对弱导数的几点思考

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$L^p = \mathcal{L}^p / \sim$
when we talk about a funtion $u \in L^1_{loc}(\Omega)$ which has a ith weak derivative g also in $L^1_{loc}(\Omega)$, we wrte

$$g = \frac{\partial u}{\partial x_i}$$

Note that since g is unique, we can write $\frac{\partial u}{\partial x_i}$ without ambiguity. Then we have a operator

$$\partial_i : W^{1,p} \rightarrow L^p$$

actually ,we can see that there is a mapping

$$\partial_i : \mathbb{W}^{1,p} \rightarrow L^p$$

The latter descends to the former operator, and we used the same notation

Thus for $[u] \in W^{1,p}$, we define $\partial_i[u] := \partial_i u$ you should check ion is well defined which means doesnot depend On the representative u !
Then we can dm the linearity of the latter operator !
Hence for $u \in W^{1,p}$, let's say $u = [v]$, hence $u^+ = [v^+], u^- = [v^-]$, and

$$\partial_i u = \partial_i (u^+ - u^-) = \partial_i v + \partial_i v 1_{v=0} = \partial_i v \quad \in L^p$$

hence

$$\partial_i v = 0 \quad \text{almost everywhere on } \{x \in \Omega : v(x) = 0\}$$

you should check that this definition is well defined which means doesnot depend On th

Speaking of complex number $\mathbb{C}$, the precise definition is as follows

定义

Complex number is a set denoted by $\mathbb{C}$ whose elements are ordered pairs

$$\mathbb{C} := \{[a, b] : a, b \in \mathbb{R}\}$$

With the convention that $[a, b] = [c, d]$ iff $a = c, b = d$.

Then we define the following addition and mulptication operation to make $\mathbb{C}$ become a field.

$$[a, b] + [c, d] = [a + c, b + d]; [a, b] \times [c, d] = [ad - bc, ac + bd]$$

Then we define i denoting the elemtent $[0, 1]$, it is clear that $[0, 0]$ and $[1, 0]$ are the zero element and unity element in the field.

There is a natural injection $\mathbb{J}$ between $\mathbb{R} \to \mathbb{C}$

$$J : \mathbb{R} \to \mathbb{C} \quad r \mapsto [r, 0]$$

Using J, we can think of $\mathbb{R}$ as a subset of $\mathbb{C}$, to be precise, when we talk about $a \in \mathbb{R}$ as an element in $\mathbb{C}$, we infer the unique element $J(a)$!!!!!

Also remember $i = [0, 1]$, we have

$[a, b] = [a, 0] + [0, b] = J(a) + iJ(b)$, we now make the convention that $a_{\mathbb{C}}{=}J(a)$, then we write the former expression as

$[a, b] = a_{\mathbb{C}} + b_{\mathbb{C}} i$

Let T be an operator defind on L^p, with range in$\mathbf{M}(\mu)/\sim$.
Then we define $T \circ u$ where $u \in L^q(0,T;L^p)$ as

$$Tu = T\tilde{u}, \quad [\tilde{u}] = u;$$

Then we can use the axiom of choice to define the following function

$$v(t,\bullet) =< Tu(t) >$$

Generating a funtion in $M((0,T)\times\Omega) << v(t,x) >>$
If $T\tilde{u}$ is ∂_i, then for v, we have the following characteriaztion

$$\int_\Omega v(t,x)\phi(x)dx = \int_\Omega < \tilde{u}(t) > \partial_i\phi(x)dx, \quad \text{for any } t \in (0,T), \phi \in \mathbf{D}(\Omega)$$

We donot distinguish the case when $\mathbb{M}$ and $\mathbf{L}^q$ have different underlying domain.

Deadkind 分割. 再仔细想一下, 首先我们约定 $[a,b]$, 其中 $a\in X, b\in Y$ 中的相等, 此时再定义笛卡尔乘积为遍历 $a\in X$ 和 $b\in Y$

你他妈的如果再这样废话的话, 真的有点欠锤了哈, 我说实话, 真的得考虑拿个 macbook 出去了

Let $q \in (1, \infty]$, there is a countable dense subset $(\{v_j\} \subset C_c(\mathbb{R}^n))$ in the separable space $L^{q'}(\mathbb{R}^n)$. If $f \in L^r(\mathbb{R}^n)$ where $1 < r < q$ and $g \in L^q(\mathbb{R}^n)$ satisfies the following

$$\int_{\mathbb{R}^n} f v_j = \int_{\mathbb{R}^n} g v_j \quad \text{for all } j$$

Then whether $f = g$ a.e.?

Consider two σ-finite measure space (X, A, μ), (Y, B, ν) For the completion of two space$(X \times Y, (A \otimes B)^*, (\mu \otimes \nu)^*)$. There is a simple observation that for any nonnegative function $f : X \times Y \to [0, +\infty]$ we have if there is a measurable set $X_0 \in A$ such that f_x is measurable for all $x \in X_0$ Then the function defined by

$$\phi_x = \int_Y f(x, y) d\nu$$

is measurable on X_0

Proof.

Note that there exists a funtion $g : X \times Y \to [0, +\infty]$ which is $A \otimes B$ measurable such that for almost all x, $f(x, \cdot) = g(x, \cdot)$ a.e. on Y. $\square$

Thus there is null set N contained in X_0 such that $\phi_x = \int_Y g(x, y) d\nu$ on $X_0 \setminus N$

When we have two Banach space X and Y, where $X \hookrightarrow Y$, then the notation $L^p(0,T;X) \hookrightarrow L^p(0,T;Y)$ is a little absurbing, as we know in general $X \neq Y$, hence the eqivalence class may arise an issue!
The correct understaning is that we use the following inclusion mapping i, and define the mapping

$$\tilde{i} : L^p(0,T;X) \to L^p(0,T;Y)$$
$$[u]_X \mapsto [iu]_Y$$

To simplify our notation, we often ignore the subscript and the inclusion mapping i, just write $L^p(0,T;X) \hookrightarrow L^p(0,T;Y)$ or $L^p(0,T;X) \subset L^p(0,T;Y)$ even more. But we should know that this is unserious!! But it is always harmless as we donot change the heart of this idea or this property!