Dadas las siguientes matrices:

$$A=\begin{pmatrix}4 & -2 & 2\\ -1 & 3 & 1\\ 1 & 0 & 5\end{pmatrix},\qquad B=\begin{pmatrix}-2 & 3 & 1\\ 5 & 0 & 1\\ 0 & -1 & 3\end{pmatrix},\qquad C=\begin{pmatrix}0 & 2 & -3\\ -1 & 0 & 1\\ 6 & 7 & 2\end{pmatrix}$$

a) A+B=
b) 3.C=
c) [A-C.B]=

a) $A+B=\begin{pmatrix}4 & -2 & 2\\ -1 & 3 & 1\\ 1 & 0 & 5\end{pmatrix}+\begin{pmatrix}-2 & 3 & 1\\ 5 & 0 & 1\\ 0 & -1 & 3\end{pmatrix}=\begin{pmatrix}2 & 1 & 3\\ 4 & 3 & 2\\ 1 & -1 & 8\end{pmatrix}$

b) 3.C=3. $\begin{pmatrix}0 & 2 & -3\\ -1 & 0 & 1\\ 6 & 7 & 2\end{pmatrix}=\begin{pmatrix}0 & 6 & -9\\ -3 & 0 & 3\\ 18 & 21 & 6\end{pmatrix}$

c) [A-C.B]= $\begin{pmatrix}4 & -2 & 2\\ -1 & 3 & 1\\ 1 & 0 & 5\end{pmatrix}-\begin{pmatrix}2 & 1 & 3\\ 4 & 3 & 2\\ 1 & -1 & 8\end{pmatrix}.\begin{pmatrix}-2 & 3 & 1\\ 5 & 0 & 1\\ 0 & -1 & 3\end{pmatrix}=$

$$=\begin{pmatrix}4 & -2 & 2\\ -1 & 3 & 1\\ 1 & 0 & 5\end{pmatrix}-\begin{pmatrix}1 & 3 & 12\\ 7 & 10 & 13\\ -7 & -5 & 24\end{pmatrix}=\begin{pmatrix}3 & -5 & -10\\ -8 & -7 & -12\\ 8 & 5 & -19\end{pmatrix}$$

$$B=\begin{pmatrix}-2 & 3 & 1\\ 5 & 0 & 1\\ 0 & -1 & 3\end{pmatrix}$$

$$\begin{pmatrix}-2 & 3 & 1\\ 5 & 0 & 1\\ 0 & -1 & 3\end{pmatrix}\underset{F1 \to -1/2\,F1}{\sim}\begin{pmatrix}1 & -\frac{3}{2} & -\frac{1}{2}\\ 5 & 0 & 1\\ 0 & -1 & 3\end{pmatrix}\underset{F2 \to F2-5.F1}{\sim}\begin{pmatrix}1 & -\frac{3}{2} & -\frac{1}{2}\\ 0 & \frac{15}{2} & \frac{7}{2}\\ 0 & -1 & 3\end{pmatrix}\underset{F2 \leftrightarrow F3}{\sim}\begin{pmatrix}1 & -\frac{3}{2} & -\frac{1}{2}\\ 0 & -1 & 3\\ 0 & \frac{15}{2} & \frac{7}{2}\end{pmatrix}\underset{F2 \to -1.\,F2}{\sim}\begin{pmatrix}1 & -\frac{3}{2} & -\frac{1}{2}\\ 0 & 1 & -3\\ 0 & \frac{15}{2} & \frac{7}{2}\end{pmatrix}$$

$$\underset{F1\to F1+3/2F2}{\sim}\begin{pmatrix}1 & 0 & -5\\ 0 & 1 & -3\\ 0 & \frac{15}{2} & \frac{7}{2}\end{pmatrix}\underset{F3\to F3-15/2F2}{\sim}\begin{pmatrix}1 & 0 & -5\\ 0 & 1 & -3\\ 0 & 0 & 26\end{pmatrix}\underset{F3\to 1/26\,F3}{\sim}\begin{pmatrix}1 & 0 & -5\\ 0 & 1 & -3\\ 0 & 0 & 1\end{pmatrix}\underset{F1\to F1+5.F3}{\sim}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & -3\\ 0 & 0 & 1\end{pmatrix}\underset{F2\to F2+3.F3}{\sim}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 0 & 3 \\ \frac{1}{2} & -1 & 1 \\ -5 & 2 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 3 & 1 & 0 & 0 \\ \frac{1}{2} & -1 & 1 & 0 & 1 & 0 \\ -5 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

F1-> ½ F1

$$\left(\begin{array}{ccc|ccc} {\color{red}1} & 0 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -1 & 1 & 0 & 1 & 0 \\ -5 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

F2-> -1.F2

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & {\color{red}1} & -\frac{1}{4} & \frac{1}{4} & -1 & 0 \\ 0 & 2 & \frac{15}{2} & \frac{5}{2} & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} & -1 & 0 \\ 0 & 0 & 8 & 2 & 2 & 1 \end{array}\right)$$

F3-> 1/8.F3

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{3}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} & -1 & 0 \\ 0 & 0 & {\color{red}1} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{8} & -\frac{3}{8} & -\frac{3}{16} \\ 0 & 1 & 0 & \frac{5}{16} & -\frac{15}{16} & \frac{1}{32} \\ 0 & 0 & {\color{red}1} & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} \end{array}\right)$$

Siendo $\vec{u} = (2,4,-8)\ y\ \vec{v} = (5,3,7)$

Encontrar el módulo.

Hallar los ángulos directores.

Determinar el ángulo que forman entre ellos.

Calcula u x v.

$$|\vec{u}| = \sqrt{2^2 + 4^2 + (-8)^2} = \sqrt{4 + 16 + 64} = \sqrt{84} = 9{,}16$$

$$|\vec{v}| = \sqrt{5^2 + 3^2 + 7^2} = \sqrt{25 + 9 + 49} = \sqrt{83} = 9{,}11$$

Ángulos directores

A.D. u

$$\cos\alpha = \frac{x}{|\vec{u}|}$$

$$\cos\alpha = \frac{2}{\sqrt{84}}$$

$$\alpha = arc\ cos\left(\frac{2}{\sqrt{84}}\right)$$

$\alpha = 77°23'44''$

$$\cos\beta = \frac{y}{|\vec{u}|}$$

β =64°7'23''

$$\cos\gamma = \frac{z}{|\vec{u}|}$$

γ =150°47'38''

A.D. v

$$\cos\alpha = \frac{x}{|\vec{v}|}$$

$$\cos\alpha = \frac{5}{\sqrt{83}}$$

$$\alpha = arc\ cos\left(\frac{5}{\sqrt{83}}\right)$$

α =56° 42'49''

$$\cos\beta = \frac{y}{|\vec{v}|}$$

β =70°46'26''

$$\cos\gamma = \frac{z}{|\vec{v}|}$$

γ = 39° 47' 38''

Ángulo entre ellos.

$\vec{u}.\vec{v} = 2.5 + 4.3 + (-8).7 = 10 + 12 - 56 = -34$ por ser el valor negativo el ángulo entre ellos es obtuso.

Producto Vectorial

$$\vec{u} \times \vec{v} = \begin{vmatrix} \check{\imath} & \check{\jmath} & \check{k} \\ 2 & 4 & -8 \\ 5 & 3 & 7 \end{vmatrix} = \left(\check{\imath}.28 + 6.\check{k} + (-40)\check{\jmath}\right) - \left(20.\check{k} + (-24).\check{\imath} + 14\check{\jmath}\right) = 52\check{\imath} - 54\check{\jmath} - 14\check{k}$$

$$\begin{matrix} \check{\imath} & \check{\jmath} & \check{k} \\ 2 & 4 & -8 \\ - & - & - \end{matrix}$$

$$\vec{u} \times \vec{v} = (52;\ -54; -14)$$