Course WI4221 in the Spring Semester of 2025
Control of Discrete-Time Stochastic Systems

Homework Set 2

20 February 2025 (Date Homework Set 2 issued)
27 February 2025 (Date solution due)

1. Exercise 3.6.1, Prediction of the state of a Gaussian state process.

2. Consider a recursive system representation of a finite-valued process in the indicator representation,

$$
\begin{aligned}
i_x(t+1) &= A i_x(t) + \Delta m(t),\ i_x(0) = i_{x,0},\\
x(t) &= C_x\, i_x(t),\ \text{ where,}\\
&\quad x : \Omega \times T \to \mathbb{Z}_{n_{i_x}} \subset \mathbb{R},\ \ i_x : \Omega \times T \to \mathbb{R}^{n_{i_x}},\ n_x,\ n_{i_x} \in \mathbb{Z}_+,\\
&\quad \Delta m : \Omega \times T \to \mathbb{R}^{n_{i_x}},\\
0 &= E[\Delta m(t) |\ F_t^{i_x}],\ \forall\, t \in T,\\
&\quad A \in \mathbb{R}^{n_{i_x} \times n_{i_x}},\ \text{a stochastic matrix,}\\
C_x &= \begin{bmatrix} 1 & 2 & \dots & n_{i_x} \end{bmatrix} \in \mathbb{R}^{n_x \times n_{i_x}}.
\end{aligned}
$$

   Prove that x is a Markov process.

Reading Advice

Lecture 2 Presented on 20 February 2025.
Read Chapter 3 – Stochastic processes, all sections but in particular Section 3.4.

Only if you are not familiar with linear systems as treated in control and system theory, then read the following parts: Chapter 21 – Appendix Control and System Theory of Deterministic Systems, the Sections 21.1 – 21.3.

Lecture 3 To be presented on Thursday 27 February 2025. You may read the Sections 4.1, 4.2, and 4.3 before the lecture is presented. This is not required reading.