Identification of Nonlinear LFR Systems starting from the Best Linear Approximation

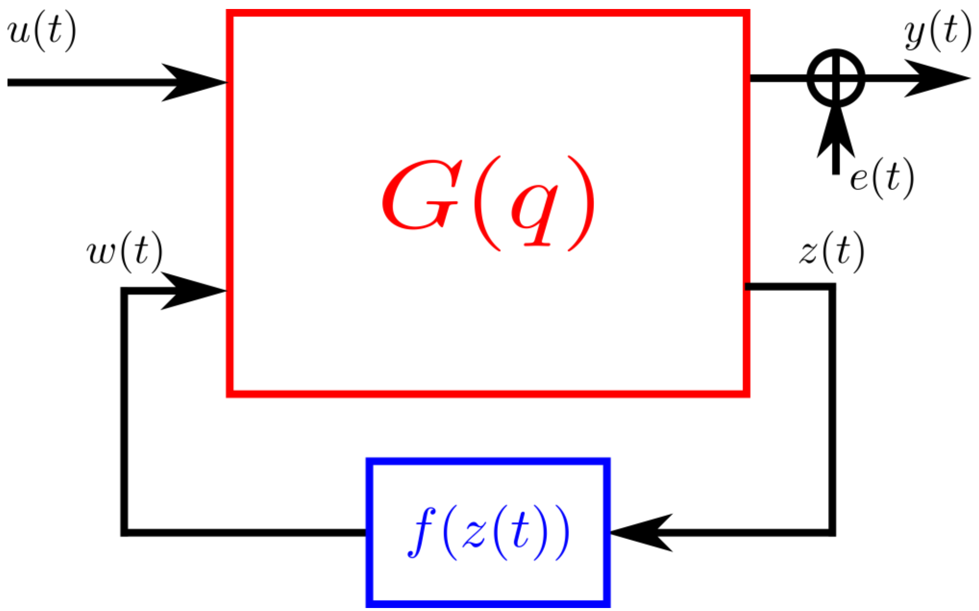
M. Schoukens and R. Tóth

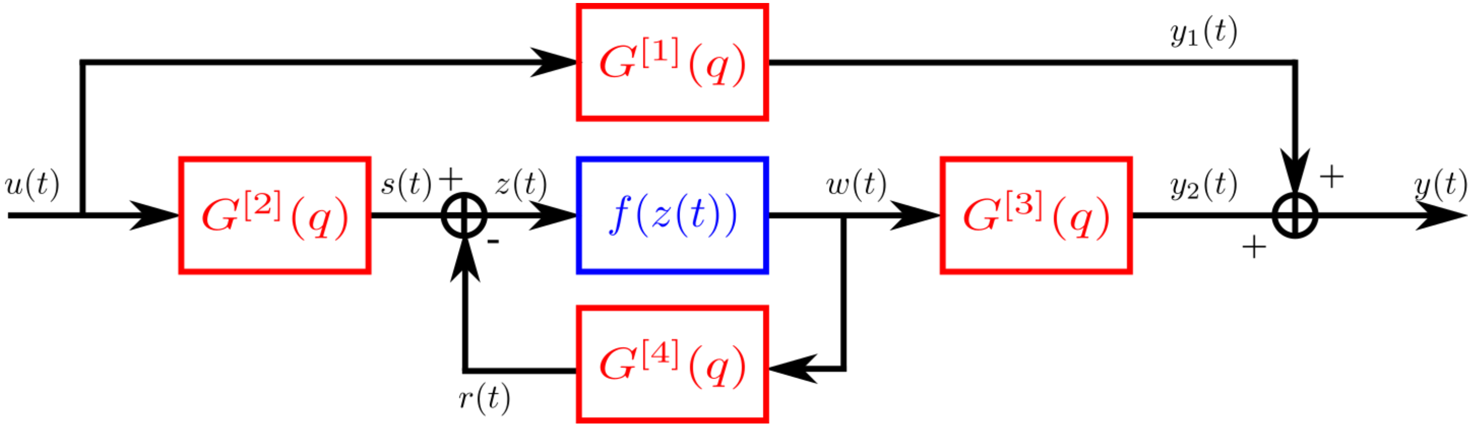
MARIE CURIE ACTIONS

Technische Universiteit Eindhoven University of Technology

EE / Control Systems

Nonlinear System Class





Outline

Nonlinear System Class

Initialization & Estimation

Examples

Conclusions

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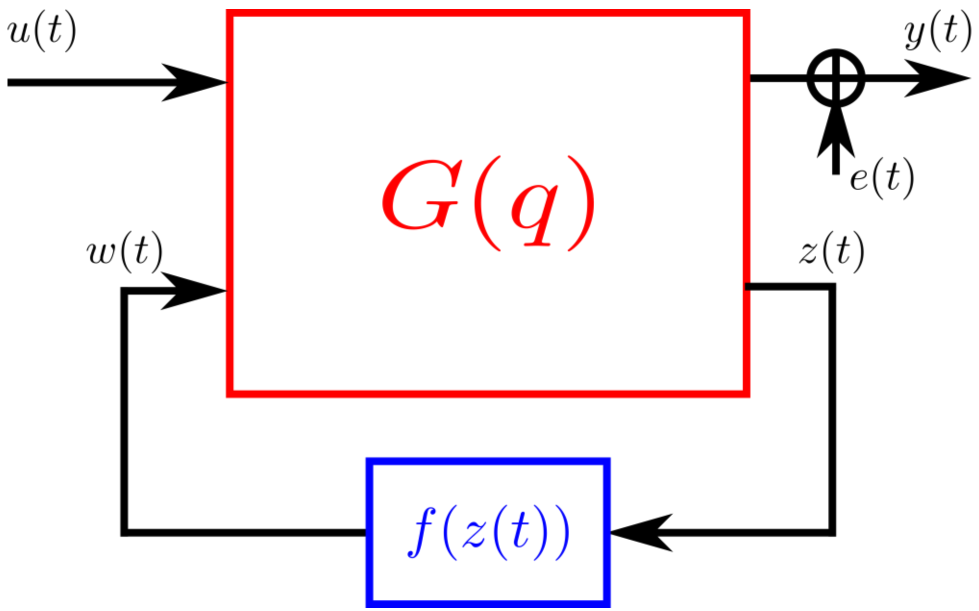
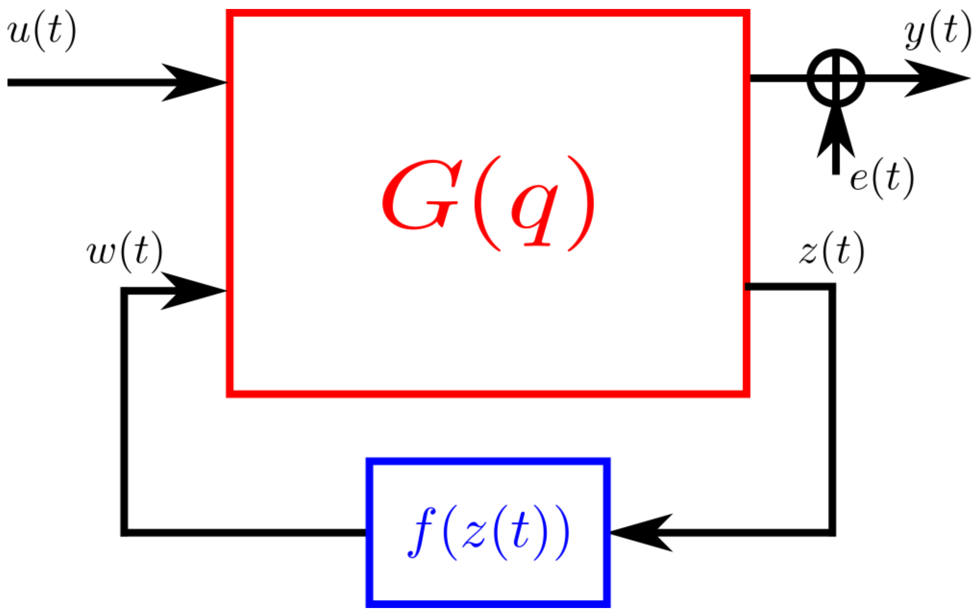
Nonlinear System Class



$$
\begin{aligned}
x(t+1) &= Ax(t) + B_{\mathrm{u}}u(t) + B_{\mathrm{w}}w(t) \\
y(t) &= C_{\mathrm{y}}x(t) + D_{\mathrm{yu}}u(t) + D_{\mathrm{yw}}w(t) + e(t) \\
z(t) &= C_{\mathrm{z}}x(t) + D_{\mathrm{zu}}u(t) + D_{\mathrm{zw}}w(t)
\end{aligned}
$$

$$w(t) = f(z(t))$$

Nonlinear System Class



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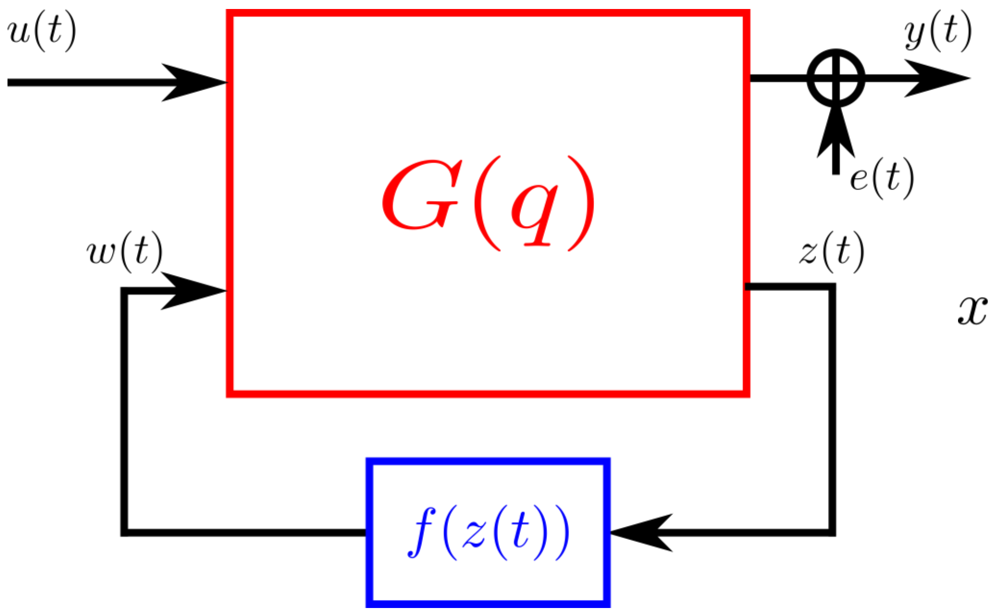
\end{aligned}
$$

$$w(t) = f(z(t))$$

$$\Downarrow \quad D_{\mathrm{zw}} = 0$$

$$
\begin{aligned}
x(t+1) &= Ax(t) + B_{\mathrm{u}}u(t) + B_{\mathrm{w}}f(C_{\mathrm{z}}x(t) + D_{\mathrm{zu}}u(t)), \\
y(t) &= C_{\mathrm{y}}x(t) + D_{\mathrm{yu}}u(t) + D_{\mathrm{yw}}f(C_{\mathrm{z}}x(t) + D_{\mathrm{zu}}u(t)),
\end{aligned}
$$

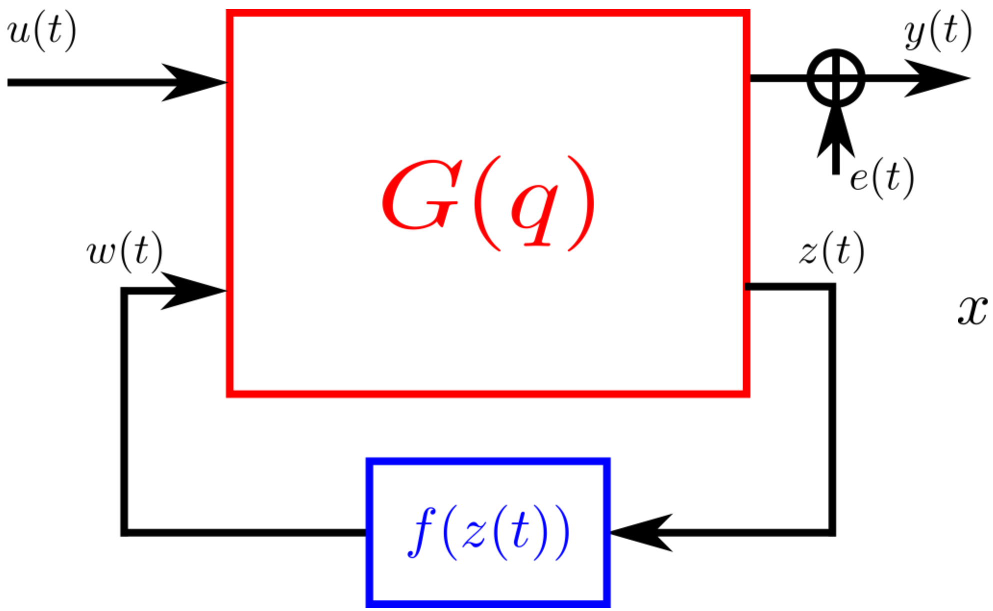
Nonlinear LFR vs Nonlinear SS



$$
\begin{aligned}
x(t+1) &= Ax(t) + B_{\mathrm{u}}u(t) \boxed{+ B_{\mathrm{w}}f(C_{\mathrm{z}}x(t) + D_{\mathrm{zu}}u(t)),} \\
y(t) &= C_{\mathrm{y}}x(t) + D_{\mathrm{yu}}u(t) \boxed{+ D_{\mathrm{yw}}f(C_{\mathrm{z}}x(t) + D_{\mathrm{zu}}u(t)),}
\end{aligned}
$$

→ Structured NL State-Space

Nonlinear LFR vs Nonlinear SS



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y(t) &= C_{\mathrm{y}}x(t) + D_{\mathrm{yu}}u(t) \boxed{+ D_{\mathrm{yw}}f(C_{\mathrm{z}}x(t) + D_{\mathrm{zu}}u(t)),}
\end{aligned}
$$

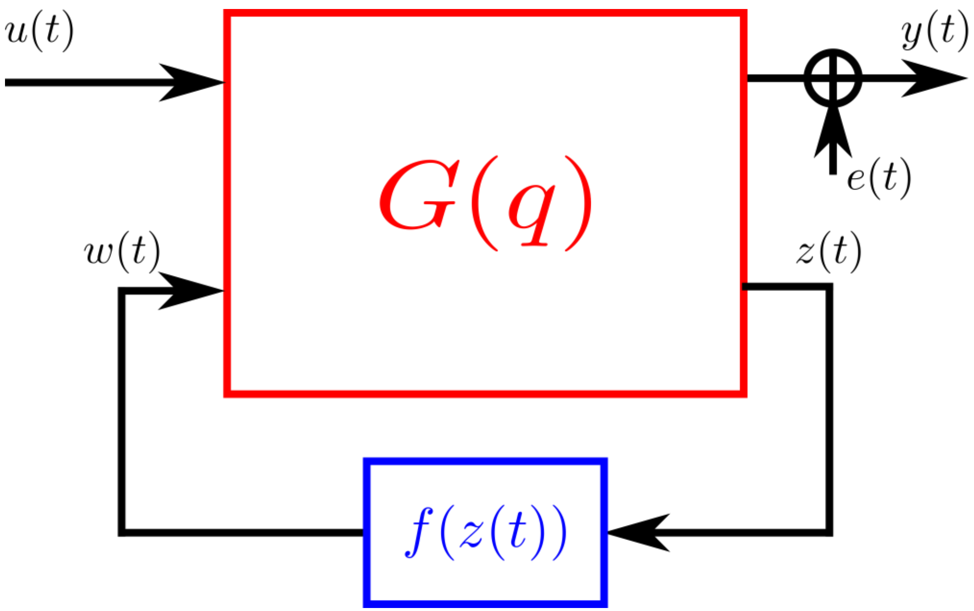
$B_{\mathrm{w}} = Inx$

$D_{\mathrm{yw}} = Iny$

$C_{\mathrm{z}} = \begin{bmatrix} I_{nx} \\ 0_{nu} \end{bmatrix}$ $\quad D_{\mathrm{zu}} = \begin{bmatrix} 0_{nx} \\ I_{nu} \end{bmatrix}$

→ Full NL State-Space

Uniqueness of the Representation



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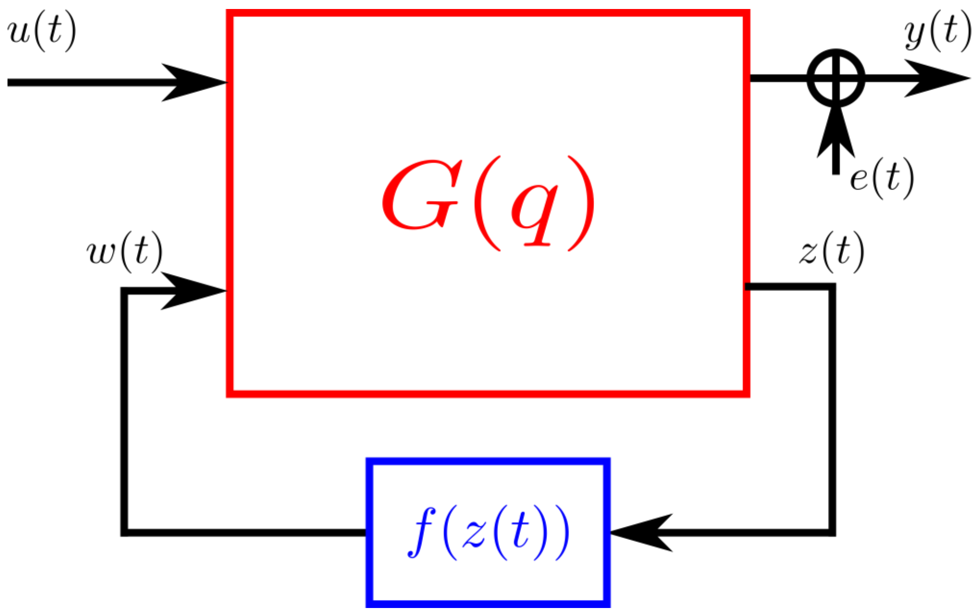
\begin{aligned}
x(t+1) &= Ax(t) + B_{\mathrm{u}}u(t) + B_{\mathrm{w}}w(t) \\
y(t) &= C_{\mathrm{y}}x(t) + D_{\mathrm{yu}}u(t) + D_{\mathrm{yw}}w(t) + e(t) \\
z(t) &= C_{\mathrm{z}}x(t) + D_{\mathrm{zu}}u(t) + D_{\mathrm{zw}}w(t)
\end{aligned}
$$

$$w(t) = f(z(t))$$

Uniqueness of the Representation



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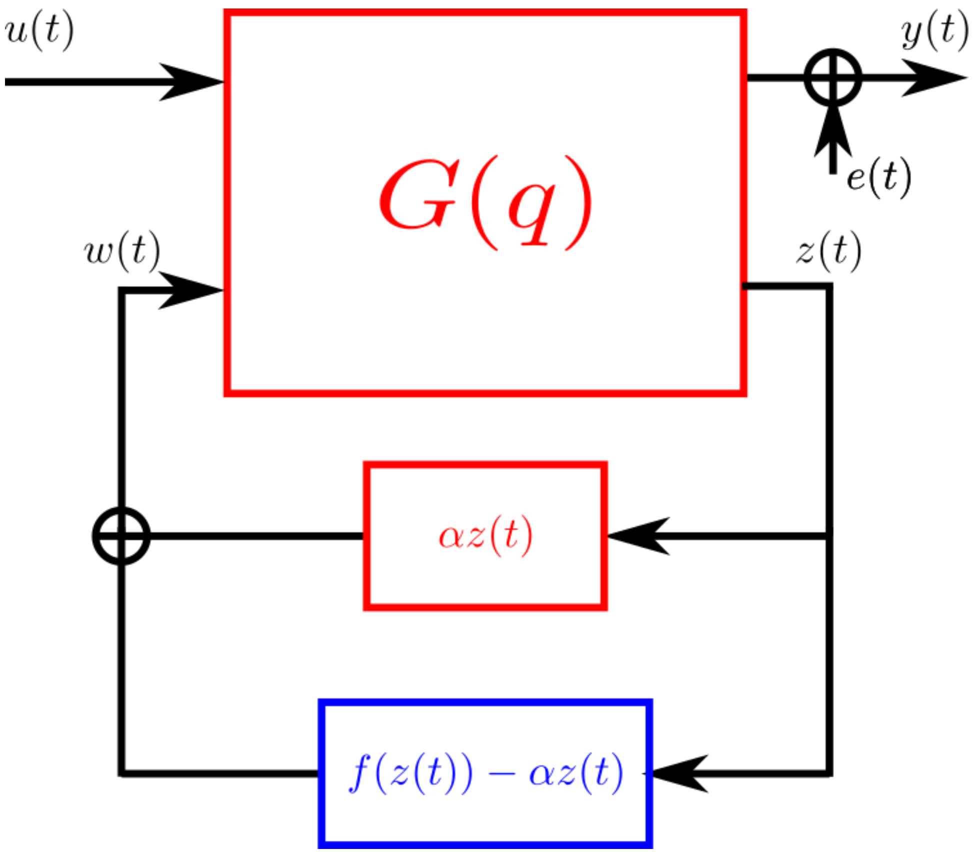
$$w(t) = f(z(t))$$

All the problems of linear state-space representation

Uniqueness of the Representation



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$$

$$w(t) = f(z(t))$$

All the problems of linear state-space representation

Additional exchange of a linear gain between the nonlinearity and the linear dynamics

Outline

Nonlinear System Class

Initialization & Estimation

Examples

Conclusions

Initialization & Estimation

Step 1: Estimate the Best Linear Approximation

$$G_{bla}(q) = \underset{G(q)}{\arg\min}\, E_u \left\{ \left| \tilde{y}(t) - G(q)\tilde{u}(t) \right|^2 \right\}$$

Initial estimate of:

$$\begin{matrix} A & B_{\mathrm{u}} \\ C_{\mathrm{y}} & D_{\mathrm{yu}} \end{matrix}$$

Frequency Domain Nonparametric BLA

↓

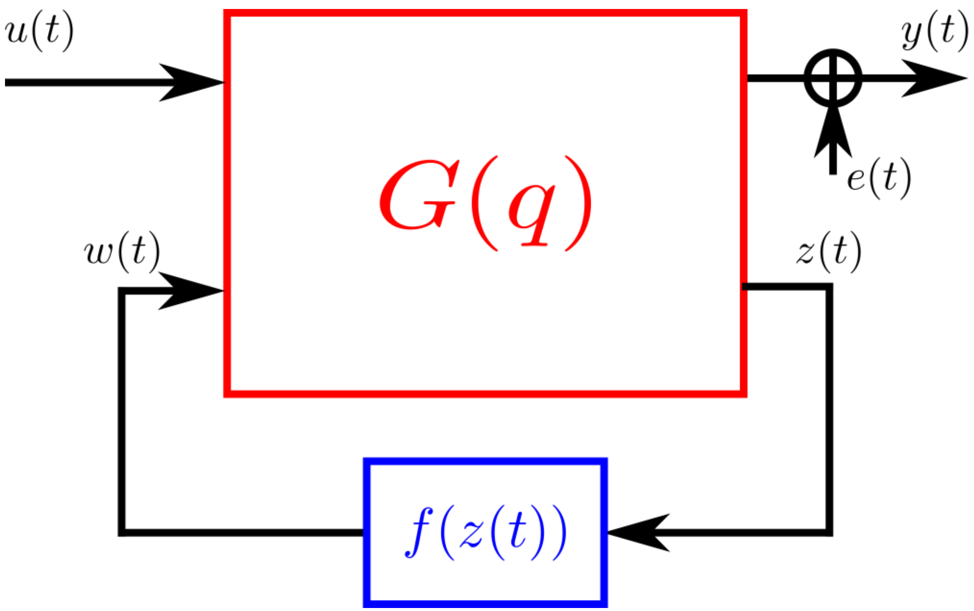
Rational Transfer Function

↓

State-Space Realization

Initialization & Estimation

Step 1: Estimate the Best Linear Approximation



$$
\begin{aligned}
x(t+1) &= \boxed{Ax(t) + B_{\mathrm{u}}u(t) +} B_{\mathrm{w}}w(t) \\
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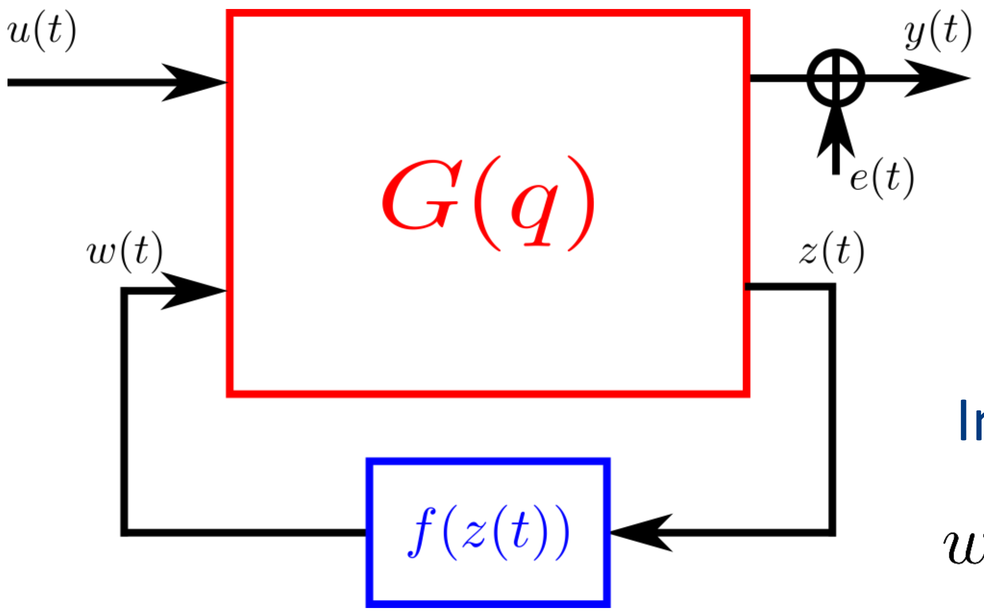
\end{aligned}
$$

$$w(t) = f(z(t))$$

For a good initial estimate, all the states should be ‘visible’ for the best linear approximation of the system

Initialization & Estimation

Step 2: Nonlinear optimization of all the parameters together



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w(t) &= f(z(t))
\end{aligned}
$$

Initializing Nonlinearity, *w* and *z* Matrices

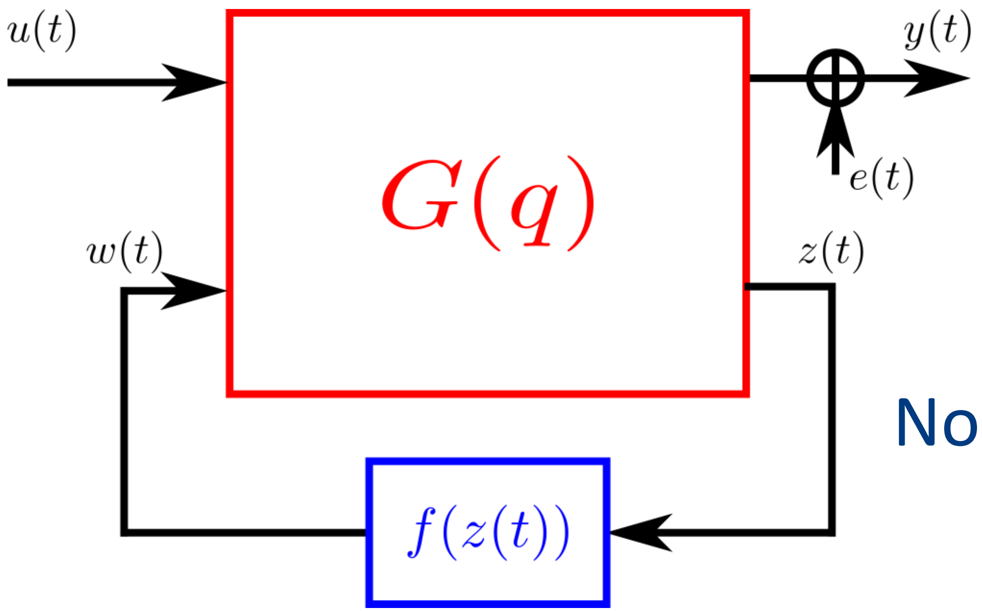
$$
\begin{aligned}
w(t) &= f(z(t)) \\
&= \sum_{i=0}^{n_f} \alpha_i f_i(z(t))
\end{aligned}
$$

$$
\begin{aligned}
D_{\mathrm{zu}} &= 0 \\
D_{\mathrm{yw}} &= 0 \\
C_{\mathrm{z}} &= 1 \\
B_{\mathrm{w}} &= 1 \\
\alpha_i &= 0
\end{aligned}
$$

Nonlinearity can be replaced in a 3rd step

Initialization & Estimation

Step 2: Nonlinear optimization of all the parameters together



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\end{aligned}
$$

$$w(t) = f(z(t))$$

Nonlinear Optimization

$$V_N(\theta) = \frac{1}{N}\sum_{t=1}^{N}\left(y(t) - \hat{y}(t,\theta)\right)^2 \longrightarrow$$ simulation error

Levenberg-Marquardt Optimization

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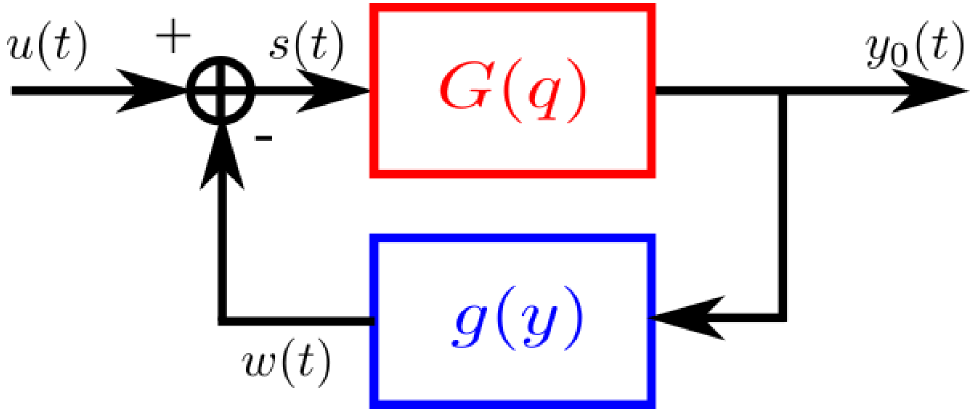
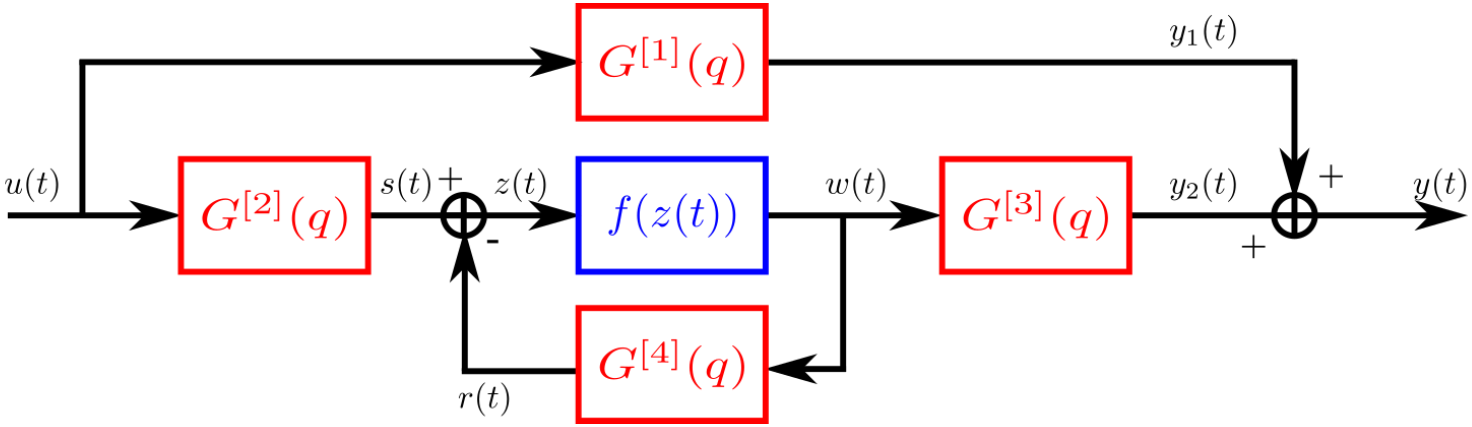
Silverbox Benchmark





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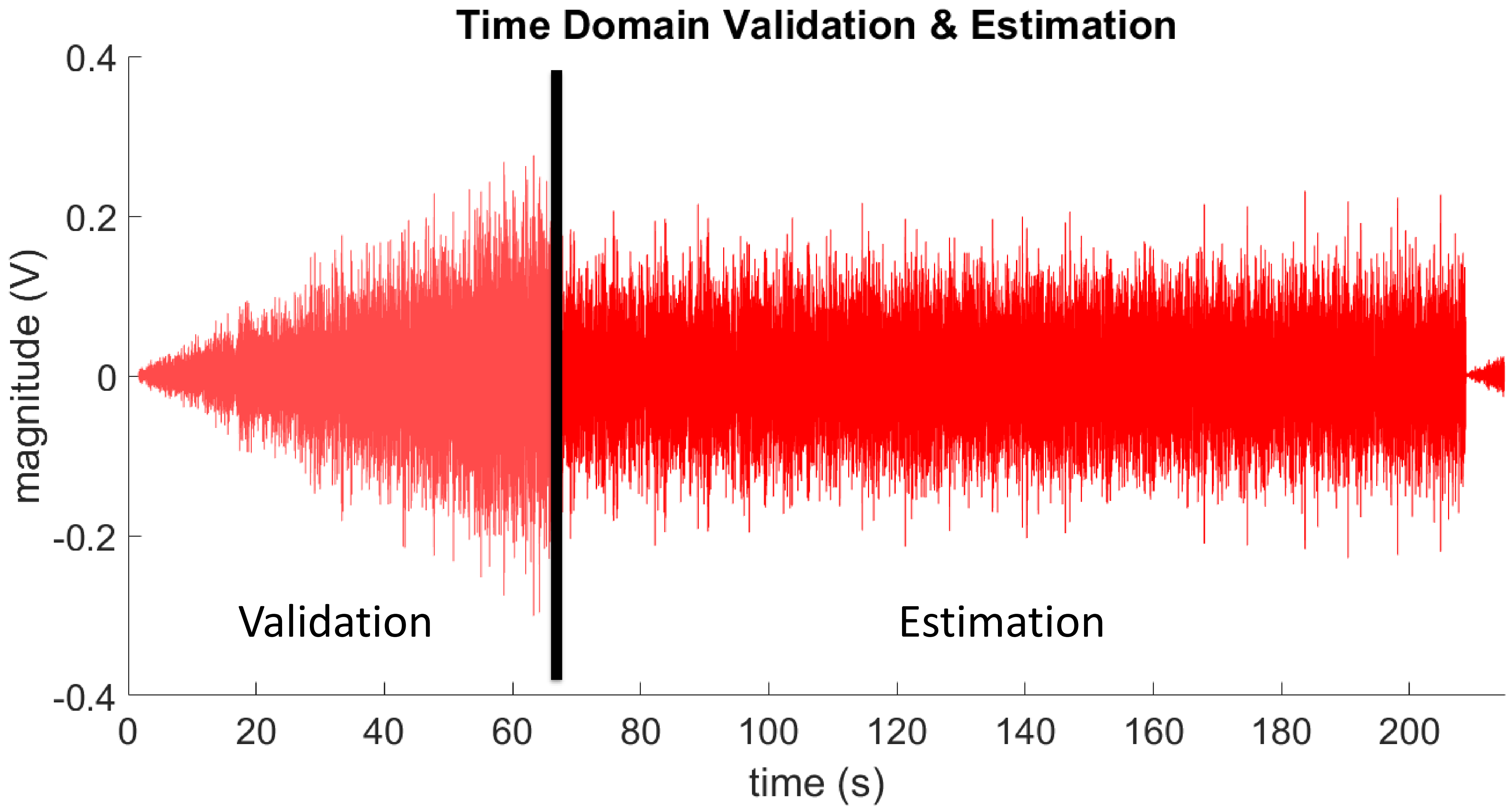

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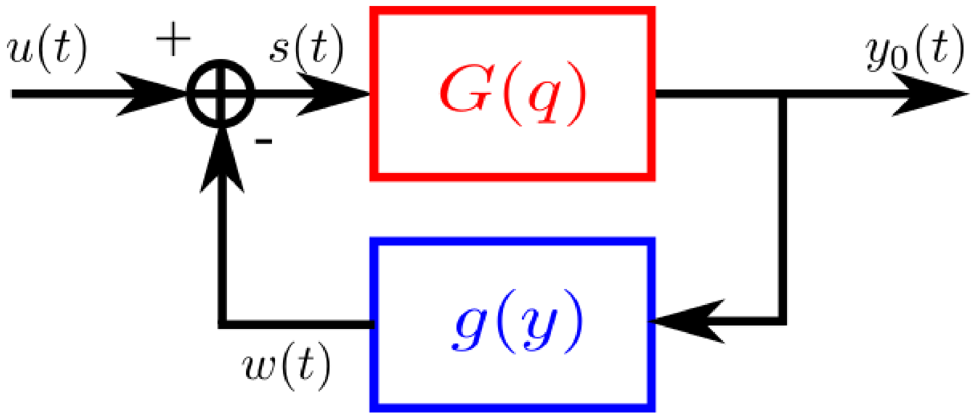
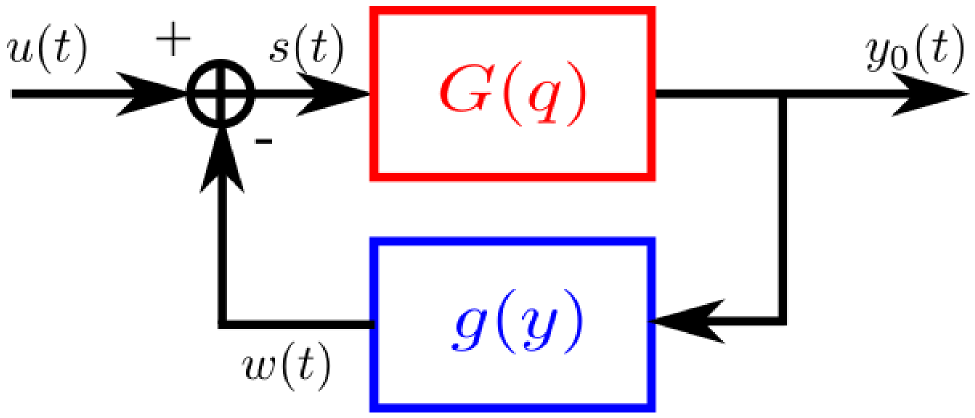


$n_x = 2$

3rd degree polynomial nonlinearity

Silverbox Benchmark



$n_x = 2$

3rd degree polynomial nonlinearity

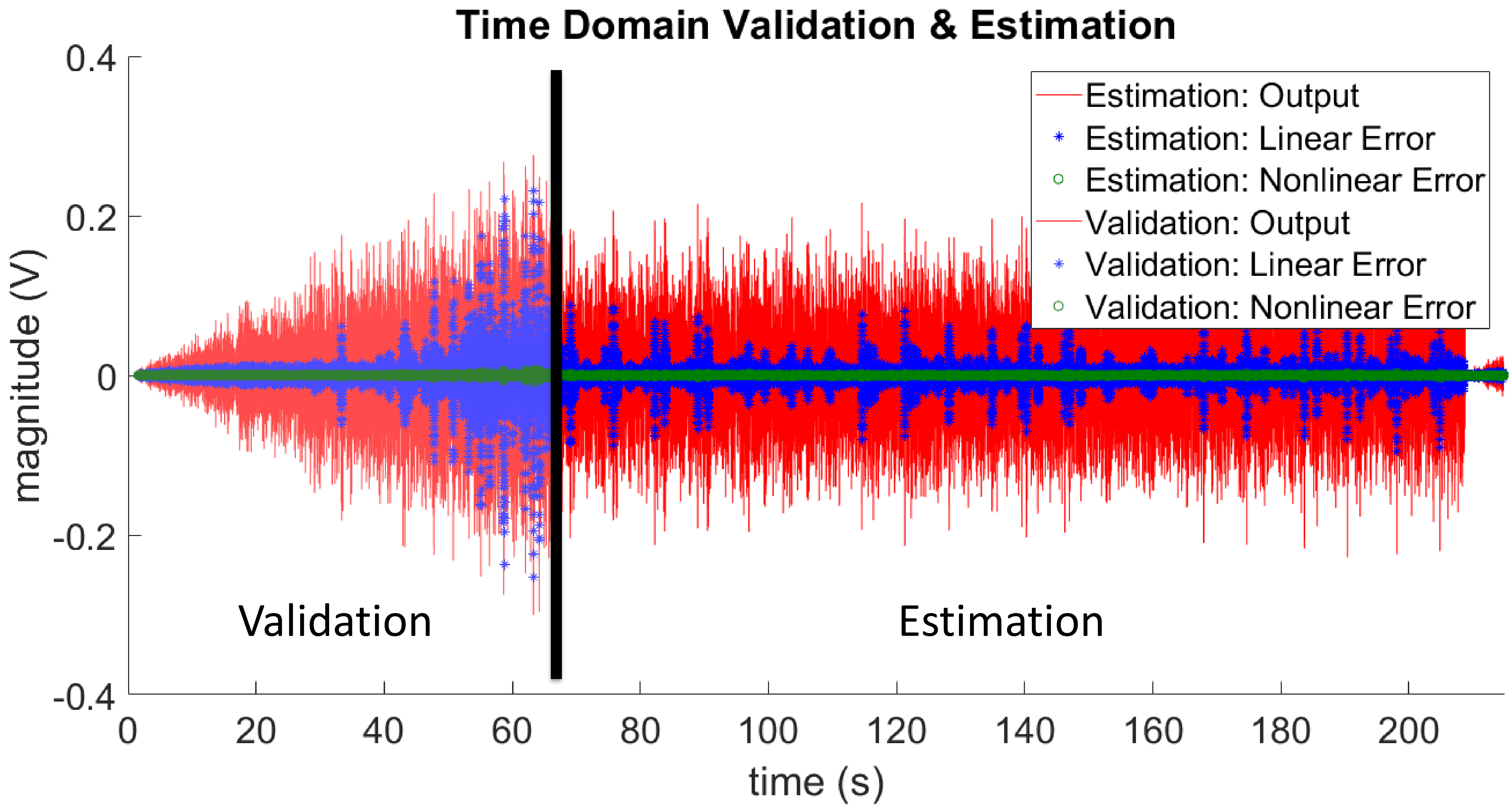
rms errors on estimation data

linear model error:	6.62 mV
NL-LFR error:	0.25 mV

rms errors on validation data

linear model error:	14.5 mV
NL-LFR error:	0.38 mV

Silverbox Benchmark



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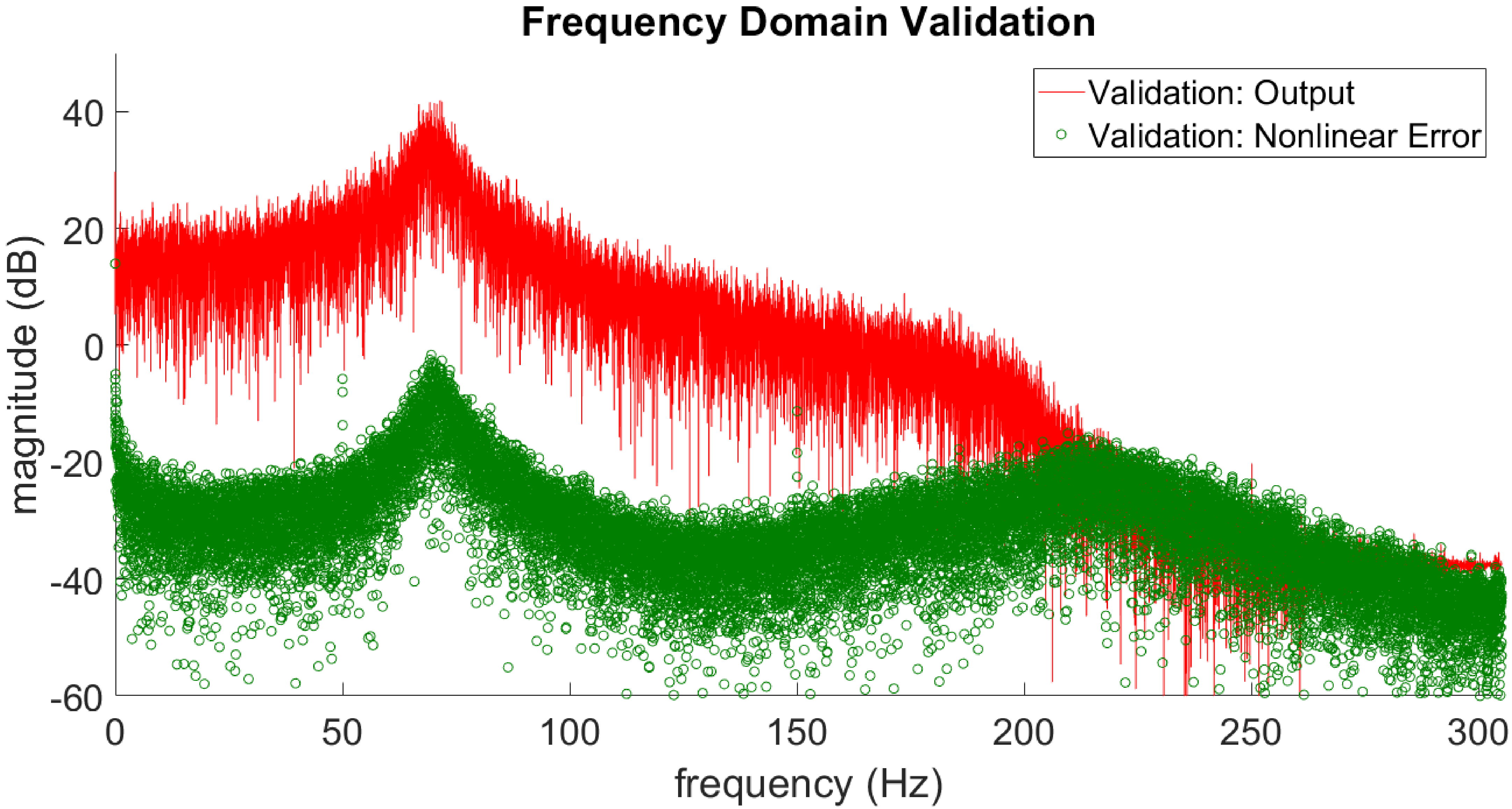

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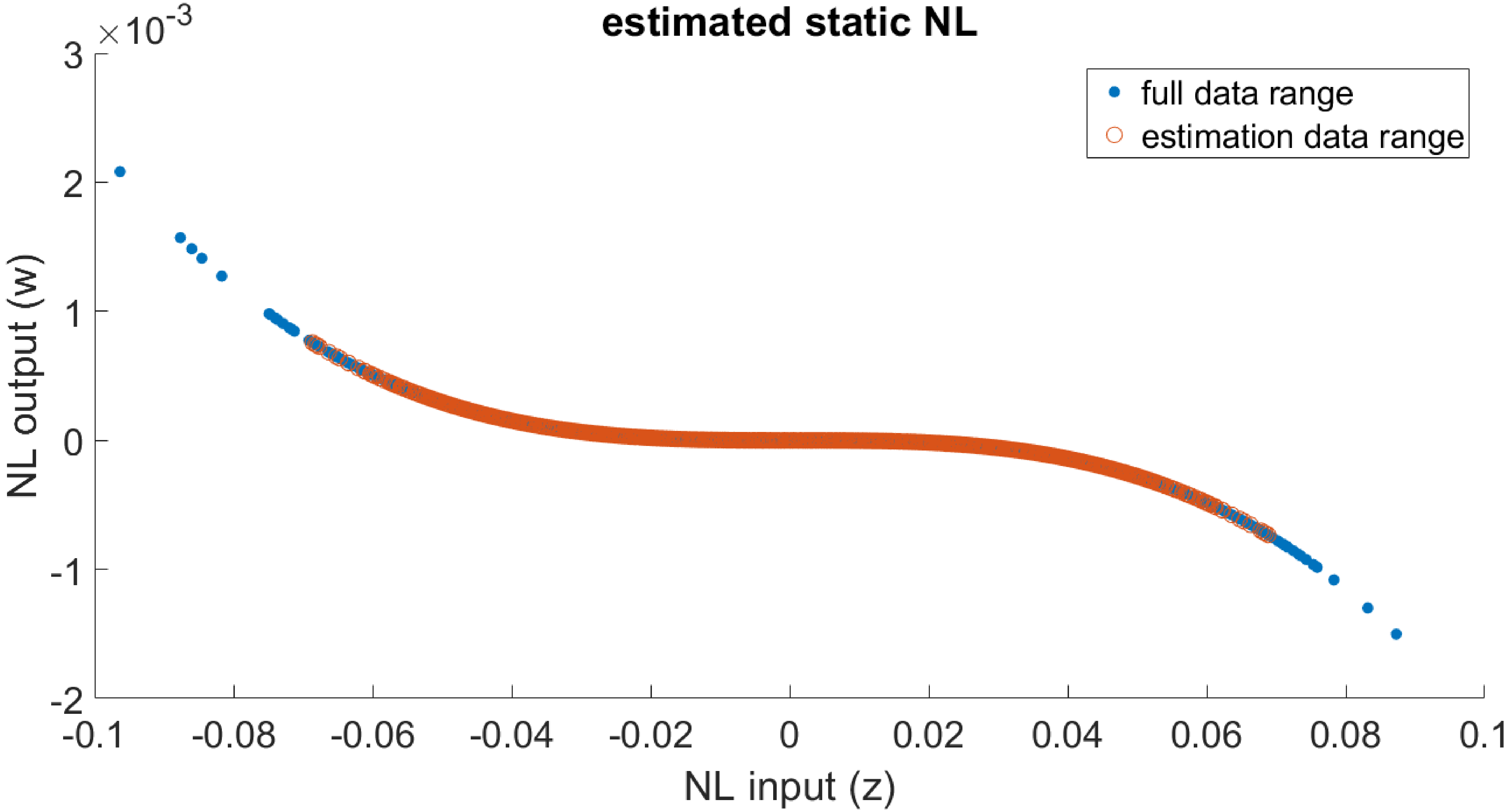

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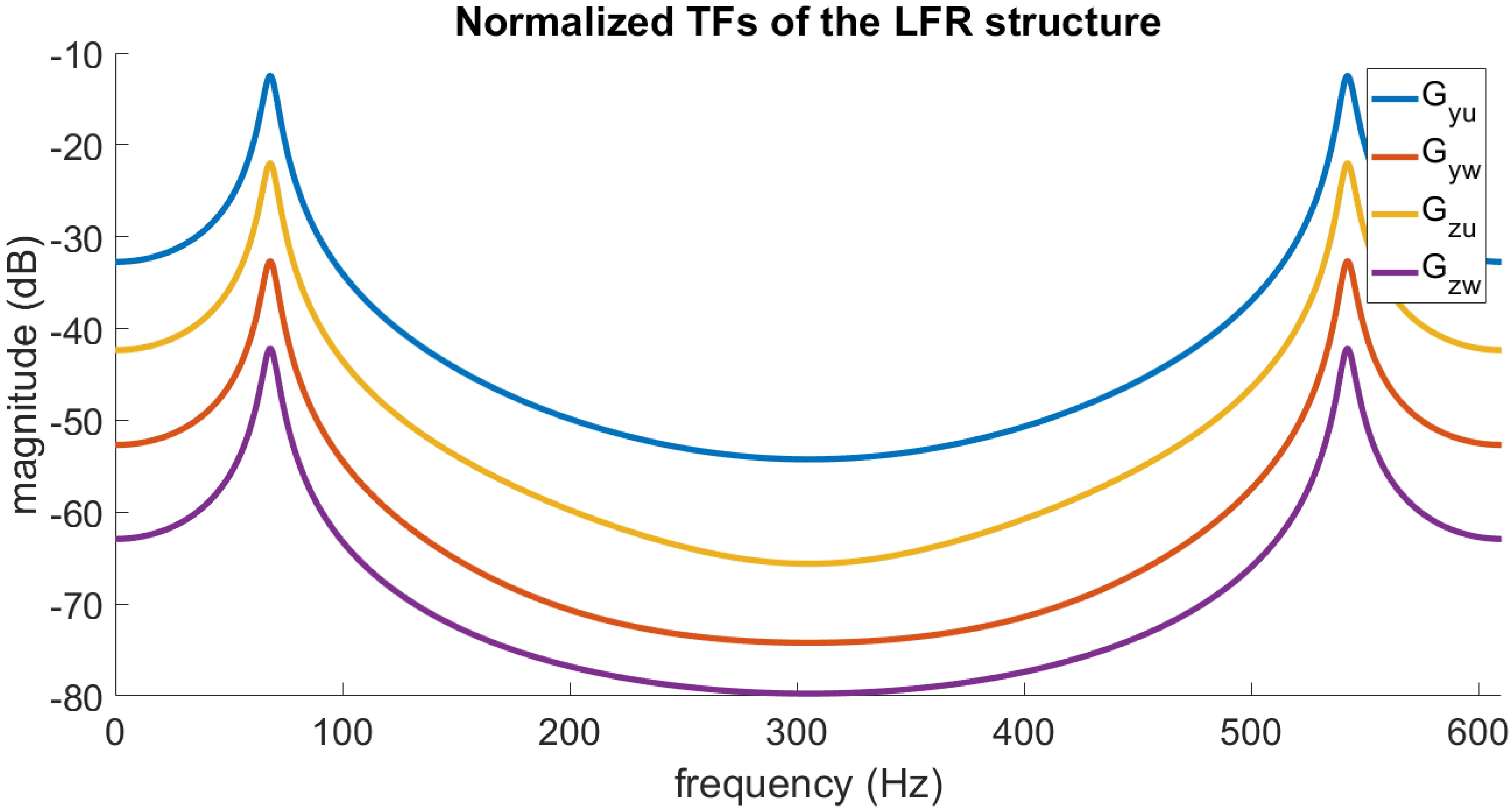
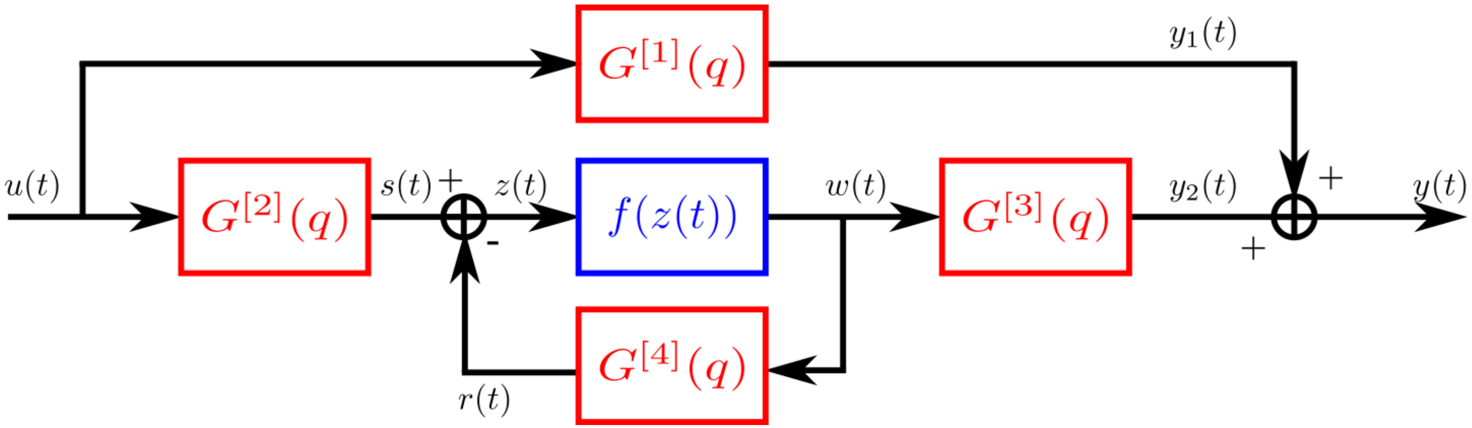


Wiener-Hammerstein Benchmark



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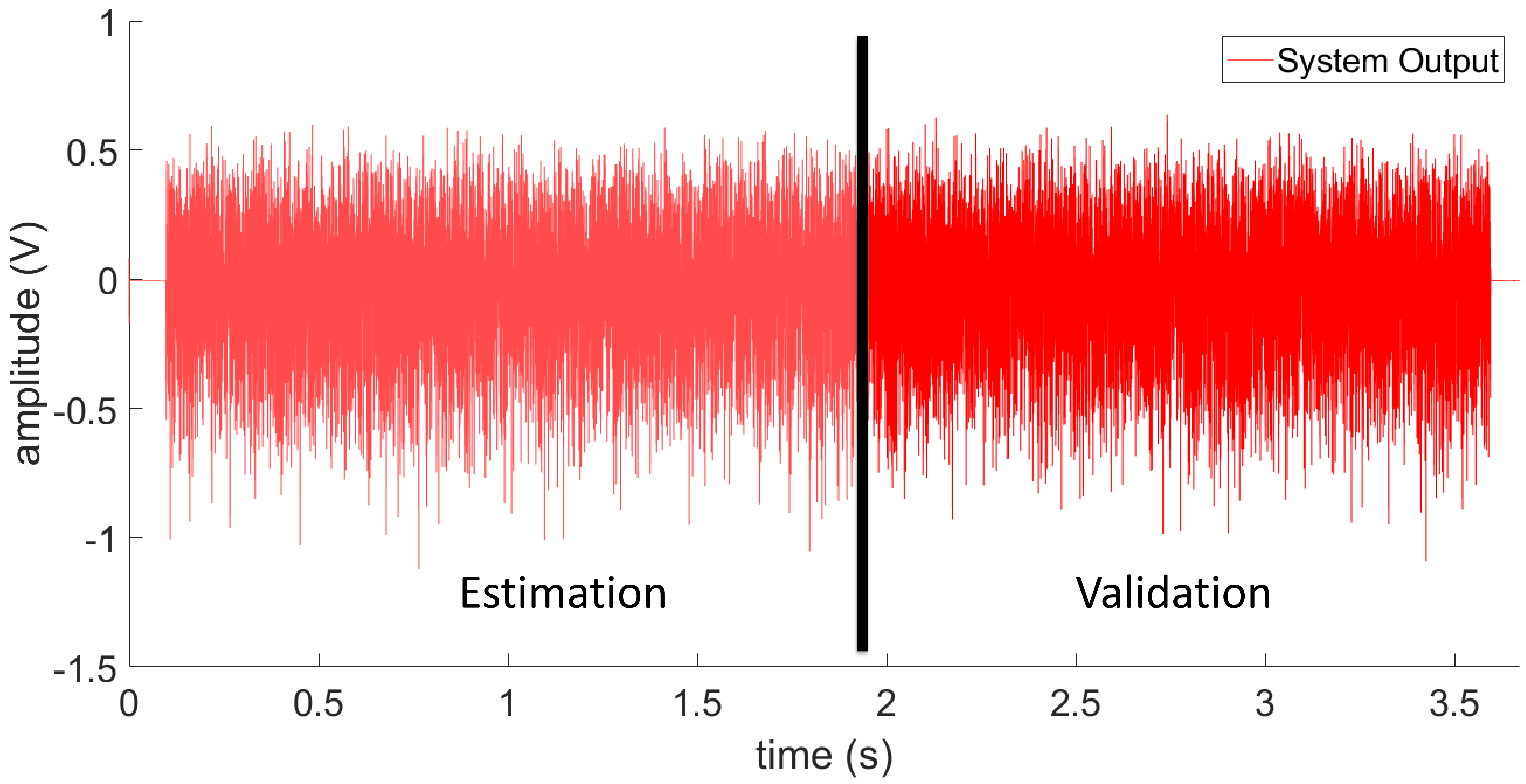
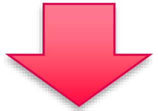


Wiener-Hammerstein Benchmark

$n_x = 6$

5^{th} degree polynomial nonlinearity



Neural network
20 neurons – 1 hidden layer - tansig

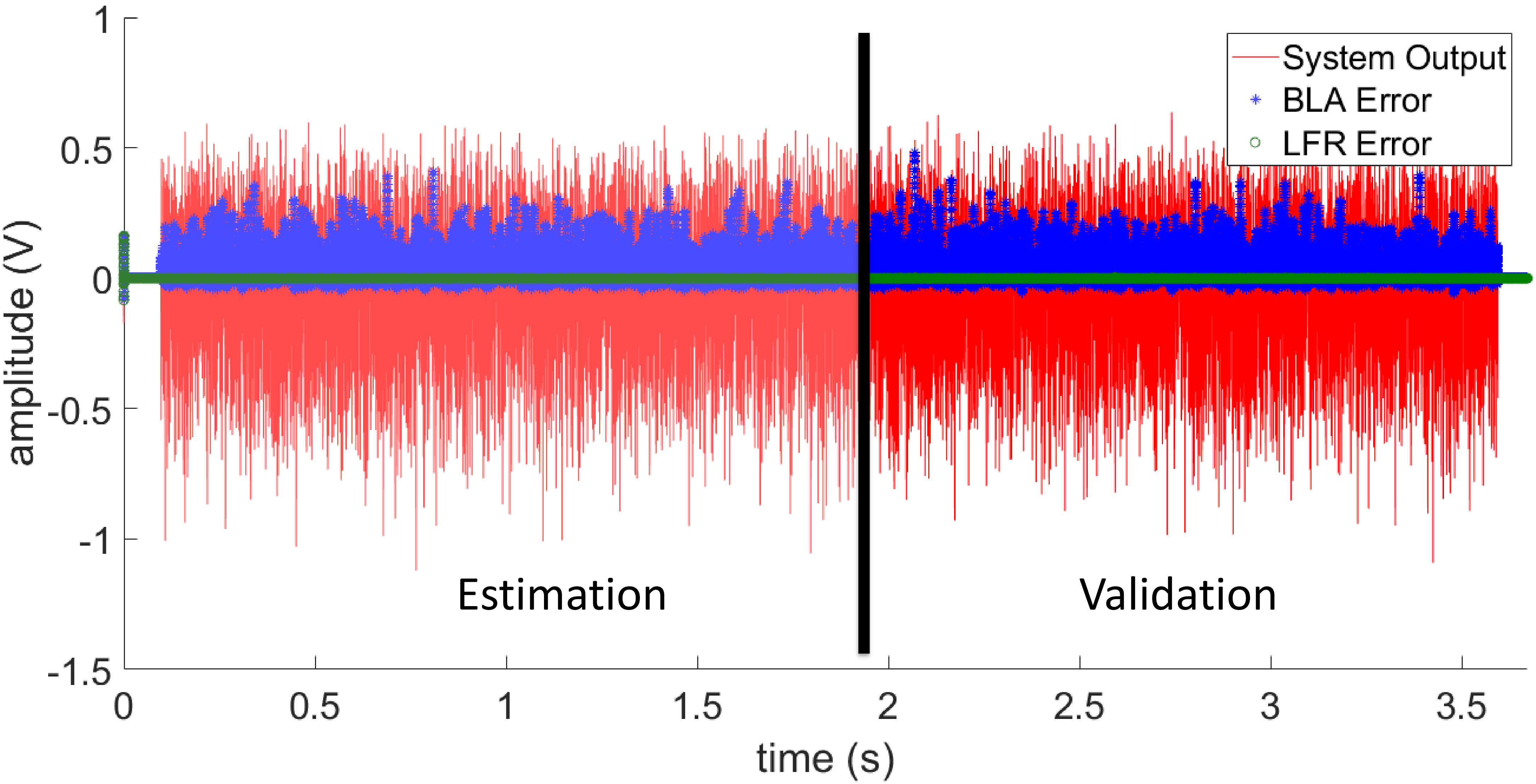
rms errors on estimation data

linear model error:	55.8 mV
NL-LFR error:	0.29 mV

rms errors on validation data

linear model error:	56.1 mV
NL-LFR error:	0.30 mV

Wiener-Hammerstein Benchmark



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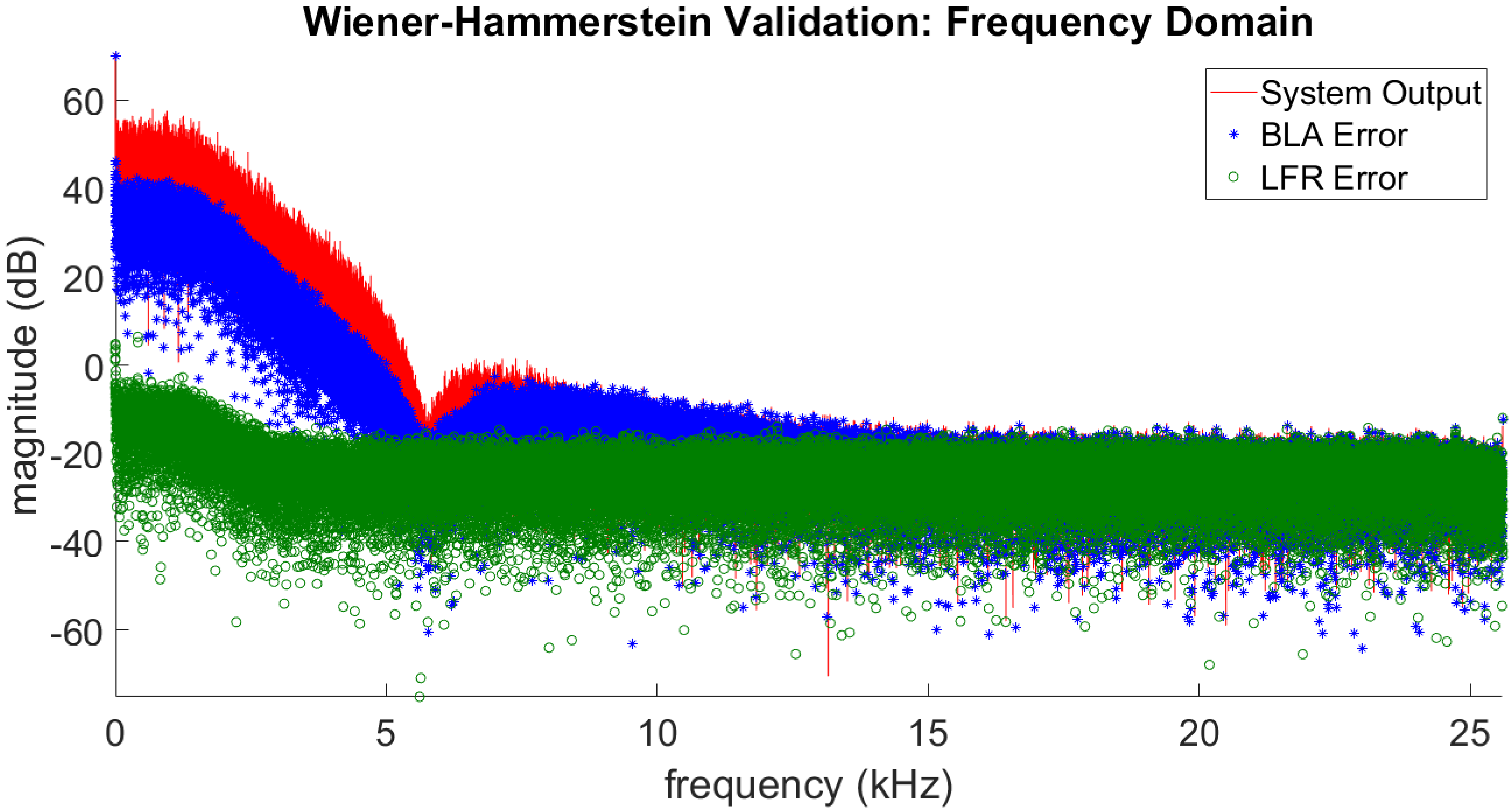

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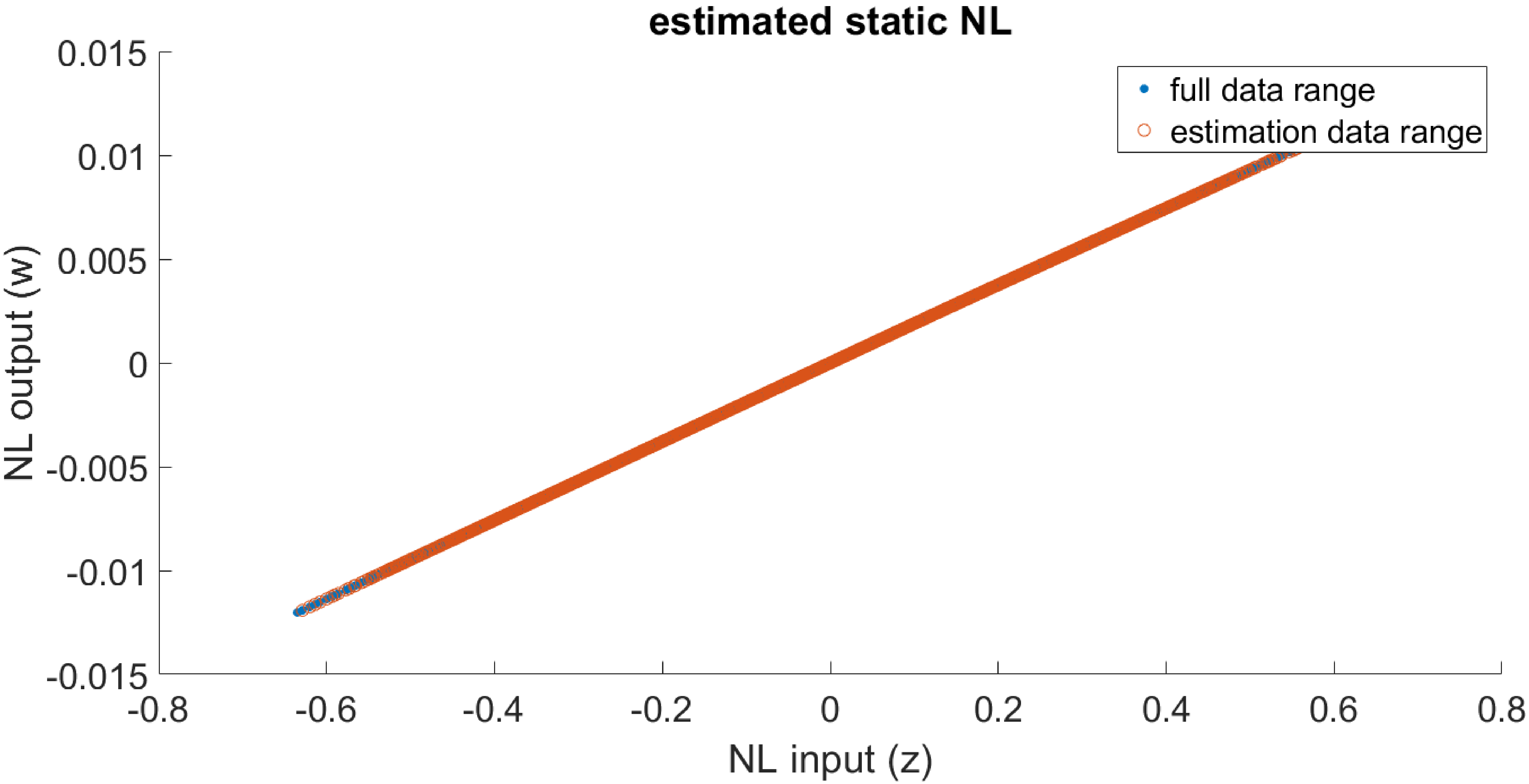

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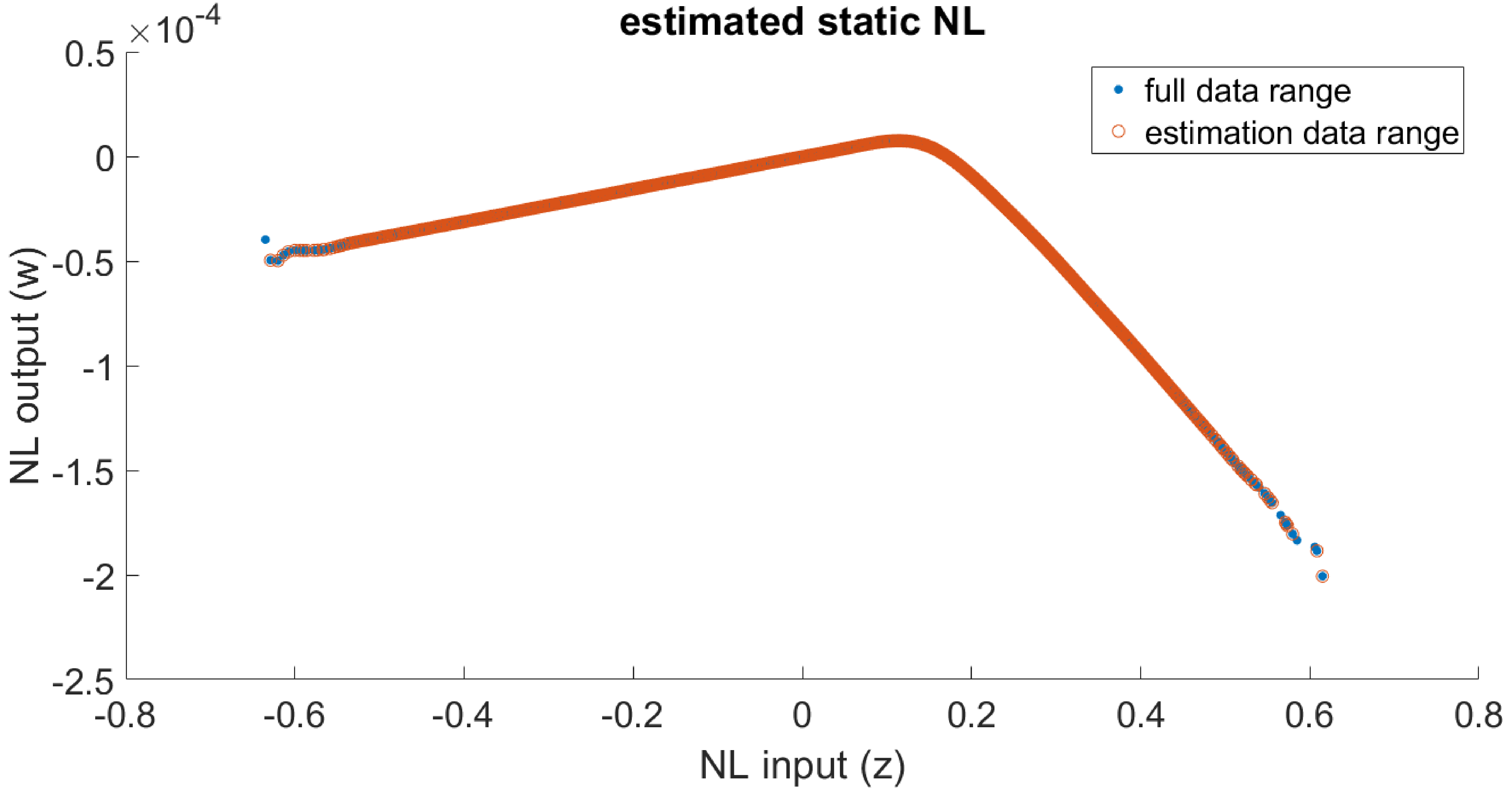

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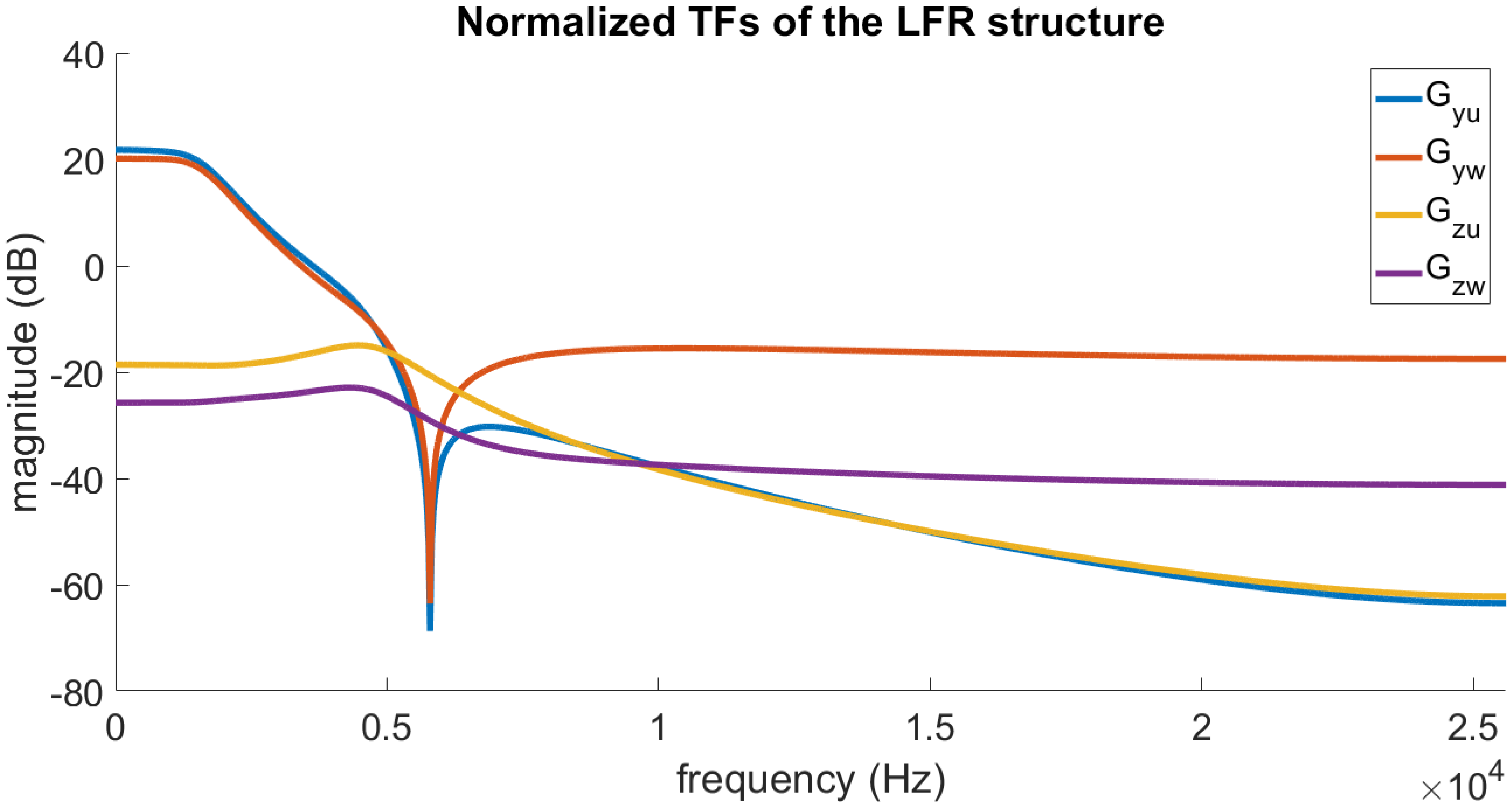

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Structured model directly from the data

Linear initial model followed by NL optimization

Good results on simple benchmark examples

Future work: MIMO NL, MIMO LTI

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