SIGNAL BASED METHODS

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Lecture 4.1a-b
14/05/24

OVERVIEW

What are we going to talk about today ?

> Main ideas and some examples

> Time based methods

> Frequency based methods

> Compression based methods: PCA

RECAP

Let us put the method in context

- What does it mean to do signal-based fault diagnosis
- Main steps and “characters”

RECAP

Remember the parallel with medical diagnosis?

SYSTEM

observations

FEATURES EXTRACTION

SYMPTOMS GENERATION

SYMPTOMS EVALUATION

DIAGNOSIS

Today

Change Detect.

KNOWLEDGE BASE

> **Nominal** behaviour
  > **Physiological** uncertainty
> Class of *N* **faulty** behaviours
  > **Pathological** limits

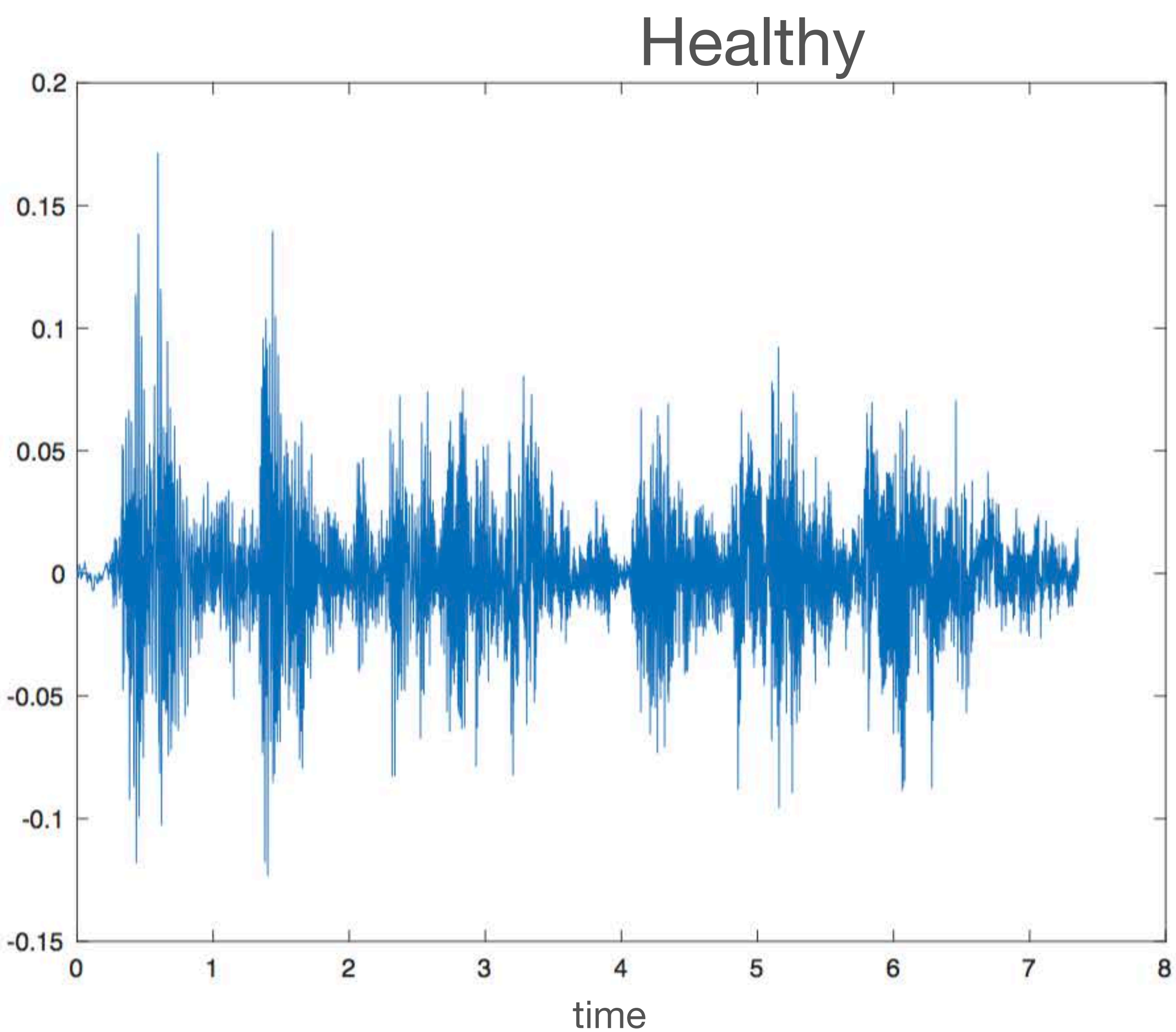
INTRODUCTION

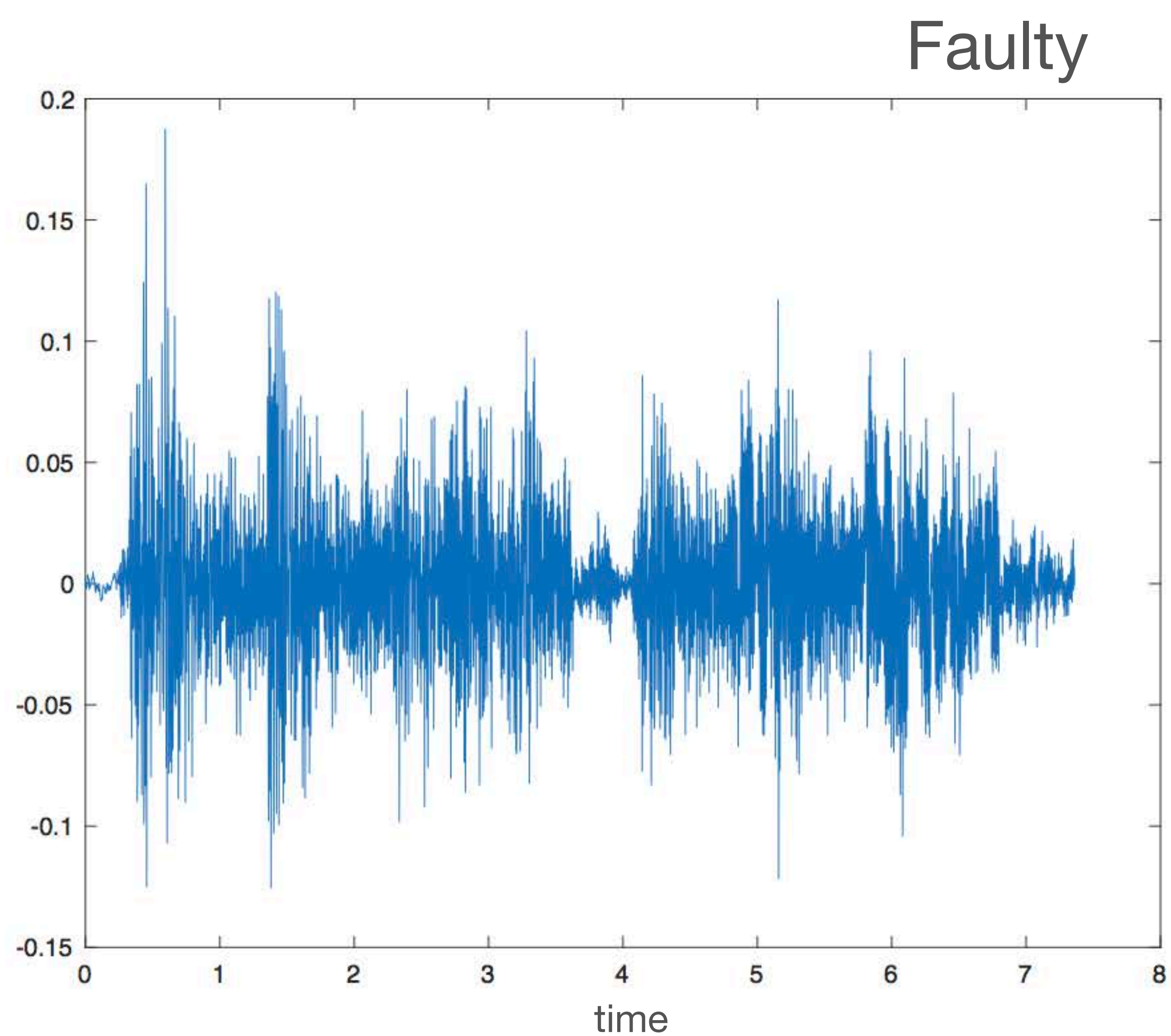
Let us present some intuitive examples

> Signals that we can hear and see
>

INTRODUCTION

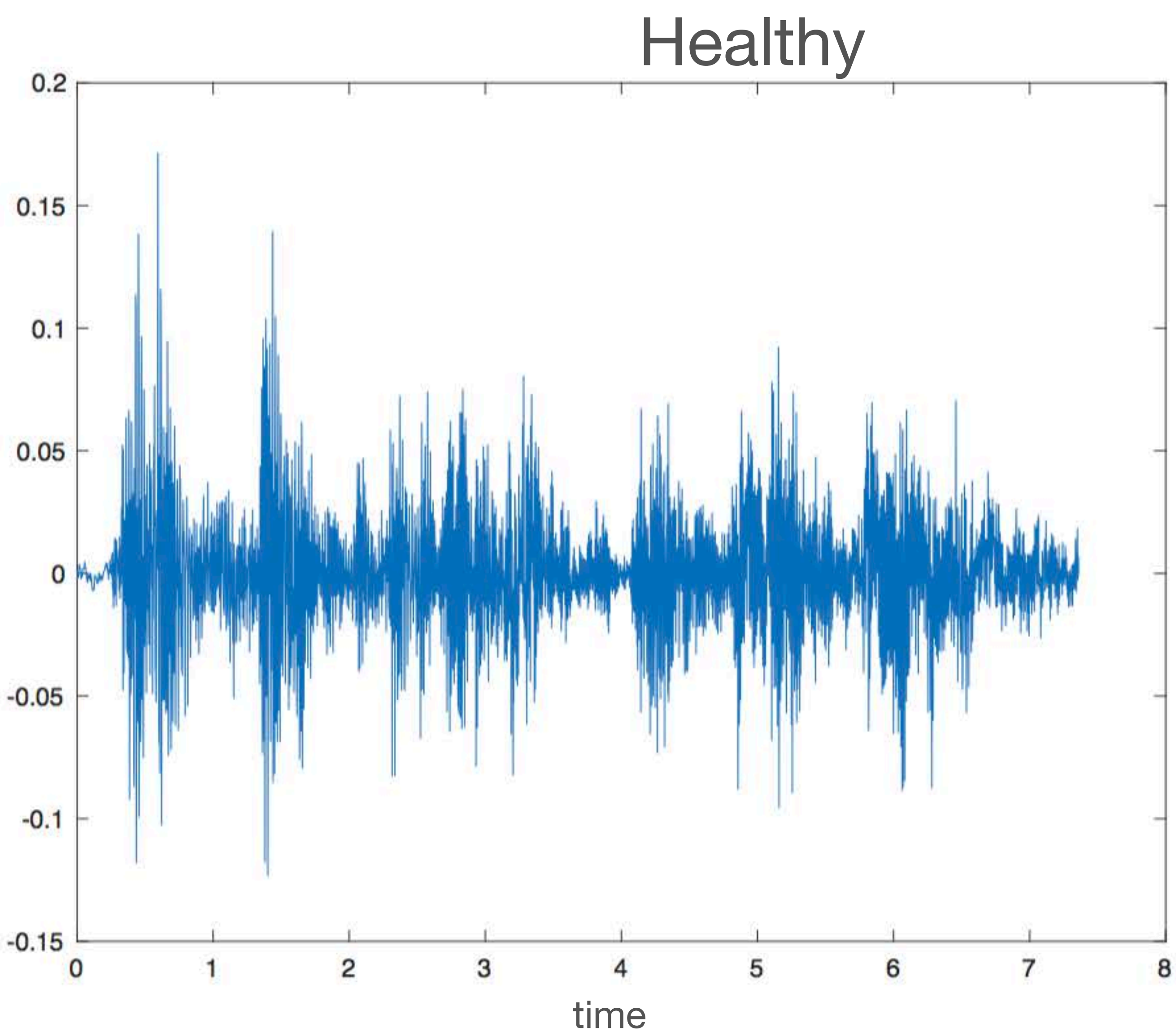
Example: can you spot a fault in these waveforms?





INTRODUCTION

No? Try listening



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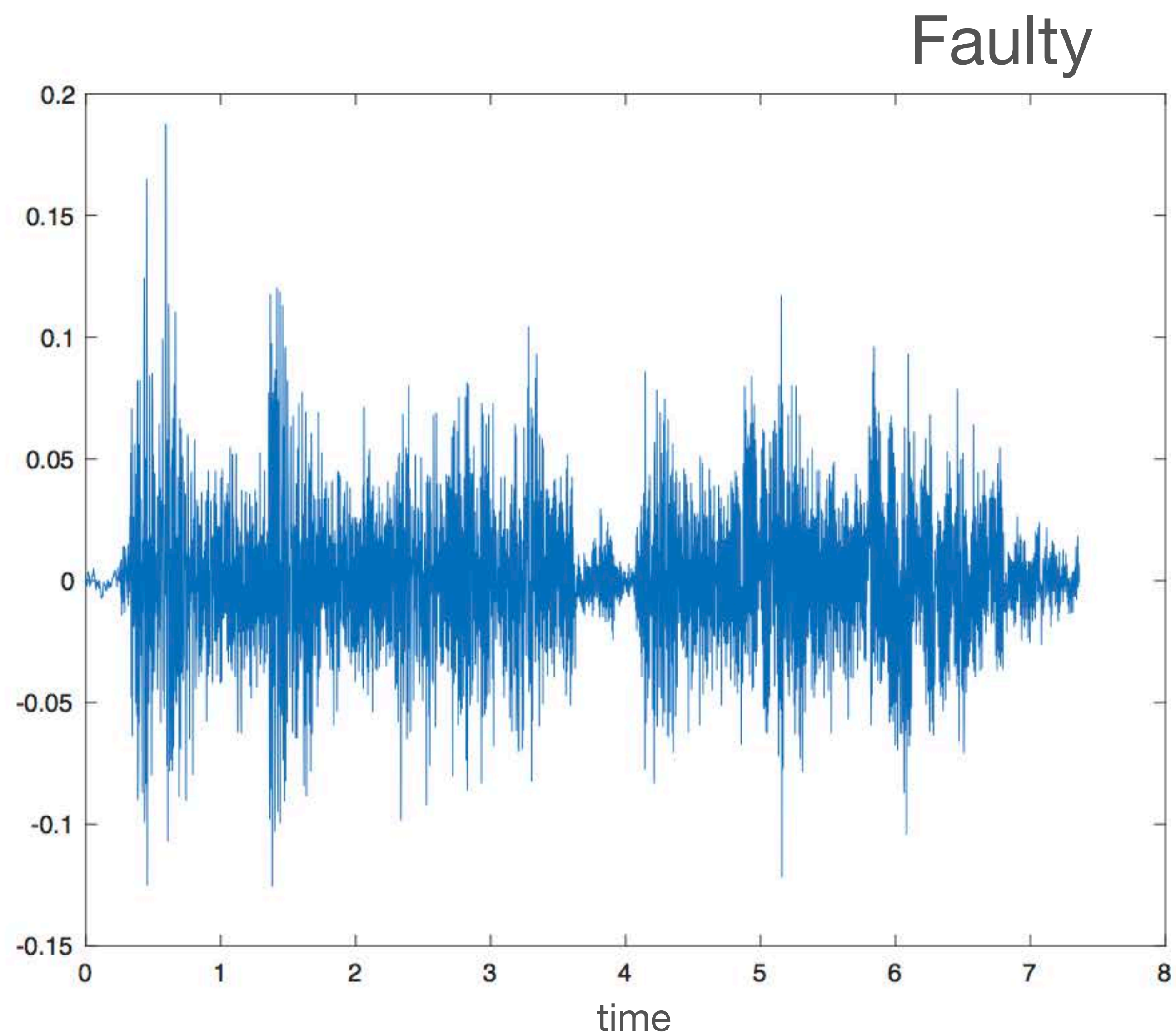

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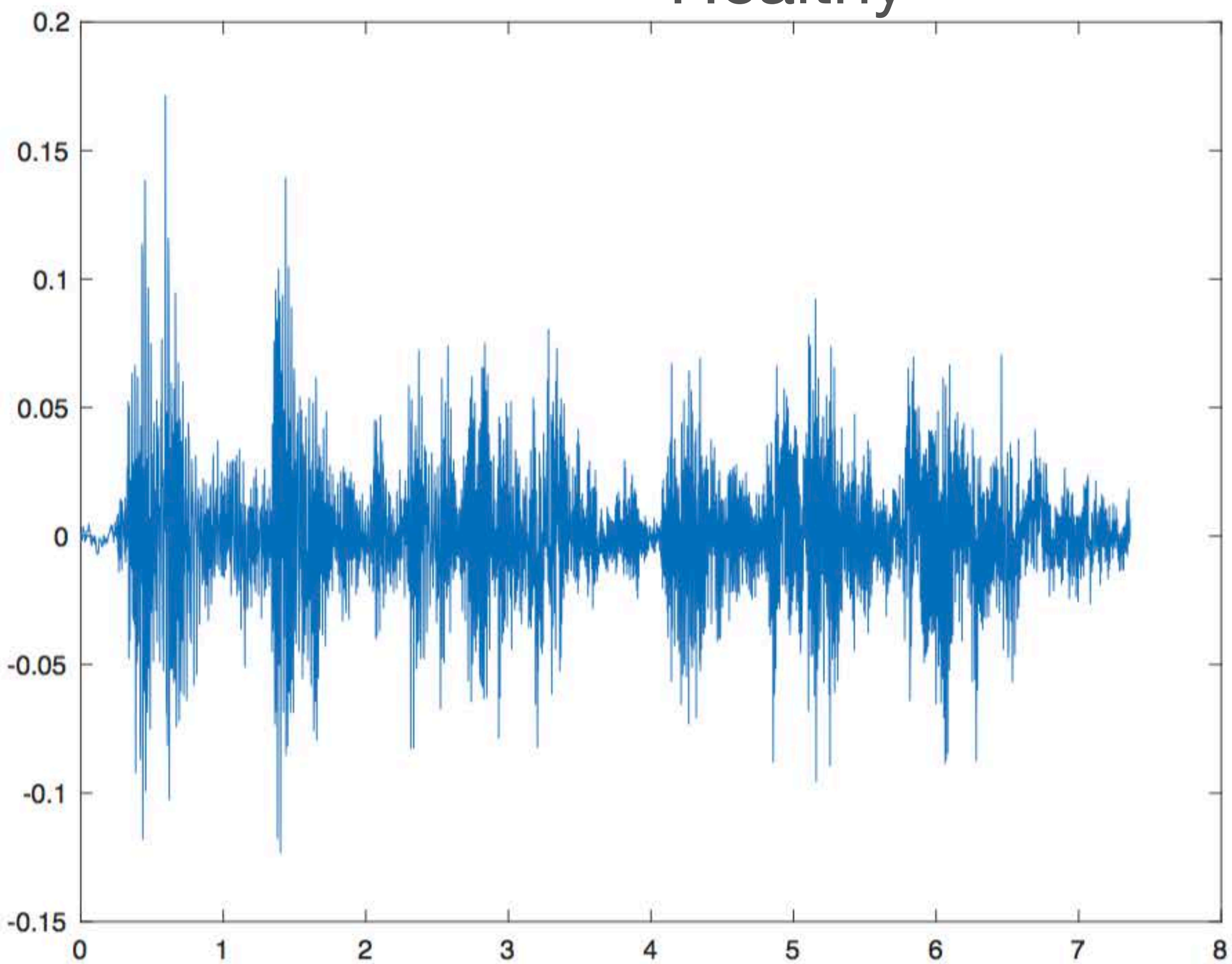
INTRODUCTION

No? Try listening

Healthy



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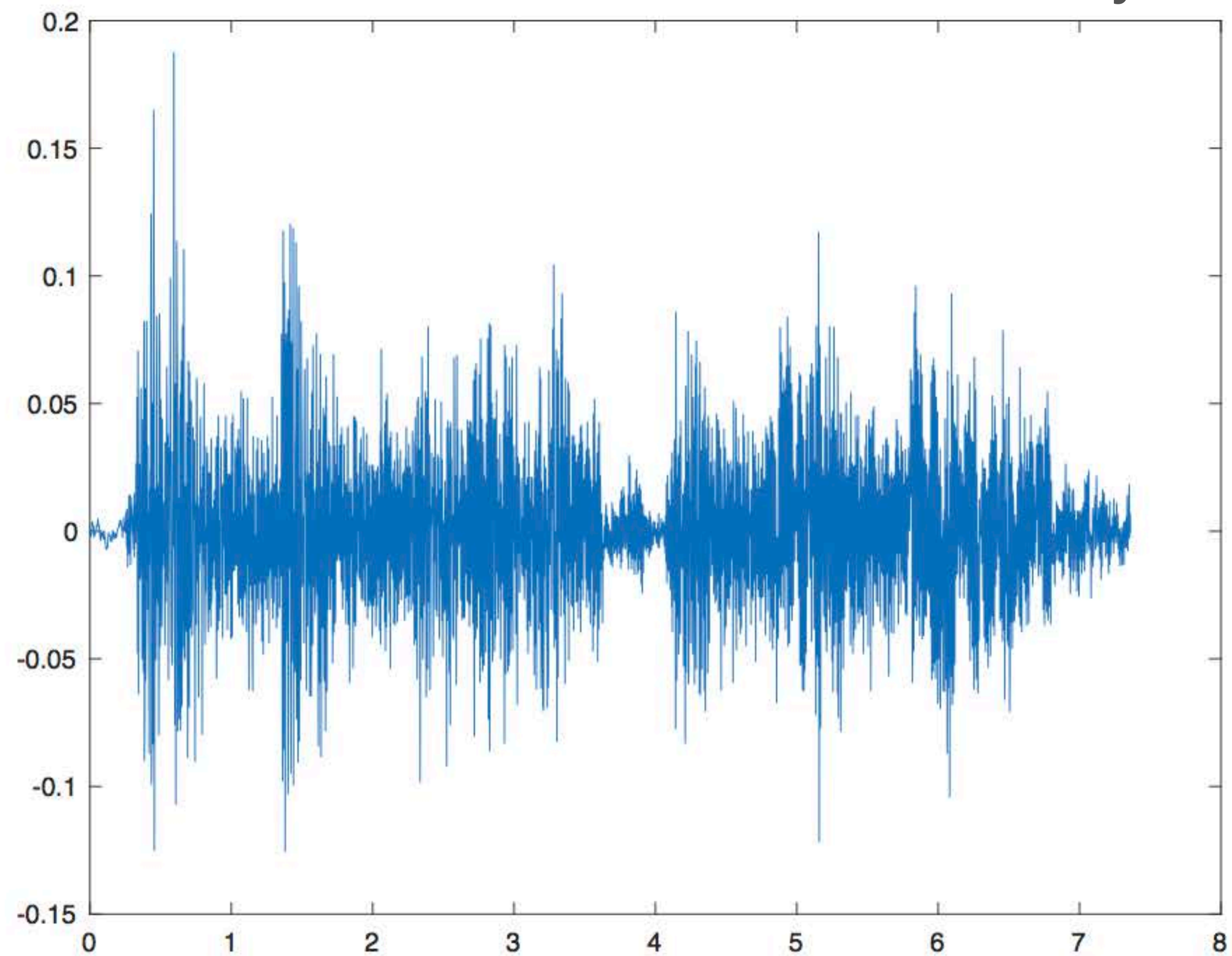
Faulty



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INTRODUCTION

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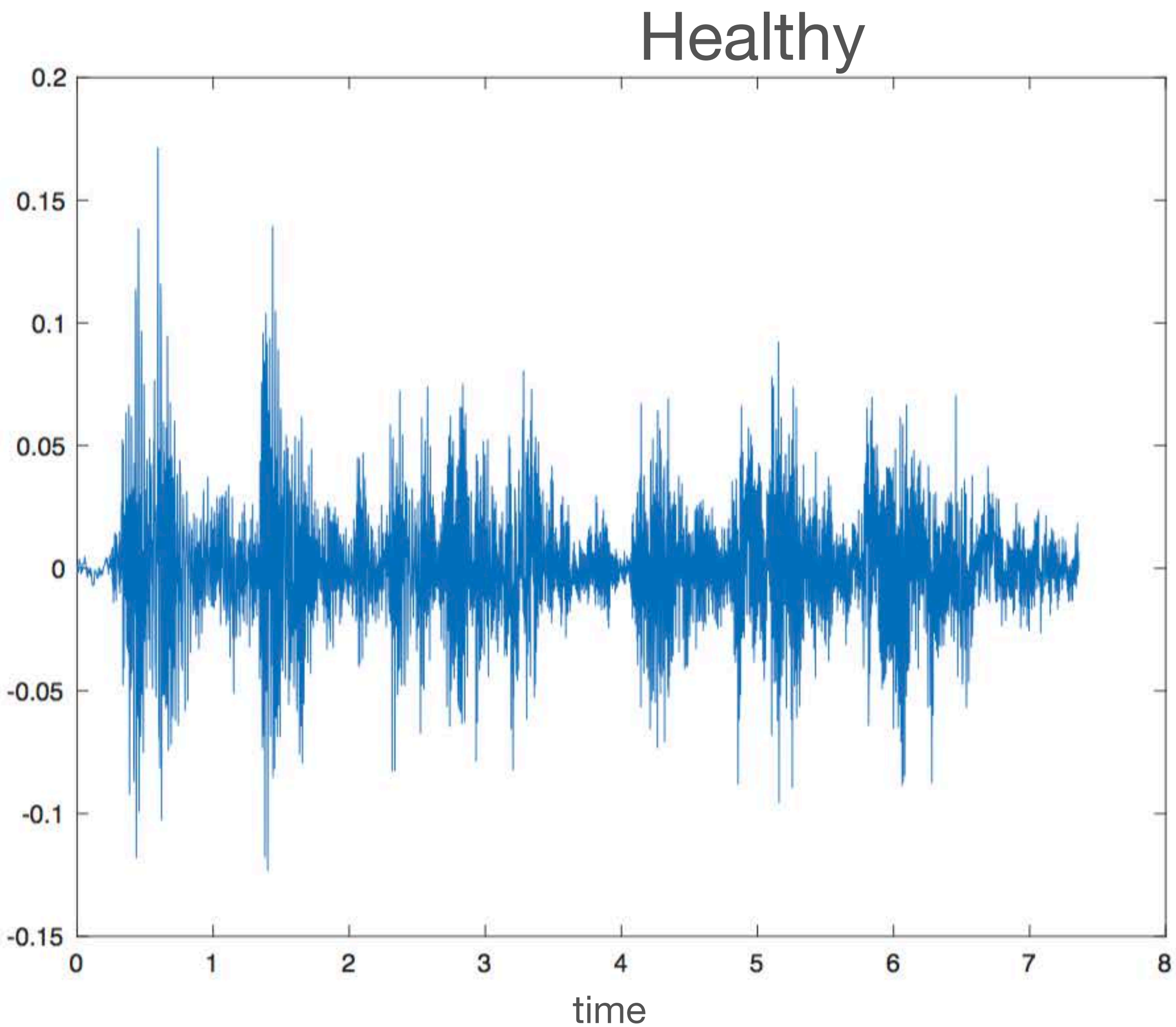

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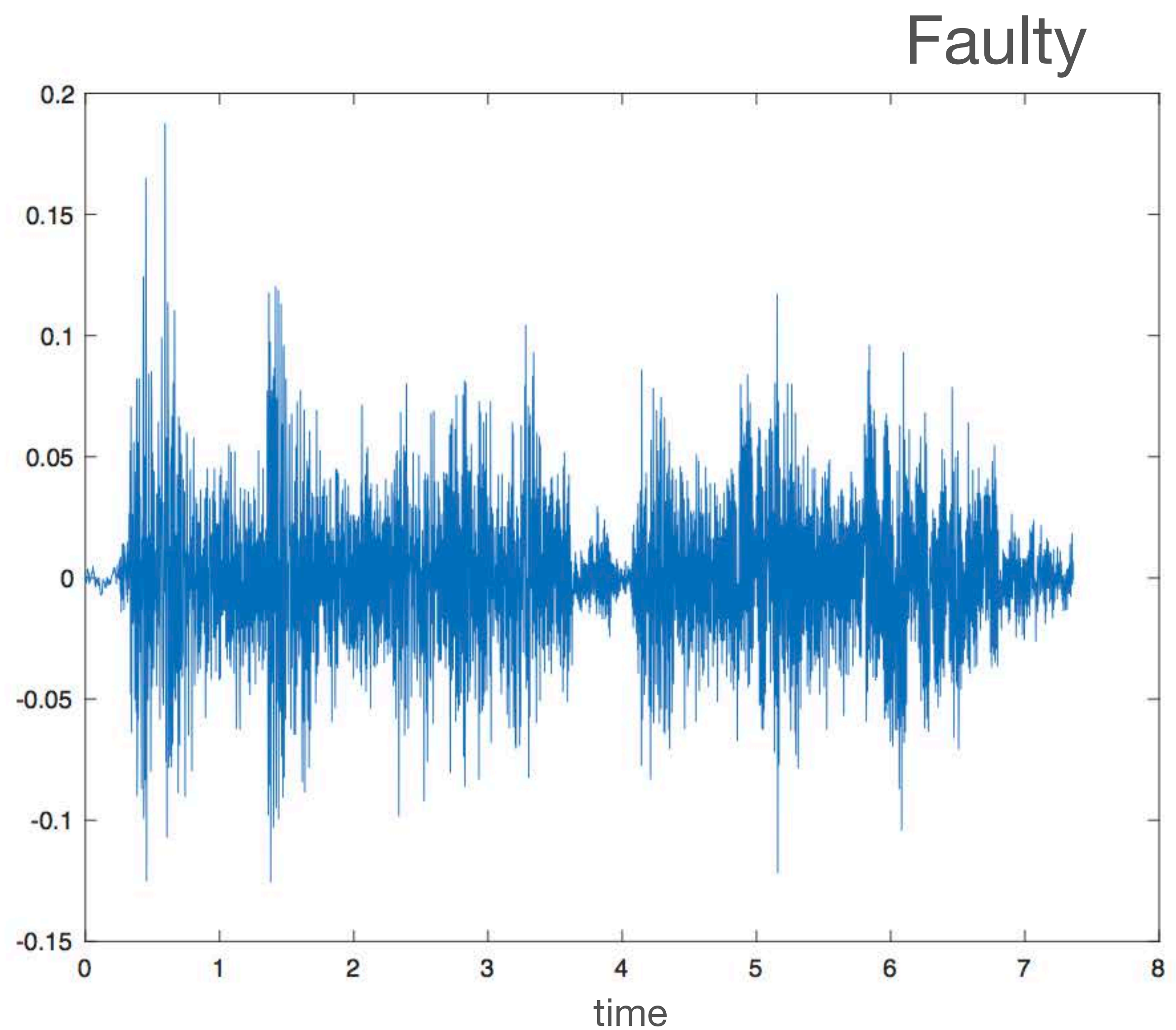

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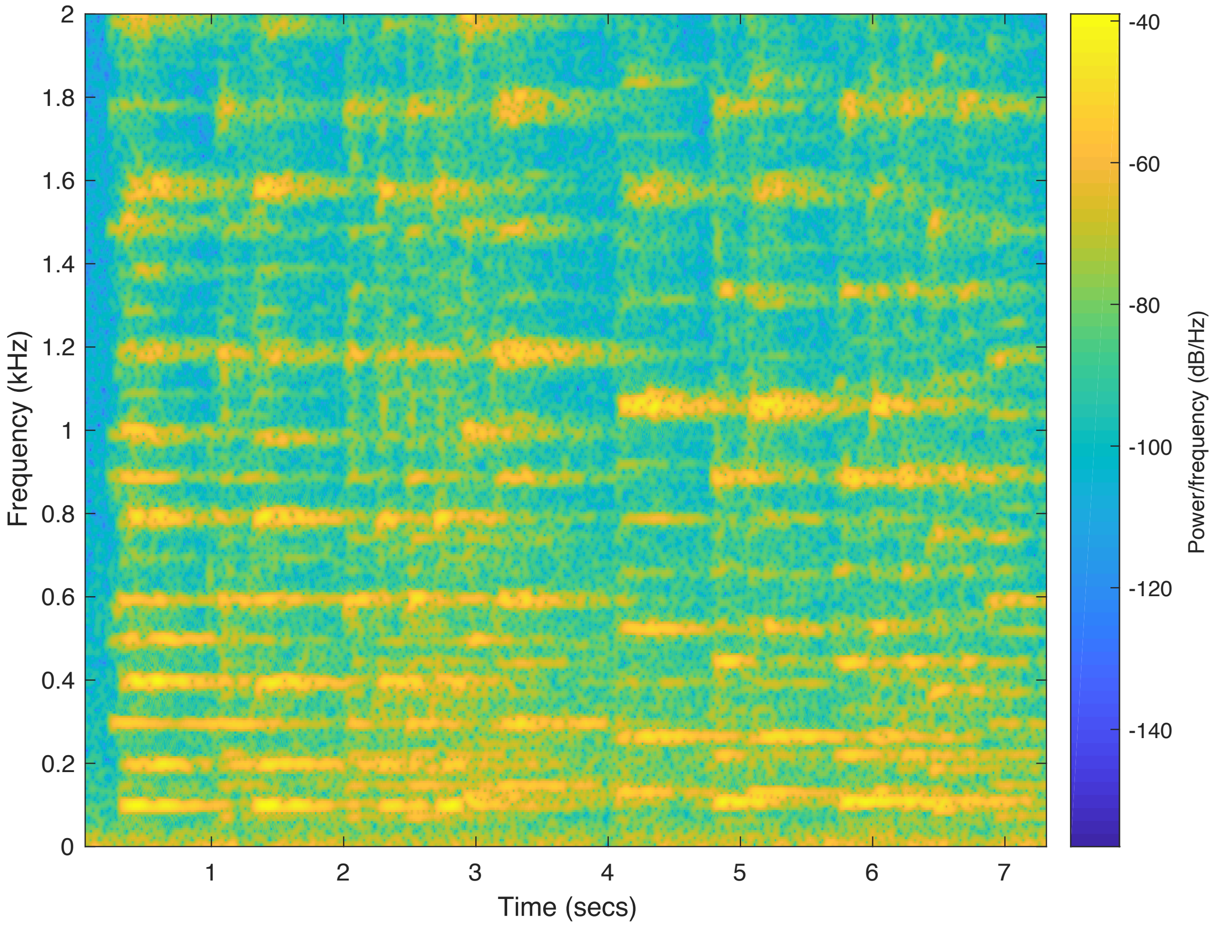
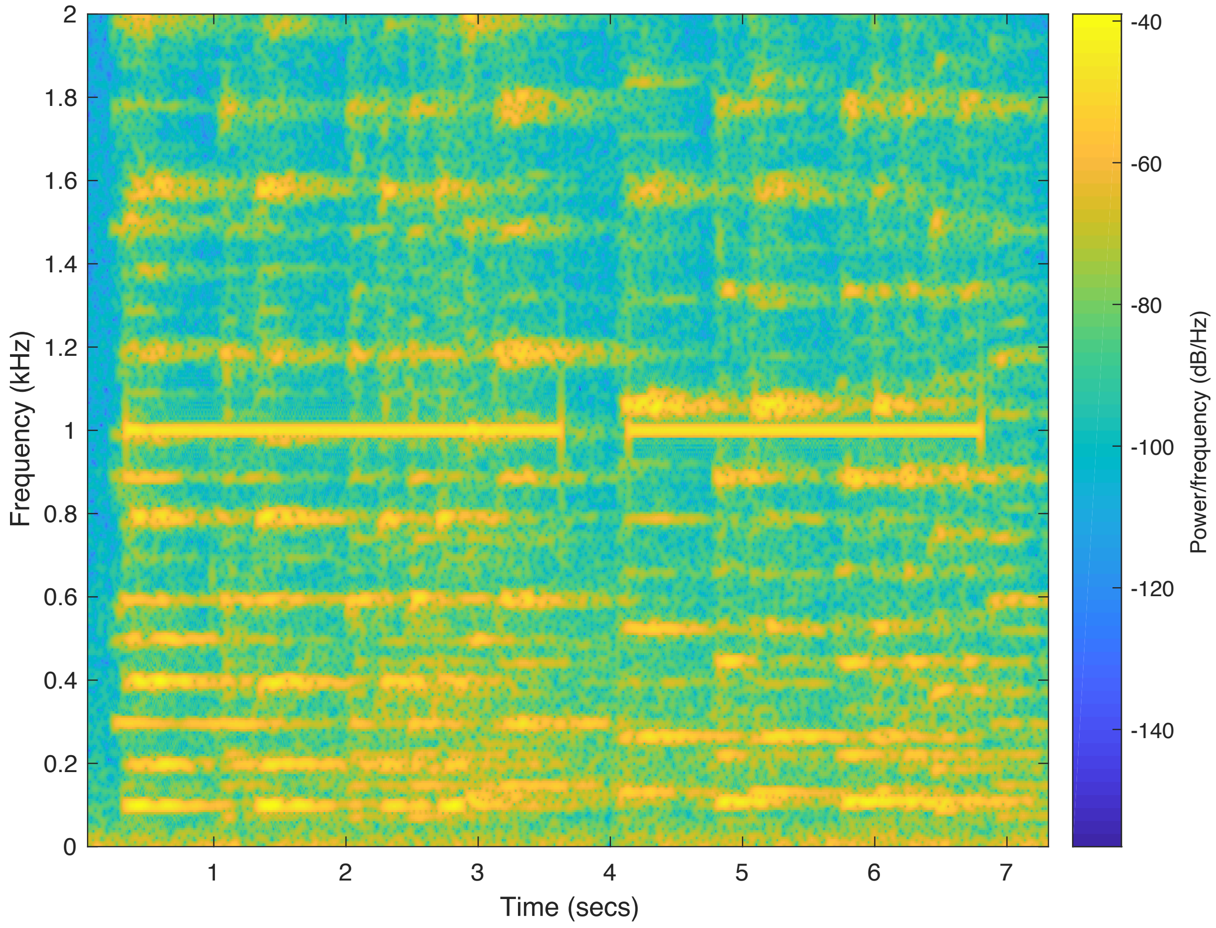
INTRODUCTION

Now, let us see what you just heard

Healthy



Faulty



INTRODUCTION

Another example

https://www.youtube.com/watch?v=M0FaKg0RZVA

INTRODUCTION

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INTRODUCTION

Luckily self driving cars are aware of the problem …

http://fcauthority.com/2017/05/see-waymos-self-cleaning-lidar-system-in-action/

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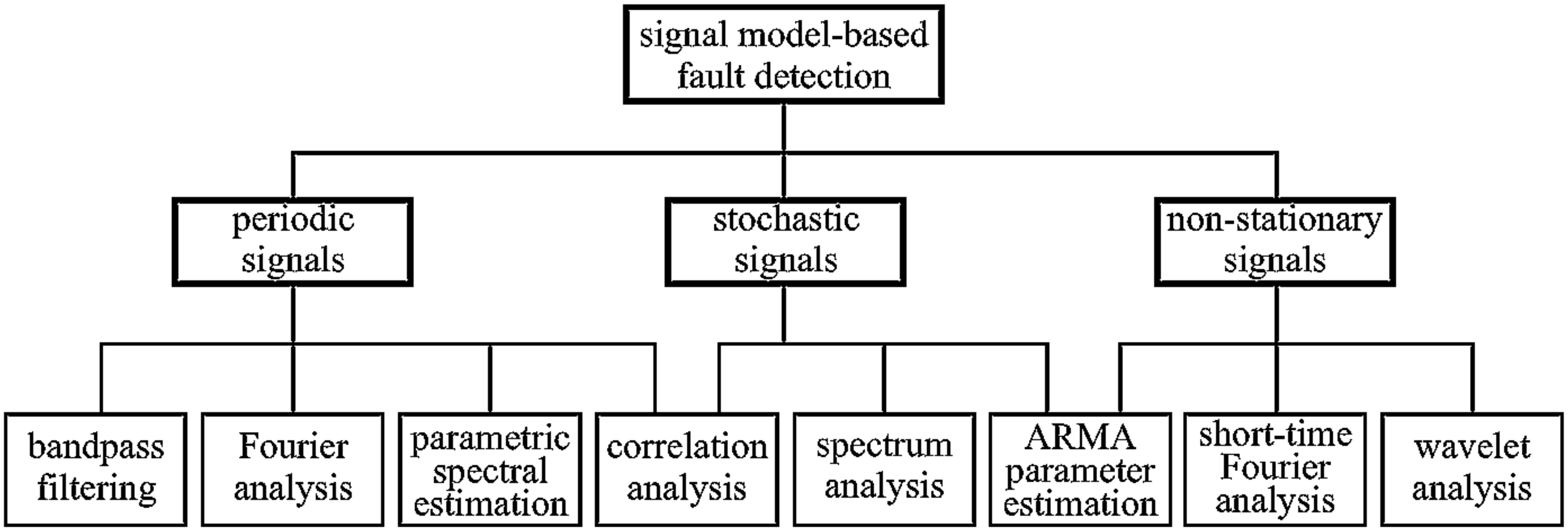
INTRODUCTION

Open question: think about how to write an algorithm that detects those two “faults”

OVERVIEW

Taxonomy of methods



[**IS06**] R. Isermann, *Fault-diagnosis systems: an introduction from fault detection to fault tolerance*. Springer Science & Business Media, 2006.

TIME ANALYSIS

Methods for analysis in time domain

TIME ANALYSIS

List of approaches

- All of change detection methods seen in previous lecture
  - applied to instantaneous raw values
  - applied to some evaluation function
- Some specific examples
  - Band-pass filtering
  - Cross-correlation
  - Auto-correlation

TIME ANALYSIS

Visual explanation

TIME ANALYSIS

Auto-correlation

$$\boxed{R_{yy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T y(t)\, y(t+\tau)\, dt}$$

Example

$$y_u(t) = y_{0\nu} \sin(\omega_\nu t + \varphi_\nu) + n(t)$$

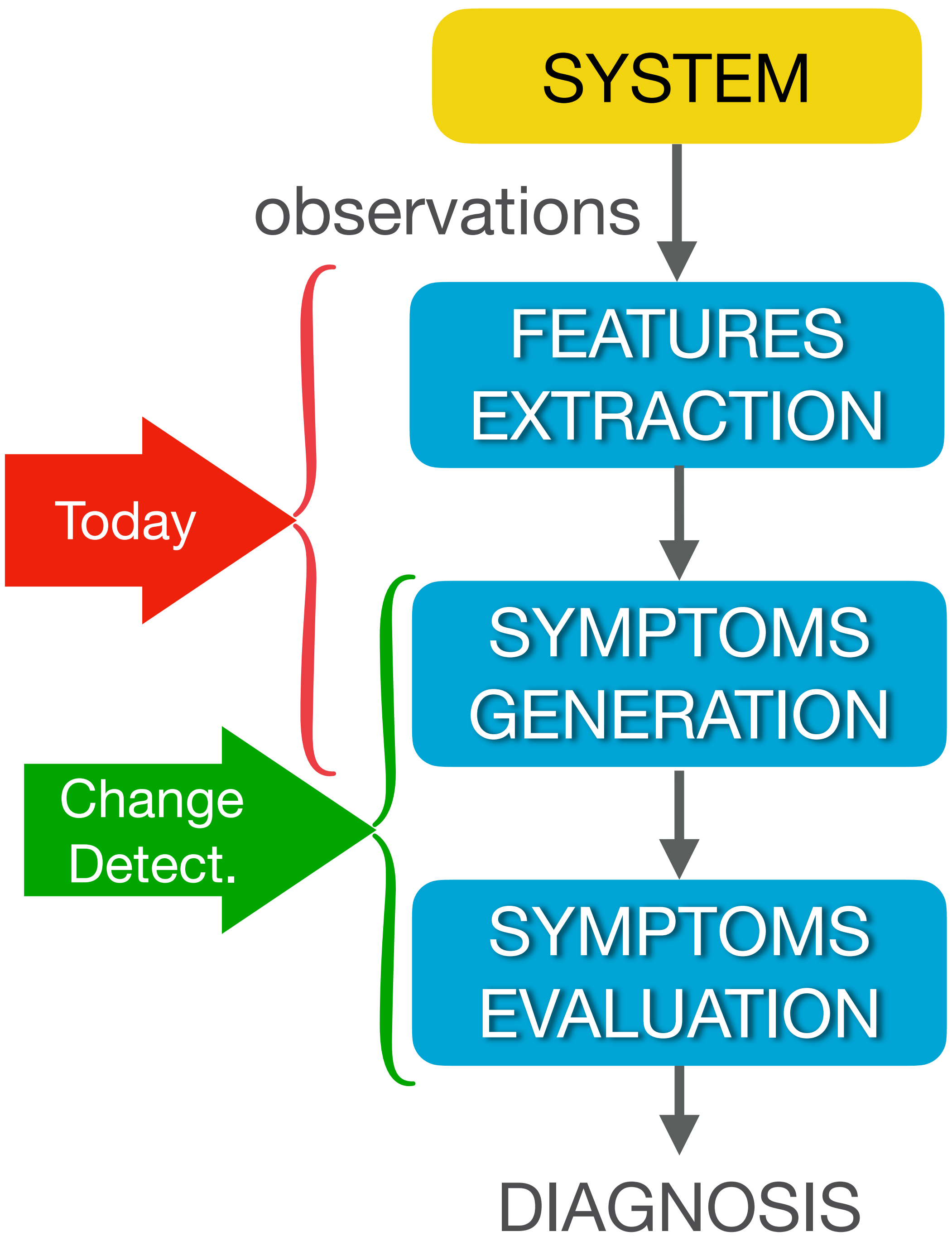
$$R_{yy}(\tau) = \frac{y_{0\nu}^2}{2} \cos\omega_\nu \tau$$

FREQUENCY ANALYSIS

A brief introduction

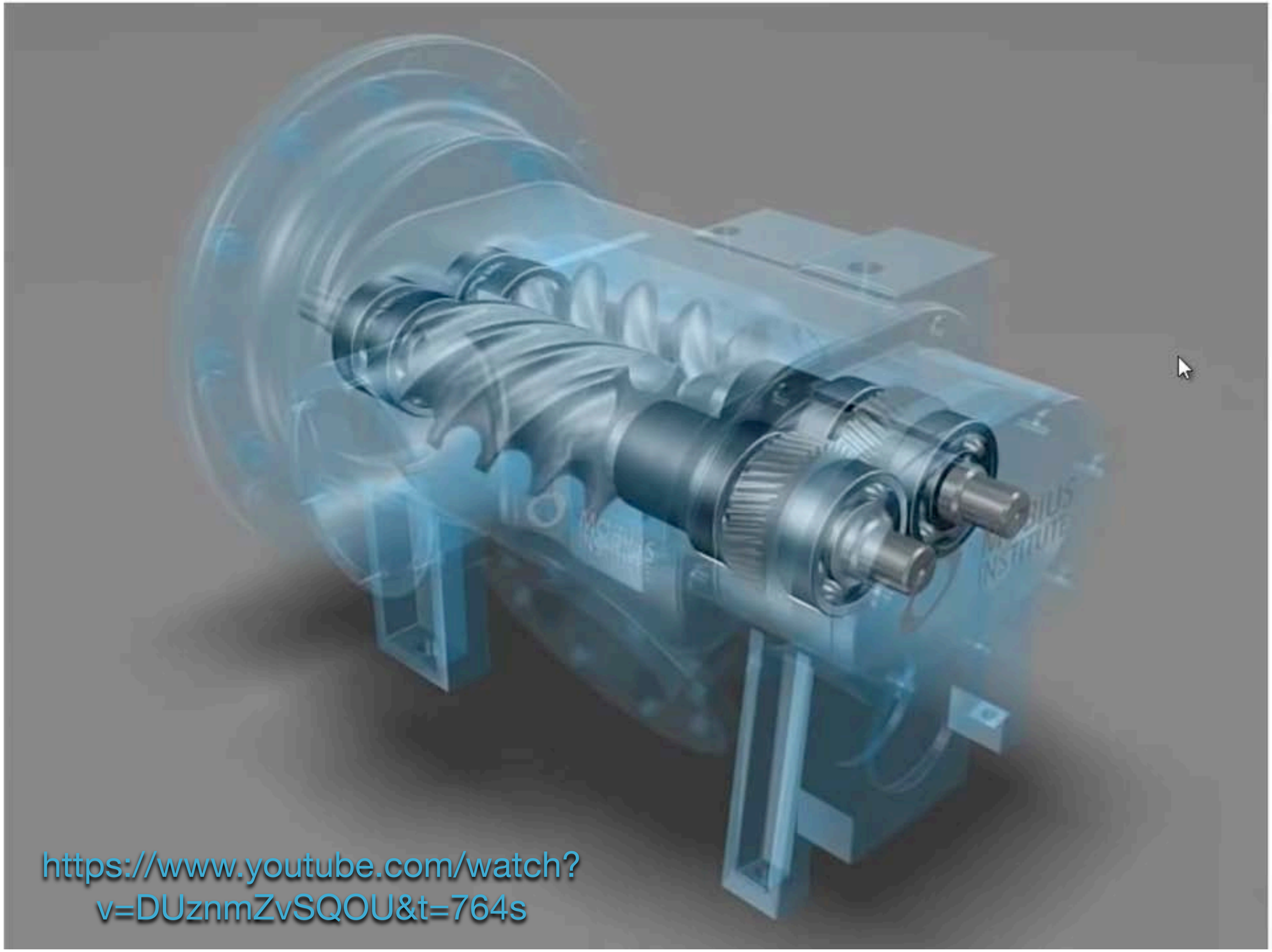
RECAP

Remember the parallel with medical diagnosis?



FREQUENCY ANALYSIS

Interest was mainly fuelled by vibration analysis



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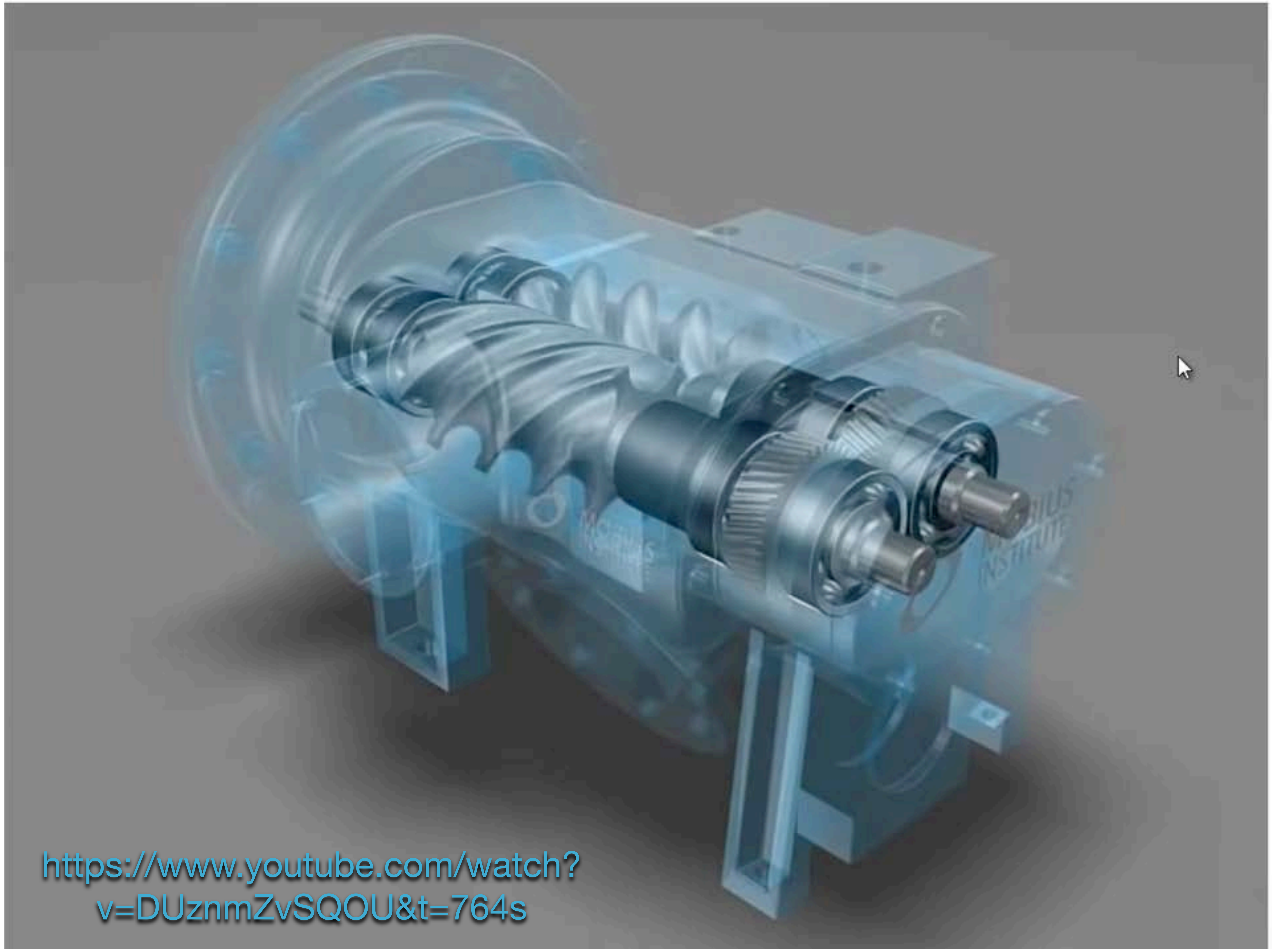

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FREQUENCY ANALYSIS

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Vibration basics

- A waveform comes from the sensor (e.g. accelerometer)
- A single motion generates a sine wave with a single frequency and amplitude
- Vibration from additional components adds additional frequencies

FREQUENCY ANALYSIS

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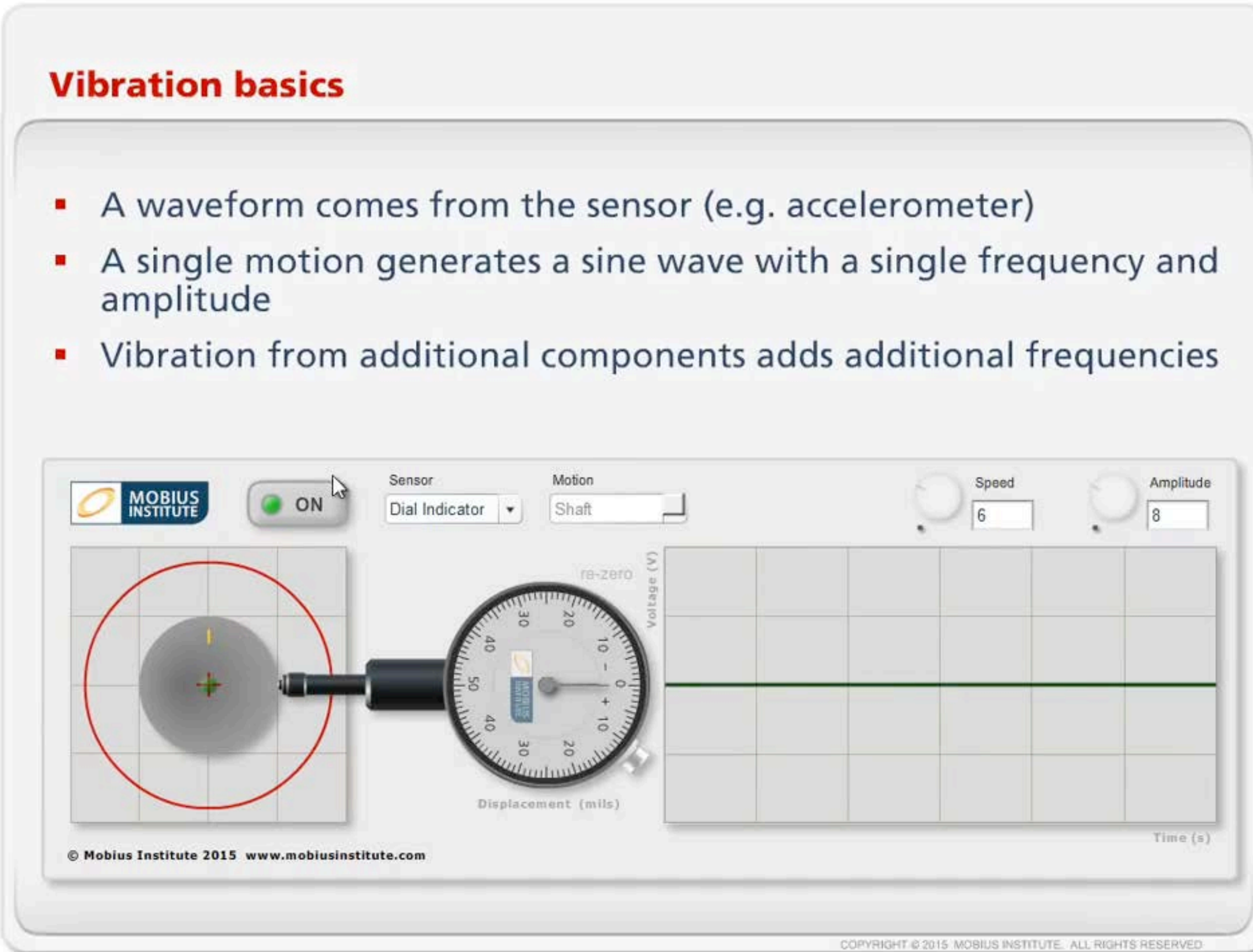

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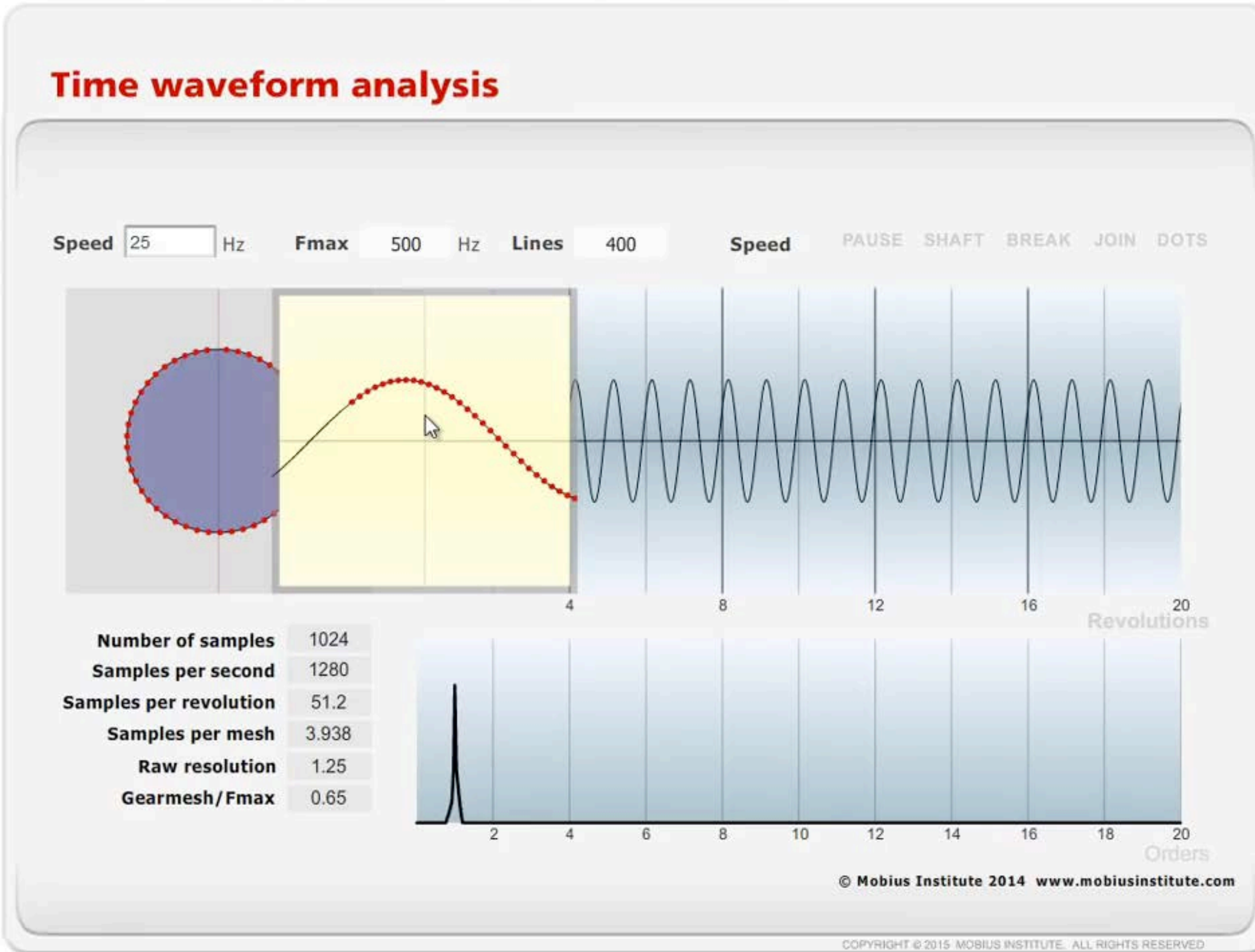

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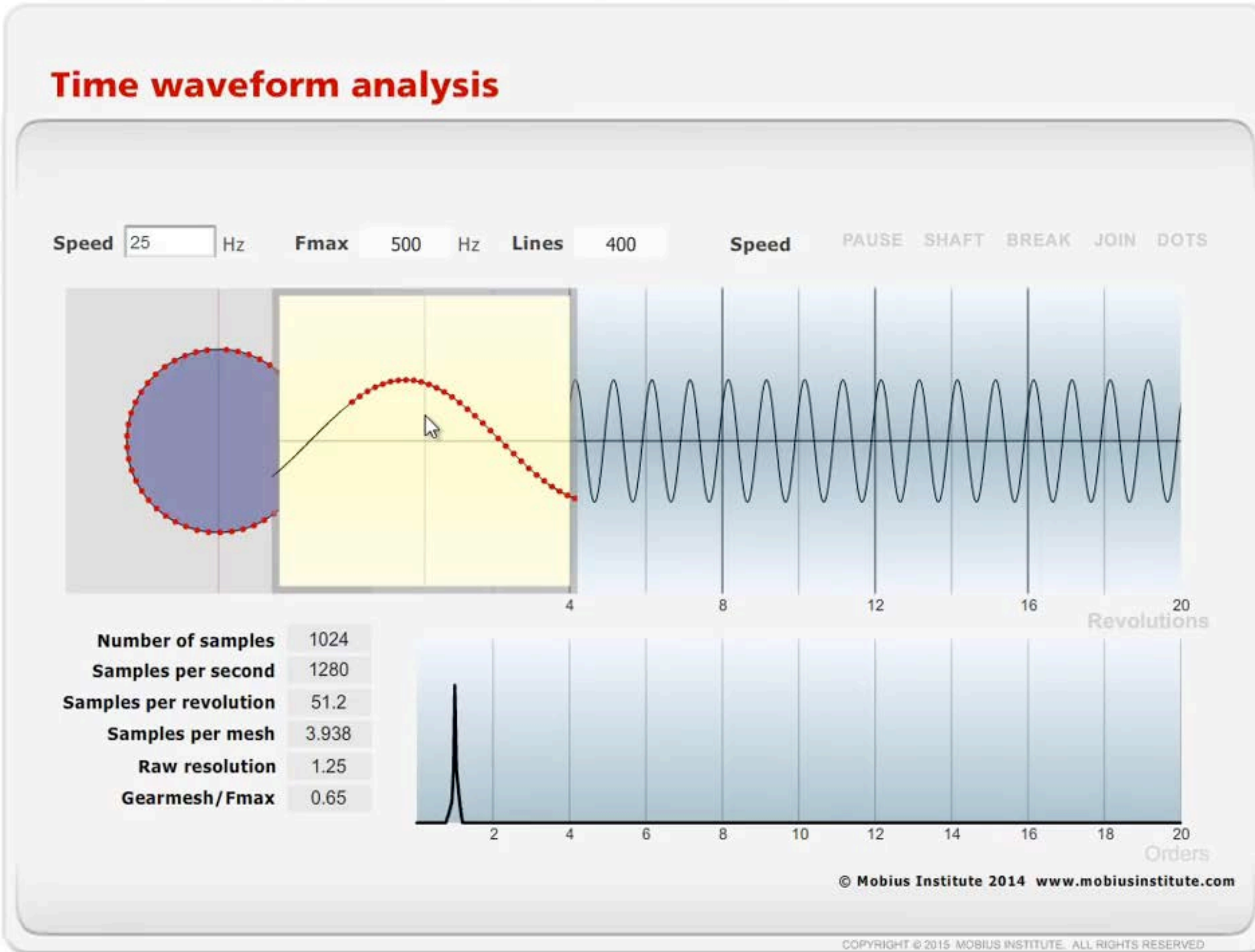

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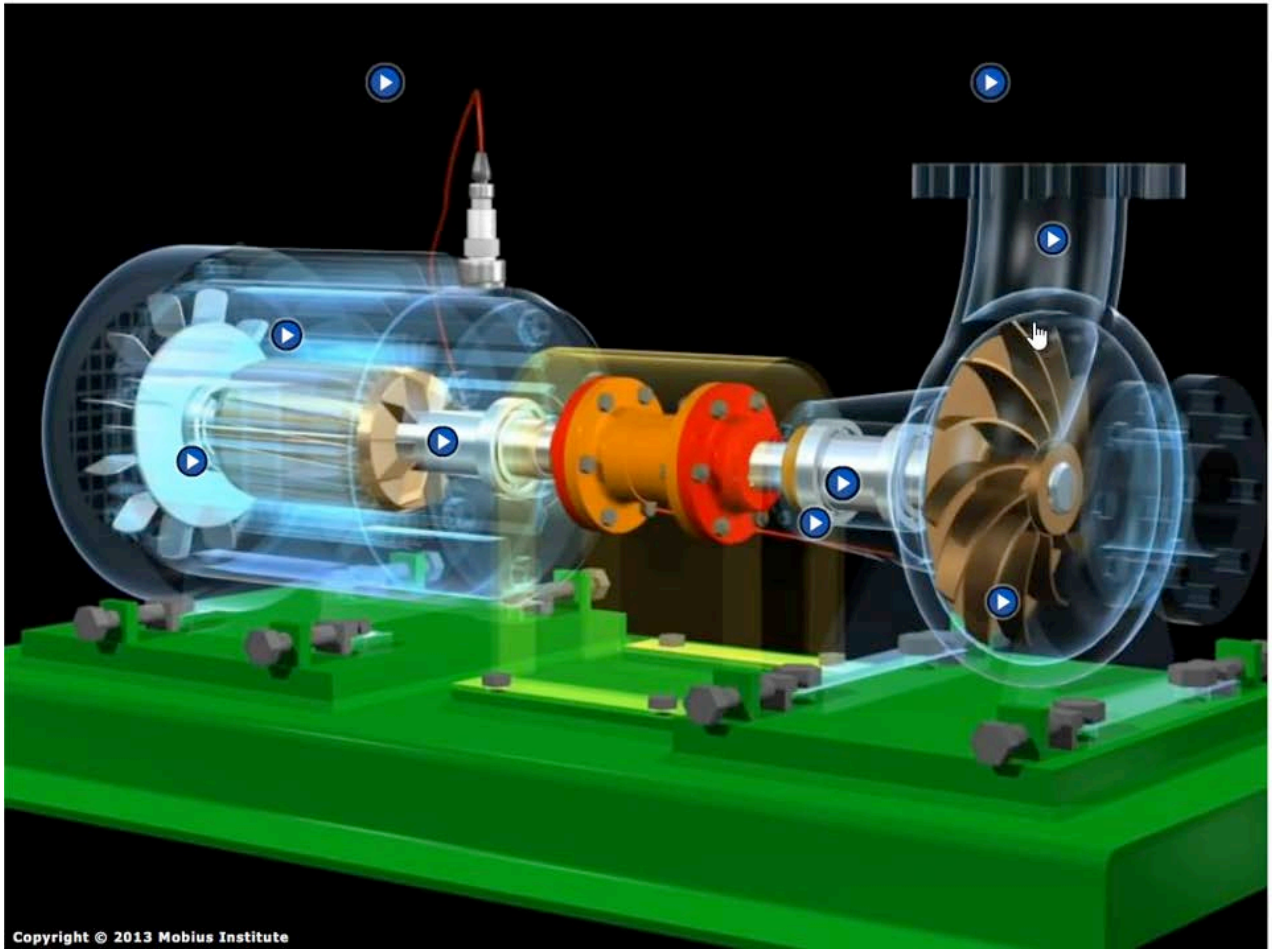

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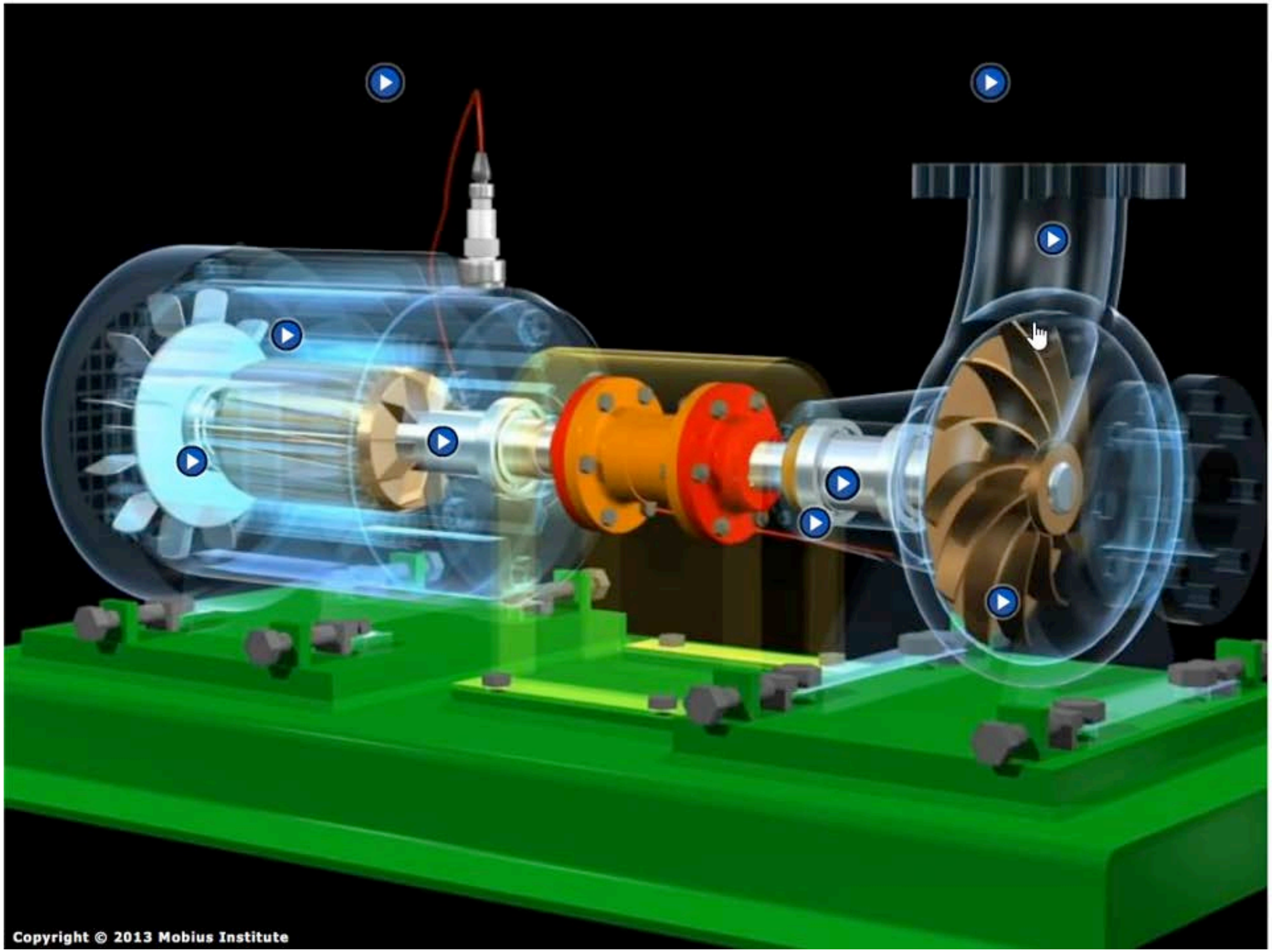


FREQUENCY ANALYSIS

Approaches to frequency analysis

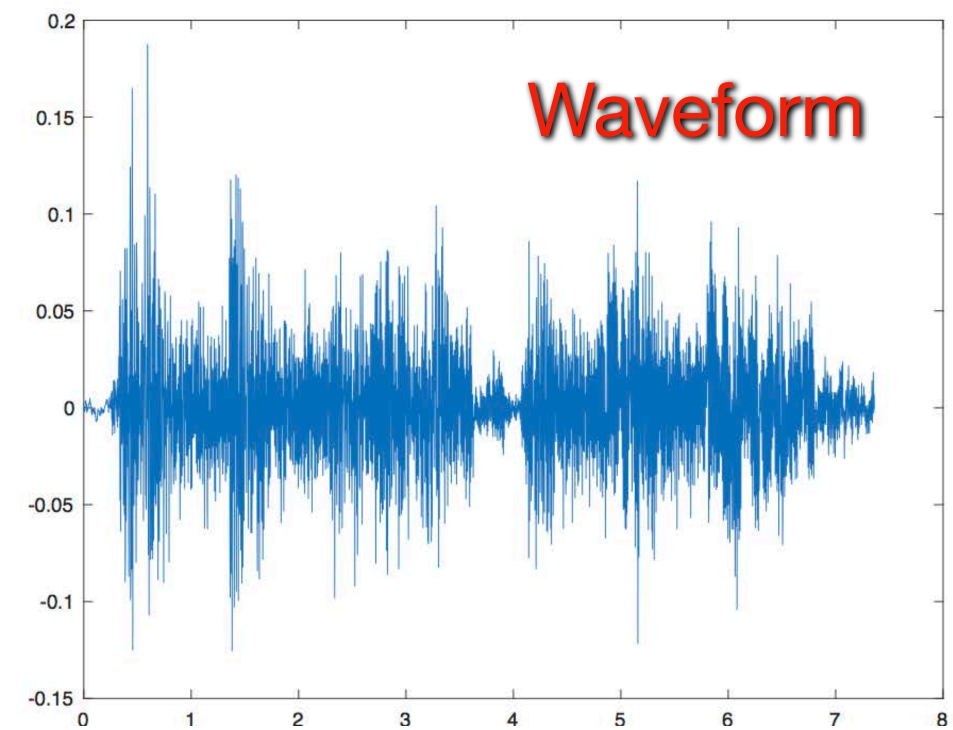
- **Fourier transform** $Y(f) = \mathcal{F}\{y(t)\}$
- **Short time Fourier Transform**

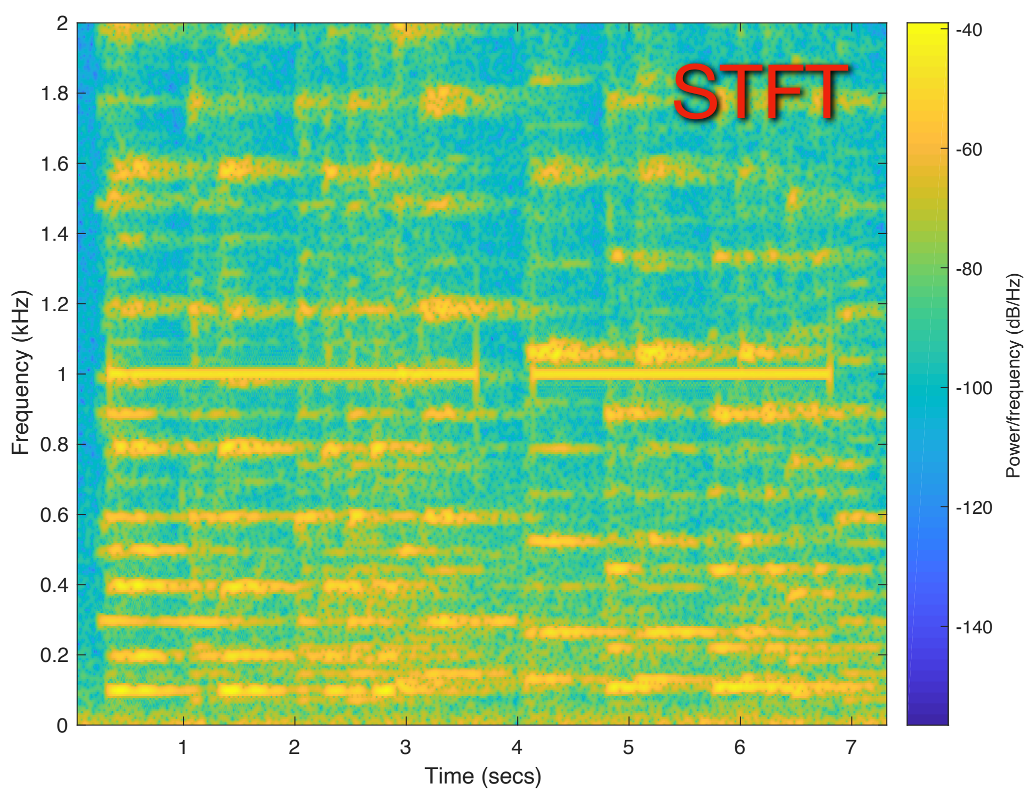
$$Y(f,\tau) = \mathcal{F}\{y(t)w(t-\tau)\}$$

- **Wavelet analysis [GR95]**
- **Power Cepstrum**

$$Y'(q) = |\mathcal{F}^{-1}\left\{\log\left(|\mathcal{F}\{y(t)\}|^2\right)\right\}|^2$$

  - it **converts convoluted signals** (e.g. input and filter impulse response) **into sums** of their cepstra, for linear separation
- …





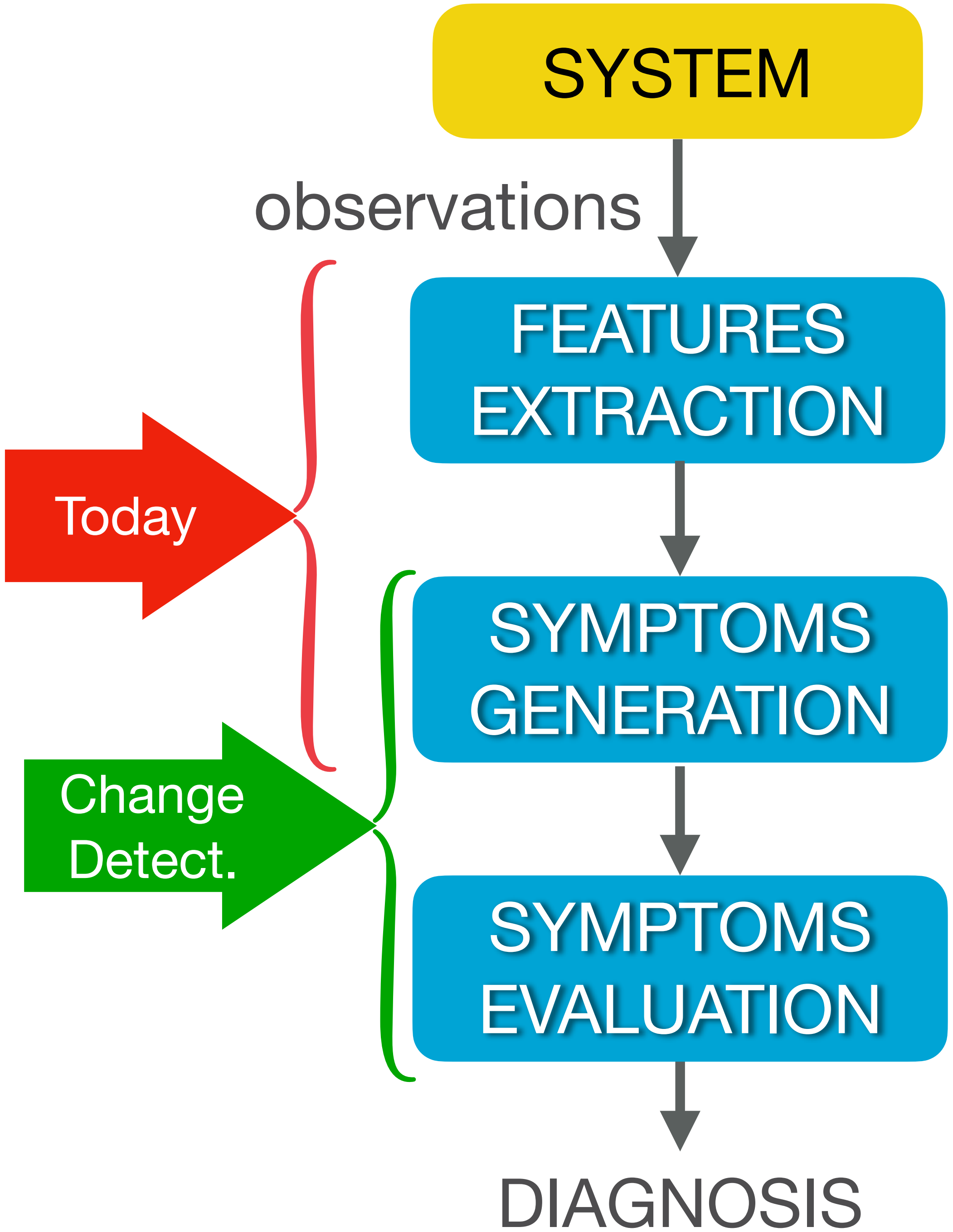
[GR95] Graps, A., 1995. An introduction to wavelets. IEEE computational science and engineering, 2(2), pp.50-61.

PCA

An introduction to Principal Component Analysis

RECAP

Remember the parallel with medical diagnosis?



PCA

Main idea

- Assume huge set of measurements from "slow" process (e.g. Chemical Reactor)
- Reduce dimensionality of data
- Express data in terms of projection on a few direction of "maximum variance"

[YI12] Yin, S., Ding, S.X., Haghani, A., Hao, H. and Zhang, P., 2012. A comparison study of basic data-driven fault diagnosis and process monitoring methods on the benchmark Tennessee Eastman process. Journal of Process Control, 22(9), pp.1567-1581.

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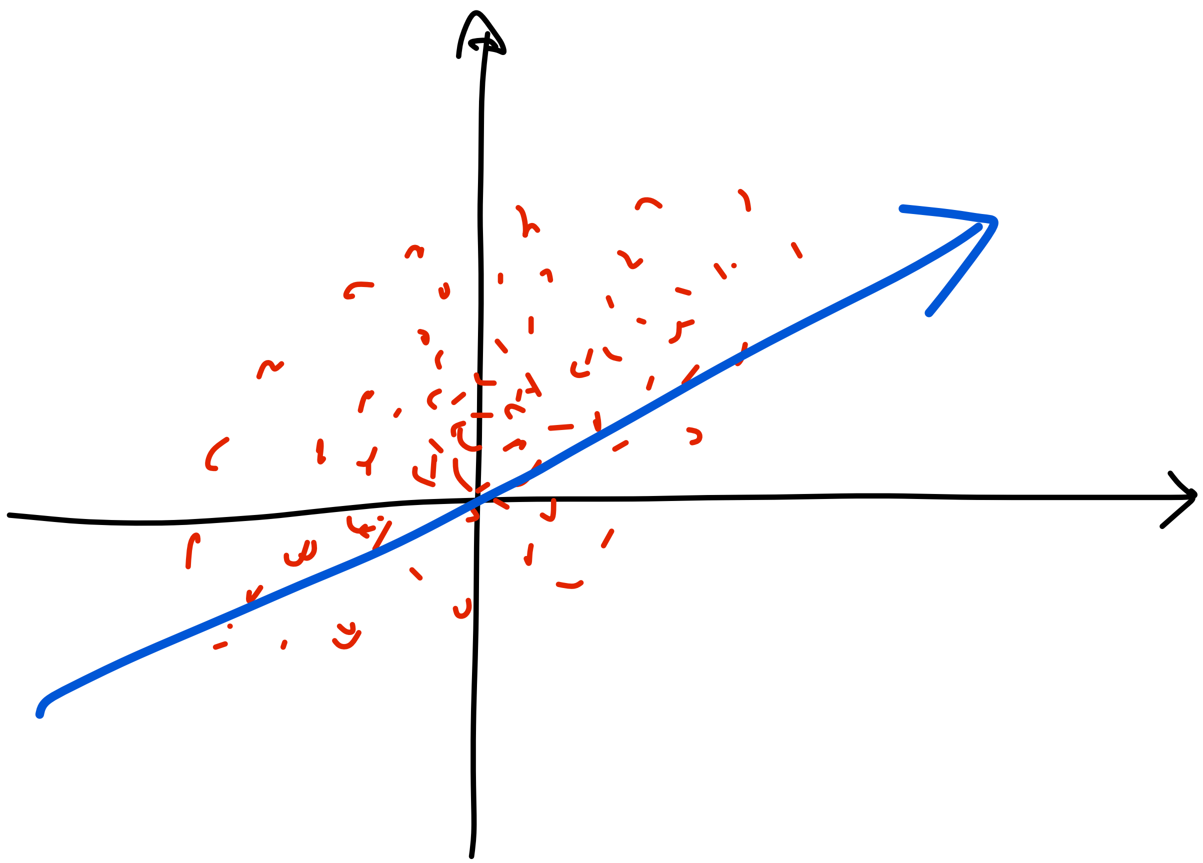

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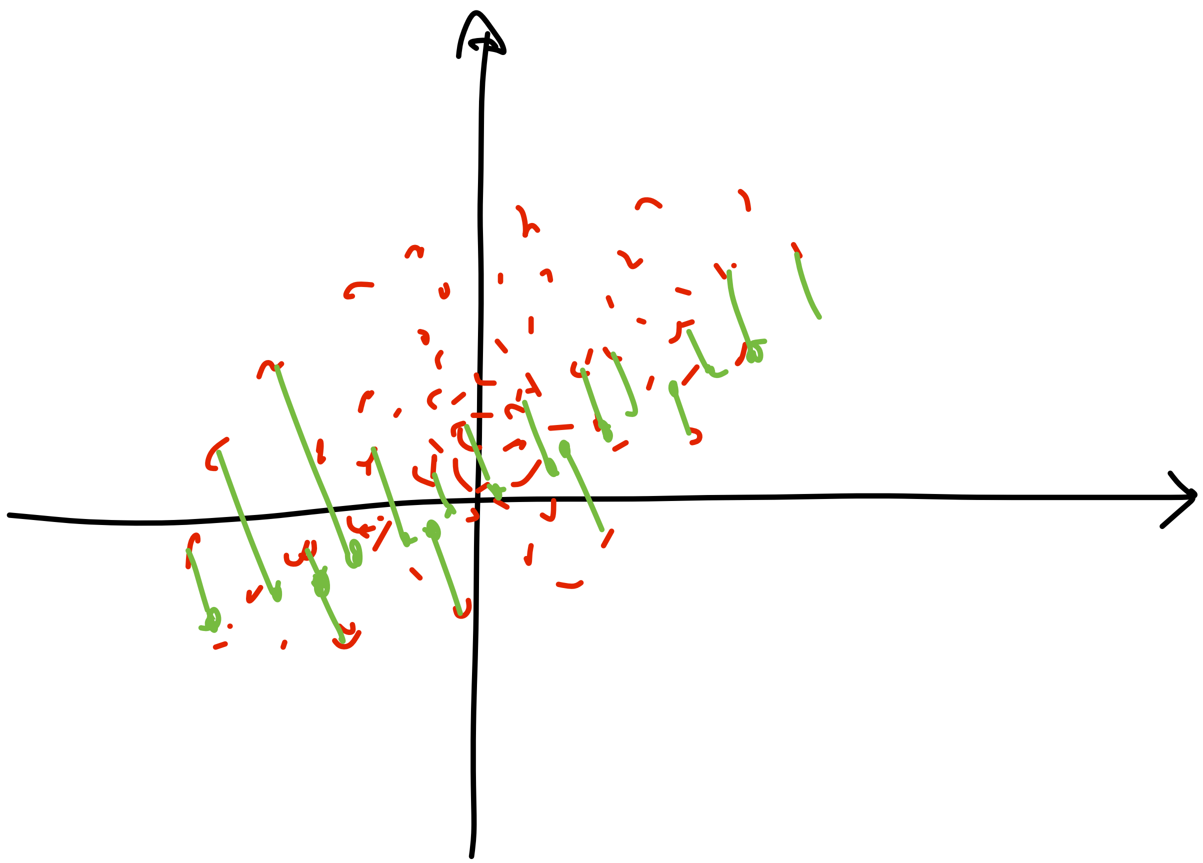

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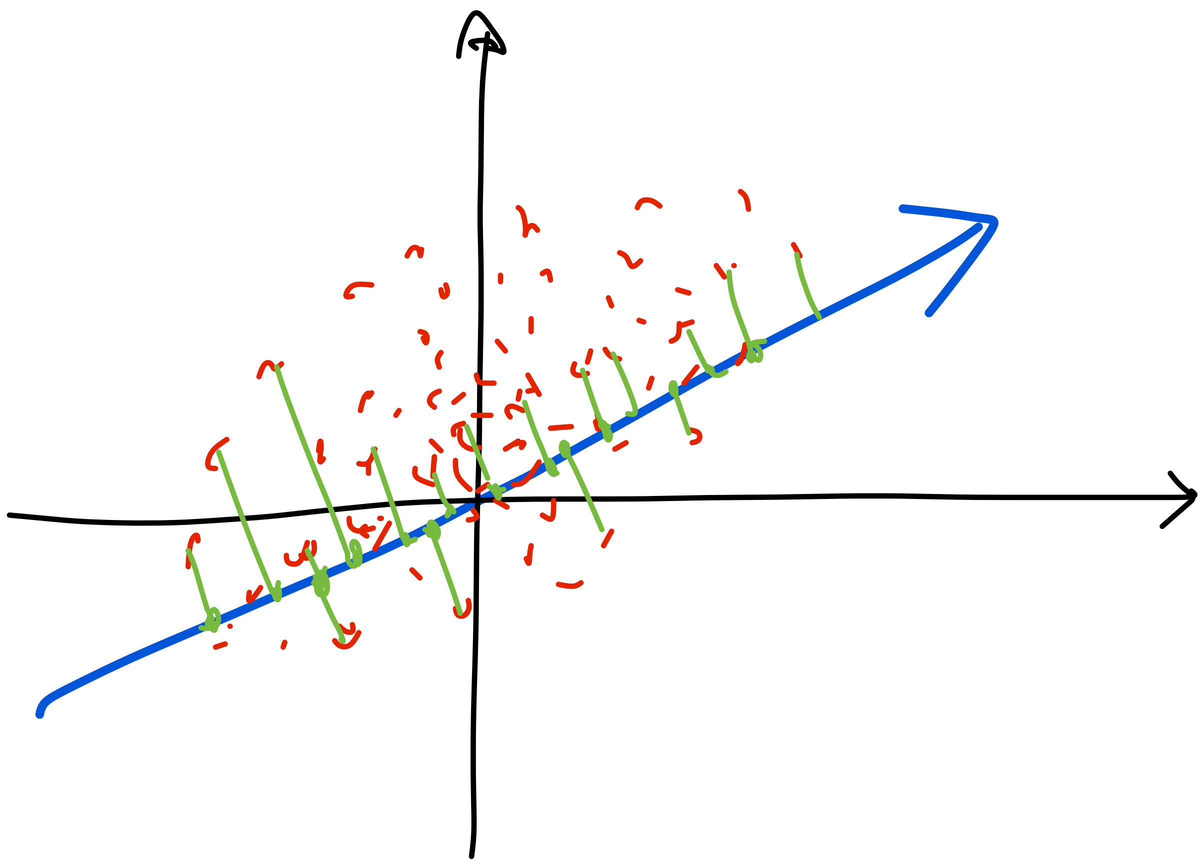

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PCA

Decomposition

- Data set: N **normalised** samples for each $$Z^\top = [z_1, z_2, \dots z_N] \in \mathcal{R}^{m \times N}$$
- Compute covariance matrix $$C = \frac{1}{N-1} Z^\top Z$$
- Compute singular value decomposition $$C = P \Lambda P^\top$$

$$\Lambda = diag(\lambda_1, \dots, \lambda_m), \quad \lambda_1 \geq \cdots \geq \lambda_m > 0.$$

PCA

Decomposition

> Keep at most l components

$$\Lambda = \begin{bmatrix} \Lambda_{pc} & 0 \\ 0 & \Lambda_{res} \end{bmatrix}, \quad \Lambda_{pc} = diag(\lambda_1, \ldots, \lambda_l),$$

$$\Lambda_{res} = diag(\lambda_{l+1}, \ldots, \lambda_m), \qquad P = [\, P_{pc} \quad P_{res} \,], \quad P_{pc} \in \mathcal{R}^{m \times l}, P_{res} \in \mathcal{R}^{m \times (m-l)}$$

PCA

Detection of changes in new data

> Compute the following limits

$$J_{th,SPE} = \theta_1 \left(\frac{c_\alpha \sqrt{2\theta_2 h_0^2}}{\theta_1} + 1 + \frac{\theta_2 h_0 (h_0 - 1)}{\theta_1^2} \right)^{1/h_0}, \qquad h_0 = 1 - \frac{2\theta_1\theta_3}{3\theta_2^2}.$$

$$J_{th,T^2} = \frac{l(N^2-1)}{N(N-l)} F_\alpha(l, N-l) \qquad \theta_i = \sum_{j=l+1}^{m} (\lambda_j)^i, \quad i = 1, 2, 3$$

> Normalize every new data sample z and compute the following statistics

$$SPE = z^T P_{res} P_{res}^T z, \qquad SPE \leq J_{th,SPE} \text{ and } T^2 \leq J_{th,T^2} \Rightarrow \text{fault free},$$

$$T^2 = z^T P_{pc} \Lambda_{pc}^{-1} P_{pc}^T z. \qquad \text{otherwise faulty}$$

CONCLUSION

Recap of today and plan for next lecture

> **TODAY**
>
> > A very brief description of some signal based detection methods
>
> **NEXT LECTURE**
>
> > Model Based Fault Diagnosis

CONCLUSION

Recap of today and plan for next lecture

> **HOMEWORK**

1. Implement a random signal, and make its mean change at an instant k0
   > Use a deterministic limit check to detect the change. Is it working well? Can you think of a simple way to improve that?

   > Use a probabilistic method to detect the change. Verify numerically that you get the level of significance you expect from theory
2. Generate a synthetic mult-isinusoidal signal: one fundamental frequency and some upper harmonics. Make the fundamental and harmonics frequency vary in time (to simulate vibrations from a machine rotating at different speeds).
   > Introduce a “fault” by introducing either a spurious new frequency component, or alter the magnitude or phase of some upper harmonics.

   > Write an algorithm to detect this

CONCLUSION

Thank you for your attention !

For further information:

Course page on Brightspace

or

r.ferrari@tudelft.nl