Задание 1

1) $\lim\limits_{x\to\infty}\left(\frac{x^2+x+1}{(x-1)^2}\right)=\lim\limits_{x\to\infty}\left(\frac{x^2+x+1}{x^2-2x+1}\right)=\lim\limits_{x\to\infty}\left(\frac{1+\frac{1}{x}+\frac{1}{x^2}}{1-\frac{2}{x}+\frac{1}{x^2}}\right)=$

$=\frac{1+0+0}{1-2\cdot0+0}=1$

Ответ: 1

2) $\lim\limits_{x\to5}\left(\frac{5+14x-3x^2}{x^2-2x-15}\right)=\lim\limits_{x\to5}\left(\frac{-3x^2+15x-x+5}{x^2+3x-5x-15}\right)=\lim\limits_{x\to5}\left(\frac{-3x(x-5)-(x-5)}{(x+3)(x-5)}\right)=$

$=\lim\limits_{x\to5}\left(\frac{-3x+1}{x+3}\right)=\frac{15+1}{8}=-2$

Ответ: -2

3) $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2-x+1}\right)=\lim\limits_{x\to\infty}\left(\frac{(x-\sqrt{x^2-x+1})(x+\sqrt{x^2-x+1})}{(x+\sqrt{x^2-x+1})}\right)=$

$=\lim\limits_{x\to\infty}\left(\frac{x^2-(x^2-x+1)}{x+\sqrt{x^2-x+1}}\right)=\lim\limits_{x\to\infty}\left(\frac{x-1}{x+\sqrt{x^2-x+1}}\right)=$

$=\lim\limits_{x\to\infty}\left(\frac{1-\frac{1}{x}}{1+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}\right)=\frac{1-0}{1+\sqrt{1-0+0}}=\frac{1}{2}$

Ответ: $\frac{1}{2}$

4) $\lim\limits_{x\to0}\left(\frac{\sin^2(5x)}{\operatorname{tg}^2(3x)}\right)=\lim\limits_{x\to0}\left(\frac{\sin^2(5x)\cdot\cos^2(3x)}{\sin^2(3x)}\right)$

$\lim_{x\to 0}\left(\cos^2(3x)\right) = 1$

$\lim_{x\to 0}\left(\frac{\sin^2(5x)}{\sin^2(3x)}\right) = \lim_{x\to 0}\left(\frac{\sin(5x)}{\sin(3x)}\right)^2$

4) $\lim_{x\to 0}\left(\frac{\sin^2(5x)}{tg^2(3x)}\right) = \lim_{x\to 0}\left(\frac{\sin^2(5x)\cdot\cos^2(3x)}{\sin^2(3x)}\right) = 1\cdot\frac{25}{9} = \frac{25}{9}$

$\lim_{x\to 0}\left(\cos^2(3x)\right) = 1$

$\lim_{x\to 0}\left(\frac{\sin^2(5x)}{\sin^2(3x)}\right) = \lim\left(\frac{\sin(5x)}{\sin(3x)}\right) = \frac{25}{9}$

+

Ответ $\frac{25}{9}$

5) $\lim_{x\to\infty}\left(\frac{3x+4}{3x+2}\right)^{x+2} = \lim_{x\to\infty}\left(1+\frac{2}{3x+2}\right)^{\frac{3(x+2)}{3}} =$

$= \lim_{x\to\infty}\left(1+\frac{2}{3x+2}\right)^{\frac{1}{3}(3x+2)} = e^{\frac{2}{3}}$

Ответ $e^{\frac{2}{3}}$