PHY180: Testlet 2 Review

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1 General Collisions

There are three types of collisions:

- **Elastic:** Energy is conserved.
- **Inelastic:** Some energy is lost.
- **Completely Inelastic:** Maximum loss of energy. Objects stick together afterwards.

The **coefficient of restitution** is defined as:

$$e = \frac{v_{2,f} - v_{1,f}}{v_{1,i} - v_{2,i}} \tag{1}$$

For an elastic collision, $e = 1$. The **conservation of momentum** applies when the net external force is zero:

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}. \tag{2}$$

The conservation of energy applies when no energy is being transferred in or out of the system (e.g. friction / explosions could make it lose energy):

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2. \tag{3}$$

Oftentimes, problems are easier to deal with in the **center of mass frame**, as the net momentum is zero. The velocity of the center of mass is:

$$v_{\text{cm}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \tag{4}$$

We can write the energy of the system as:

$$K_{\text{tot}} = K_{\text{cm}} + K_{\text{conv}} \tag{5}$$

where:

$$K_{\text{cm}} = \frac{1}{2}(M + m) v_{\text{cm}}^2 \tag{6}$$

For a totally inelastic collision, the energy dissipated (which so happens to be the max) is given by the **convertible kinetic energy**:

$$K_{\text{conv}} = \frac{1}{2} \mu v_{12}^2 \tag{7}$$

where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is the **reduced mass** and v_{12} is the relative velocity of the two objects.

1.1 Tips and Tricks

When solving a general collisions problem, there are two ways (that are equally as valid) to do so.

1. Use conservation of momentum and the coefficient of restitution equation. Avoid using conservation of energy unless absolutely needed, since that will give a quadratic.
2. View things in the center of mass frame where the total momentum is zero. In this frame, the objects simply collide, switch directions, and have their speeds scaled by a factor of e. Switching back to the lab frame gives us the answer.

Example 1: As an example and for practice, I will solve the most general problem. A mass m_1 has velocity v_i and collides with a mass m_2 with velocity u_i. If the coefficient of restitution is e, what are their final velocities v_f and u_f?

We have two equations:

$$u_f - v_f = ev_i - eu_i \tag{8}$$
$$m_1 v_i + m_2 u_i = m_1 v_f + m_2 u_f \tag{9}$$

Making the substitution $u_f = ev_i - eu_i + v_f$ gives us:

$$m_1 v_i + m_2 u_i = m_1 v_f + m_2(ev_i - eu_i + v_f) \tag{10}$$
$$m_1 v_i + m_2 u_i = (m_1 + m_2)v_f + m_2 ev_i - m_2 eu_i \tag{11}$$
$$v_f = \frac{m_1 v_i + m_2 u_i - em_2(u_i - v_i)}{m_1 + m_2} \tag{12}$$

and similarly for u_f, we have:

$$u_f = \frac{m_1 v_i + m_2 u_i - em_1(v_i - u_i)}{m_1 + m_2} \tag{13}$$

Example 2: We can also solve this from the center of mass frame. The CoM has a velocity of $v_{\text{cm}} = \dfrac{m_1 v_i + m_2 u_i}{m_1 + m_2}$, so in this frame, the two objects have a velocity of $v_i' = v_i - v_{\text{cm}}$ and $u_i' = u_i - v_{\text{cm}}$. When they collide, their velocities change by a factor of $-e$ to become:

$$v_f' = -e(v_i - v_{\text{cm}}) \tag{14}$$
$$u_f' = -e(u_i - v_{\text{cm}}) \tag{15}$$

and shifting back to the lab frame, they have velocities of:

$$v_f' = -e(v_i - v_{\text{cm}}) + v_{\text{cm}} = \frac{m_1 v_i + m_2 u_i}{m_1 + m_2}(1 + e) - ev_i \tag{16}$$
$$u_f' = -e(u_i - v_{\text{cm}}) + v_{\text{cm}} = \frac{m_1 v_i + m_2 u_i}{m_1 + m_2}(1 + e) - eu_i \tag{17}$$

which you can verify is the same result as in example 1.

2 Energy

For a closed system, the change in **mechanical energy** is zero:

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0 \tag{18}$$

The potential energy near Earth's surface is:

$$U_g = mgh \tag{19}$$