2 \times 3}=',zeros(2,3),r', \ 0_{2 \times 2}=',zeros(2,2))
```

Out[44]:
$$0_{2\times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ 0_{2\times 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Matrices iguales

Dos matrices son iguales si tienen el mismo tamaño y los mismos elementos.

In [45]:
```
zeros(2,3) == zeros(2,2)
```

Out[45]: `False`

In [46]:
```
0*eye(2) == zeros(2,2)
```

Out[46]: `True`

Escalar, vector, matriz video (cap02a.mp4) html (cap02a\cap00present.html)

Operaciones entre matrices

[Nakos, Sec 3.1 y 2.1]

Escalar por matriz

video (cap02b.mp4) html (cap02b\cap00present.html)

In [47]:
```
D=c*A
imprimir('D=',c,A,'=',D)
```

Out[47]:
$$D = c\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} a_{00}c & a_{01}c & a_{02}c \\ a_{10}c & a_{11}c & a_{12}c \\ a_{20}c & a_{21}c & a_{22}c \\ a_{30}c & a_{31}c & a_{32}c \end{bmatrix}$$

In [48]:
```
B=(1/c)*A
imprimir('B=',1/c,A,'=',B)
```

Out[48]:
$$B = \frac{1}{c}\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} \frac{a_{00}}{c} & \frac{a_{01}}{c} & \frac{a_{02}}{c} \\ \frac{a_{10}}{c} & \frac{a_{11}}{c} & \frac{a_{12}}{c} \\ \frac{a_{20}}{c} & \frac{a_{21}}{c} & \frac{a_{22}}{c} \\ \frac{a_{30}}{c} & \frac{a_{31}}{c} & \frac{a_{32}}{c} \end{bmatrix}$$

Suma de matrices

video (cap02c.mp4) html (cap02c\cap00present.html)

In [49]:
```
C=A+B
imprimir('C=',A,'+',B,'=',C)
```

Out[49]:

$$C = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} + \begin{bmatrix} \frac{a_{00}}{c} & \frac{a_{01}}{c} & \frac{a_{02}}{c} \\ \frac{a_{10}}{c} & \frac{a_{11}}{c} & \frac{a_{12}}{c} \\ \frac{a_{20}}{c} & \frac{a_{21}}{c} & \frac{a_{22}}{c} \\ \frac{a_{30}}{c} & \frac{a_{31}}{c} & \frac{a_{32}}{c} \end{bmatrix} = \begin{bmatrix} a_{00} + \frac{a_{00}}{c} & a_{01} + \frac{a_{01}}{c} & a_{02} + \frac{a_{02}}{c} \\ a_{10} + \frac{a_{10}}{c} & a_{11} + \frac{a_{11}}{c} & a_{12} + \frac{a_{12}}{c} \\ a_{20} + \frac{a_{20}}{c} & a_{21} + \frac{a_{21}}{c} & a_{22} + \frac{a_{22}}{c} \\ a_{30} + \frac{a_{30}}{c} & a_{31} + \frac{a_{31}}{c} & a_{32} + \frac{a_{32}}{c} \end{bmatrix}$$

In [50]:
```
C1=C.applyfunc(cancel)
imprimir('C=',C,'=',C1)
```

Out[50]:

$$C = \begin{bmatrix} a_{00} + \frac{a_{00}}{c} & a_{01} + \frac{a_{01}}{c} & a_{02} + \frac{a_{02}}{c} \\ a_{10} + \frac{a_{10}}{c} & a_{11} + \frac{a_{11}}{c} & a_{12} + \frac{a_{12}}{c} \\ a_{20} + \frac{a_{20}}{c} & a_{21} + \frac{a_{21}}{c} & a_{22} + \frac{a_{22}}{c} \\ a_{30} + \frac{a_{30}}{c} & a_{31} + \frac{a_{31}}{c} & a_{32} + \frac{a_{32}}{c} \end{bmatrix} = \begin{bmatrix} \frac{1}{c}(a_{00}c + a_{00}) & \frac{1}{c}(a_{01}c + a_{01}) & \frac{1}{c}(a_{02}c + a_{02}) \\ \frac{1}{c}(a_{10}c + a_{10}) & \frac{1}{c}(a_{11}c + a_{11}) & \frac{1}{c}(a_{12}c + a_{12}) \\ \frac{1}{c}(a_{20}c + a_{20}) & \frac{1}{c}(a_{21}c + a_{21}) & \frac{1}{c}(a_{22}c + a_{22}) \\ \frac{1}{c}(a_{30}c + a_{30}) & \frac{1}{c}(a_{31}c + a_{31}) & \frac{1}{c}(a_{32}c + a_{32}) \end{bmatrix}$$

Matriz opuesta

video (cap02d.mp4) html (cap02d\cap00present.html)

In [51]:
```
imprimir('-A=-',A,'=',-A)
```

Out[51]:

$$-A = -\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} -a_{00} & -a_{01} & -a_{02} \\ -a_{10} & -a_{11} & -a_{12} \\ -a_{20} & -a_{21} & -a_{22} \\ -a_{30} & -a_{31} & -a_{32} \end{bmatrix}$$

Resta de matrices

video (cap02e.mp4) html (cap02e\cap00present.html)

In [52]:
```
D=A-B
imprimir('C=',A,'-',B,'=',D,'=',D.applyfunc(cancel))
```

Out[52]:

$$C = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} - \begin{bmatrix} \frac{a_{00}}{c} & \frac{a_{01}}{c} & \frac{a_{02}}{c} \\ \frac{a_{10}}{c} & \frac{a_{11}}{c} & \frac{a_{12}}{c} \\ \frac{a_{20}}{c} & \frac{a_{21}}{c} & \frac{a_{22}}{c} \\ \frac{a_{30}}{c} & \frac{a_{31}}{c} & \frac{a_{32}}{c} \end{bmatrix} = \begin{bmatrix} a_{00} - \frac{a_{00}}{c} & a_{01} - \frac{a_{01}}{c} & a_{02} - \frac{a_{02}}{c} \\ a_{10} - \frac{a_{10}}{c} & a_{11} - \frac{a_{11}}{c} & a_{12} - \frac{a_{12}}{c} \\ a_{20} - \frac{a_{20}}{c} & a_{21} - \frac{a_{21}}{c} & a_{22} - \frac{a_{22}}{c} \\ a_{30} - \frac{a_{30}}{c} & a_{31} - \frac{a_{31}}{c} & a_{32} - \frac{a_{32}}{c} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{c}(a_{00}c - a_{00}) & \frac{1}{c}(a_{01}c - a_{01}) & \frac{1}{c}(a_{02}c - a_{02}) \\ \frac{1}{c}(a_{10}c - a_{10}) & \frac{1}{c}(a_{11}c - a_{11}) & \frac{1}{c}(a_{12}c - a_{12}) \\ \frac{1}{c}(a_{20}c - a_{20}) & \frac{1}{c}(a_{21}c - a_{21}) & \frac{1}{c}(a_{22}c - a_{22}) \\ \frac{1}{c}(a_{30}c - a_{30}) & \frac{1}{c}(a_{31}c - a_{31}) & \frac{1}{c}(a_{32}c - a_{32}) \end{bmatrix}$$

Combinación lineal y matriz por vector

video (cap02f.mp4) html (cap02f\cap00present.html)

In [53]:
```
b=Matrix(symbols('b0:3'))
imprimir('b=',b)
```

Out[53]:

$$b = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

In [54]:
```
u=Matrix(symbols('u0:4'))
v=Matrix(symbols('v0:4'))
w=Matrix(symbols('w0:4'))
imprimir('u=',u,', v=',v,'w=',w)
```

Out[54]:

$$u = \begin{bmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix}, v = \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

In [55]:
```
y=b[0]*u+b[1]*v+b[2]*w
imprimir('y=b_0 u + b_1 v + b_2 w= ',y)
```

Out[55]:

$$y = b_0 u + b_1 v + b_2 w = \begin{bmatrix} b_0 u_0 + b_1 v_0 + b_2 w_0 \\ b_0 u_1 + b_1 v_1 + b_2 w_1 \\ b_0 u_2 + b_1 v_2 + b_2 w_2 \\ b_0 u_3 + b_1 v_3 + b_2 w_3 \end{bmatrix}$$

In [56]:
```
A=juntar(u,v,w)
imprimir('A=',A)
```

Out[56]:

$$A = \begin{bmatrix} u_0 & v_0 & w_0 \\ u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix}$$

In [57]:
```
y=A*b
imprimir('y=',A,b,'=',y)
```

Out[57]:

$$y = \begin{bmatrix} u_0 & v_0 & w_0 \\ u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} b_0 u_0 + b_1 v_0 + b_2 w_0 \\ b_0 u_1 + b_1 v_1 + b_2 w_1 \\ b_0 u_2 + b_1 v_2 + b_2 w_2 \\ b_0 u_3 + b_1 v_3 + b_2 w_3 \end{bmatrix}$$

Multiplicación matricial o matriz por vectores

html (cap02g\cap00present.html)

In [58]:
```
A=Matrix(symbols('a:4:3')).reshape(4,3)
imprimir('A=',A)
```

Out[58]:

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix}$$

In [59]:
```
B=Matrix(symbols('b:3:2')).reshape(3,2)
imprimir('B=',B)
```

Out[59]:

$$B = \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \\ b_{20} & b_{21} \end{bmatrix}$$

In [60]:
```
C=A*B
imprimir('C=',A,B,'=',C)
```

Out[60]:

$$C = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \\ b_{20} & b_{21} \end{bmatrix} =$$

$$\begin{bmatrix} a_{00}b_{00} + a_{01}b_{10} + a_{02}b_{20} & a_{00}b_{01} + a_{01}b_{11} + a_{02}b_{21} \\ a_{10}b_{00} + a_{11}b_{10} + a_{12}b_{20} & a_{10}b_{01} + a_{11}b_{11} + a_{12}b_{21} \\ a_{20}b_{00} + a_{21}b_{10} + a_{22}b_{20} & a_{20}b_{01} + a_{21}b_{11} + a_{22}b_{21} \\ a_{30}b_{00} + a_{31}b_{10} + a_{32}b_{20} & a_{30}b_{01} + a_{31}b_{11} + a_{32}b_{21} \end{bmatrix}$$

Propiedades

Dadas las matrices A, B, C de tamaños adecuados y los escalares a, b, c:

	$Reales$	$Matrices$
$Suma\ conmutativa$	$a+b=b+a$	$A+B=B+A$
$Mult.\ conmutativa$	$ab=ba$	$a\left[\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right]$ $pero$ $\left[\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right]\cancel{a}$ $\left[\begin{smallmatrix}2&0\\0&0\end{smallmatrix}\right]\left[\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right] \neq \left[\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right]\left[\begin{smallmatrix}2&0\\0&0\end{smallmatrix}\right]$
$Suma\ asociativa$	$(a+b)+c=a+(b+c)$	$(A+B)+C=A+(B+C)$
$Mult.\ asociativa$	$(ab)c=a(bc)$	$(ab)C=a(bC)$ $(aB)C=a(BC)$ $(AB)C=A(BC)$
$Modulativa$	$a+0=a$	$A+0_{m\times n}=A$
$Identidad$	$1a=a$	$1A=A$ $IA=A$ $AI=A$
$Nulidad$	$0a=0$	$0A_{m\times n}=0_{m\times n}$ $0_{m\times k}A_{k\times n}=0_{m\times n}$ $A_{m\times k}0_{k\times n}=0_{m\times n}$
$Opuesto$	$a+(-a)=0$	$A+(-A)=0_{m\times n}$
$Distributiva\ I$	$(a+b)c=ac+bc$	$(a+b)C=aC+bC$ $(A+B)C=AC+BC$
$Distributiva\ D$	$a(b+c)=ab+ac$	$a(B+C)=aB+aC$ $A(B+C)=AB+AC$

Potencia de una matriz

Para una matriz cuadrada A se definen las potencias:

- $A^0 = I$,
- $A^n = AA \dots A$, n veces para $n \in \{1, 2, 3, \dots\}$,

Teorema:

Si A es una matriz cuadrada, c es un escalar distinto se cero y $r, s \in \mathbb{N}$ se tiene que se cumplen las siguientes igualdades.

- Suma de Exponentes: $A^r A^s = A^{r+s}$
- Producto de exponentes: $((A)^r)^s = A^{rs}$
- El escalar sale con exponente: $(cA)^r = c^r (A)^r$