

MSE160 Notes

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Contents

1 Introduction	1
1.1 Types of Material	1
1.2 Elastic Behaviour	1
1.3 Simple Model for Bonding in solids	2
1.4 Getting a stress-strain curve	2
1.5 Poisson's Ratio and Shear	3
2 Inelastic Behaviour	3

1 Introduction

1.1 Types of Material

- There are three classes of material (though not all materials fall under these categories):
 - Metals
 - Ceramics
 - Polymers
- **Metals** (e.g. Fe, Cr, Cu, Zn, Al) are held together with *metallitic* bonds and is described by **bond theory**.
- **Ceramics** (e.g. porcelain, concrete) are held together with *ionic* bonds and are *brittle*. A lot of them are metal oxides.
- **Polymer** (Teflon®, Gore-tex®, polyethylene) *tend* to be from *covalent bonds*

Warning: The word plastic actually describes a material property, and not a material type. There are plastics that are not polymers.

- Examples of materials that do not fall under this classification scheme include wood, skin, superconductors, and more.

1.2 Elastic Behaviour

- Hooke's law tells us that $F = -k\Delta x$, where Δx is the displacement from equilibrium.
- **Engineering stress** is defined as $\sigma = \frac{F}{A_0}$ where A_0 is the *initial* (unloaded) cross-sectional area.

Warning: Due to material properties, the cross sectional area of a spring can change as it elongates or compresses, so the engineering stress only refers to the initial cross sectional area. The *true stress* refers to the force divided by the real area.

- **Engineering strain** is defined as $\varepsilon = \frac{\Delta \ell}{\ell_0}$ and the two are related via the **Young's Modulus**:

$$\sigma = E\varepsilon \quad (1)$$

- There are two possible definitions for elastic deformation. When viewing it from a macroscopic perspective:

Definition: During elastic deformation, the sample dimensions return to their original dimensions upon unloading.

but it is also possible to view it from a microscopic perspective:

Definition: During elastic deformation, atoms return to their original positions upon unloading.

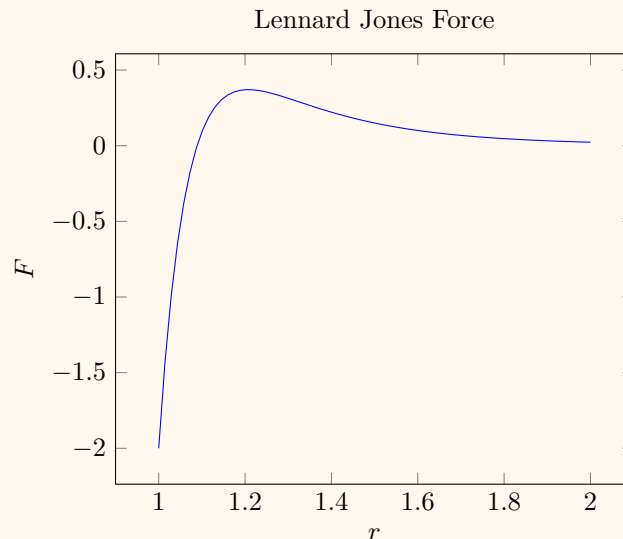
1.3 Simple Model for Bonding in solids

- A crude (but quite accurate) model is to assume nearby atoms in a solid are connected by springs. (This is actually Einstein's model of solid, except he modeled the interactions as quantum harmonic oscillators)

Idea: A more realistic model would be using the Lennard-Jones potential, which gives the force between two atoms as:

$$V = -\frac{a_1}{r^{13}} + \frac{a_2}{r^7} \quad (2)$$

and is graphically represented below (here, $a_1 = 5$ and $a_2 = 3$ for illustration purposes only)



When the two atoms are close to each other, the force scales roughly linearly with displacement, which is exactly the description of Hooke's Law.

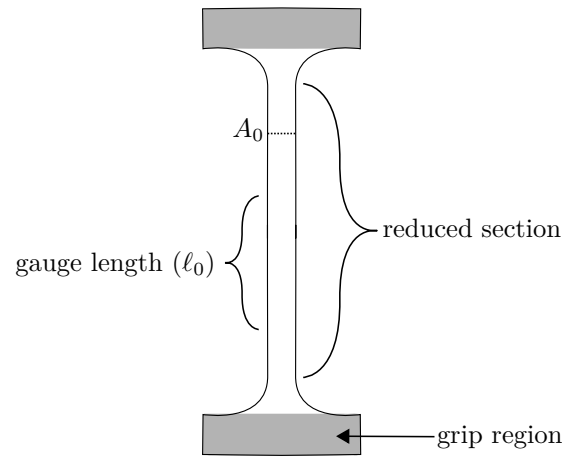
- Specifically, the Young's Modulus can be recovered by defining it as:

$$E \propto \left. \frac{dF}{dr} \right|_{r=r_0} \quad (3)$$

where r_0 is the equilibrium distance and is only dependent on the material. Permanently deforming a metal will not change its Young's Modulus.

1.4 Getting a stress-strain curve

- The tensile specimen is in a **dogbone** shape as illustrated below:



1.5 Poisson's Ratio and Shear

- When a material deforms, it does not deform in only one direction. The **poisson's ratio** ν relates the strain in all three directions:

$$\nu = -\frac{\varepsilon_R}{\varepsilon_Z} = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z} \quad (4)$$

for a cylindrically symmetrical material.

- Shear stress is defined as

$$\tau = \frac{F}{A_0} \quad (5)$$

and shear strain is defined as:

$$\gamma = \frac{\Delta \ell}{\ell_0} \quad (6)$$

- Similarly, shear stress and strain is related via the shear modulus G :

$$\tau = G\gamma \quad (7)$$

- The Young's modulus and the shear modulus is related via the poisson ratio:

$$E = 2G(1 + \nu) \quad (8)$$

2 Inelastic Behaviour

- Permanent deformation can be defined in three ways:
 - Upon unloading, sample does not return to original dimensions
 - Strain does not return to zero
 - Atoms move to new positions
 - Occurs near the end of linear behaviour
- Plastic comes from the greek word *plastikos*, which means to shape or to sculpt. In this course, plastic does not refer to the material type but instead the permanent deformation.
- The **strength** of a material describes when the permanent deformation occurs.
- The stress strain curve for different materials resemble different shapes.
 - Polymers have a distinct yielding region
 - Metals start a concave down behaviour as soon as elastic deformation ends
 - Ceramics have linear behaviour all the way until they fracture

- For polymers, the use of *Young's Modulus* is misleading since the elastic behaviour depends on several different types of bonds, while Young's Modulus is related to the behaviour of a single bond. As a result, the term **elastic modulus** is used to describe polymers and composite materials.