

8 Lecture 8 (05/03/2020)

8.1 Angular momentum

Imagine a mass on a table that is connected to the middle of the table with a cable. This mass is rotating on the table. Figure 1 (a) and (b) contain the top and isometric view of this situation.

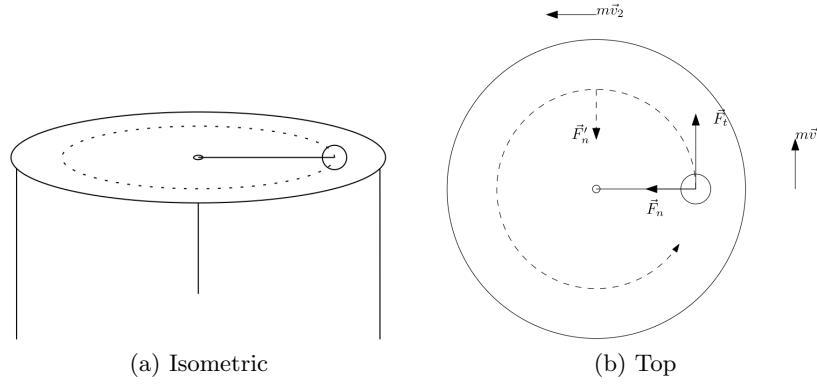


Figure 1: The situation of the mass rotating on the table

This figure leads to the following question: Does conservation of momentum apply to this situation? Recall that conservation of momentum looks like the following if described mathematically:

$$\int_{t_1}^{t_2} \vec{F} dt = m\vec{v}_2 - m\vec{v}_1 = 0 \quad (1)$$

If we look at the Free Body Diagram of the rotating mass we will see that the direction of the forces changes relative to time. This means that the direction of the momentum changes. Since momentum is a vector, a change in direction also implies a change in momentum even if the magnitude of the momentum stays the same. Because of this we instead look at the moment:

$$\vec{M} = \vec{r} \times \vec{F} \quad (2)$$

$$\int_{t_1}^{t_2} \vec{r} \times \vec{F} dt = \vec{r} \times m\vec{v}_2 - \vec{r} \times m\vec{v}_1 \quad (3)$$

The integral $\int \vec{r} \times \vec{F} dt$ is referred to as angular impulse (stootmoment in het Nederlands) and the term $\vec{r} \times m\vec{v}_2 - \vec{r} \times m\vec{v}_1$ is referred to as angular momentum (impulsmoment in het Nederlands). Looking at equation (2) and noting that the resulting moment should be 0 since the forces pass through the center of rotation, we can conclude the following:

$$\int \vec{r} \times \vec{F} dt = 0 \quad (4)$$

Hence angular momentum is conserved.

8.2 Steady fluid flow

Recall Newton's formulation of his second law:

$$\Sigma \vec{F} = \frac{d(m\vec{v})}{dt} \quad (5)$$

if the flow is considered to be a steady flow we know that the velocity does in fact not change. This takes the following form:

$$\Sigma \vec{F} = \vec{v}_2 \cdot \frac{dm}{dt} - \vec{v}_1 \cdot \frac{dm}{dt} \quad (6)$$

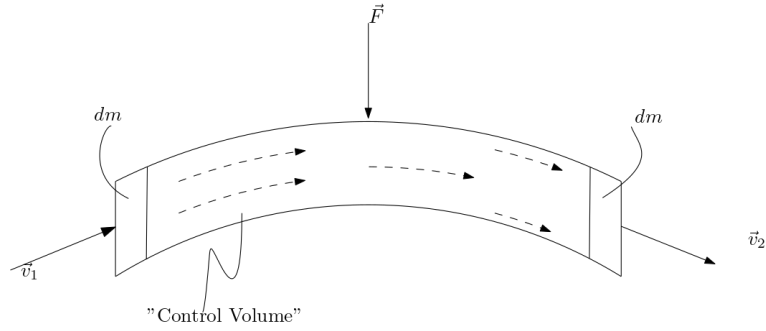


Figure 2: Steady flow of a fluid in a pipe

Since we consider a flow through a pipe we know that the fluid going out is the same as the amount of fluid going in, thus the flow through the pipe is described as an input of dm and an output of the same amount of mass dm . See also figure 3.

The control volume can be considered as an amount of momentum going in and an amount of momentum going out. This implies that a force is required to keep a pipe in place if a fluid is flowing through a pipe. The resulting force \vec{F} from equation (6) represents the amount of force needed to keep the pipe in place. This subject will be discussed in much greater detail in a later course¹.

¹WB1530-14, Thermofluids, TU Delft