PHY180: Classical Mechanics
Tutorial #1

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Problem 1

We can model velocity as a linear function:

$$a(t) = 1 - \frac{1}{2}t \tag{1}$$

where each term has units of acceleration. Then the velocity as a function of time is given by:

$$v(t) = \int \left(1 - \frac{1}{2}t\right) \mathrm{d}t = t - \frac{1}{4}t^2 + C \tag{2}$$

Here $C = 0$ since $v(0) = 0$. Integrating again, we find:

$$x(t) = \int \left(t - \frac{1}{4}t^2\right) \mathrm{d}t = \frac{1}{2}t^2 - \frac{1}{12}t^3 + C \tag{3}$$

Again, $C = 0$ since $x(0) = 0$. To find the position at $t = 4$s, we plug this in to get:

$$x(4) = 2.7\text{m} \tag{4}$$

To find the average velocity, we have:

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x(2)}{2\text{s}} = 0.67\text{m s}^{-1} \tag{5}$$

Problem 2

The average velocity in the first part of the journey is $\frac{v_f}{2}$ and the average velocity in the second part of the journey is v_f. If the first part of the journey takes a time $\Delta t = 3$s, then we can write the total distance traveled as:

$$L = \frac{1}{2}v_f\Delta t + (10 - \Delta t)v_f = v_f\left(10 - \frac{1}{2}\Delta t\right) \tag{6}$$

Solving for v_f then gives:

$$v_f = \frac{L}{10 - \frac{1}{2}\Delta t} = 11.76\text{m s}^{-1}. \tag{7}$$

Problem 3

From conservation of energy (or the $v_f^2 = v_i^2 + 2ad$ equation), we can find that the impact velocity on the ground is $v_i = \sqrt{2gh_0}$ and the exit velocity from the impact as $v_f = -\sqrt{2gh'}$ where the down direction as taken as the positive direction. Then the change in velocity is:

$$\Delta v = \sqrt{2g}\left(\sqrt{h_0} + \sqrt{h'}\right) \tag{8}$$

and the time it takes is thus:

$$\Delta t = 0.89 - \left(\sqrt{\frac{2h_0}{g}} + \sqrt{\frac{2h'}{g}}\right) \tag{9}$$

and the average acceleration is thus:

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{\sqrt{2g}\left(\sqrt{h_0} + \sqrt{h'}\right)}{0.89 - \left(\sqrt{\frac{2h_0}{g}} + \sqrt{\frac{2h'}{g}}\right)} = 344\text{m s}^{-2}. \tag{10}$$