ESC195 Notes

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1 Hyperbolic Functions

- Sometimes, combinations of e^x and e^{-x} are given certain names, for example:
  - **Hyperbolic sine:** $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$
  - **Hyperbolic cosine:** $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$
- They have the following properties:

$$\frac{d}{dx}\sinh x = \cosh x \tag{1}$$
$$\frac{d}{dx}\cosh x = \sinh x \tag{2}$$

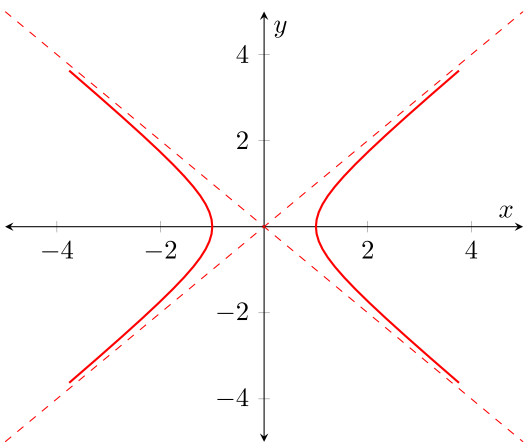
- They are related via:

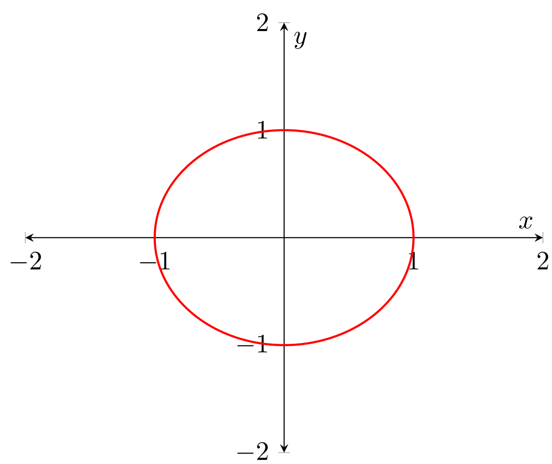
$$\cosh^2 x - \sinh^2 x = 1 \tag{3}$$

- Both the area of a circular sector and that of a hyperbolic sector is described by:

$$A = \frac{1}{2}t \tag{4}$$

  where t is the subtended angle, and the figures are parametized by $(\cos t, \sin t)$ and $(\cosh t, \sinh t)$.





- The catenary

$$y = a\cosh\left(\frac{x}{a}\right) + C \tag{5}$$

  describes the shape of a free hanging rope between two walls separated by a width a.

- The hyperbolic tangent is given by $\tanh x = \frac{\sinh x}{\cos hx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. and its derivative is given by:

$$\frac{d}{dx}\tanh x = \text{sech}^2\, x \tag{6}$$

- The inverse of $y = \sinh x$ is given by:

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right) \tag{7}$$

> **Tip**: A table of integrals and derivatives revolving around hyperbolic trig functions can be found in the textbook.