TP1 Analisis Matemático Matrizes y determinantes

1) $A = \begin{vmatrix} 1 & 2 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 4 & 1 & 2 \end{vmatrix}$ $\quad B = \begin{vmatrix} \frac{1}{2} & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & \frac{1}{4} \end{vmatrix}$

2) $A = \begin{vmatrix} 1 & 2 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 4 & 1 & 2 \end{vmatrix}$

$\begin{matrix} 1 & 2 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \end{matrix}$

$|A| = 2 + \frac{1}{2} + 4 - (4 + \frac{1}{2} + 2)$

$|A| = 6{,}5 - 6{,}5$

$|A| = 0 \longrightarrow$ NO tiene inversa

$B = \begin{vmatrix} \frac{1}{2} & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & \frac{1}{4} \end{vmatrix}$

$|B| = \frac{1}{8} + 24 + 16 - (16 + 3 + 1)$

$|B| = 40{,}125 - 20$

$|B| = 20{,}125 \rightarrow$ Tiene inversa

inversa de B $= B^{-1}$

$$\left[\begin{array}{ccc|ccc} \frac{1}{2} & 2 & 4 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 4 & 3 & \frac{1}{4} & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} F1 = 2.F1 \\ F2 = F2 - 2.F1 \\ F3 = F3 - 4.F1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 8 & 2 & 0 & 0 \\ 0 & -7 & -14 & -4 & 1 & 0 \\ 0 & -13 & -31{,}75 & -8 & 0 & 1 \end{array}\right] \quad \begin{array}{l} F1 = F1 - 4.F2 \\ F2 = -\frac{1}{7}.F2 \\ F3 = F3 + 13.F2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{2}{7} & \frac{4}{7} & 0 \\ 0 & 1 & 2 & \frac{4}{7} & -\frac{1}{7} & 0 \\ 0 & 0 & -\frac{23}{4} & -\frac{4}{7} & -\frac{13}{7} & 1 \end{array}\right] \quad \begin{array}{l} \\ F2 = F2 - 2F3 \\ F3 = \frac{1}{-\frac{23}{4}}.F3 \end{array}$$

$$\left|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right| \left| \begin{matrix} -\frac{2}{7} & \frac{4}{7} & 0 \\ \frac{60}{7} & -\frac{127}{7} & \frac{8}{23} \\ \frac{16}{161} & \frac{52}{161} & -\frac{4}{23} \end{matrix}\right| = B^{-1}$$

RAngo de A

$$\left|\begin{matrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/2 \\ 4 & 1 & 2 \end{matrix}\right|$$

Paso1

$F2 = F2 - \frac{1}{2} . F1$

$$\left|\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 4 & 1 & 2 \end{matrix}\right|$$

Paso2

$F3 = F3 - 4F1$

$$\left|\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & -7 & -2 \end{matrix}\right|$$

Paso3

$F3 = F3 . \left(-\frac{1}{7}\right)$

$$\left|\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & \frac{2}{7} \end{matrix}\right|$$

Forma escalonada

EL rango de la matriz A es 2

Rango de B

$$\left|\begin{matrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{matrix}\right|$$

Paso1

$F1 = F1 . 2$

$F2 = F2 - 4 . F1$

$F3 = F3 - 8 . F1$

$$\left|\begin{matrix} 1 & 4 & 8 \\ 0 & -7 & -14 \\ 0 & -13 & -31.75 \end{matrix}\right|$$

Paso2

$F3 = F3 - \frac{13}{1} . F1$

$F2 = F2 : (-7)$

$$\left|\begin{matrix} 1 & 4 & 8 \\ 0 & 1 & 2 \\ 0 & 0 & -5.75 \end{matrix}\right|$$

Forma escalonada

EL rango de la matriz B es 3

3) Propiedades de las matrices

A- $(A \cdot B)^T = B^T \cdot A^T$

$$\left(\begin{vmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/2 \\ 4 & 1 & 2 \end{vmatrix} \cdot \begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix}\right)^T =$$

$$\begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix}^T \cdot \begin{vmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/2 \\ 4 & 1 & 2 \end{vmatrix}^T =$$

Ambos dan el mismo Resultado:

$$\begin{vmatrix} \frac{17}{2} & \frac{17}{4} & 12 \\ 7 & \frac{1}{2} & 15 \\ \frac{33}{4} & \frac{33}{8} & \frac{37}{2} \end{vmatrix}$$

B- $A + B = B + A$

$$\begin{vmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/2 \\ 4 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix} =$$

$$\begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/2 \\ 4 & 1 & 2 \end{vmatrix} =$$

Ambos dan el mismo Resultado:

$$\begin{vmatrix} \frac{3}{2} & 4 & 5 \\ \frac{5}{2} & 2 & \frac{5}{2} \\ 8 & 4 & \frac{9}{4} \end{vmatrix}$$

C- $a\,(A + B) = a.A + B.a$

$$2 \cdot \left(\begin{vmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/2 \\ 4 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix}\right) =$$

$$2 \cdot \begin{vmatrix} 1 & 2 & 1 \\ 1/2 & 1 & 1/2 \\ 4 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix} . 2 =$$

Ambos dan el mismo Resultado

$$\begin{vmatrix} 3 & 8 & 10 \\ 5 & 4 & 5 \\ 16 & 8 & 9/2 \end{vmatrix}$$

D- $B \cdot B^{-1} = I$

$$\begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix} \cdot \begin{vmatrix} -2/7 & 4/7 & 0 \\ \frac{60}{161} & -\frac{127}{7} & \frac{8}{23} \\ \frac{16}{161} & \frac{52}{161} & -\frac{4}{23} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = I$$

La matriz conseguida es

E- $(B^{-1})^T = (B^T)^{-1}$

$$\left(\begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix}^{-1}\right)^T = \begin{vmatrix} -\frac{2}{7} & \frac{60}{161} & \frac{16}{161} \\ \frac{4}{7} & -\frac{127}{161} & \frac{52}{161} \\ 0 & \frac{8}{23} & -\frac{4}{23} \end{vmatrix}$$

$$\left(\begin{vmatrix} 1/2 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 3 & 1/4 \end{vmatrix}^T\right)^{-1} = \begin{vmatrix} -\frac{2}{7} & \frac{60}{161} & \frac{16}{161} \\ \frac{4}{7} & -\frac{127}{161} & \frac{52}{161} \\ 0 & \frac{8}{23} & -\frac{4}{23} \end{vmatrix}$$

Matrizes iguales cumplen la igualdad