Cosine Kernel Approximation using Box-filtered Mip-maps

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If the cosine function were expressed by the sum of weighted box functions, our goal would be achieved:

$$w_G(\ell) \approx -\frac{4^\ell df(\ell)}{d\ell}$$

Here, the cosine function can be expressed as a function of mip-level L:

$$f(L) = \cos(\theta) = \cos\left(\frac{2^L\pi}{4s}\right) \Rightarrow \begin{cases} f(\ell) = \cos\left(\frac{2^\ell\pi}{4s}\right) \\ \frac{df(\ell)}{d\ell} = -\frac{2^\ell\pi\ln 2}{4s}\sin\left(\frac{2^\ell\pi}{4s}\right) \end{cases}$$

$$\therefore w_G(\ell) \approx \frac{8^\ell\pi\ln 2}{4s}\sin\left(\frac{2^\ell\pi}{4s}\right)$$

In order to evaluate the blending weight between two mip levels:

$$\alpha(L) = \frac{w_G(L)}{\int_L^{\log_2 2s} w_G(\ell)d\ell} \approx \frac{\frac{8^L\pi\ln 2}{4s}\sin\left(\frac{2^L\pi}{4s}\right)}{\int_L^{\log_2 2s}\frac{8^\ell\pi\ln 2}{4s}\sin\left(\frac{2^\ell\pi}{4s}\right)d\ell}$$

$$= \frac{\frac{8^L\pi\ln 2}{4s}\sin\left(\frac{2^L\pi}{4s}\right)}{\left.\frac{(32s^2-4^\ell\pi^2)\cos\left(\frac{2^\ell\pi}{4s}\right)+2^{\ell+3}s\pi\sin\left(\frac{2^\ell\pi}{4s}\right)}{\pi^2}\right|_L^{\log_2 2s}}$$

$$= \frac{8^L\pi^3\sin\left(\frac{2^L\pi}{4s}\right)\ln 2}{\left.(128s^3-4^{\ell+2}s\pi^2)\cos\left(\frac{2^\ell\pi}{4s}\right)+2^{\ell+5}s^2\pi\sin\left(\frac{2^\ell\pi}{4s}\right)\right|_L^{\log_2 2s}}$$

$$= \frac{8^L\pi^3\sin\left(\frac{2^L\pi}{4s}\right)\ln 2}{64s^3\pi+(128s^3-4^{L+2}s\pi^2)\cos\left(\frac{2^L\pi}{4s}\right)-2^{L+5}s^2\pi\sin\left(\frac{2^L\pi}{4s}\right)}$$

$$\therefore \alpha(L) \approx \frac{8^L\pi^3\sin\left(\frac{2^L\pi}{4s}\right)\ln 2}{(128s^3-4^{L+2}s\pi^2)\cos\left(\frac{2^L\pi}{4s}\right)-2^{L+5}s^2\pi\sin\left(\frac{2^L\pi}{4s}\right)+64s^3\pi}$$