

CSC 211: Computer Programming

(Recursive) Backtracking

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Images and material from web.stanford.edu

Administrative Announcements

- **A03 due 11/01**
- **MC05 due 11/05**
- **Exam# 02 -11/12**

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Recursion Reminder

- Problem solving technique in which we solve a task by reducing it to smaller tasks (**of the same kind**)
 - then use same approach to solve the smaller tasks
- Technically, a recursive function is one that **calls itself**
- General form:
 - ✓ **base case**
 - solution for a **trivial case**
 - it can be used to stop the recursion (prevents "stack overflow")
 - every recursive algorithm needs at least one base case
 - ✓ **recursive call(s)**
 - divide problem into **smaller instance(s)** of the **same structure**

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Recursion Reminder

- Recursive Checklist:
 - ✓ **Find what information we need to keep track of.** What inputs/ outputs are needed to solve the problem at each step?
 - ✓ **Find our base case(s).** What are the simplest (nonrecursive) instance(s) of this problem?
 - ✓ **Find our recursive step.** How can this problem be solved in terms of one or more simpler instances of the same problem that lead to a base case?
 - ✓ **Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?

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Recursion Reminder

Recursive Checklist:

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- ✓ **Ensure every input is handled.** Do we cover all possible cases? Do we need to handle errors?

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Backtracking

- Write a recursive function `printAllBinary` that accepts an integer number of digits and prints all binary numbers that have exactly that many digits, in ascending order, one per line

`printAllBinary(2);`

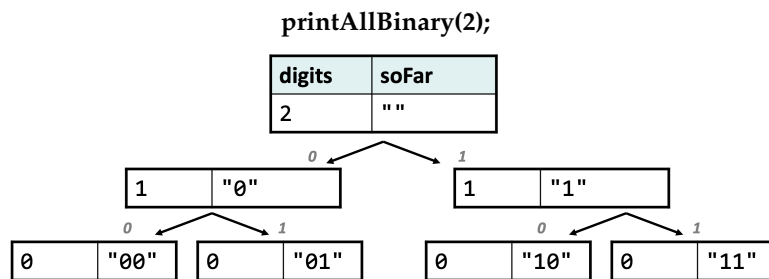
00
01
10
11

`printAllBinary(3);`

000
001
010
011
100
101
110
111

6

Decision Trees

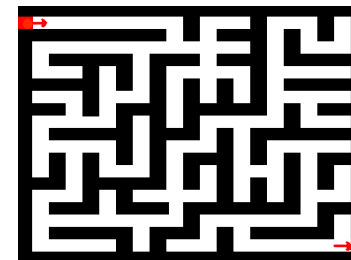


- This kind of diagram is called a **call tree** or **decision tree**
- Think of each call as a choice or decision made by the algorithm:
 - Should I choose 0 as the next digit?
 - Should I choose 1 as the next digit?
- The idea is to try every permutation. For every position, there are 2 options, either '0' or '1'. **Backtracking** can be used in this approach to try every possibility or permutation to generate the correct set of strings.

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Backtracking

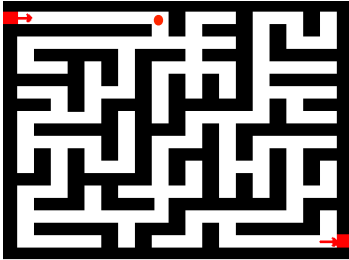
- **Recursive Backtracking:** using recursion to explore solutions to a problem and abandoning them if they are not suitable



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Backtracking

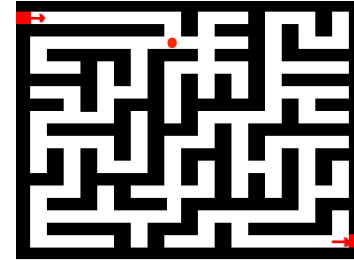
- **Recursive Backtracking:** using recursion to explore solutions to a problem and abandoning them if they are not suitable



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Backtracking

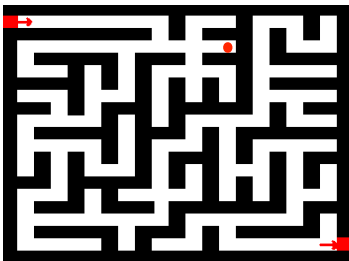
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Backtracking

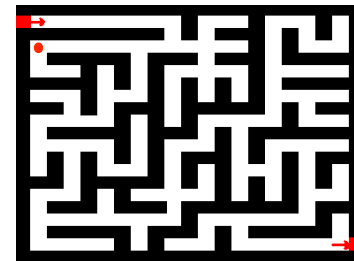
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Backtracking

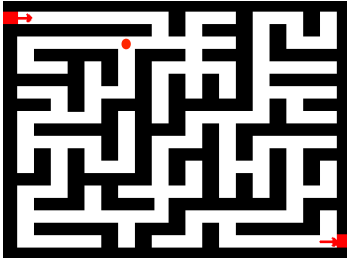
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Backtracking

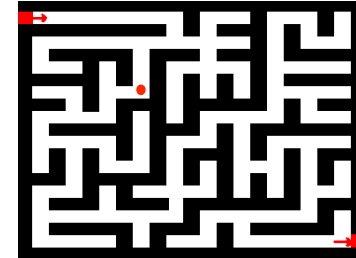
- **Recursive Backtracking:** using recursion to explore solutions to a problem and abandoning them if they are not suitable



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Backtracking

- **Recursive Backtracking:** using recursion to explore solutions to a problem and abandoning them if they are not suitable



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Backtracking

- Let's take a look at a problem similar to the binarySequence problem.

- Write a recursive function diceRoll that accepts an integer representing a number of 6-sided dice to roll, and output all possible permutations of values that could appear on the dice.

diceRoll(2)

{1,1}	{3, 1}	{5, 1}
{1, 2}	{3, 2}	{5, 2}
{1, 3}	{3, 3}	{5, 3}
{1, 4}	{3, 4}	{5, 4}
{1, 5}	{3, 5}	{5, 5}
{1, 6}	{3, 6}	{5, 6}
{2, 1}	{4, 1}	{6, 1}
{2, 2}	{4, 2}	{6, 2}
{2, 3}	{4, 3}	{6, 3}
{2, 4}	{4, 4}	{6, 4}
{2, 5}	{4, 5}	{6, 5}
{2, 6}	{4, 6}	{6, 6}

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Backtracking

- Backtracking Checklist:

- ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

For each valid choice:

- **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
- **Undo it after exploring.** Restore everything to the way it was before making this choice.

- ✓ **Find our base case(s).** What should we do when we are out of decisions?

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Backtracking

Backtracking Checklist:

- ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

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- **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
- **Undo it after exploring.** Restore everything to the way it was before making this choice.

- ✓ **Find our base case(s).** What should we do when we are out of decisions?

What die value should I choose next?

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Backtracking

Backtracking Checklist:

- ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

For each valid choice:

- **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).

- **Undo it after exploring.** Restore everything to the way it was before making this choice.

- ✓ **Find our base case(s).** What should we do when we are out of decisions?

We need to communicate the dice chosen so far to the next recursive call

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Backtracking

Backtracking Checklist:

- ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

For each valid choice:

- **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
- **Undo it after exploring.** Restore everything to the way it was before making this choice.

- ✓ **Find our base case(s).** What should we do when we are out of decisions?

We need to be able to remove the die we added to our first roll so far

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Backtracking

Backtracking Checklist:

- ✓ **Find what choice(s) we have at each step.** What different options are there for the next step?

For each valid choice:

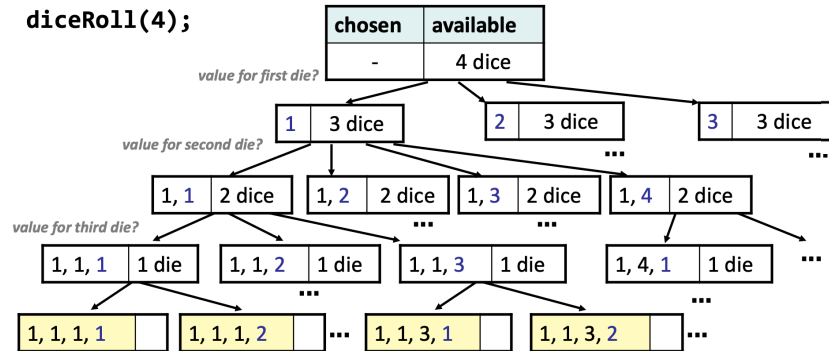
- **Make it and explore recursively.** Pass the information for a choice to the next recursive call(s).
- **Undo it after exploring.** Restore everything to the way it was before making this choice.

- ✓ **Find our base case(s).** What should we do when we are out of decisions?

We have no dice left to choose, print them out

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Backtracking



- Observations?
- This is a really big search space.
- Depending on approach, we can make wasteful decisions. Can we optimize it? Yes. Will we right now? No.

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Backtracking

- Let's us write flexible code, allowing us to make a decision and "backtrack" if we need to

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

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Backtracking

- Pseudocode**
- function diceRolls(dice, chosenArr):
 - if dice == 0:
 - Print current roll.
 - else:
 - // handle all roll values for a single die; let recursion do the rest.
 - for each die value i in range [1..6]:
 - choose that the current die will have value i
 - // explore the remaining dice
 - diceRolls(dice-1, chosenArr)
 - un-choose (backtrack) the value i

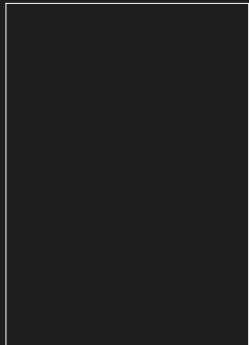
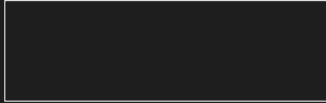
** Need to keep track of our choices somehow

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Code Demo

Recursive Backtracking Trace

Output for diceRolls (2):



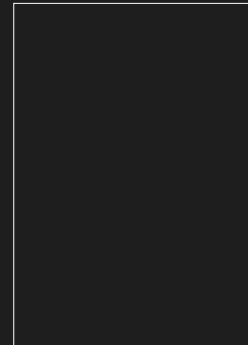
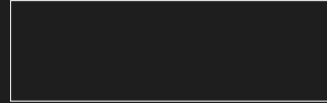
Stack

```
//create vector and call driver function
void diceRolls(int dice) { // dice = 2
    std::vector<int> chosen; // chosen = {}
    diceRollHelper(dice, chosen);
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):



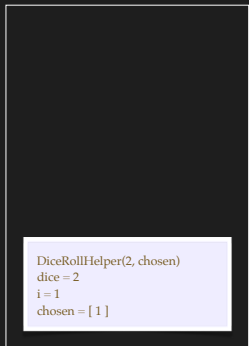
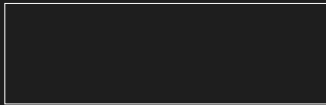
Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):



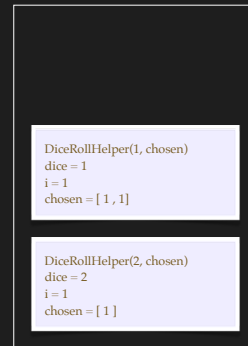
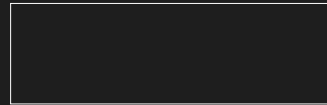
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Recursive Backtracking Trace

Output for diceRolls (2):



Stack

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            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 1]

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1, 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
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            }
        }
        std::cout << "}" << "\n";
    }
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    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}

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dice = 0
chosen = [1, 1]

DiceRollHelper(1, chosen)
dice = 1
i = 1
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DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

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void diceRollHelper(int dice, std::vector<int>& chosen) {
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            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1, 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

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void diceRollHelper(int dice, std::vector<int>& chosen) {
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            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [1, 1]

DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]

Stack

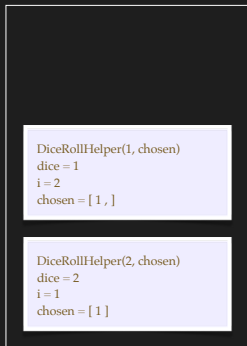
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            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}



Stack

```

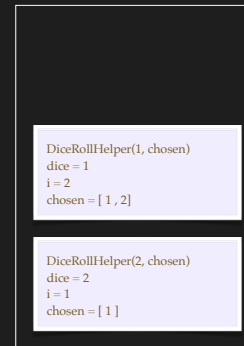
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                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
    
```

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Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}



Stack

```

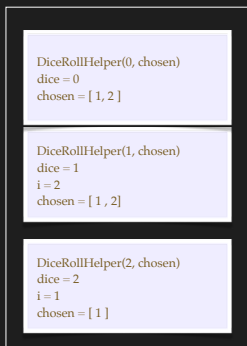
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            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
    
```

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Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}



Stack

```

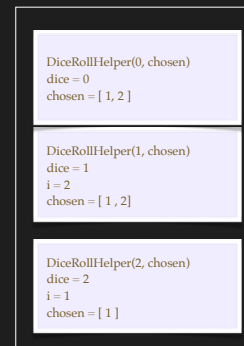
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            if(i < chosen.size() -1){
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            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
    
```

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Recursive Backtracking Trace

Output for diceRolls (2):

{1, 1}
{1, 2}



Stack

```

void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
    
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
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            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
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            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = {1, 2}
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = {1}
```

Stack

37

Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 2
chosen = {1, }
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = {1}
```

Stack

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = {1, }
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = {1}
```

Stack

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = {1, 3}
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = {1}
```

Stack

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 3]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [1, 3]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
{1, 3}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 3]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [1, 3]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1}
{1, 2}
{1, 3}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 3
chosen = [1, 3]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Fastforward...

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 6]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [1, 6]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [1, 6]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [1, 6]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [1, 6]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 6
chosen = [1, 6]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 1
chosen = [1]
```

Stack

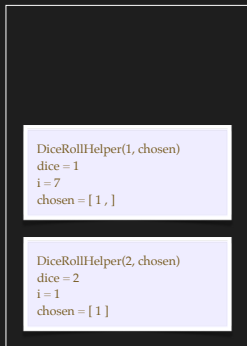
```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```



Stack

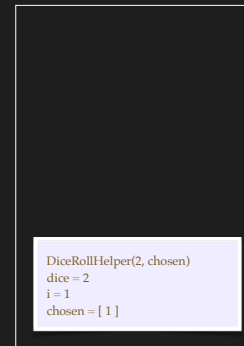
```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```



Stack

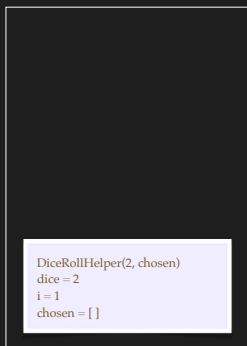
```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```



Stack

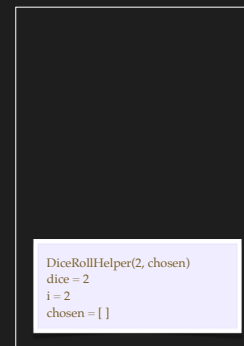
```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```



Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2, ]

DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2, 1]

DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3}
{1, 4}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [2, 1]

DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2, 1]

DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3} {2, 1}
{1, 4}
```

```
DiceRollHelper(0, chosen)
dice = 0
chosen = [2, 1]
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2, 1]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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Recursive Backtracking Trace

Output for diceRolls (2):

```
{1, 1} {1, 5}
{1, 2} {1, 6}
{1, 3} {2, 1}
{1, 4}
```

```
DiceRollHelper(1, chosen)
dice = 1
i = 1
chosen = [2, 1]
```

```
DiceRollHelper(2, chosen)
dice = 2
i = 2
chosen = [2]
```

Stack

```
void diceRollHelper(int dice, std::vector<int>& chosen) {
    // Base Case
    if (dice == 0) {
        //Print out contents of vector {1,1}
        std::cout << "{";
        for(int i=0; i < chosen.size(); i++){
            std::cout << chosen.at(i);
            if(i < chosen.size() -1){
                std::cout << ",";
            }
        }
        std::cout << "}" << "\n";
    }
    //Recursive case
    else {
        for (int i = 1; i <= 6; i++) {
            chosen.push_back(i);
            diceRollHelper(dice - 1, chosen);
            chosen.pop_back();
        }
    }
}
```

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