Part IB Paper 6: Information Engineering

LINEAR SYSTEMS AND CONTROL

Example Paper 6/2

Straightforward questions are marked †
*Tripos standard questions are marked **

Pole locations

1. Classify the following transfer functions as asymptotically stable, marginally stable, or unstable:

(a)$1/(s+3)$	(b)$(s-1)/(s+3)$	(c)$1/(s-3)$
(d)$1/s$	(e)$1/s^2$	(f)$(2s+3)/(s^2+8)$
(g)$(2s+3)/(s^2-8)$	(h)$10/[(s+5)(s^2-s+1)]$	(i)$10e^{-2s}/[(s+5)(s^2+s+1)]$

In each case draw the pole–zero diagram.

2. † Figure 1 shows, in the left–hand column, the pole locations of some second–order systems. The right–hand column shows the impulse responses of these systems, but *not in the same order*. Match the diagrams in the left–hand column with the corresponding diagrams in the right–hand column.

3. * The U.S. military specification 'Flying Qualities of Piloted Airplanes' (MIL–F–8785C) lays down the following requirements for the damping factor (ζ) and undamped natural frequency (ω_n) of the so–called 'Dutch Roll' mode of oscillation:

Flight phase A: $\zeta \geq 0.4$, $\omega_n \geq 1.0$ rad/sec, $\zeta\omega_n \geq 0.35$ rad/sec.
Flight phase B: $\zeta \geq 0.08$, $\omega_n \geq 0.4$ rad/sec, $\zeta\omega_n \geq 0.15$ rad/sec.

Assume that, for this mode of oscillation, an aircraft can be modelled as a second–order transfer function, with an atmospheric disturbance as input, and the aircraft's heading as output. (Flight phase A includes reconnaisance and terrain following, while Flight Phase B includes cruising and in-flight refuelling.)

For each flight phase show, on a diagram of the complex plane, the corresponding regions where the poles of the transfer function may *not* lie in order to satisfy each constraint individually. Describe the behaviour of the aircraft's heading in response to a short–term disturbance, in each of the flight phases (if the full specification is satisfied).

Frequency response

4. † A sinusoidal input is applied to a system with transfer function

$$\frac{1}{(s+1)(s+0.1)}$$

At which frequency does the output lag behind the input by 90°? If the input amplitude is X, what is the output amplitude at this frequency?

5. Heat propagation in a semi-infinite metal rod is described by the partial differential equation

$$\frac{\partial \theta}{\partial t} = a\frac{\partial^2 \theta}{\partial x^2} - \mu\theta$$

where $\theta(x,t)$ is the temperature, $a = \frac{\lambda}{\rho C}$ is the thermal diffusivity of the metal and μ represents the thermal losses.

The transfer function from temperature at the end of the rod to temperature at at distance l along it is given by

$$G(s) = e^{-l\sqrt{(s+\mu)/a}}.$$

Show that

$$\ln|G(j\omega)| \arg G(j\omega) = \frac{l^2\omega}{2a}.$$

Hint: Recall that $\ln x = \ln|x| + j\arg(x)$ *for complex* x.

How would you exploit this result to measure experimentally the thermal diffusivity of a given metal? (This method was discovered by Angstrom.)

6. Draw the Bode diagrams for the following transfer functions on the attached log-lin graph paper:

$$G_1(s) = \frac{(1+sT)}{(1+asT)} \quad \text{for} \quad \begin{matrix} a = 0.1 \\ T = 1 \end{matrix}, \qquad G_3(s) = \frac{1}{(1+2\zeta sT + s^2T^2)} \quad \text{for} \quad \begin{matrix} \zeta = 0.1 \\ T = 10 \end{matrix}$$

$$G_2(s) = \frac{(1+sT)}{(1+asT)} \quad \text{for} \quad \begin{matrix} a = 4 \\ T = 2.5 \end{matrix}, \qquad G_4(s) = \frac{1}{sT(1+2\zeta sT + s^2T^2)} \quad \text{for} \quad \begin{matrix} \zeta = 0.01 \\ T = 0.1 \end{matrix}$$

7. Substitute the parameters from Table 1 in Examples Paper 1 into the transfer function of the Mars Lander, $P(s)$ (found in Q3(d) of Paper 1) to justify the use of the approximation $P(s) = \frac{1}{ms^2} = \frac{1}{200s^2}$ which will be used from now on. For the engine model, assume $H(s) = H_0 = 1121.0$

Sketch the Bode diagram for $L(s) = P(s)H(s)K(s)$ (known as the 'return ratio' of the system) using

(a) $K(s) = k_P$

(b) $K(s) = k_p + k_d s$,

for $k_p = 0.01$ and $k_d = 0.01$

Test: Initialise the lander at 510 m and 700 m, trying different values for k_p and k_d (see 'Simulation Notes' below). What role do each of these parameters appear to perform?

8. In question 6 on Example Paper 1, the transmission path through a telephone network was modelled as a system with impulse response $\beta e^{-\beta t}$. A better model is given by the transfer function
$$\frac{22000s}{(s+300)(s+22000)}$$
Sketch Bode diagrams for the two models, taking $\beta = 22000$. If a sinusoidal signal of frequency ω rad/sec is transmitted through the network, for which values of ω will the received signal be attenuated by less than $1/\sqrt{2}$, according to each of the two models?

   Hint: First put the transfer function into the standard form given in lectures (and used in Q6 above).

   Check your answer using Python (see 'Simulation Notes' below).

9. * Figures 2 and 3 give the magnitude and phase plots of the following transfer functions (note that two of the magnitude plots are identical):
$$G_1(s) = \frac{(s+30)}{(s+1)(s+10)(s+100)} \qquad G_2(s) = \frac{s}{(1+0.5s+s^2)}$$
$$G_3(s) = \frac{s}{(1+0.01s)(1+0.5s+s^2)} \qquad G_4(s) = \frac{s(1-0.1s)}{(1+0.1s)(1+0.5s+s^2)}$$
$$G_5(s) = \frac{(1+0.001s+0.01s^2)(1+10s)}{(1+0.01s)^4}$$
Associate the various plots with the correct $G_i(s)$ by considering straight line approximations to each Bode diagram.

Suitable questions on past Tripos papers 2005 Q1, 2007 Q3, 2008 Q1, 2009 Q1, 2010 Q3, 2011 Q2, 2012 Q3, 2013 Q1, 2014 Q2, 2015 Q1, 2017 Q1 (a)-(b).

Simulation Notes for Q7

The controller can be implemented by editing the function defining `throttle` and using the variables `altitude`, `target_altitude` and `speed`. Note that you will always need to add the constant value of F_{eq}/`MAX_THRUST` that defines the equilibrium throttle: remember that all the theory relates to changes about this equilibrium, and that in the simulator H_0 = `MAX_THRUST`. For (i), start by trying $K_p = 0.01$ and for (ii) start with $K_p = K_d = 0.01$.

Simulation Notes for Q8

Python and its control theory module python-control can easily be used online without any local installation:

1. Go to: `https://notebooks.azure.com/thiagoburghi/libraries/ib-control`.
2. Click on the relevant template file.

3. Click on the 'clone' button (near top left).
4. If needed: log in to Azure using your @cam institutional e-mail (you will be redirected to the Raven log-in page).
5. Agree to creating a clone when prompted.
6. Click on the relevant template file again: this will start a working iPython Notebook that you can run and edit.
7. To run a cell, select it, and press shift + enter
8. You only need to run the first cell once. It will automatically install and import all necessary files.

You can also run the files locally by installing Python. The most straightforward way is to download the Anaconda distribution from: `https://www.anaconda.com/download/` (make sure you choose the latest version of Python). Once Anaconda is installed, install python-control by opening the application Anaconda Prompt and typing: `conda install -c conda-forge control`. The full documentation for python-control can be found in `https://python-control.readthedocs.io/`.

To check your answer using Python, first type (this code can also be found in the relevant Azure notebook template):

```
import control as ctr
import numpy as np
import matplotlib.pyplot as plt
```

Note that those commands create aliases (`plt`, `ctr`, and `np`) which are useful when calling functions from the corresponding packages. To create a transfer function G_1, type

```
G1 = ctr.tf([22000],[1,22000])
```

You can type `help(ctr.tf)` to understand how this function works. Now, plot a bode diagram by typing

```
fig1 = plt.figure()
mag, phase, omega = ctr.bode(G1)
fig1.set_size_inches(10,10)
```

The first line creates a figure object where the diagram is plotted, and the last line rescales the figure. Now, type

```
G2 = ctr.tf([22000],np.convolve([1,300],[1,22000]).astype(float))
fig2 = plt.figure()
mag, phase, omega = ctr.bode(G2)
```

In addition,

```
t1,y1 = ctr.step_response(G1)
fig3 = plt.figure()
plt.plot(t1,y1)
plt.grid()
```

and

```
t2,y2 = ctr.step_response(G2)
fig4 = plt.figure()
plt.plot(t2,y2)
plt.grid()
```

will display the step responses of the first and second model, respectively, and add grids to the plots. Are the step responses of the two models consistent with their frequency responses? Type help(`np.convolve`), `help(ctr.step_response)`, and `help(ctr.bode)` to see why the commands given above work, and for further options.

Answers:

1. Asymptotically stable: (a), (b), (i).
   Marginally stable: (d), (f).
   Unstable: (c), (e), (g), (h).

2. —

3. —

4. $\omega = 1/\sqrt{10}$ rad/sec (or $f = 0.0503$ Hz).
   $\frac{X\sqrt{10}}{1.1}$.

5. —

6. —

7. —

8. First–order model: $\omega < 22000$ rad/sec (or $f < 3501$ Hz).
   Second–order model: $300 < \omega < 22000$ rad/sec (or $47.7 < f < 3501$ Hz).

9. —

Michaelmas 2017

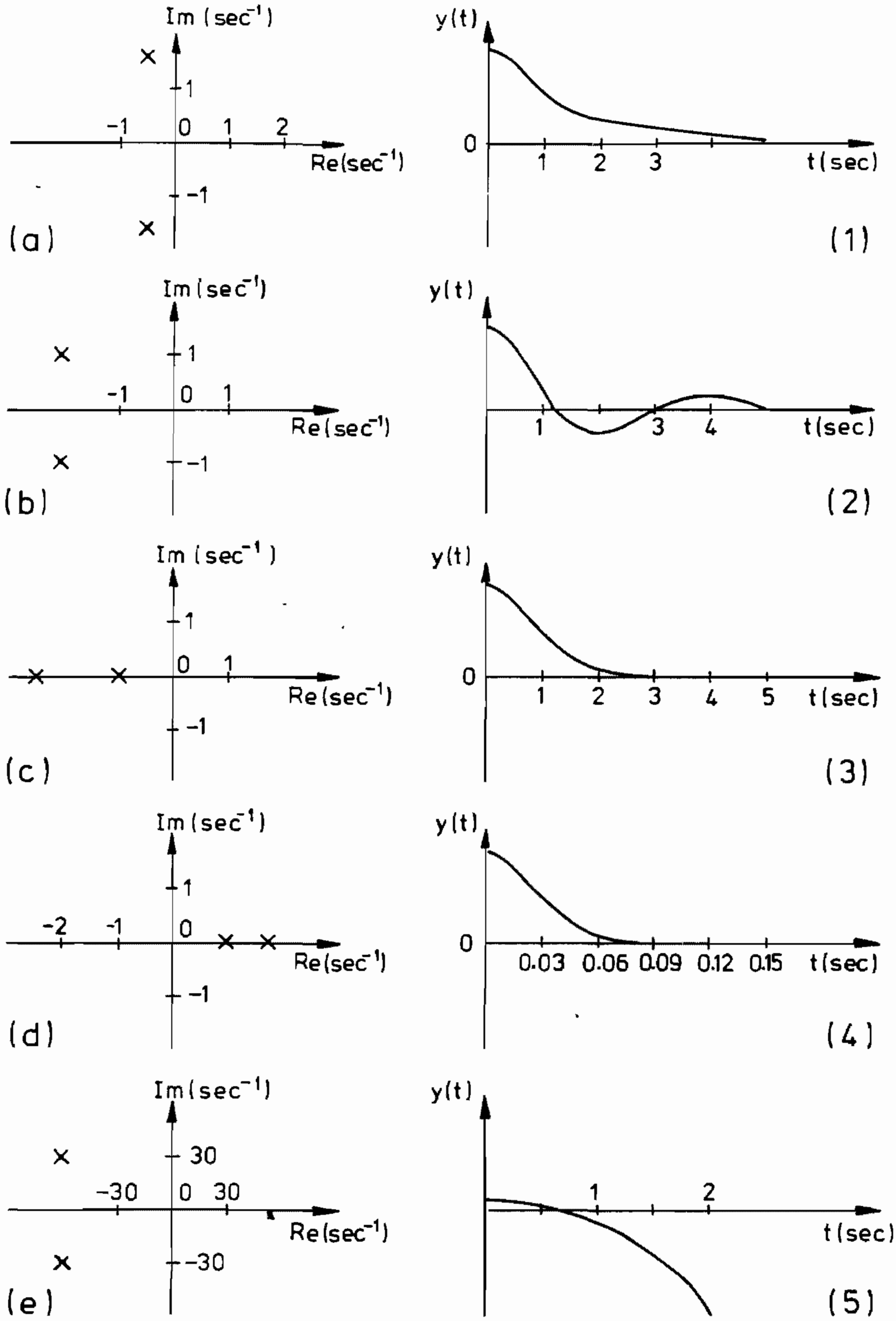


Fig.1

Figure 2

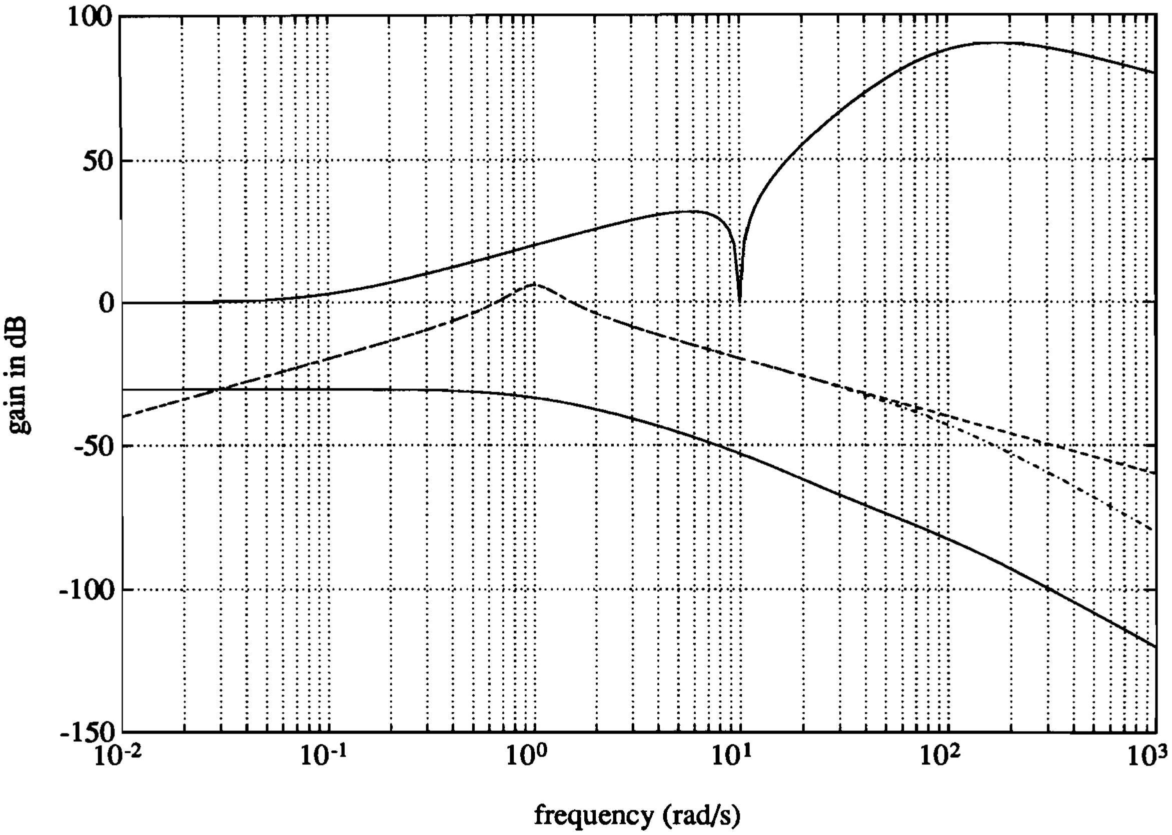
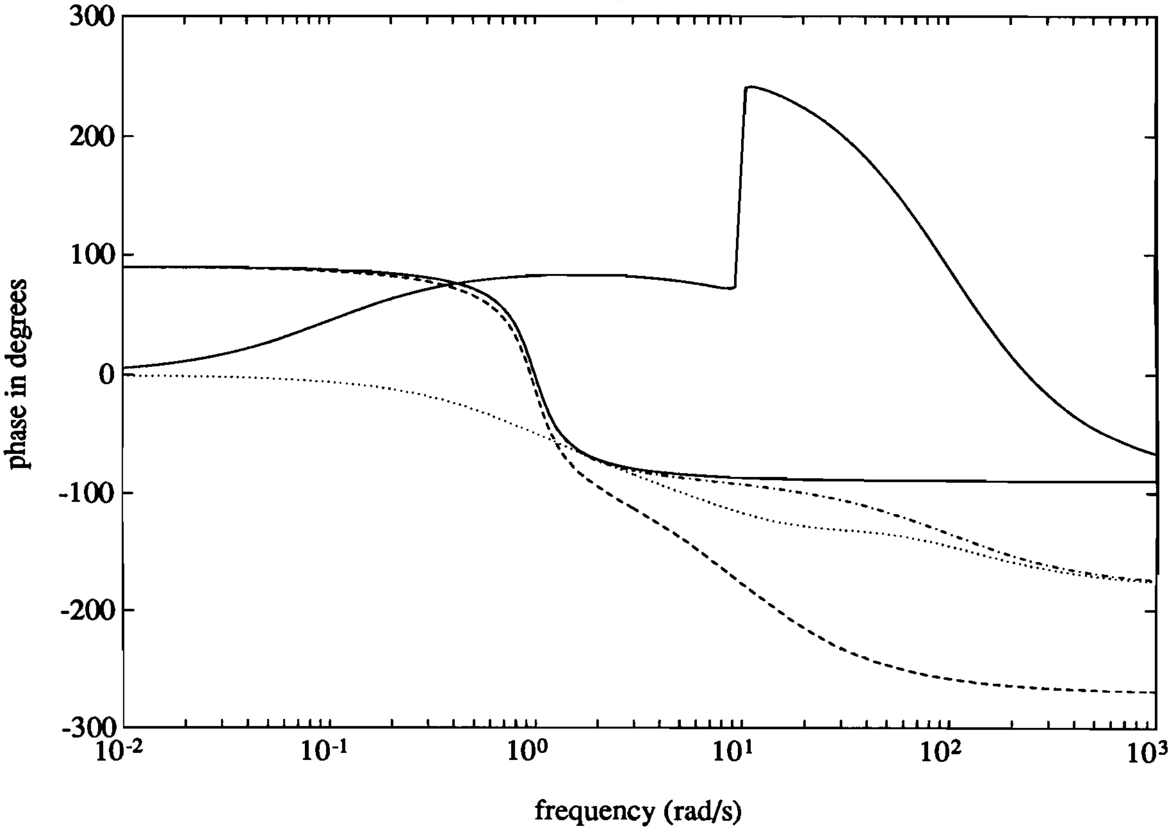
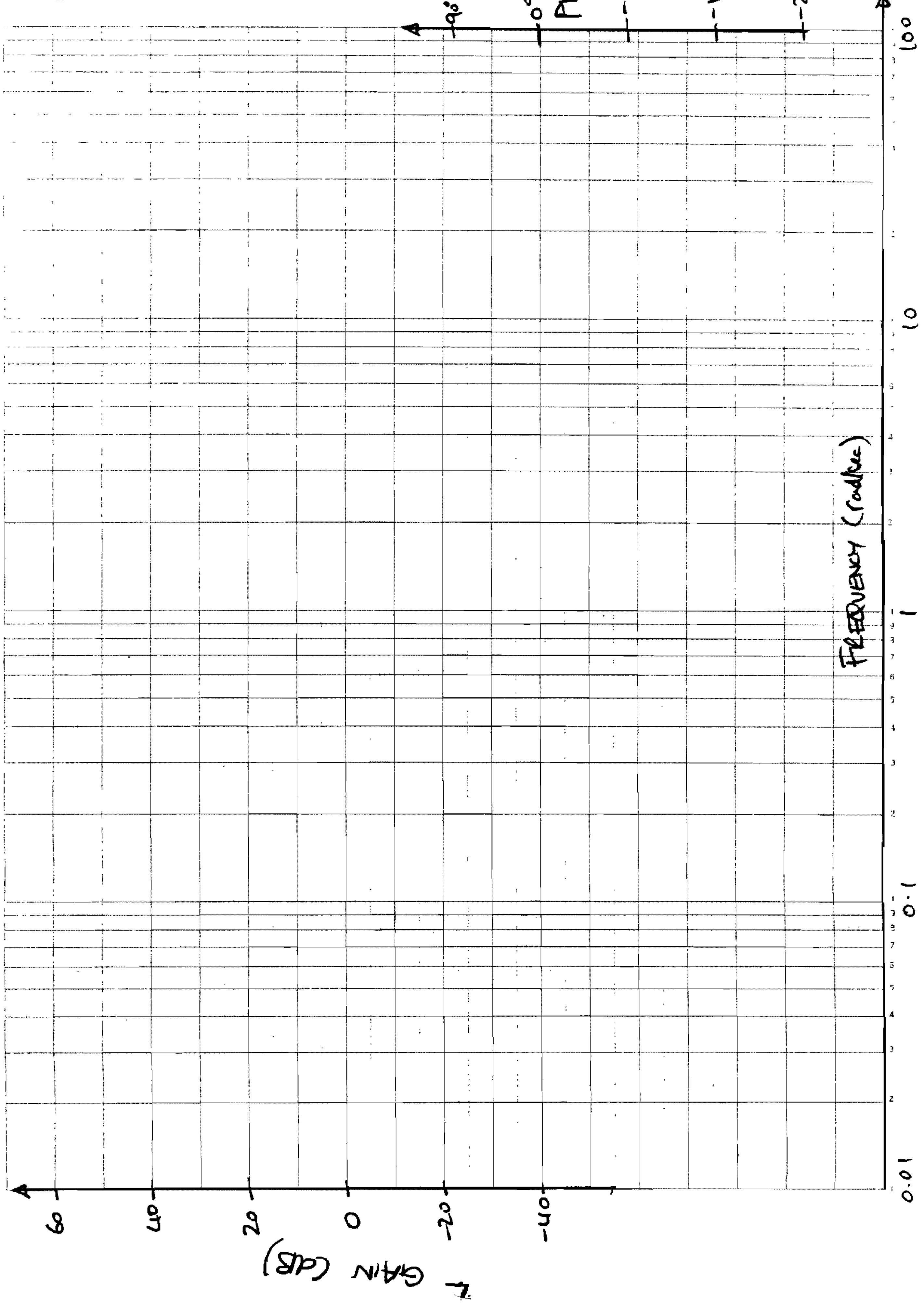


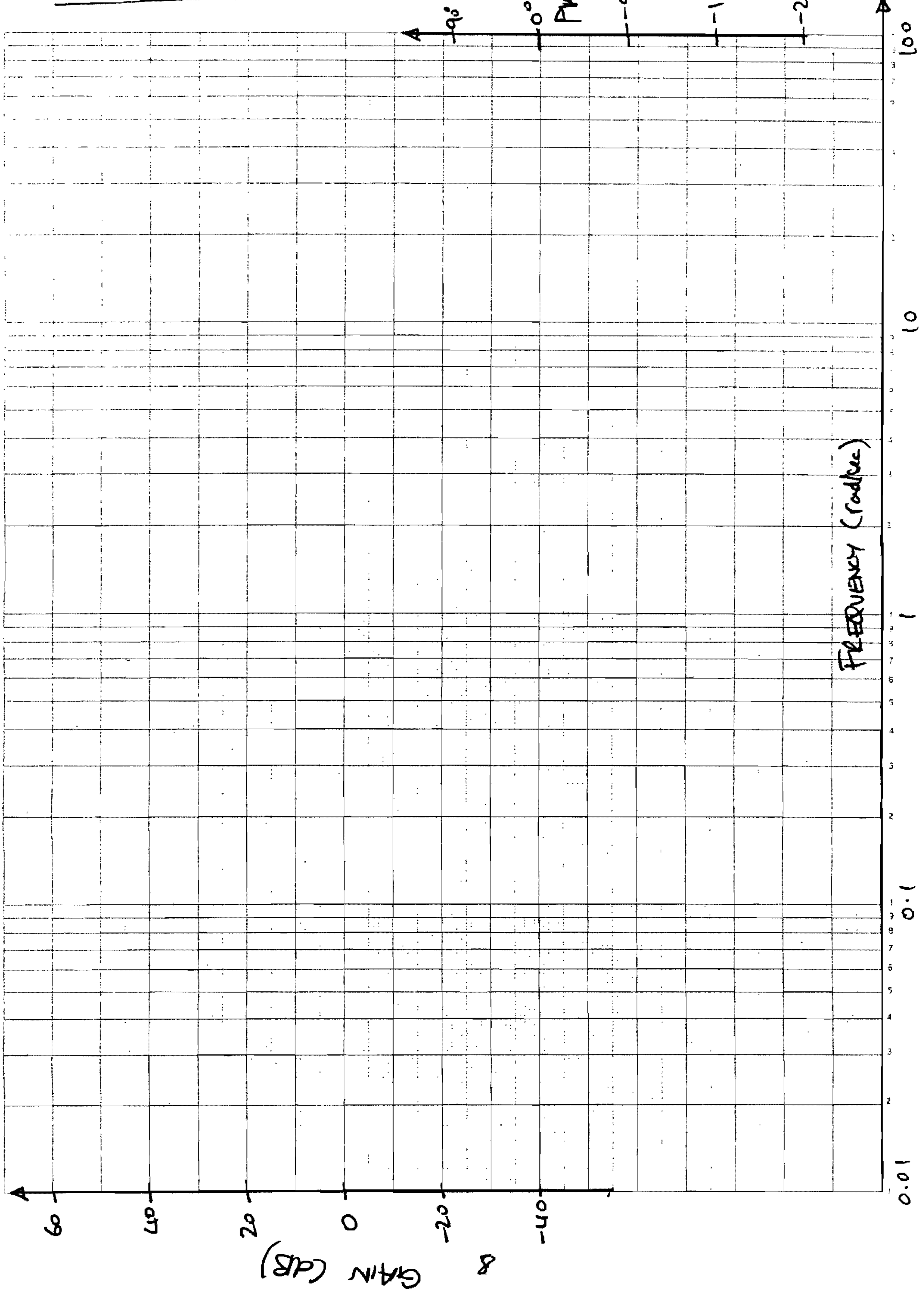
Figure 3



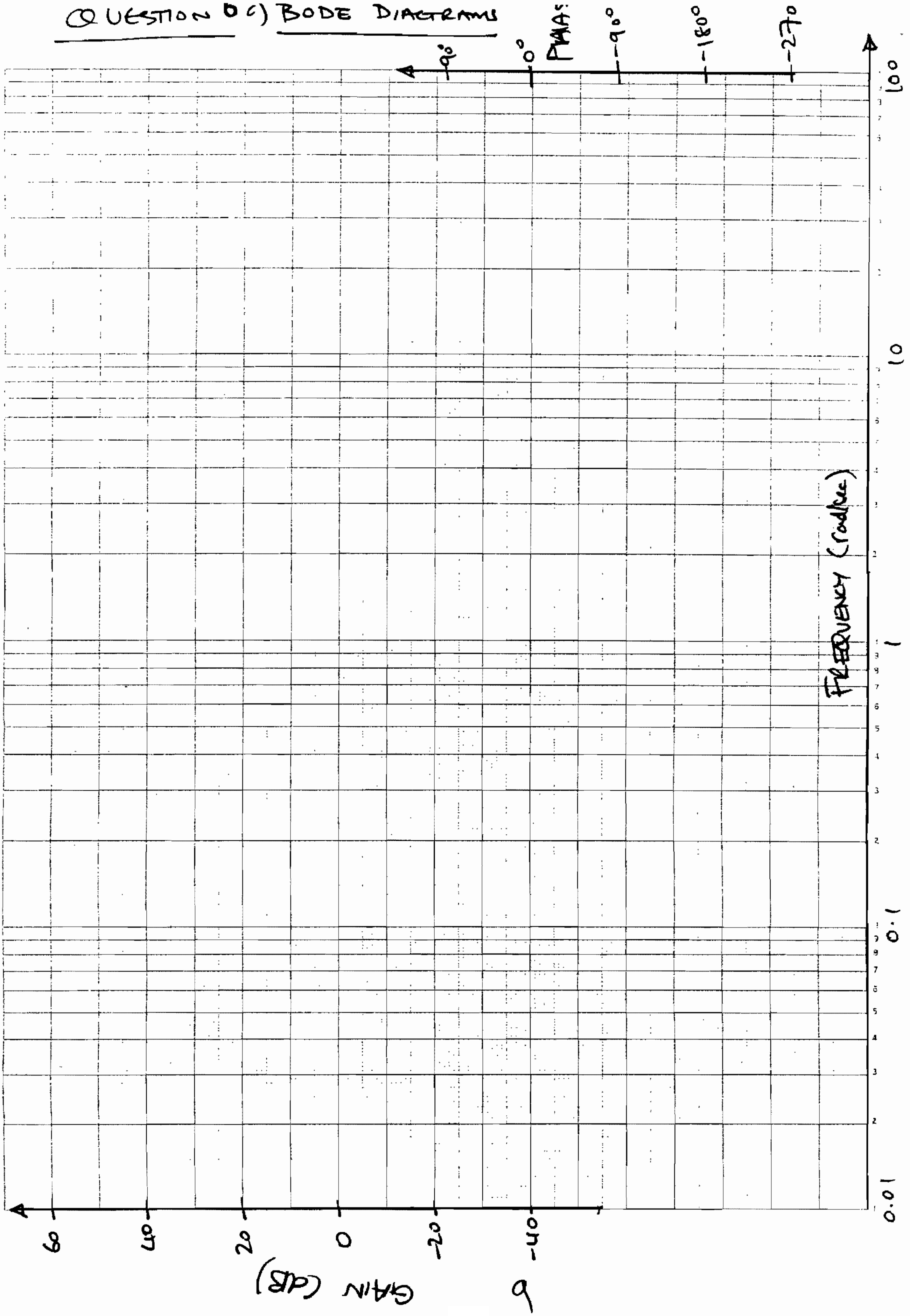
QUESTION 6 a) BODE DIAGRAMS



QUESTION 6 b) BODE DIAGRAMS



QUESTION 0 c) BODE DIAGRAMS



QUESTION 0d) BODE DIAGRAMS