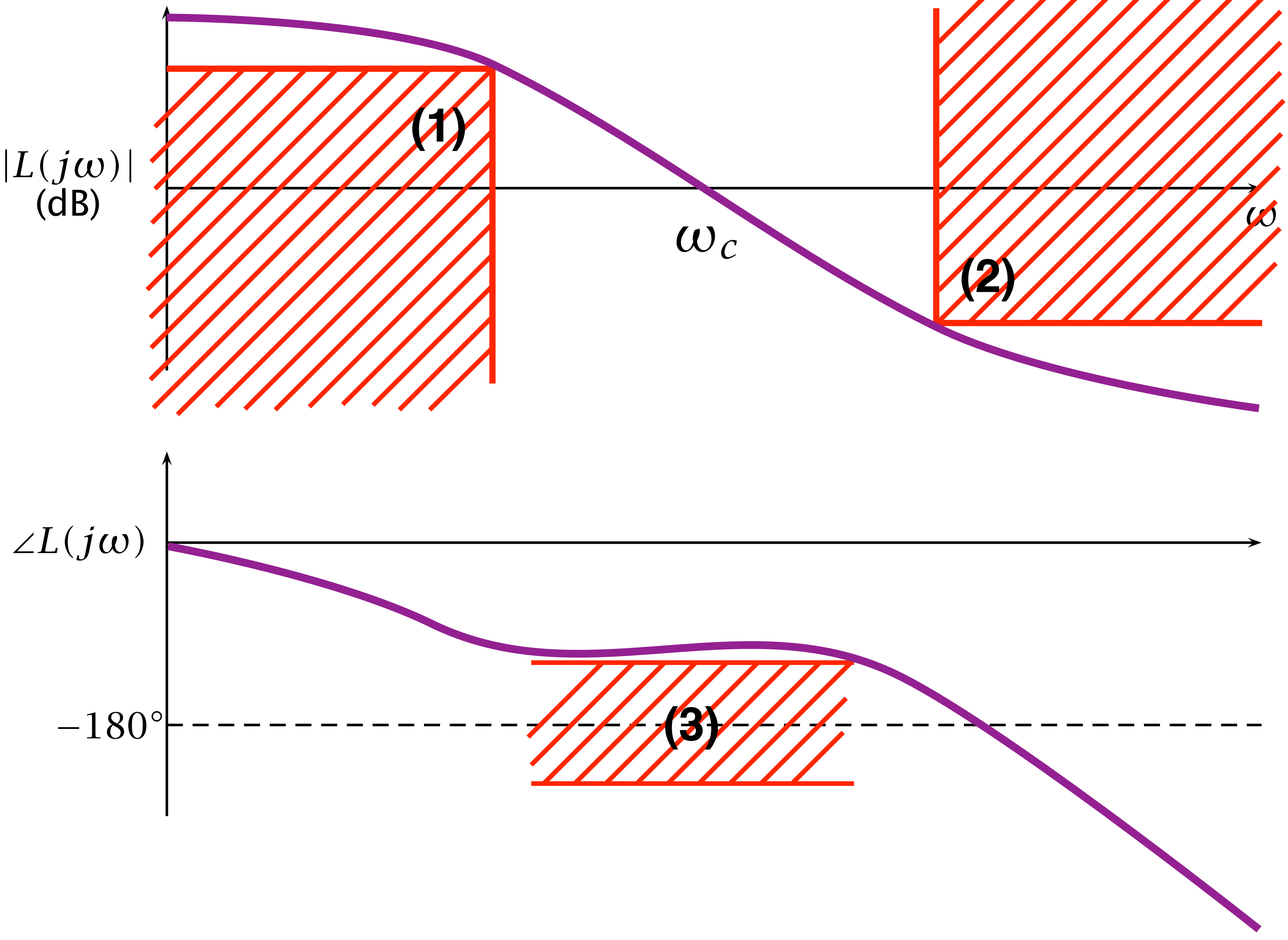
Part IB Paper 6: Information Engineering

LINEAR SYSTEMS AND CONTROL

Ioannis Lestas

HANDOUT 7

“The design of feedback systems – an introduction”



$|L(j\omega)|$
(dB)
(1)
ω_c
ω
(2)
$\angle L(j\omega)$
$-180°$
(3)

7.1 Feedback system design, a loop-shaping approach

- This consists of choosing $K(s)$ to shape $L(s) = K(s)G(s)$ such that

1. $|K(j\omega)G(j\omega)| \gg 1$ for frequency ranges where the benefits of feedback are sought (typically $\omega < \omega_C$)

   (in order to ensure that the sensitivity function $\left|S(j\omega)\right| \ll 1$ at those frequencies.)

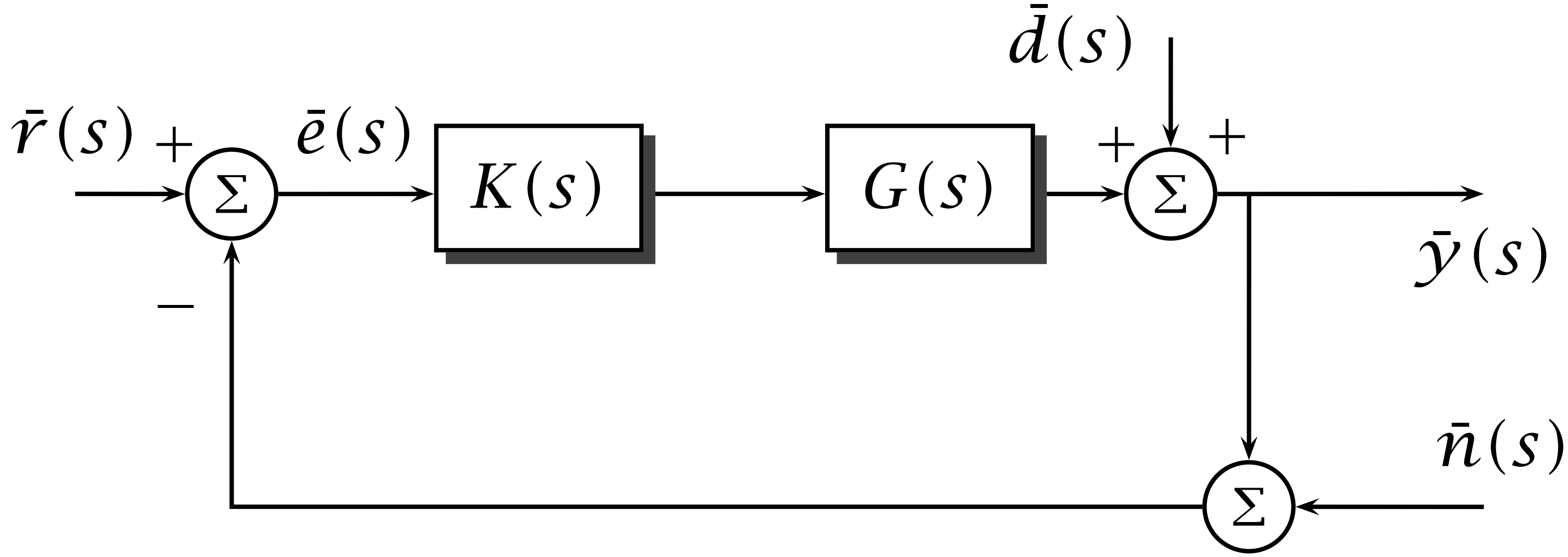
2. $|K(j\omega)G(j\omega)| \ll 1$ at other frequencies (typically high frequencies $\omega \gg \omega_C$)

   (ensuring that the complementary sensitivity function $\left|T(j\omega)\right| \ll 1$ at those frequencies.)

3. $K(j\omega)G(j\omega)$ satisfies the Nyquist stability criterion, with adequate gain and phase margins. (ensuring that neither $S(j\omega)$ or $T(j\omega)$ have a large peak in the *crossover* region in between)

Recall that $S(s) = \frac{1}{1+L(s)} = T_{d\mapsto y}$ and $T_{r\mapsto e}$

and $T(s) = \frac{L(s)}{1+L(s)} = -T_{n\mapsto y}$ and $T_{r\mapsto y}$

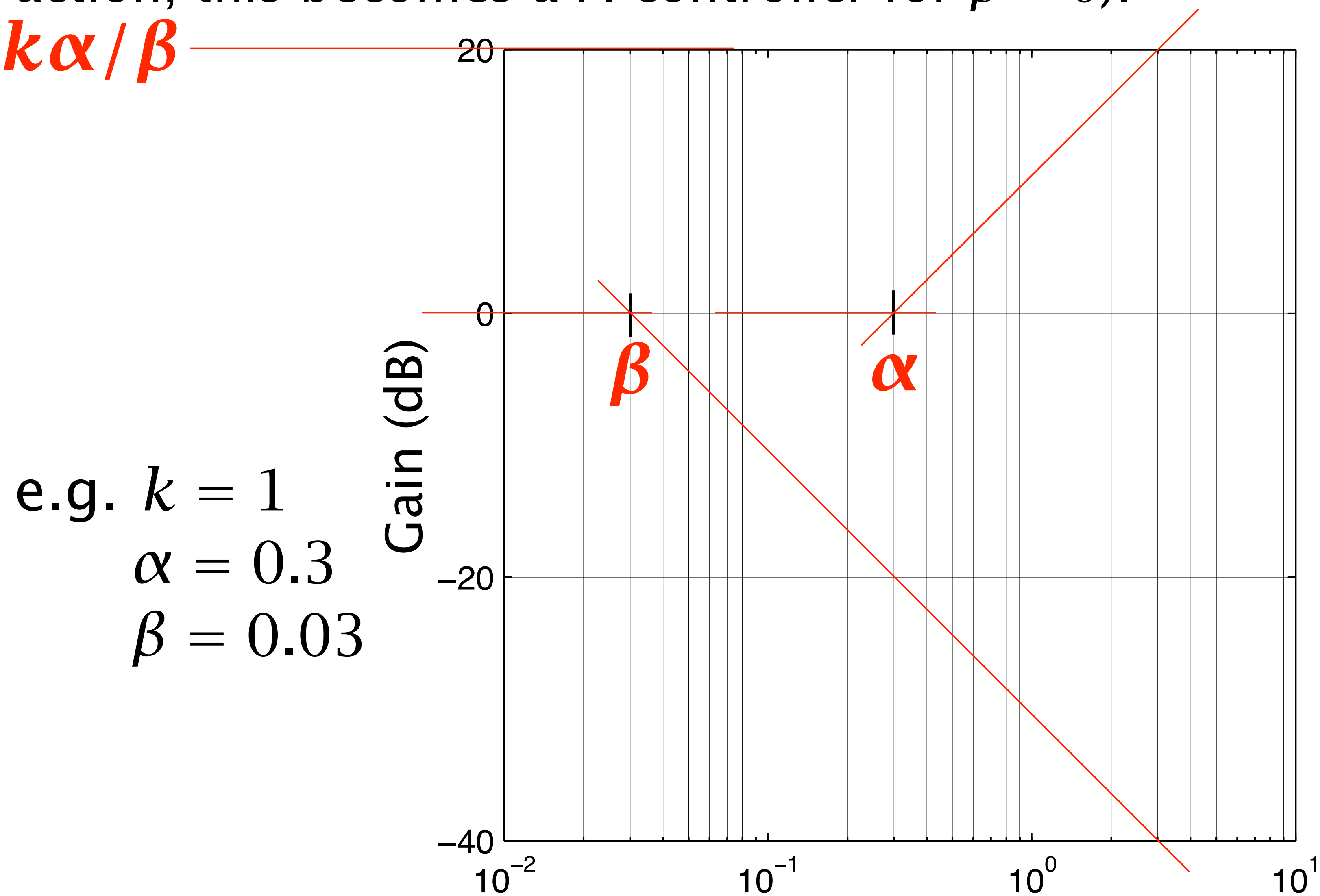


To achieve this, we can use combinations of phase lag and phase lead compensators. Phase lag compensators are a generalized form of P+I action and phase lead compensators are a generalized form of P+D action.

Note: Example 2 of Handout 4 is a phase lead compensator and Example 3 is a PI controller (and so a special case of a phase lag compensator).

- $\boxed{K(s) = k\dfrac{s+\alpha}{s+\beta}}$ for $\beta < \alpha$ ($< \omega_C$ typically) $= \dfrac{k\alpha}{\beta}\dfrac{1+s/\alpha}{1+s/\beta}$

– Phase lag compensator (a generalized form of proportional+integral action, this becomes a PI controller for $\beta = 0$).



e.g. $k = 1$
$\alpha = 0.3$
$\beta = 0.03$

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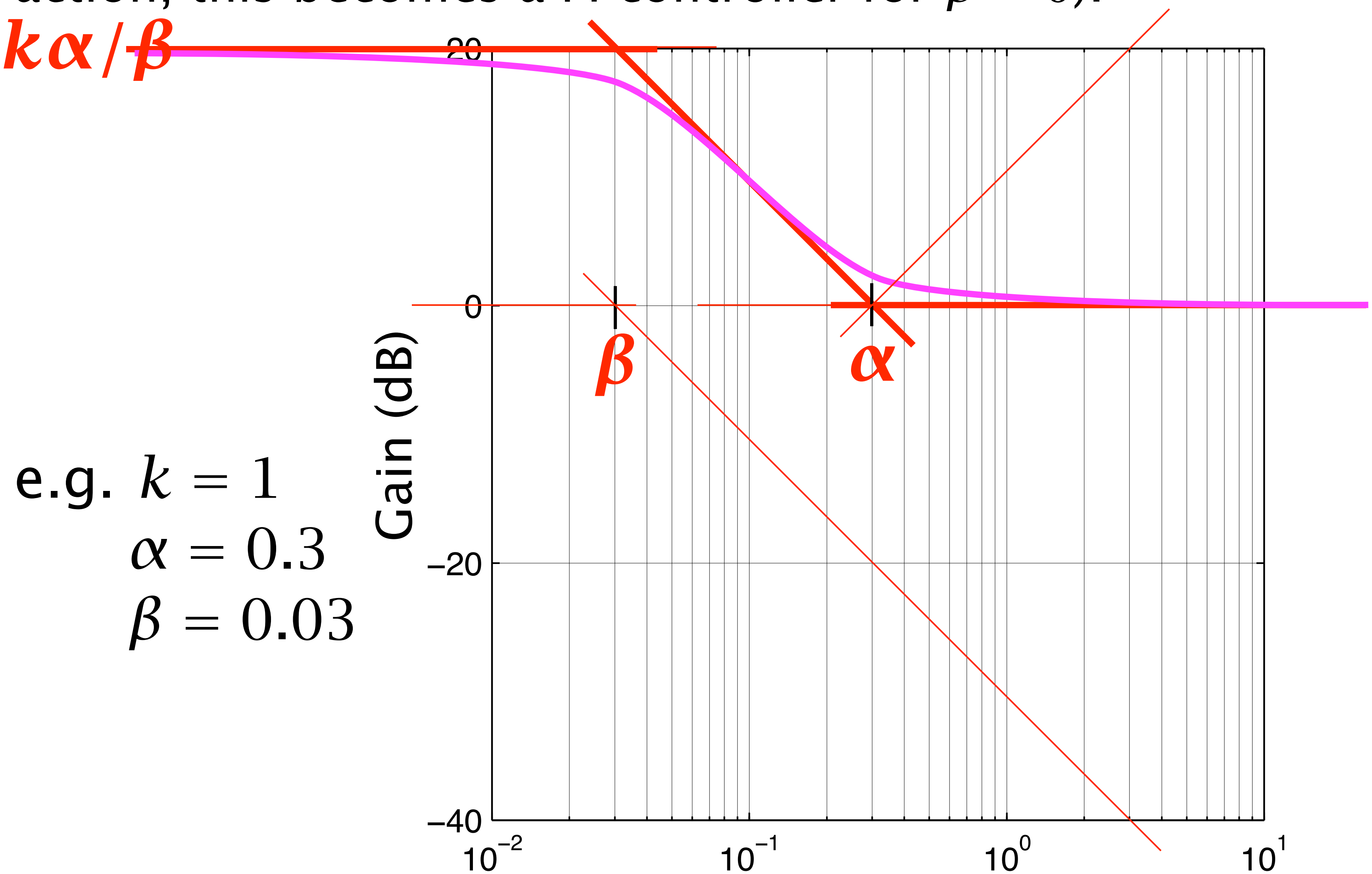

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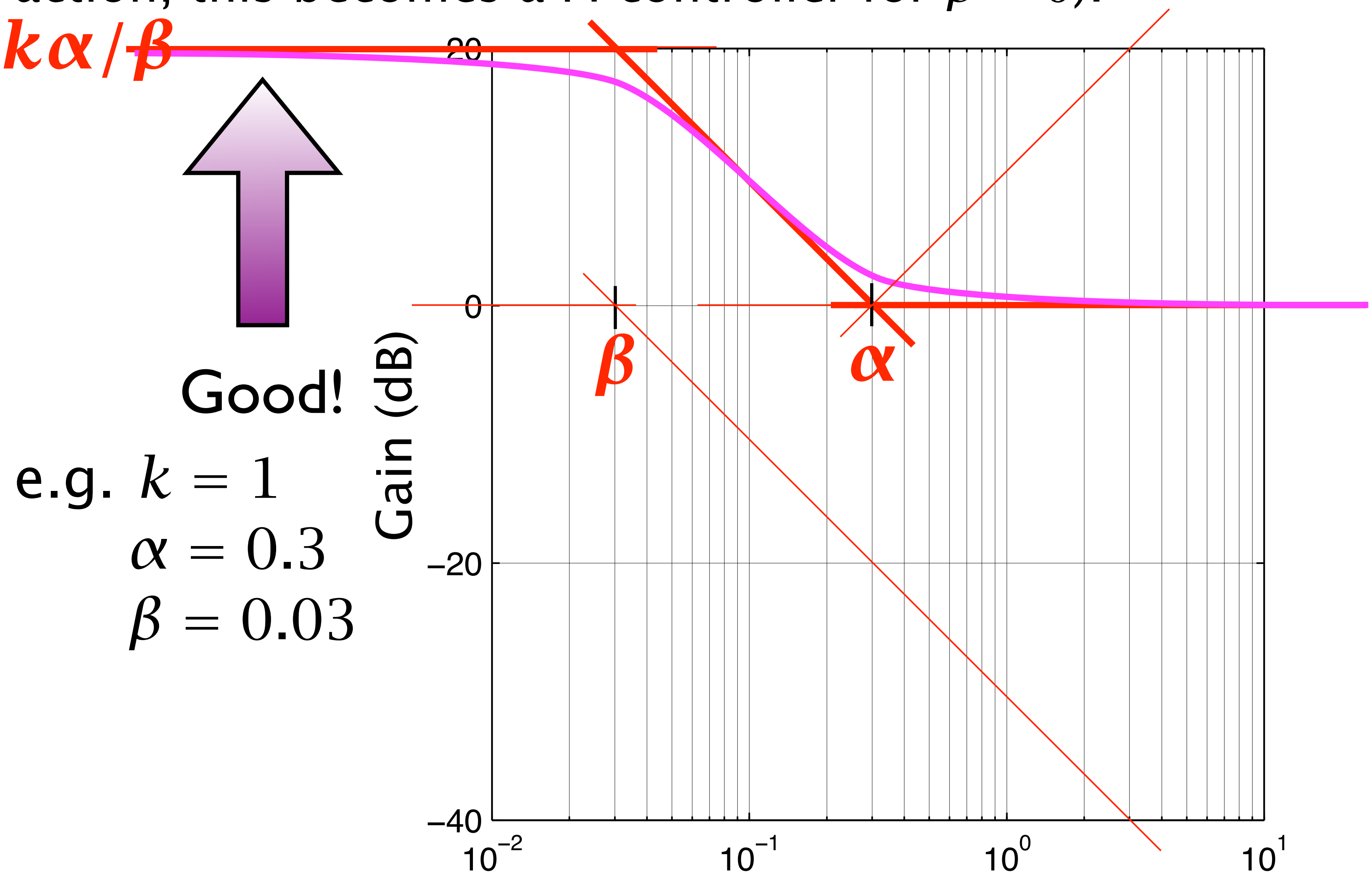

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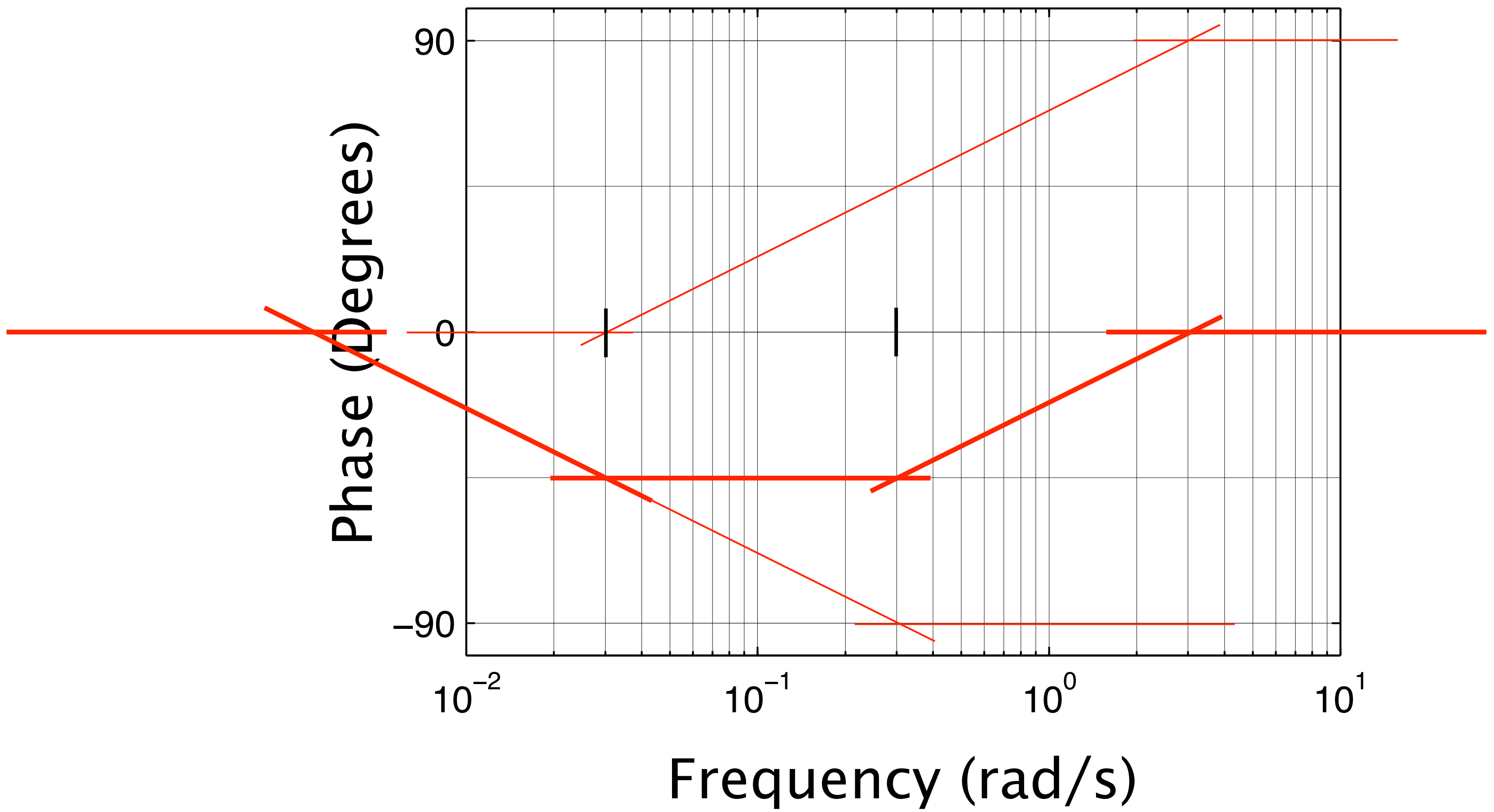
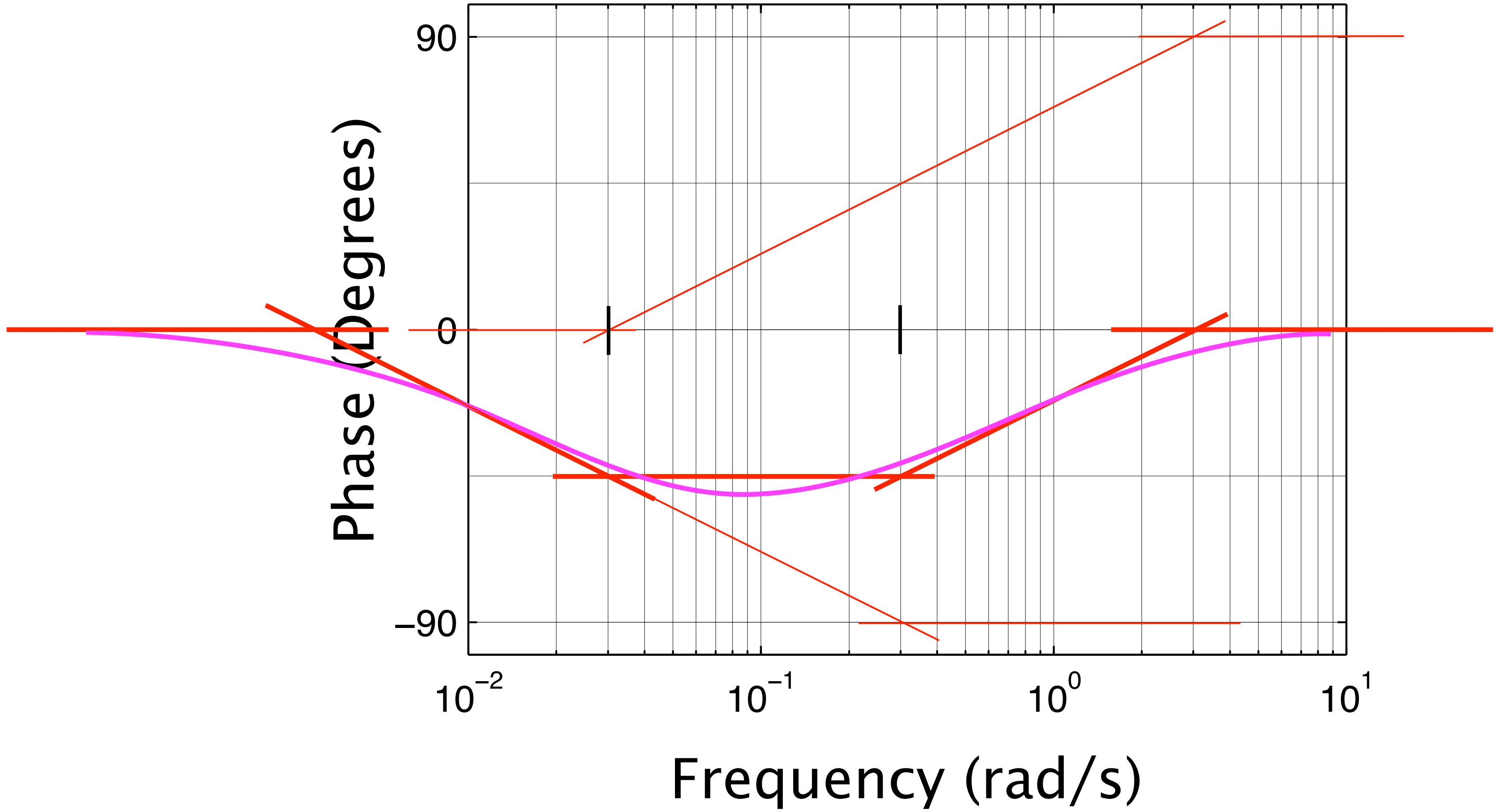


– improves low frequency gain (and so reduces steady-state errors) at the expense of introducing phase lag at frequencies between $\omega \approx \beta$ and $\omega \approx \alpha$ (although this is not an issue if $\alpha \ll \omega_c$) .



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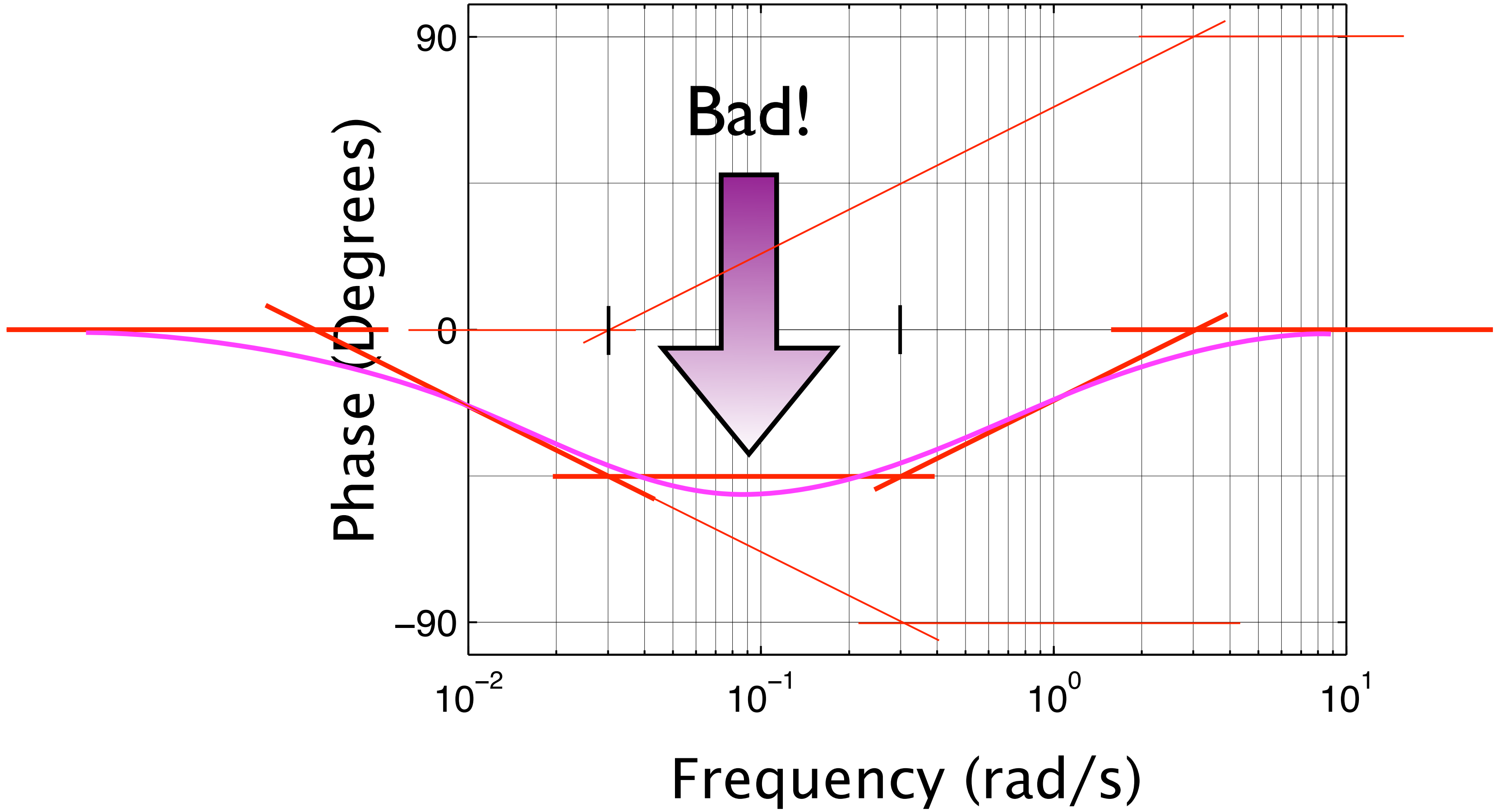

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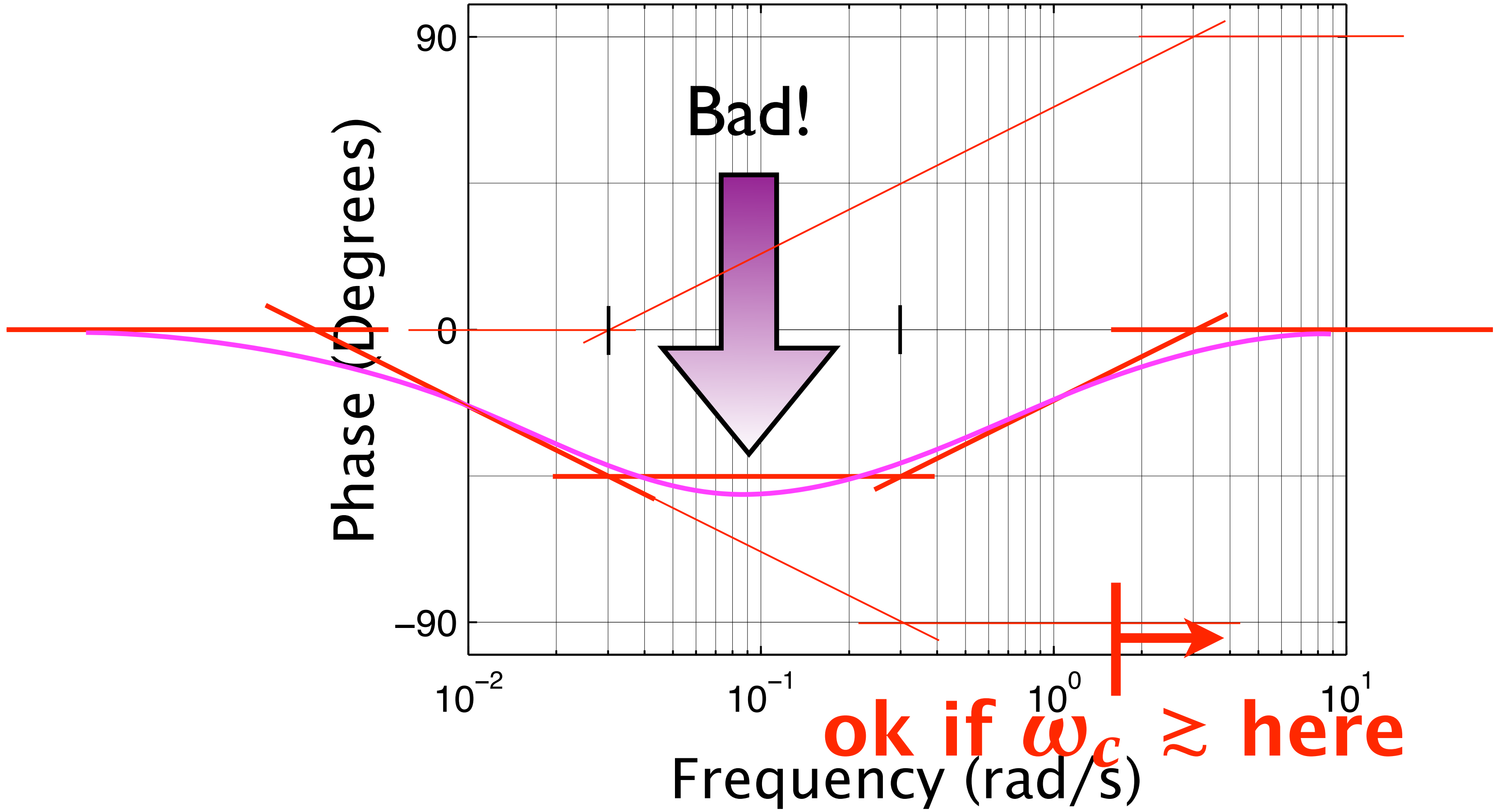

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- $\boxed{K(s) = k\frac{s+\alpha}{s+\beta}}$ for $\alpha < \beta$ (and $\alpha < \omega_C < \beta$ typically)

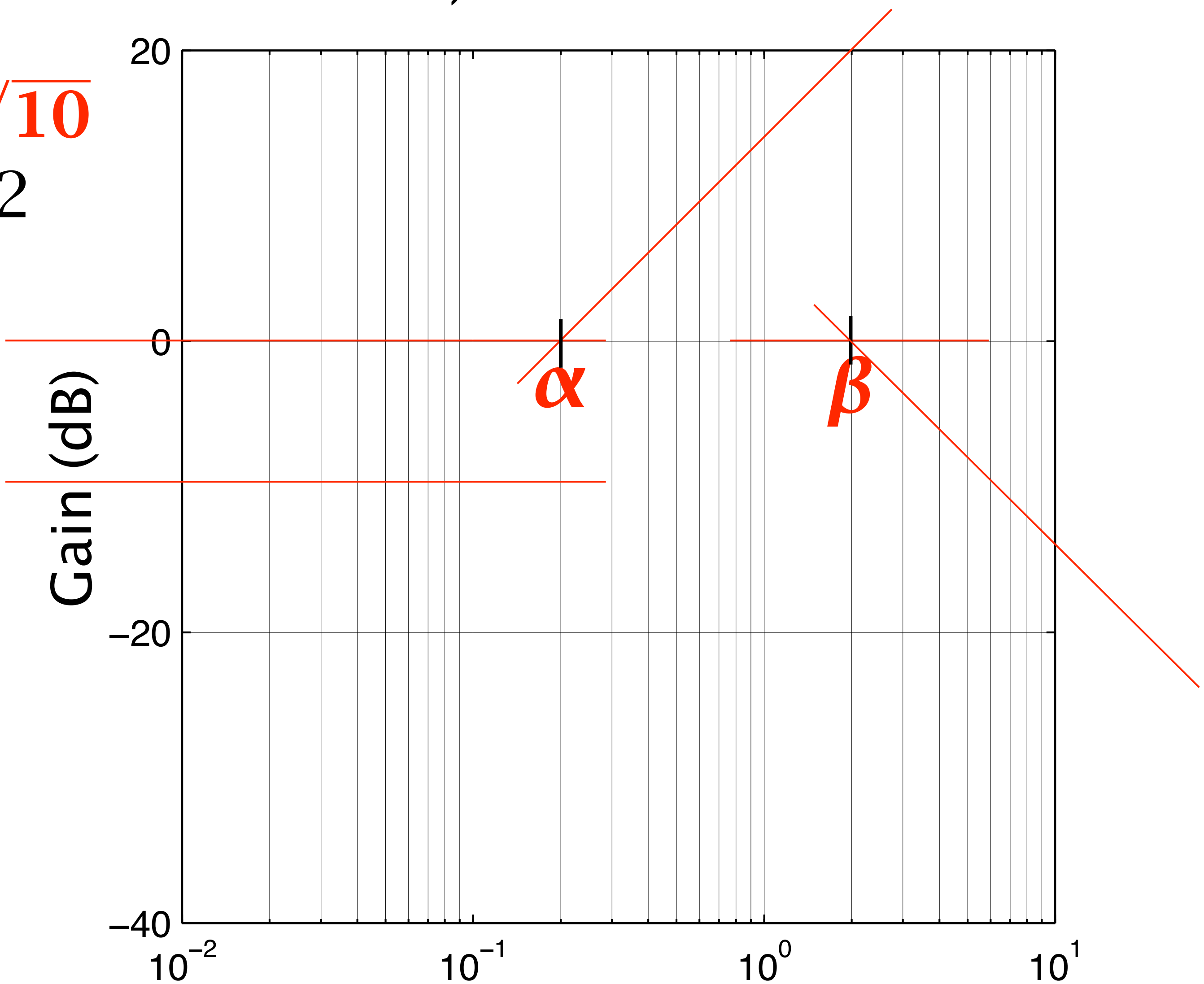
– Phase lead Compensator (a generalized form of proportional+derivative action).

e.g. $k = \sqrt{10}$

$\alpha = 0.2$

$\beta = 2$

$k\alpha/\beta$



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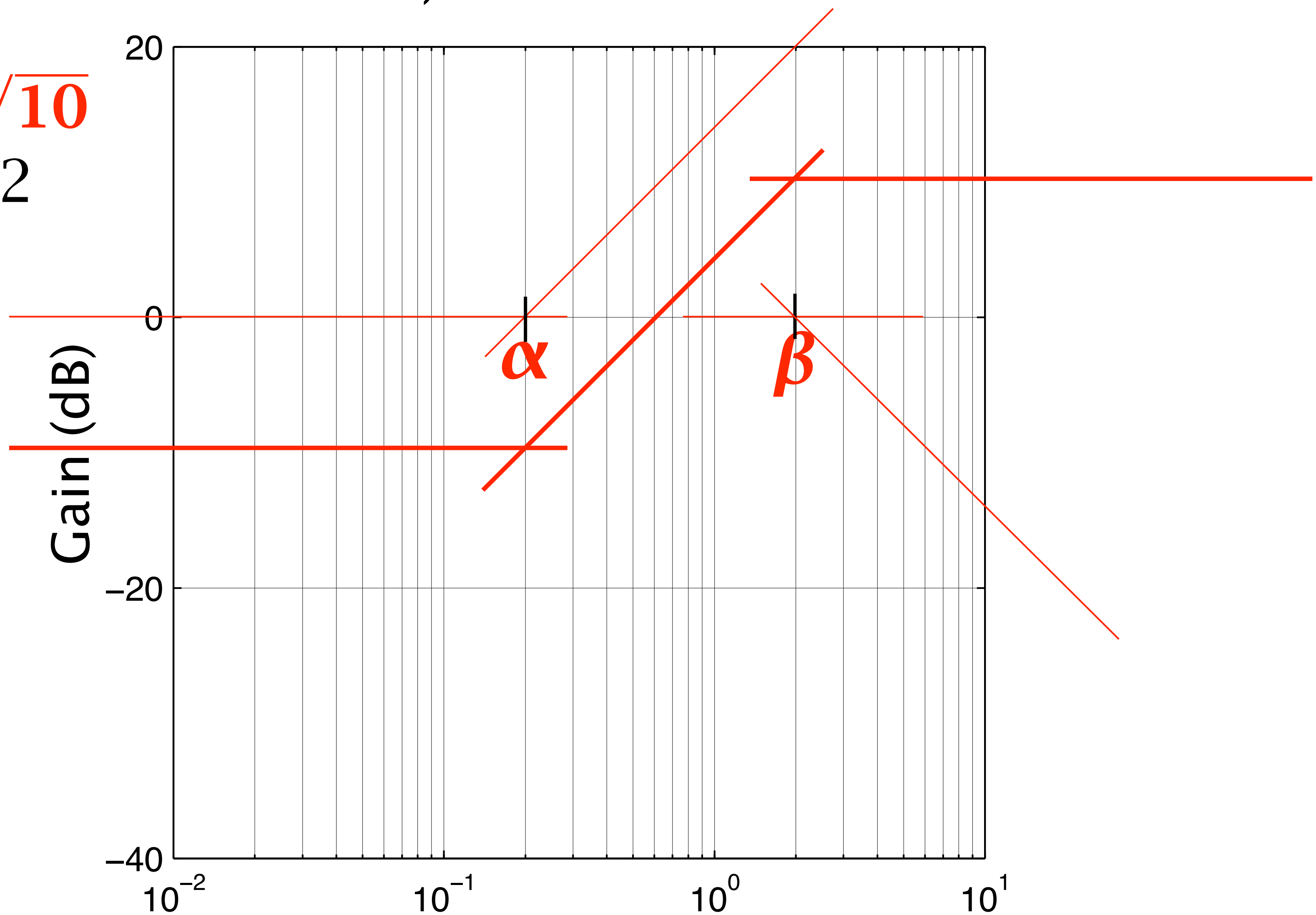

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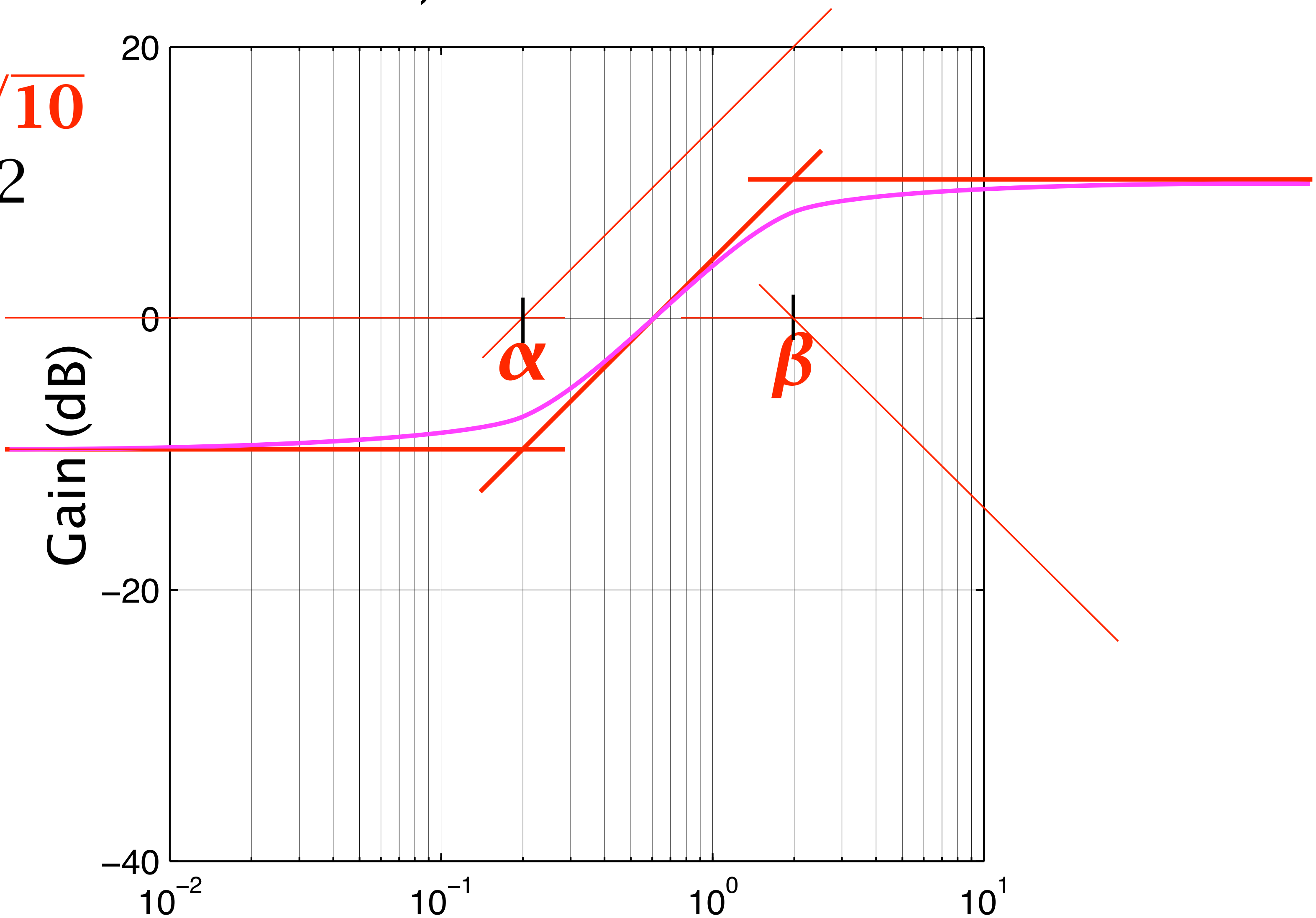

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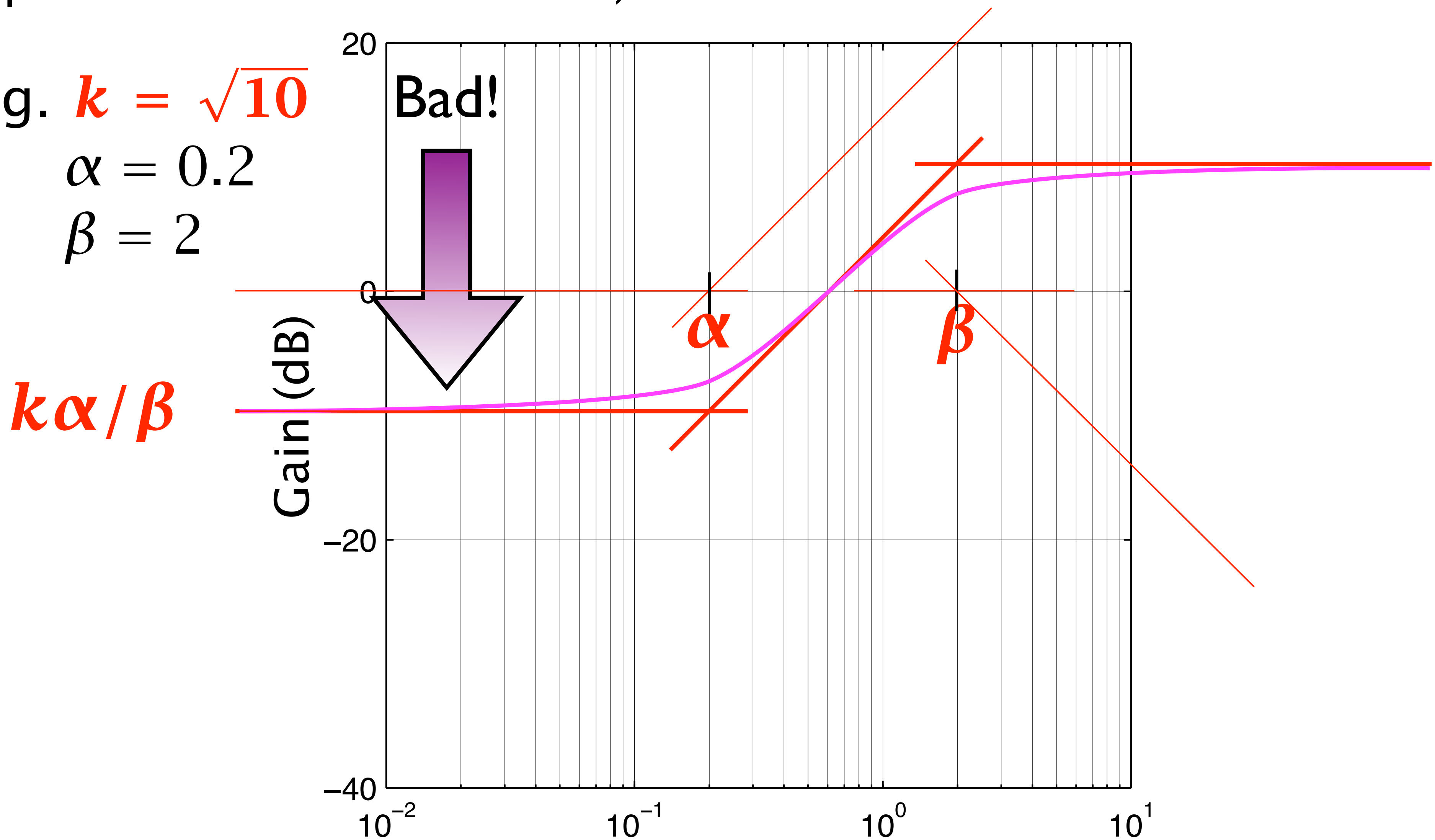

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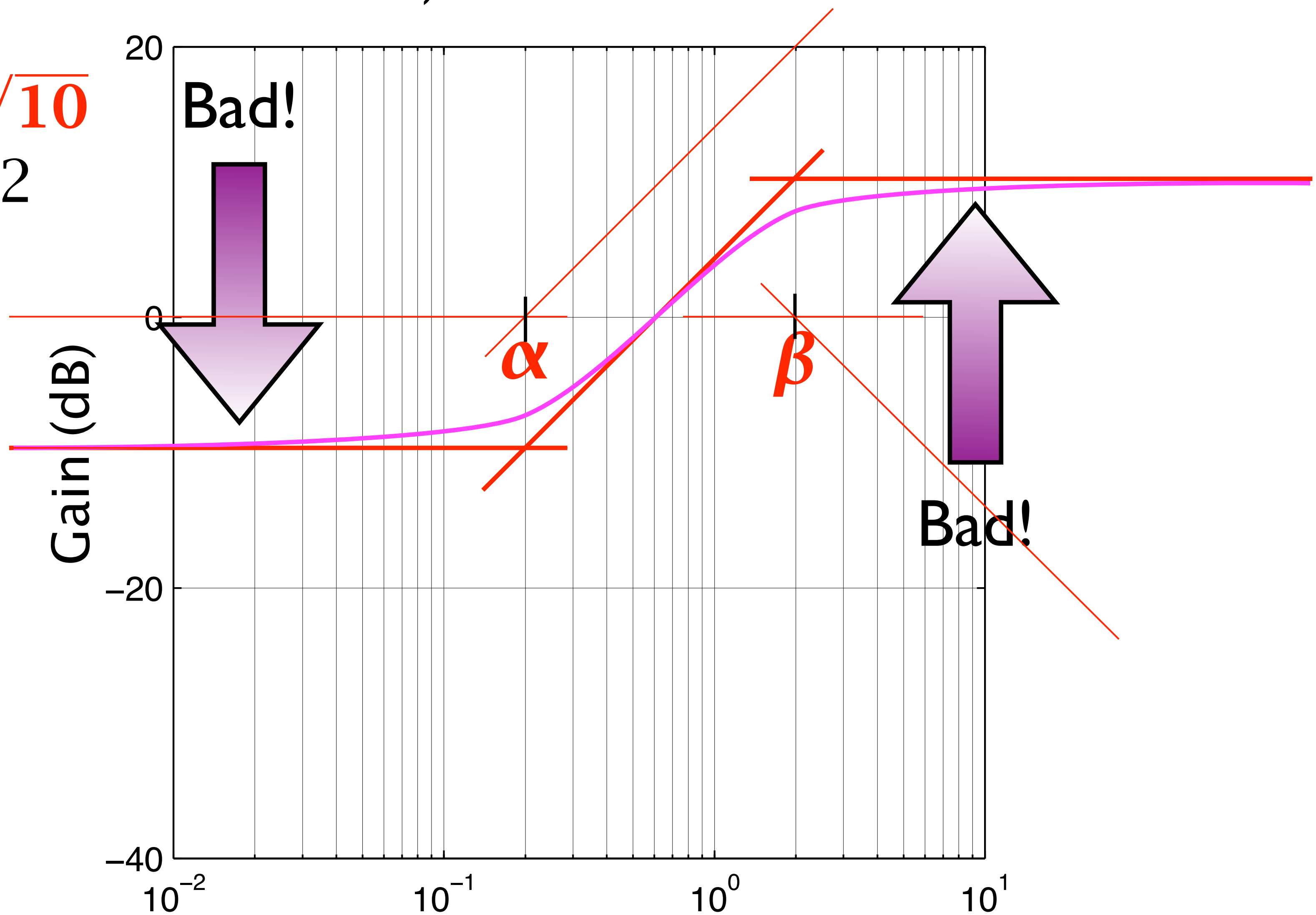
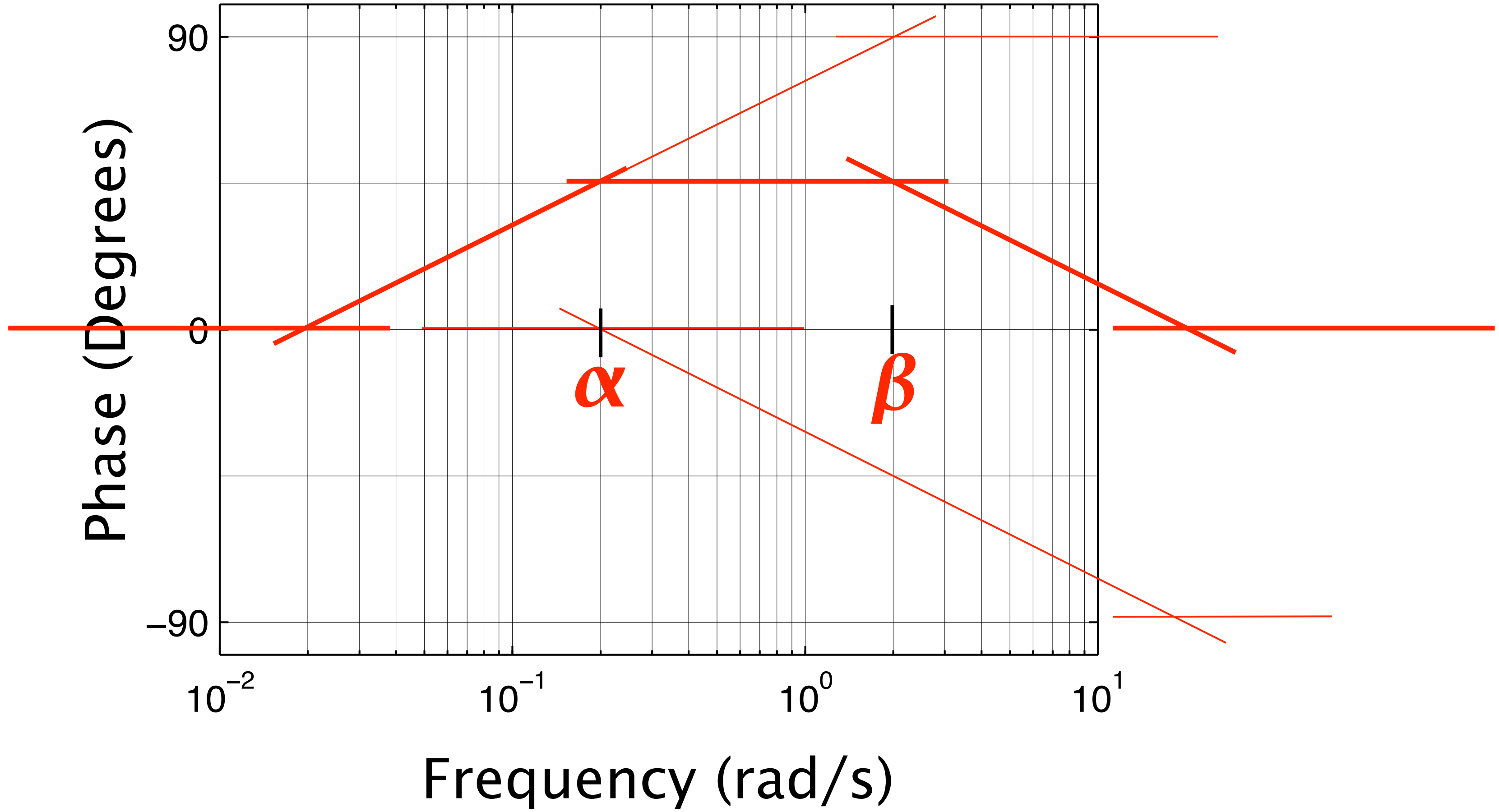
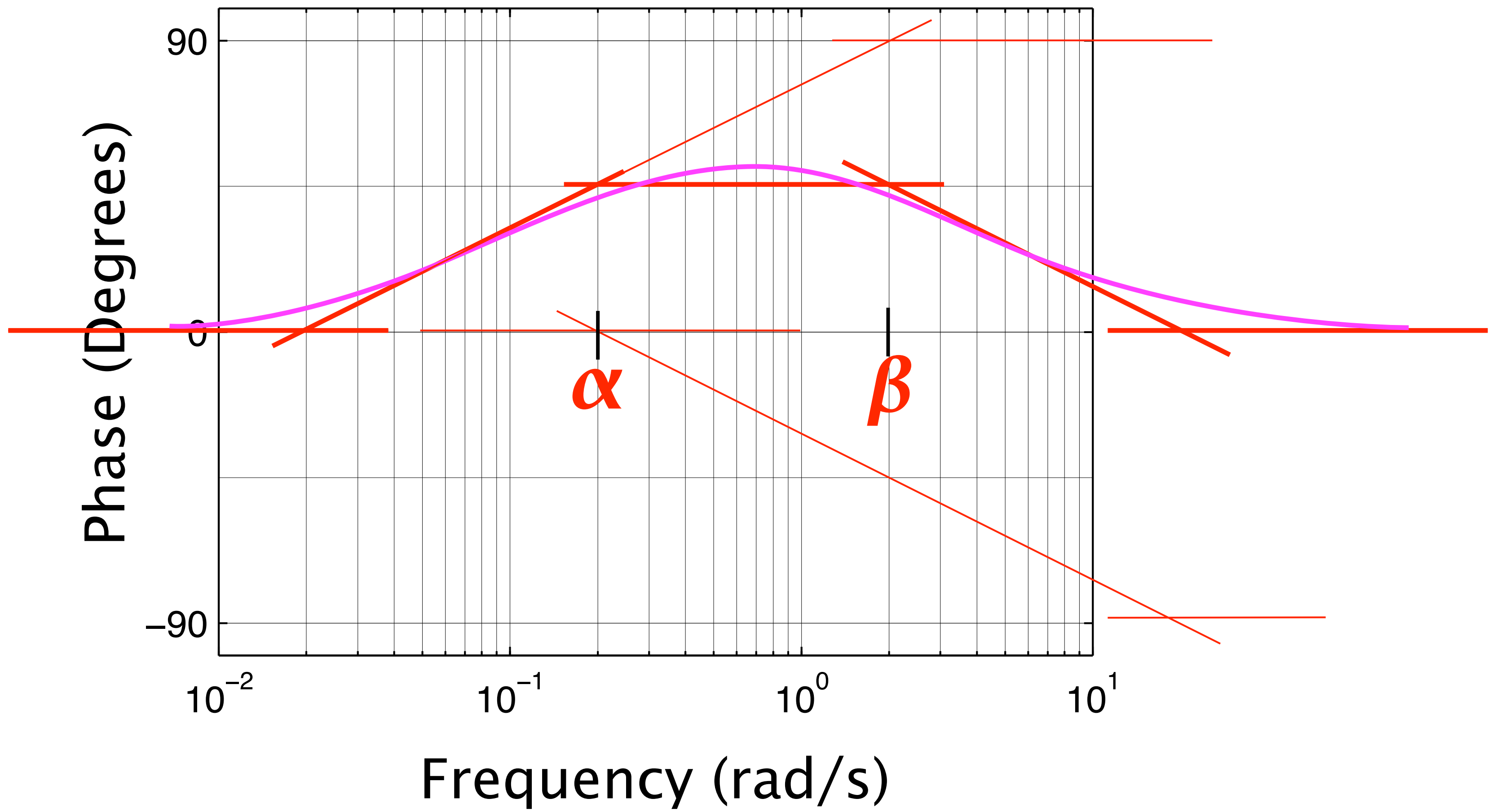




– can improve gain and phase margins (improving robustness and damping) at the expense of decreasing the low frequency gain (increasing steady-state errors) and increasing the high frequency gain (increasing sensitivity to noise).

NOTE: WE CAN USE A COMBINATION OF PHASE LEAD AND PHASE LAG.



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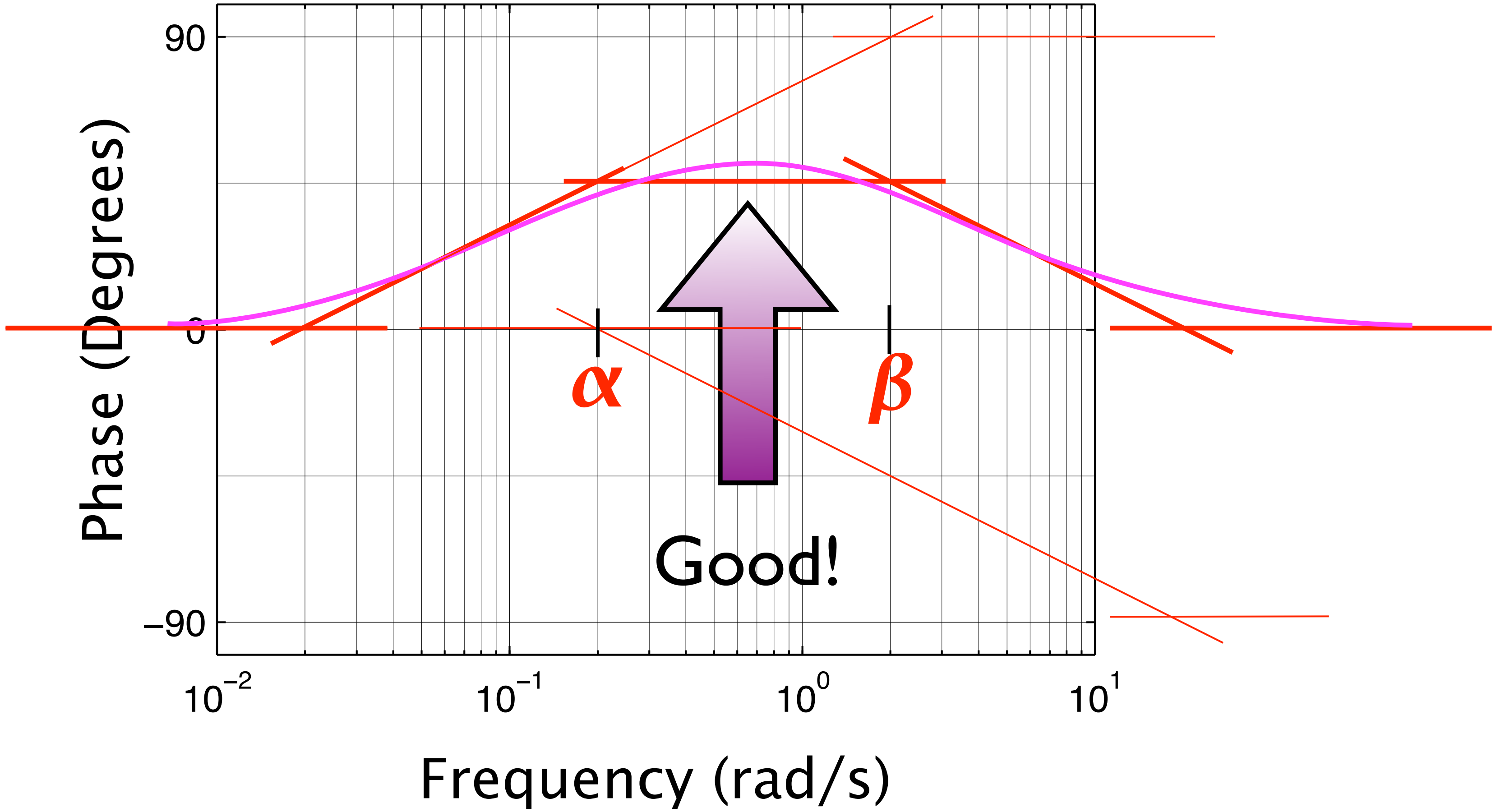

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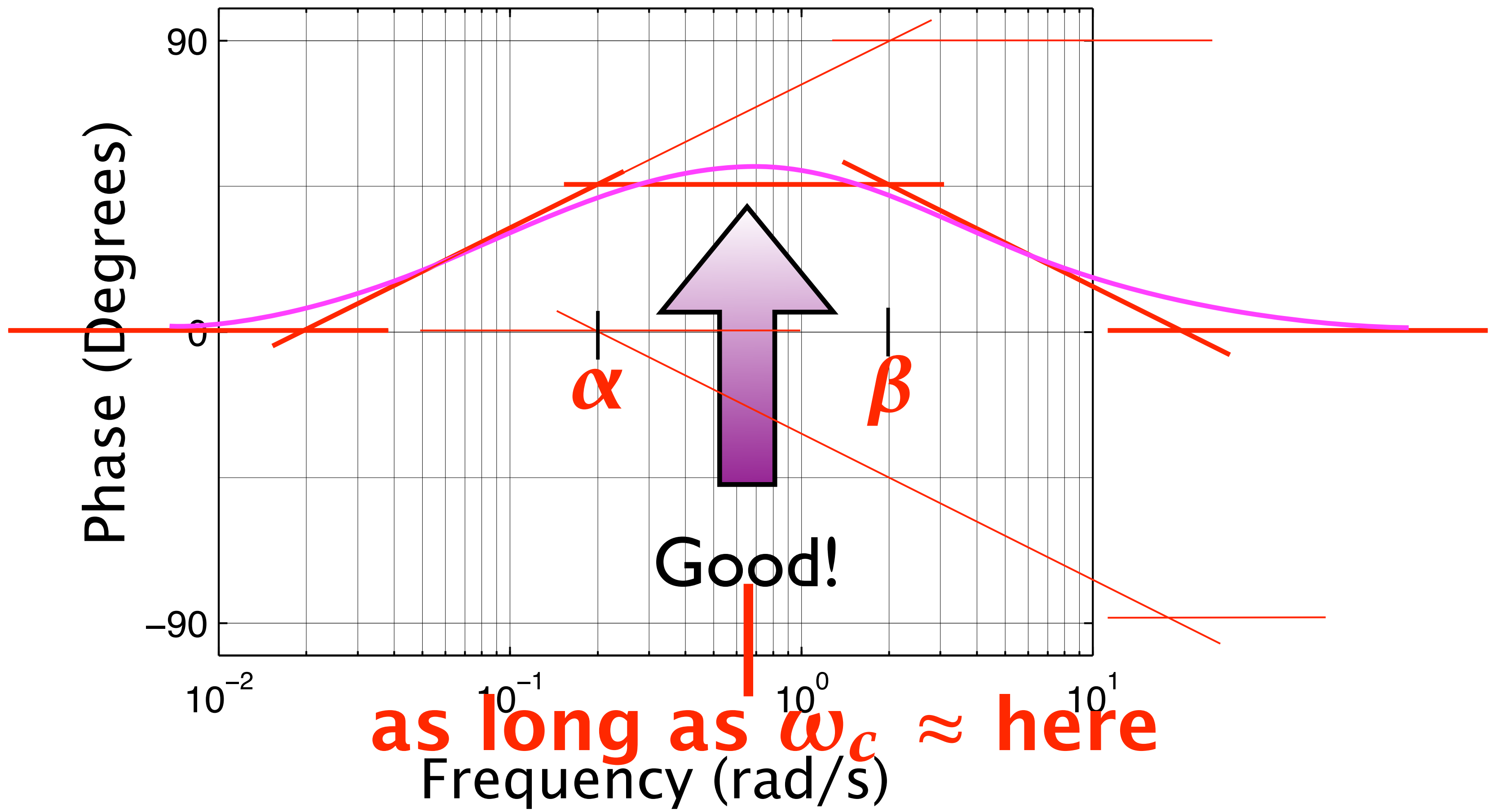

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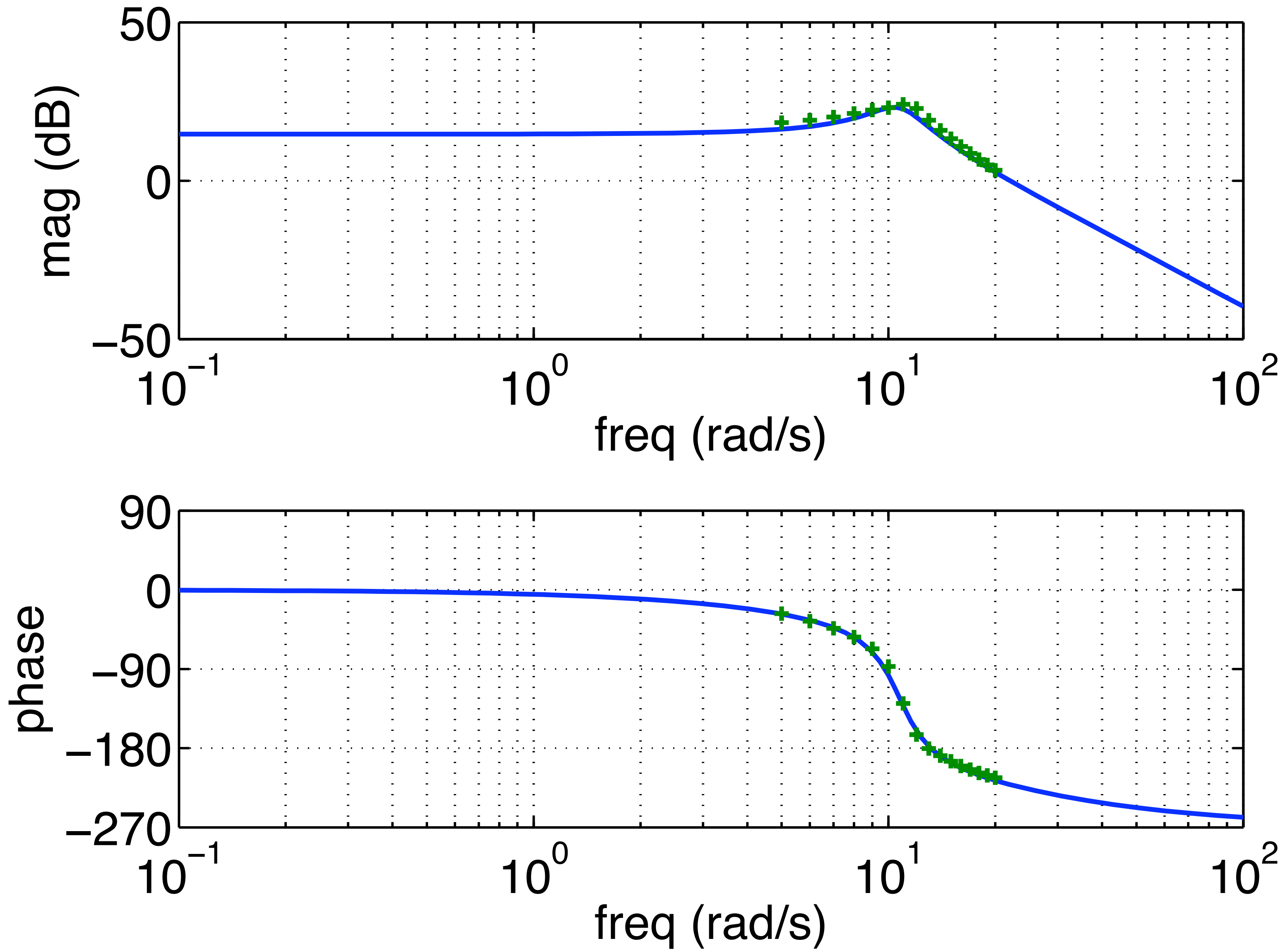
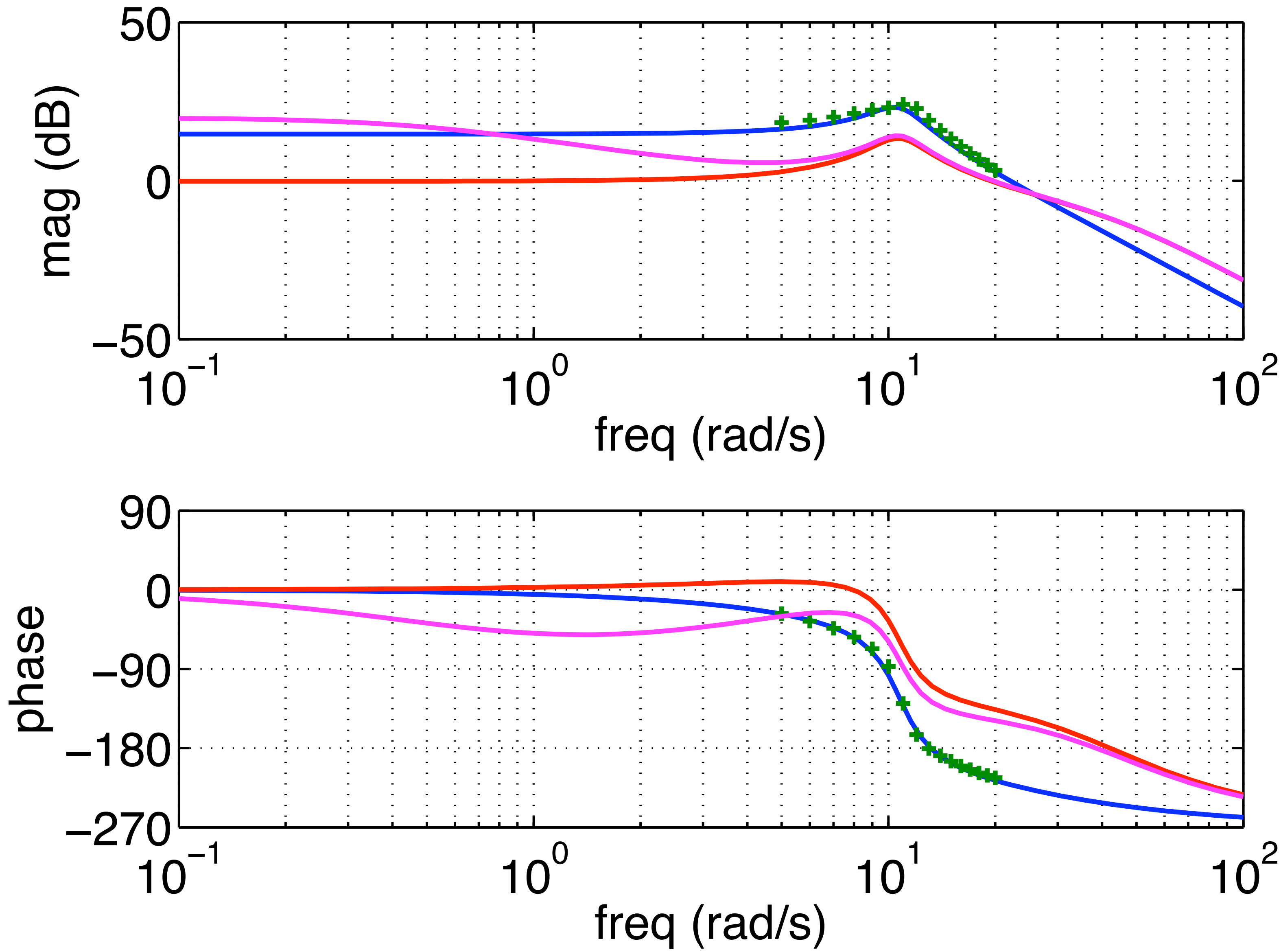
Lead/lag controller design



$$G(s) = \frac{5.431}{\left(\frac{s^2}{10.745^2} + 2\times 0.1613\frac{s}{10.745} + 1\right)(s/16.54+1)}$$

Lead/lag controller design



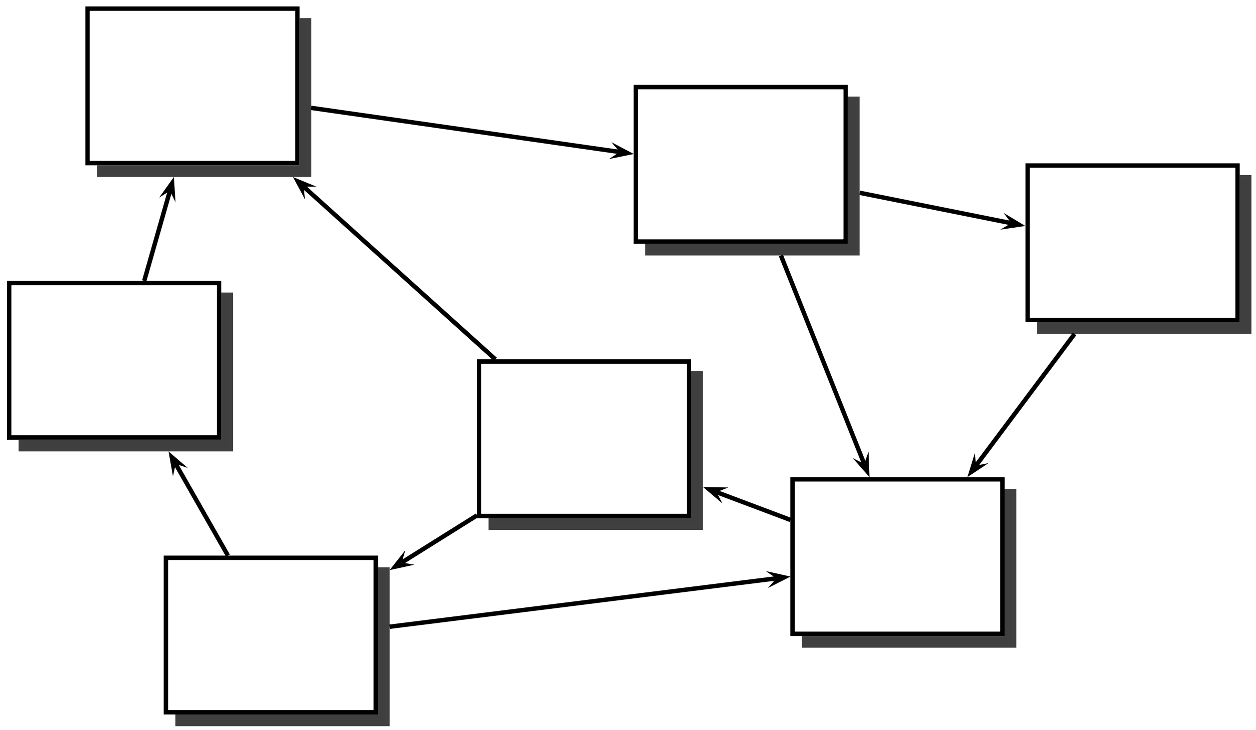
($L(s)$ is plotted in each case)

What the course was about:

The keys to understanding the course are the following *two* relationships:

- The relationship between the time and frequency (Laplace) domains:
  - Steady state responses (to both constant and sinusoidal inputs).
  - Pole locations.
- The relationship between open and closed loop properties (i.e. predicting properties of the feedback system from its return ratio).
  - Nyquist stability criterion (predicting stability of the feedback system).
  - Gain and phase margins (predicting the robustness of that stability and, indirectly, closed loop pole locations).
  - Understanding the map $L(j\omega) \mapsto L(j\omega)/(1+L(j\omega))$ and $L(j\omega)/(1+L(j\omega))$ (predicting the *performance* of the feedback system – which involves reading off $L(j\omega)$ and $1+L(j\omega)$ from the Nyquist diagram).

What the course is *really* about



- Understanding complex systems as an interconnection of simpler subsystems.
- Relating the behaviour of the interconnected system to the behaviour of the subsystems.
- (although we'll only consider the feedback interconnection in detail)