1 Lecture 1 (10/02/2020)

1.1 The basics

In general any dynamics problem is solved in three steps. The first and last of these is not unlike the problems one would solve in statics. The second step is however new to dynamics.

First Draw a Free Body Diagram of the problem

Second Set up equations of motion, Note that $\Sigma F \neq 0$, instead $\Sigma F = ma$ (a can still possibly be 0)

Third Check if there are enough equations of motion to solve for all unknown variables.

Notation for accelaration, velocity and displacement may vary depending on personal prefference. $\ddot{x}$ is used as a means of writing accelaration and $\dot{x}$ is used in place of velocity. A single dot is defined as a single derrivative with respect to time, two dots in turn refers to the second derrivative with respect to time.

$$a = \dot{v} = \ddot{x}$$

$$v = \dot{x}$$

1.2 An example problem

Below will be an example problem of a single cup on an angled plane. The angle between the x-axis and the plane shall be called θ. Our coordinate system will consist of 2 axes, an u- and v-axis, where u is parrallel to the plane and v perpendicular to the plane. See figure 1 for a free body diagram of the problem. Friction will be ignored for this problem.

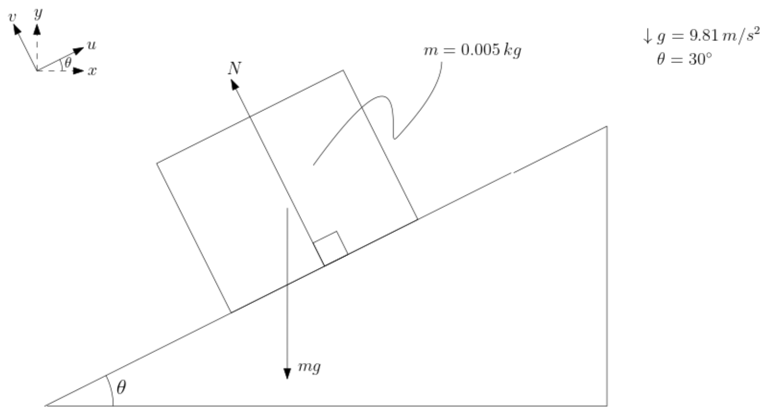

Figure 1: The Free Body Diagram of the cup on a plane

$$\Sigma F_u = mg\,\sin(\theta) = m\ddot{u} \tag{1}$$

$$\Sigma F_v = N - mg\,\cos(\theta) = m\ddot{v} \tag{2}$$

$$\ddot{v} = 0 \tag{3}$$

Note that equation (3) is typically called a "Constrain Equation"

Subsituting equation (3) into (2) will give:

$$N = mg\,cos(\theta) \tag{4}$$

Subsituting equation (4) into (1) and solving for $\ddot{u}$:

$$\ddot{u} = g \cdot \sin(\theta) = a_u = 4,91\,m/s^2 \tag{5}$$

Equation (5) is often (and much to the annoyance of mathematicians) reffered to as the "differetial equation". This equation can be used to determin the velocity and displacement as a function of time. The initial for this problem will be $x_0 = 0$ and $v_0 = 0$.

$$\dot{u} = \int \ddot{u}\, dt = a_u t + C_1 = a_y t \tag{6}$$

$$u = \int \dot{u}\, dt = \int a_u t\, dt = \frac{1}{2} a_u t^2 + C_2 = \frac{1}{2} a_u t^2 \tag{7}$$

thus when subsituting in equation (5) in (6) and (7) the final answer becomes:

$$\dot{u} = 4,91 \cdot t$$

$$u = \frac{1}{2} \cdot 4,91 \cdot t^2$$

1.3 Friction with an example

The equation for kinetic friction is much the same as the equation for static friction with a few key differences. Note that in general $\mu_k < \mu_s$.

$$F_w = \mu_k N$$
$$F_w \leq \mu_s N$$

Solving a dynamics problem involving friction forces to make an assumption at some point. Either the object is moving in which case dry friction does occur, or the object is not moving in which case the accelaration parallel to the plane will be zero.

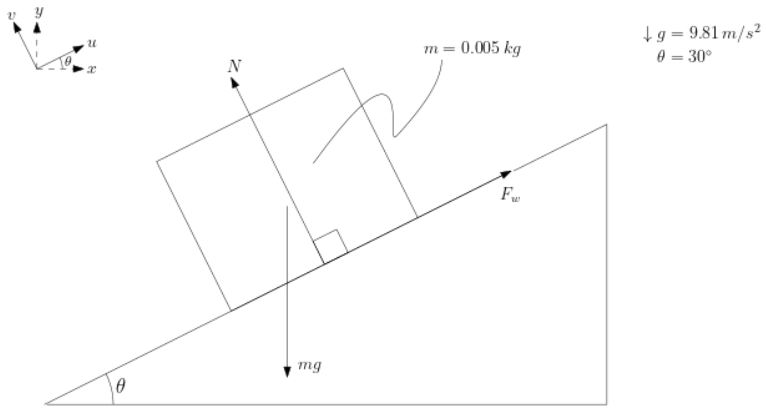

Figure 2: The Free Body Diagram of the cup on a plane

$$\Sigma F_u = -F_w + mg\,\sin(\theta) = m\ddot{u} \tag{1}$$

$$\Sigma F_v = N - mg\,\cos\theta = m\ddot{v} \tag{2}$$

$$\ddot{v} = 0 \tag{3}$$

This is the point where we assume whether dry friction will or will not occur. For this example both assumptions will be shown.

Dry Friction does not occur:

$$\ddot{u} = 0 \tag{4}$$
$$F_w \leq \mu_s N \tag{5}$$

the object must be static since dry friction does not occur. In this case an extra constraint equation is introduced to the system. Solving the system with $\ddot{u} = 0$ will give a value for F_w which should be filled in equation (12) to make sure the assumption was correct. If the inequality holds the assumption was correct.

Dry friction does occur:

$$F_w = \mu_k N \tag{4}$$

In case dry friction does occur the object is sliding across a surface. In this case the object is sliding down the plane. After solving the value $\ddot{u}$ should be checked for direction. If the direction is sliding down the plane like we would expect due to gravity excisting the assumption holds. If the direction is opposite to the expected direction the assumption was false. This would imply the object is magically sliding upwards across the surface due to friction. This does not make sense due to the fact that friction always is a reactionary force rather then an external one.

1.4 Applications of calculus

Calculus is among one of the most important tools for any problem involving a dynamical system. In general the posistion and velocity of any system can be described by integration of the "differential equation" that follows from solving the equations of motion. Vice versa differentiation can be used to determin the accelaration, and by extension the forces acting on a given particle or body, from the position. The most important aspects of describing motion with calculus will be listed below. Note that all of these are time dependent.

$$\begin{aligned}
\ddot{x} &= \frac{d\dot{x}}{dt} \quad or \quad a = \frac{dv}{dt} \\
\ddot{x} &= \frac{d^2x}{dt^2} \quad or \quad a = \frac{d^2x}{dt^2} \\
\dot{x} &= \frac{dx}{dt} \quad or \quad v = \frac{dx}{dt} \\
\dot{x} &= \int \ddot{x}\, dt \quad or \quad v = \int a\, dt \\
x &= \int \dot{x}\, dt \quad or \quad x = \int v\, dt
\end{aligned}$$