2 Reliability

2.1 Level 1: Probability analysis

Modelling up untill now has generally be done deterministically. This means some amount of neccesary variables are assumed, found in a table or found empirically. Then a worst case scenario is modeled to find some exact value. This is however not a very realistic approach as worst case scenario's never occur more then in 5% of cases at the very most. It is instead sometimes usefull to convert our deterministic model to a probabilistic one. In this case we often model using a normal distribution (sometimes also called a Bell curve or Gaussian distribution.)
For our case study we will be looking at a part[1] which has a variable coefficient of friction. The models 2 types of models for this are given in the figure below. We want to find the minimal coefficient of

(a) Uniform distribution (b) Gaussian distribution

Figure 2.1: The distribution of possible coefficients of friction of the bearing modeled in 2 different types of distribution

friction of with a maximum of 5% chance to exceed this value. On a uniform distribution we can easily see that this value is 0.16, howver this is very unrealistic. Normally we expect the amount of samples closer the mean value μ to be larger then the amount of samples closer to some amount of standard deviations away from the mean value. When modeling using a normal distribution we know the following values:

$$\mu = 0.25 \quad \sigma = \frac{0.1}{3} \quad f(t) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(\frac{(-t-\mu)^2}{2\sigma^2}\right)$$

We also know that:

$$x_{min} = 0.15$$
$$x_{max} = 0.35$$

We want to find the point t_1 where the area under the bell curve is less then 5%:

$$F(t_1) = \int_{-\infty}^{t_1} f(t)\,\mathrm{d}t$$

When solving this we find that $t_1 = 0.1953$. Having to constantly eveluate the integral at different values to find probabilities is quiet cumbersome. Instead we can apply a standard model of a Gaussian distribution with mean value $\mu = 0$ and standard deviation $\sigma = 1$ and tabulate the results. We call these z-tables. We can find some value for z in the table and then convert that to apply it to our

[1] use your imagination for which part this is.

situation with different μ and σ. An example of a z-table is given below. The equations that apply to a z-table are as follows:

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-z^2}{2}\right)$$

Where:

$$x_{min} = -3$$
$$x_{max} = 3$$

In our case we for finding the coefficient of friction we had:

Table 1: A z-table. This can be expanded using the equations listed above.

$F(z)$	0.5	0.75	0.80	0.90	0.925	0.95
z	0.0	0.67	0.84	1.03	1.44	1.64

$$\mu = 0.5 \quad \sigma = \frac{0.1}{3}$$

For a probability of 5% we find that $z_1 = 1.64$. Converting this back to our value t_1 we want to find is done with the following equation:

$$t' = \mu - z_1\sigma \tag{2.1}$$

Which in our case evaluates back to 0.1953 which is the same as what we found using the intgral. Thus we can conclude that the coefficient of friction will be higher then 0.1953 in 95% of the cases.

2.2 Level 2: Reliability optimization techniques