2 Continuation of introduction to Rigid Body Dynamics

2.1 Convention for unit notation

Units are denoted after the quantity, not in square brackets. Square brackets denote an operator which extracts the unit from a physical quantity. Examples of correct notation of units are:

$$F = m \cdot \vec{a} = 3\,kg \cdot \vec{a}$$
$$[F] = N$$

2.2 Verification of answers

Assume the following FBD was given: Someone has derrived the equation of motion as:

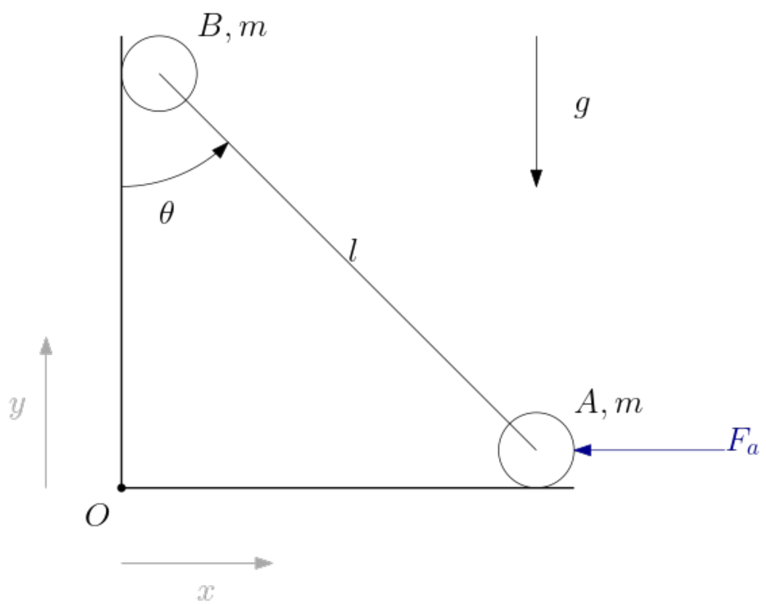

Figure 1: The FBD from which the equation of motion we want to check is derrived

$$\ddot{\theta} = \frac{g}{l}\sin(\theta) - \frac{F_a}{m \cdot l}\cos(\theta)$$

We want to quickly verify whether this is correct, without derriving the EoM ourselves. The first check we can do is assume a special case for which the expected answer is easy to predict. Let's look at the case where $\theta = 0\,\text{rad}$. In this case we also assume $F_a = 0\,N$. Because of this we also expect the value for $\ddot{\theta}$ to be 0 as the ladder will stand upright against the wall. Filling this in we get:

$$\ddot{\theta} = \frac{g}{l}\sin(0\,rad) - \frac{0\,N}{m \cdot l}\cos(0\,\text{rad}) = 0\,\text{rad}/s^2$$

Which checks out with our expectation. As a different type of verification we can also use static equilibrium to find a relation between F_a and the angle θ. This works because the relation between F_a and θ should be the same in both static and dynamic situations. The expression according to statics works out as:

$$F_a = \frac{1}{2}g\tan(\theta)$$

If we find the same relation between F_a and θ we can assume our relation is correct.
Furthermore we can use dimensional analysis as a quick check.

$$
\begin{aligned}
[\ddot{\theta}] &= \frac{[g]}{[l]}[\sin(\theta)] - \frac{[F_a]}{[m]\cdot[l]}[\cos(\theta)] \\
\frac{\text{rad}}{s^2} &= \frac{\cancel{m}/s^2}{\cancel{m}}\cdot 1 - \frac{\cancel{kg}\cdot\cancel{m}/s^2}{\cancel{kg}\cdot\cancel{m}}\cdot 1 \\
\frac{\text{rad}}{s^2} &= \frac{1}{s^2} - \frac{1}{s^2}
\end{aligned}
$$

Which checks out because $1\,\text{rad} = 1\frac{m}{m} = 1$.

2.3 The eqiopllence principle

An equipollent system of force and torque resultants has the same effect on a rigid body as the original system of forces and torques. The equipollent system with all the forces and moments situated at some point O can be found using:

$$
\begin{aligned}
\vec{F}_r &= \sum_{i=1}^{N} \vec{F}_i \\
\vec{M}_r &= \sum_{j=1}^{k} \vec{M}_j + \sum_{i=1}^{N} \vec{r}_i \times \vec{F}_i
\end{aligned}
$$