IB Paper 6: Information Engineering

COMMUNICATIONS

Examples Paper 8: Analogue Modulation and Digitisation

1. Show from first principles the following properties of the Fourier transform

   (a) Convolution: $f(t) * g(t) \longleftrightarrow F(\omega)G(\omega)$

   (b) Modulation: $f(t)\cos(\omega_0 t) \longleftrightarrow \frac{1}{2}[F(\omega-\omega_0)+F(\omega+\omega_0)]$

   (c) Parseval's Theorem: $\int |f(t)|^2 dt = \frac{1}{2\pi}\int |F(\omega)|^2 d\omega$

2. The Medium Waveband covers frequencies from 500 to 1500kHz. Calculate the number of simultaneous AM transmissions that can be accommodated in this band, if the bandwidth of each modulating signal is 4kHz and a gap of 3kHz is required between sidebands of adjacent signals to prevent crosstalk interference. How many extra transmissions could be accommodated if the modulation format were changed to SSB?

3. An amplitude modulated signal has a waveform defined by

$$s_{\mathrm{AM}} = [10 + a\cos(2\pi f_x t)]\cos(18\times 10^6 \pi t)$$

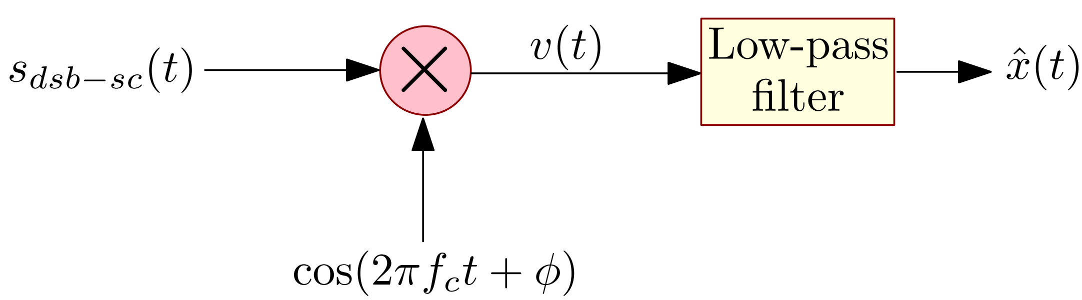
   (a) What are the amplitude and frequency (in Hz) of the unmodulated carrier?

   (b) If $a = 3$ and $f_x = 1.5$ kHz calculate the modulation index and sketch the waveform.

4. Consider a DSB-SC wave

$$s_{\text{dsb-sc}}(t) = x(t)\ \cos(2\pi f_c t)$$

   where $x(t)$ is the information signal. Recall that the receiver consists of a product modulator followed by a low-pass filter. The product modulator multiplies the received waveform by a locally generated sinusoid of frequency f_c. Suppose that the locally generated sinusoid has a phase difference of ϕ with the original carrier, as shown in the figure below.



   Show that the demodulated signal is $\hat{x}(t) = x(t)\cos\phi$, when the gain of the low-pass filter is appropriately chosen. What happens when $\phi = 90°$?

5. A simple diode-resistor *square-law* circuit can be made with input x V and output y V such that $y = x^2$. The input to the square-law circuit is

$$x(t) = 2 + a_1(t) + a_2(t)$$

where $a_1(t) = b\cos(2\pi f_x t)$ is a sinusoidal modulating signal and $a_2(t) = \cos(2\pi f_c t)$ is a unit amplitude carrier signal. Show that if the output $y(t)$ is fed into a filter which passes only a suitable frequency band centered on f_c, then the filter output is an AM signal with carrier frequency f_c. What is the modulation index of the AM signal?

6. Consider the FM modulation of a tone $x(t) = a_x\cos(2\pi f_x t)$. As discussed in the lecture notes, the modulated FM signal is

$$s_{\mathrm{FM}}(t) = A_c\cos\left(2\pi f_c t + \beta\sin(2\pi f_x t)\right),$$

where β is the modulation index.

(a) Show that $s_{\mathrm{FM}}(t) = \mathsf{Re}\left[\tilde{s}(t)e^{j2\pi f_c t}\right]$, where $\tilde{s}(t) = A_c e^{j\beta\sin(2\pi f_x t)}$

(b) Show that $\tilde{s}(t)$ is periodic. What is the fundamental frequency?

(c) Express $\tilde{s}(t)$ using Fourier series: $\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n\, e^{j2\pi n f_x t}$, and show that the Fourier coefficients are

$$c_n = \frac{A_c}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin u - nu)}du = A_c\, J_n(\beta),$$

where

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin u - nu)}du.$$

(d) Show that $J_n(\beta)$ is real, and can therefore be written as

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos(\beta\sin u - nu)du.$$

(e) Using the Fourier expansion of $\tilde{s}(t)$ in part (a), show that

$$s_{\mathrm{FM}}(t) = A_c\sum_{n=-\infty}^{\infty} J_n(\beta)\cos(2\pi(f_c + nf_x)t).$$

(f) From part (d), show that the spectrum of the modulated FM signal is

$$S_{\mathrm{FM}}(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\,\delta(f - f_c - nf_x) + \delta(f + f_c + nf_x)\,\right].$$

For $\beta = 5$, use the graph of $J_n(\beta)$ in the lecture notes to estimate the strength of the impulse in the spectrum at $f = f_c$.

7. A frequency modulator converts an input voltage $x(t)$ into an output signal of frequency $f(t)$, such that an input of 1V peak causes a frequency deviation of 50 kHz from the nominal carrier frequency of 10 MHz. Calculate the modulation index and estimate the total bandwidth required for each of the following cases:

(a) $x(t)$ is a sine wave with frequency 5 kHz and 1V peak

(b) $x(t)$ is a sine wave with frequency 10 kHz and 1V peak

(c) $x(t)$ is a sine wave with frequency 10 kHz and 0.2V peak

8. Estimate the minimum sampling frequency that can be used for the following signals, if ideal low-pass (anti-aliasing) filters are used:

   (a) A strain gauge generating frequencies up to 100 Hz.

   (b) A telephone speech signal containing frequencies up to 3400 Hz.

   (c) A high quality music signal containing frequencies up to 16 kHz.

   (d) A television signal containing frequencies up to 5.5 MHz.

   Repeat the exercise assuming low-pass (anti-aliasing) filter with a transition bandwidth that is 20% of their pass-band width.

   Compute the bit rates at the output of a uniform quantiser with 12-bit resolution for (a) and (b), 16 bits for (c) and 8 bits for (d), using the minimum sampling frequencies obtained in the last question.

9. Calculate the rms quantisation noise voltage that would be produced by ideal (i.e. perfectly accurate) analogue-to-digital converters with the following characteristics:

   (a) -5 to $+5$ volt signal range and 8-bit resolution.

   (b) -10 to $+10$ volt signal range and 12-bit resolution.

   For each case calculate the signal-to-noise power ratio (in dB) that can be achieved if the input signal is (i) a sinusoid covering the full input signal range; (ii) a signal for which the peak value is $2\sqrt{2}$ times the rms value, again scaled to occupy the full input signal range.

   Why would it be inappropriate to estimate quantisation noise voltage in the same way for a square wave signal?

Answers:

1.

2. 91, 52

3. a) 10 V, 9 MHz; b) $m_A = 0.3$

4.

5. $m_A = \frac{b}{2}$

6.

7. a) 10, 110 kHz; b) 5, 120 kHz; c) 1, 40 kHz

8. a) 200 Hz; b) 6800 Hz; c) 32 kHz; d) 11 MHz
   a) 240 Hz; b) 8160 Hz; c) 38.4 kHz; d) 13.2 MHz
   a) 2.88 kbit/s; b) 97.92 kbit/s; c) 614.4 kbit/s; d) 105.6 Mbit/s

9. a) 11.28 mV, (i) 49.9 dB, (ii) 43.9 dB; b) 1.41 mV, (i) 74.0 dB., (ii) 68 dB.

   For a square wave signal, the quantisation error is not uniformly distributed in $[-\Delta/2, \Delta/2]$ where Δ is the step size of the quantiser. Rather, for a square wave signal, the quantisation error can only take one of two values, and is itself a square wave.