# PHY180: Classical Mechanics

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## 1 Lecture 2: Calculus of Motion

- The position, velocity, and acceleration are related by derivatives and integrals.

$$x(t) \xrightarrow{\frac{d}{dt}} v(t) \xrightarrow{\frac{d}{dt}} a(t) \tag{1}$$

$$x(t) \xleftarrow{\int \mathrm{d}t} v(t) \xleftarrow{\int \mathrm{d}t} a(t) \tag{2}$$

- Each quantity is consisted of certain dimensions that are not dependent on whether the quantity is a scalar or vector.
  - Dimensions of length are represented by $L$.
  - Dimensions of time are represented by $T$.
  - Dimensions of mass are represented by $M$.

**Example 1:** Suppose $x(t) = 2\text{ m} + (3\text{ m/s}^3)t^3$. What is $v(t)$?

We can take derivatives, and dropping the units, we get:

$$v = \frac{dx}{dt} \tag{3}$$

$$= \frac{d}{dt}(2 + 3t^3) \tag{4}$$

$$= 6t^2 \tag{5}$$

**Example 2:** Let $v = 3$ m/s. What is $x(t)$? suppose at $t = 0$, we have $x = 2$ m.

To solve, we need to integrate with respect to time:

$$x(t) = \int 3\,\mathrm{d}t \tag{6}$$

$$= 3t + C \tag{7}$$

We can determine the integration constant by plugging in the relationship $x(t = 0) = 3(0) + C$ which gives $C = 2$ m. Therefore:

$$x(t) = (3\text{ m/s})t + 2\text{ m} \tag{8}$$

- A definite integral represents the area under a graph between two certain points. We can compare this to the indefinite integral

$$F(t) = \int f(t)dt \tag{9}$$

such that

$$\int_a^b \mathrm{d}t = F(b) - F(a) \tag{10}$$

known as the fundamental theorem of calculus

**Example 3:** Suppose $f = 5t^2$. What is $\int_{t=1}^{t=2} f(t)\,\mathrm{d}t$?

We first integrate $f(t)$:

$$\int 5t^2 dt = \frac{5}{3}t^3 + C \equiv F(t) \tag{11}$$

We then use the fundamental theorem of calculus:

$$F(2) - F(1) = \left(\frac{5}{3}2^3 + \cancel{C}\right) - \left(\frac{5}{3}1^3 + \cancel{C}\right) = \frac{35}{3} \tag{12}$$

- We can apply this to position, velocity, and acceleration. If we want to determine:

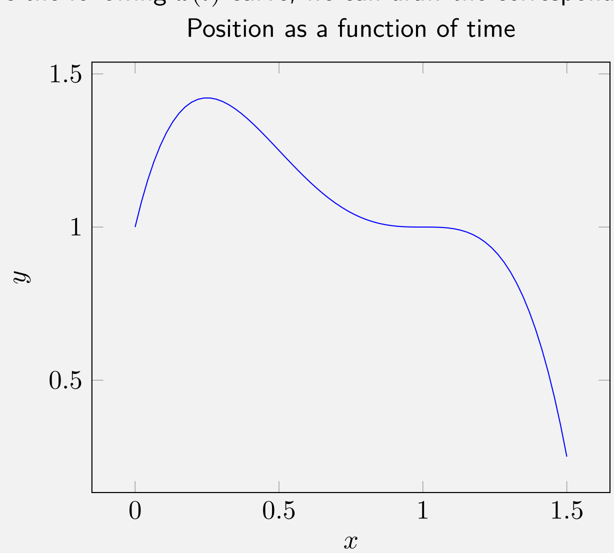
$$\int_{t_i}^{t_f} v(t)\,\mathrm{d}t = F(t_f) - F(t_i) = x_f - x_i = \Delta x \tag{13}$$

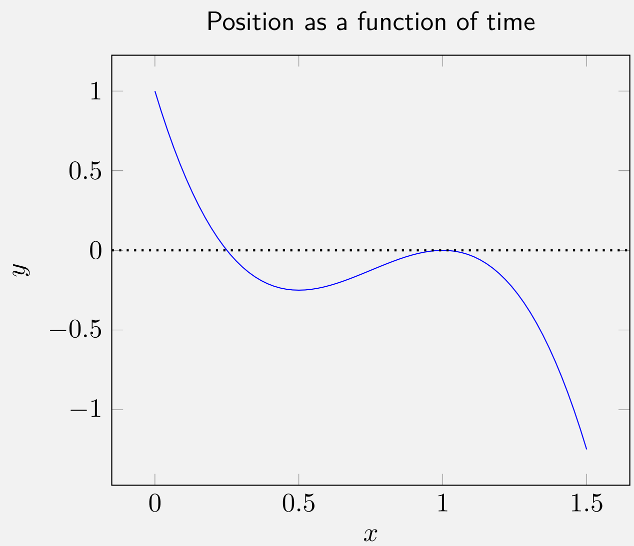
Similarly, the definite integral of acceleration gives the change in velocity:

$$\int_{t_i}^{t_f} a(t)\,\mathrm{d}t = \Delta v(t) \tag{14}$$

- Similarly, there are several position functions $x(t)$ that lead to the same $v(t)$. Information is lost.
- We can interpret these results graphically.

**Example 4:** Suppose we have the following $x(t)$ curve, we can draw the corresponding $v(t)$ curve:






Pay especially close attention to how the points line up when $v = 0$.

- We can go in the opposite direction as well by looking at how the area is changing via time graphically.
- For the special case where $v(t) = v_0$, we can differentiate to get:

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}v_0 = 0 \tag{15}$$

And integrate it to get:

$$\Delta x = \int_{t_i}^{t_f} v_0 \, \mathrm{d}t = v_0 \Delta t \implies x(t) = v_0 t + x_0 \tag{16}$$

- Similarly we can take a constant acceleration $a(t) = a_0$ so that we can determine that:

$$v(t) = \int a_0 \, \mathrm{d}t = a_0 t + v_0 \tag{17}$$

and the position is given by:

$$x(t) = \int (a_0 t + v_0) \, \mathrm{d}t = \frac{1}{2} a_0 t^2 + v_0 t + x_0 \tag{18}$$