MAT185 Notes

QiLin Xue

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1 Vector Spaces

- A vector space consistso f three things: a set V together with the two operations of vector addition and scalar multiplication. We use V to describe both the vector space and the set itself. Specifically:

> **Definition**: A **real vector space** is a set V together with two operations called *vector addition* and *scalar multiplication* such that the following *axioms* hold:
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> 1. **Additive Closure (AC):** For all vectors $\boldsymbol{x}, \boldsymbol{y} \in V$, $\boldsymbol{x}+\boldsymbol{y} \in V$.
> 2. **Scalar Closure (SC):** For all vectors $\boldsymbol{x} \in V$, and scalars $c \in \mathbb{R}$, $c\boldsymbol{x} \in V$.
> 3. **Additive Associativity (AA):** For all vectors $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in V$, $(\boldsymbol{x}+\boldsymbol{y})+\boldsymbol{z} = \boldsymbol{x}+(\boldsymbol{y}+\boldsymbol{z})$
> 4. **Zero Vector (Z):** There exists a unique vector $\mathbf{0} \in V$ with the property that $\boldsymbol{x}+\mathbf{0} = \boldsymbol{x}$ for all vectors $\boldsymbol{x} \in V$.
> 5. **Additive Inverse (AI):** For each vector $\boldsymbol{x} \in V$, there exists a unique vector $-\boldsymbol{x} \in V$ with the property that $\boldsymbol{x}+(-\boldsymbol{x}) = \mathbf{0}$.
> 6. **Scalar Multiplication Associativity (SMA):** For all vectors $\boldsymbol{x} \in V$, and scalars c, $d \in \mathbb{R}$, $(cd)\boldsymbol{x} = c(d\boldsymbol{x})$.
> 7. **Distributivity of Vector Addition (DVA):** For all vectors $\boldsymbol{x}, \boldsymbol{y} \in V$, and scalars $c \in \mathbb{R}$, $c(\boldsymbol{x}+\boldsymbol{y}) = c\boldsymbol{x}+c\boldsymbol{y}$.
> 8. **Distributivity of Scalar Addition (DSA):** For all vectors $\boldsymbol{x} \in V$, and scalars $c, d \in \mathbb{R}$, $(c+d)\boldsymbol{x} = c\boldsymbol{x}+d\boldsymbol{x}$.
> 9. **Identity (I):** For all vectors $\boldsymbol{x} \in V$, $1\boldsymbol{x} = \boldsymbol{x}$.

- Note that *anything* can be a vector space as long as we define operations such that it has properties that satisfies the above properties.
- To define a **complex vector space**, we let the scalars belong in the complex space instead.