Dynamic Programming & Stochastic Control (SC42110)
Exercise Set 0

Friday 25^{th} April, 2025

Exercise 1. Consider a function $p : \mathbf{R} \to \mathbf{R}$ defined by

$$p(\xi) = \begin{cases} ae^{-a\xi+b} & \text{if } \xi \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Find all pairs of parameters (a, b) for which $p(\cdot)$ is a probability density on the real line. Find the expectation and variance of the distribution for every such pair (a, b).

Exercise 2. Three black and three white balls are distributed between three urns. Urn 1 contains one black and one white balls, urn 2 contains two black balls, and urn 3 contains the remaining two white balls. See Figure 1. One chooses an urn at random and draws one ball from it. Suppose that this ball is white. Which of the events below is more probable?

- A: The remaining ball in the chosen urn is white.
- B: The remaining ball in the chosen urn is black.

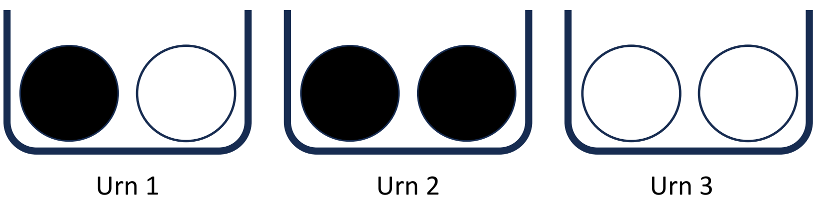

Figure 1: Three urns including three black and three white balls. See Exercise 2.

Exercise 3. Assume that 0.1% of a particular population has Covid. Also, the available Covid test has a 95% reliability, that is, the false positive rate (i.e., the probability of a positive test result for a healthy individual) and the false negative rate (i.e., the probability of a negative result for an infected individual) are both 5%. What is the probability that an individual is infected given that their test result is positive?

Exercise 4. Prove Lemmas A.5 and A.6.

Exercise 5. Prove the equality (A.1).

Exercise 6. Use induction to prove that

$$1^2 + 2^2 + \ldots + k^2 = \frac{1}{6}k(k+1)(2k+1), \quad \forall k \in \mathbf{N}.$$