PHY180: Classical Mechanics
Tutorial Questions

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Problem 1

Assume that the elevator undergoes unciform acceleration, then the scale reading as it moves up with acceleration a should be:

$$W' = m(g+a) \tag{1}$$

Letting $m = W/g$ be the mass of the student with W being the weight before the elevator moves, then:

$$W' = \frac{W}{g}(g+a) \implies a = g\left(\frac{W'}{W} - 1\right) \tag{2}$$

So the top speed would be:

$$v_{\mathsf{max}} = a\Delta t = g\Delta t\left(\frac{W'}{W} - 1\right) \approx 2.1\,\mathrm{m\,s^{-1}} \tag{3}$$

Problem 2

At steady state, each mass has the same acceleration of:

$$a = \frac{F}{2m} \tag{4}$$

For mass B, the only force acting on it is the spring force, so this must mean from Newton's second law:

$$m\left(\frac{F}{2m}\right) = k\Delta x \implies \Delta x = \frac{F}{2k} \tag{5}$$

so the distance between the two carts is $\boxed{L + \frac{F}{2k}}$. At the instant we let go, box B would still have the same acceleration and box A would have the same acceleration as box B but in the opposite direction.

After I stop pulling, I expect the carts to undergo SHM around their center of mass, which moves at a constant velocity. When cart B is moving at its fastest, the spring is not stretched.

Problem 3

(a) The quantity F_0 represents the initial force at $t = 0$.

(b) The time integral of the force gives us the change in momentum:

$$p_f - 0 = \int_0^\tau F_0 e^{-t/\tau}\,\mathrm{d}t = -F_0\tau e^{-t/\tau}\Big|_{t=0}^{t=\tau} = F_0\tau\left(1 - e^{-1}\right) \tag{6}$$

(c) We do the same thing to get:

$$p_f = \int_0^{5\tau} F_0 e^{-t/\tau}\,\mathrm{d}t = F_0\tau\left(1 - e^{-1/5}\right) \tag{7}$$

(d) As $t \to \infty$, we get:

$$p_f = F_0\tau \tag{8}$$

(e) Suppose this time occurs at $t = N\tau$, then the momentum after this time has elapsed is:

$$fF_0\tau = F_0\tau\left(1 - e^{-1/N}\right) \tag{9}$$

where $f = 95\%$. Solving this, we get:

$$e^{-1/N} = 1 - f \implies -\frac{1}{N} = \ln(1-f) \implies N = \frac{1}{\ln\left(\frac{1}{1-f}\right)} = 0.334 \tag{10}$$

for a total time of:

$$t = N\tau = 0.167\text{ms}. \tag{11}$$