Structural Analysis

Graphs and Finite State Machines

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April 24th, 2025

TU Delft

Lecture Summary

1. An introduction to graphs

Finite State Machines

2. Structural Bi-partite graphs

Analytical Redundancy Relations

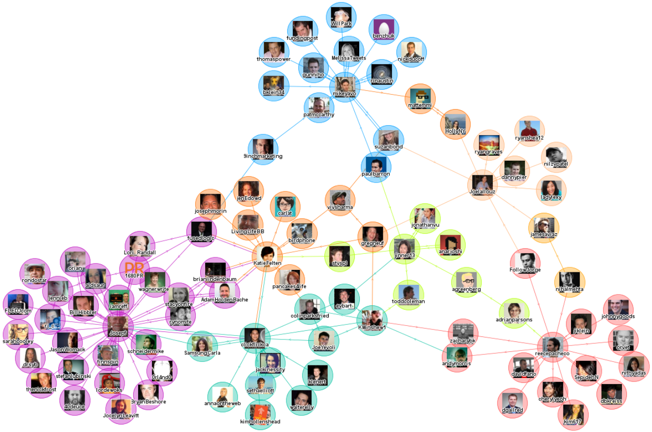
3. Conclusions

An introduction to graphs

AN INTRODUCTION TO GRAPHS

GRAPHS ARE EVERYWHERE

Social graph



AN INTRODUCTION TO GRAPHS

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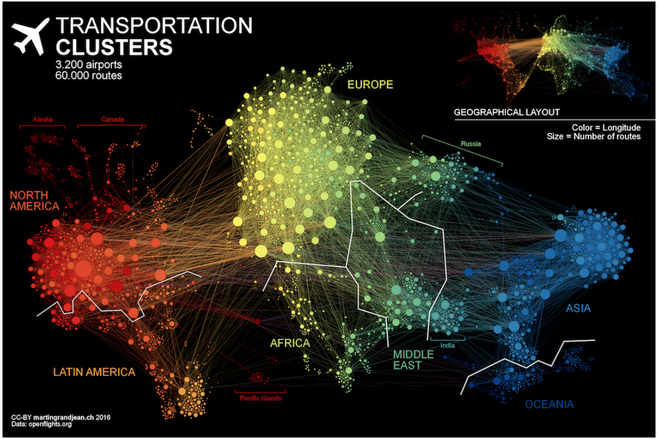
Airline routes graph



AN INTRODUCTION TO GRAPHS

WHAT IS A GRAPH?

Definition (Graph)

$$\mathcal{G} \triangleq (\mathcal{N}, \mathcal{E})$$

$$\mathcal{N} \triangleq \{n_1, \ldots, n_N\} \quad \textbf{nodes}$$

$$\mathcal{E} \triangleq \{e_1, \ldots, e_M\} \quad \textbf{edges}$$

Reinhard Diestel. *Graph Theory*. Springer, 2017

An introduction to graphs

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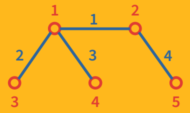
ex.:



i.e. $e_3 = (1, 4)$

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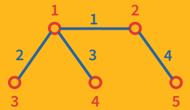


i.e. $e_3 = (1, 4)$

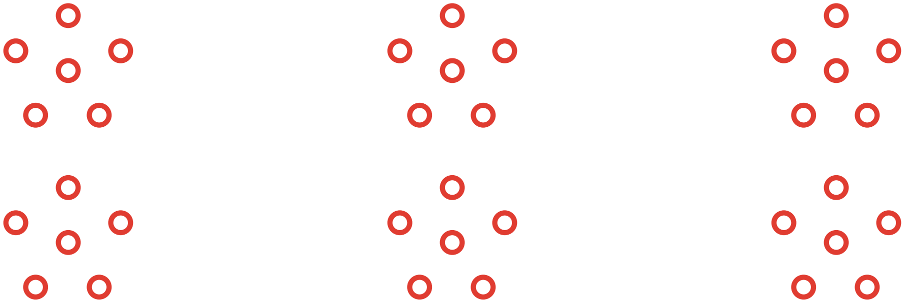
$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$ **Adjacency matrix**

Edges can be oriented!

Reinhard Diestel. *Graph Theory*. Springer, 2017

An introduction to graphs

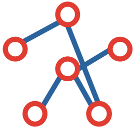
Some types of graphs



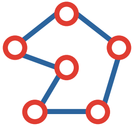
An introduction to graphs

Some types of graphs

Random

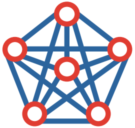


Regular

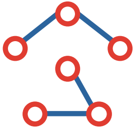


degree = 2

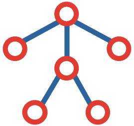
Complete



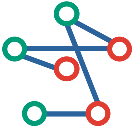
Disconnected



Tree



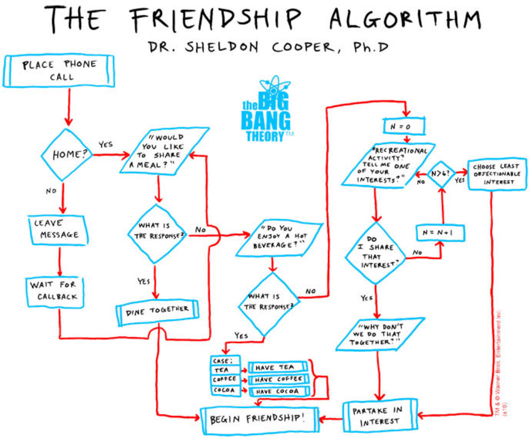
Bipartite



An introduction to graphs

Graphs can represent dynamical behaviour

A flow-chart is a graph whose edges are conditionally traversed



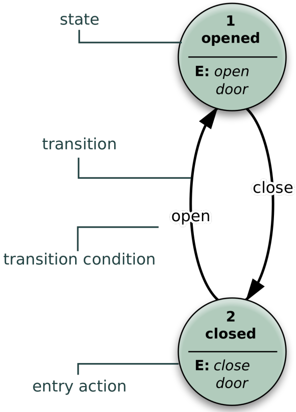
An introduction to graphs

Graphs can represent dynamical behaviour

A state diagram or state-transition graph represents a

Finite Automaton

- **Nodes** $\rightarrow$ states
- **Edges** $\rightarrow$ transitions
  - instead of a static 1 or 0 value, **boolean function** of an input
- **Extra**: output function per each state



Taylor L Booth. *Sequential machines and automata theory*. Vol. 20. Wiley New York, 1967

SC42155 - Modelling of Dynamical Systems

Structural Bi-partite graphs

Structural Bi-partite graphs

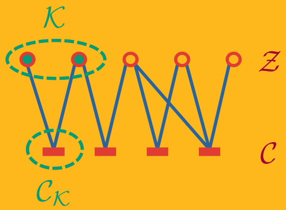
Bi-partite graphs

Definition (Bipartite Graph)

$$\mathcal{G} = ((\underbrace{\mathcal{C}}_{\text{Constraints}}, \underbrace{\mathcal{Z}}_{\text{Variables}}), \mathcal{E})$$

$$\mathcal{Z} = \{\underbrace{\mathcal{K}}_{\text{Known}}, \underbrace{\mathcal{X}}_{\text{Unknown}}\}$$

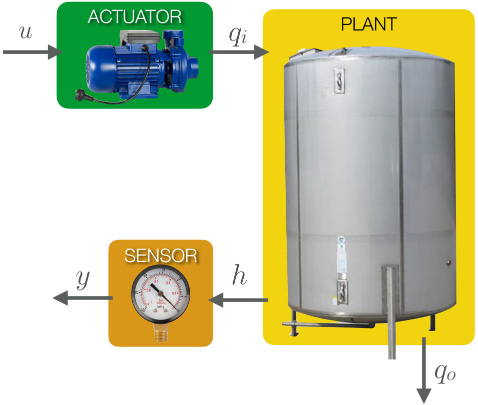
$$\mathcal{C} = \{\underbrace{\mathcal{C}_{\mathcal{K}}}_{\text{Only known variables}}, \mathcal{C}_{\mathcal{X}}\}$$



Reinhard Diestel. *Graph Theory*. Springer, 2017

STRUCTURAL BI-PARTITE GRAPHS

SINGLE TANK



Mogens Blanke et al. *Diagnosis and fault-tolerant control*. Vol. 2. Springer, 2006

STRUCTURAL BI-PARTITE GRAPHS

SINGLE TANK

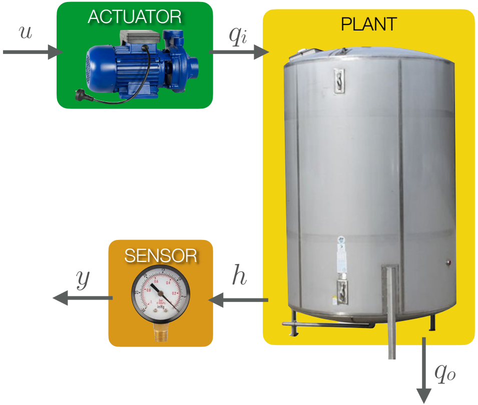
Tank c_1 : $\dot{h}(t) = q_i(t) - q_o(t)$

Input valve c_2 : $q_i(t) = \alpha u(t)$

Output pipe c_3 : $q_o(t) = k\sqrt{h(t)}$

Level sensor c_4 : $y(t) = h(t)$

Integration c_5 : $h(t) = \int \dot{h}(t)$



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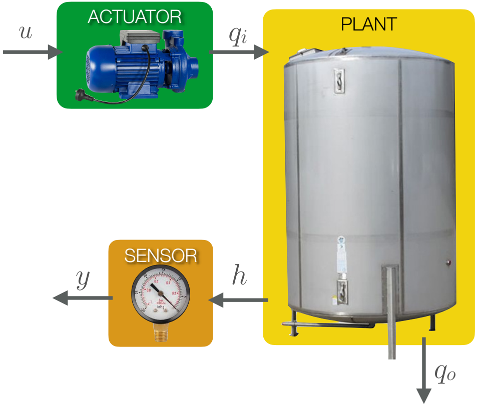
Structural Bi-partite graphs

Single tank

	Known				Unknown			
	u	α	k	y	h	$\dot{h}$	q_i	q_o
c_1						1	1	1
c_2	1	1					1	
c_3			1		1			1
c_4				1	1			
c_5					1	1		



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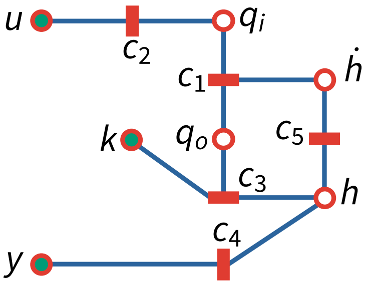

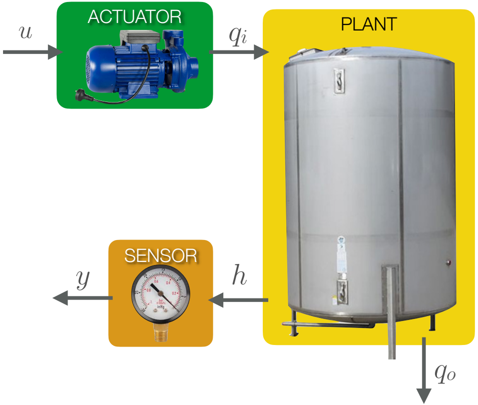

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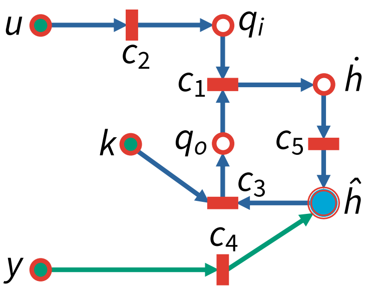
Structural Bi-partite graphs

Analytical Redundancy Relation



- We can compute two estimates $\hat{h}$ of the tank level
  - Using the **dynamical model** and the known variables u and k
  - Using the **output equation** and the known variable y
- **Analytical Redundancy Relation**

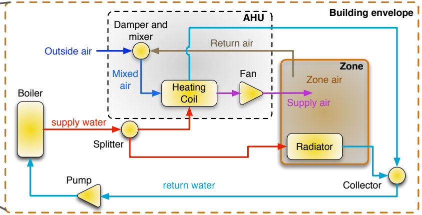
Can be used to **detect** anomalies in the constraints c_i.

Mogens Blanke et al. *Diagnosis and fault-tolerant control*. Vol. 2. Springer, 2006

Structural Bi-partite graphs

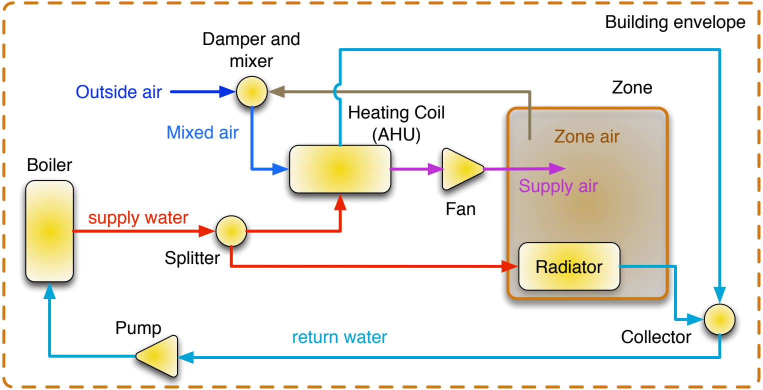
A more complex example: HVAC system

HVAC-system



Structural Bi-partite graphs

A more complex example: HVAC system



Structural Bi-partite graphs

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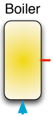
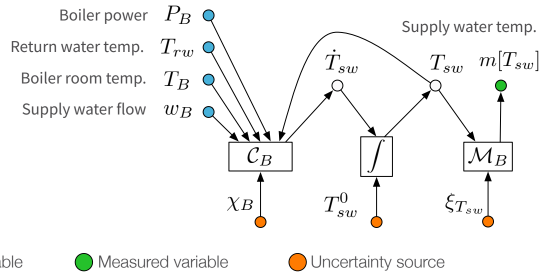


Boiler dynamics

$$\begin{cases} \dot{T}_{sw} &= (c_w \rho_w V_B)^{-1}(P_B + c_w w_B(T_{rw} - T_{sw}) \\ & +h_B A_B (T_B - T_{sw})) + \chi_B \\ m[T_{sw}] &= T_{sw} + \xi_B \end{cases}$$

Boiler bi-partite structural graph



Structural Bi-partite graphs

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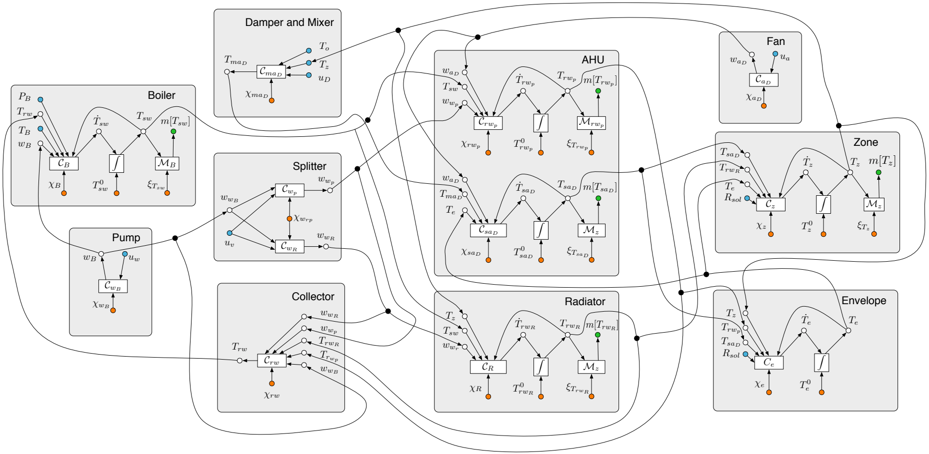

Conclusions

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In this lecture we covered

- Overview of **graphs**
- Graphs can be extended to describe a **Finite Automaton**
- **Bipartite graphs** can be used to describe the **structure** of a system
- **ARRs** can be used to **detect** anomalies, by checking consistency of subsets of constraints

Next lecture: **System structural analysis and components and services model**

CONCLUSIONS

THANK YOU FOR YOUR ATTENTION!

For further information:

Course page on **Brightspace**

or

our **MS Team**