Appendix A
Proof of Theorem 2

Proof: We first give the recursive equation for $p(\mathbf{S^r}, q^r)$:

$$\begin{aligned}
&p(\mathbf{S^r}, q^r)\\
=&p(\mathbf{S^r}|q^r)p(q^r)\\
=&p(\mathcal{S}^r|q^r)p(\mathbf{S^{r-1}}|q^r)p(q^r)\\
=&p(\mathcal{S}^r|q^r)p(\mathbf{S^{r-1}}, q^r)\\
=&p(\mathcal{S}^r|q^r)\int p(\mathbf{S^{r-1}}, q^{r-1}, q^r)\mathrm{d}q^{r-1}\\
=&p(\mathcal{S}^r|q^r)\int p(\mathbf{S^{r-1}}, q^r|q^{r-1})p(q^{r-1})\mathrm{d}q^{r-1}\\
=&p(\mathcal{S}^r|q^r)\int p(\mathbf{S^{r-1}}|q^{r-1})p(q^r|q^{r-1})p(q^{r-1})\mathrm{d}q^{r-1}\\
=&p(\mathcal{S}^r|q^r)\int p(\mathbf{S^{r-1}}, q^{r-1})p(q^r|q^{r-1})\mathrm{d}q^{r-1}.
\end{aligned} \tag{25}$$

On the other hand, we have

$$p(\mathbf{S^r}, q^r) = p(q^r|\mathbf{S^r})p(\mathbf{S^r}) = \widehat{\alpha}(q^r)\prod_{t=1}^{r} p(\mathcal{S}^t|\mathbf{S}^{t-1}). \tag{26}$$

Substituting terms $p(\mathbf{S^r}, q^r)$ and $p(\mathbf{S^{r-1}}, q^{r-1})$ in Eq. (25) using Eq. (26), we have

$$\begin{aligned}
&\widehat{\alpha}(q^r)\prod_{t=1}^{r} p(\mathcal{S}^t|\mathbf{S}^{t-1})\\
=&p(\mathcal{S}^r|q^r)\int \widehat{\alpha}(q^{r-1})\prod_{t=1}^{r-1} p(\mathcal{S}^t|\mathbf{S}^{t-1})p(q^r|q^{r-1})\mathrm{d}q^{r-1}.
\end{aligned} \tag{27}$$

After eliminating identical terms from both sides of Eq. (27) we can prove Theorem 2. □

Appendix B
Proof of Theorem 3

Before proving Theorem 3, we first give a lemma of marginal distribution for Gaussian variables:

Lemma 5. *Given a Gaussian distribution for $\boldsymbol{x}$ and a conditional Gaussian distribution for $\boldsymbol{y}$ given $\boldsymbol{x}$ in the form: $p(\boldsymbol{x}) = \mathcal{N}(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Lambda})$, $p(\boldsymbol{y}|\boldsymbol{x}) = \mathcal{N}(\boldsymbol{y}; \boldsymbol{A}\boldsymbol{x}+\boldsymbol{b}, \boldsymbol{L})$, the marginal distribution of $\boldsymbol{y}$ is given by $p(\boldsymbol{y}) = \mathcal{N}(\boldsymbol{y}; \boldsymbol{A}\boldsymbol{\mu}+\boldsymbol{b}, \boldsymbol{A}\boldsymbol{\Lambda}\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{L})$.*

The proof of Lemma 5 can be found in [19]. We then give the proof of Theorem 3 as follows:

Proof: Substituting the terms in Eq. (16) using Eq. (12), (13), and (14), we have

$$\begin{aligned}
p(\mathcal{S}^r|\mathbf{S^{r-1}})\mathcal{N}(q^r; \widehat{\mu}^r, \widehat{\sigma}^r) = &\prod_j \mathcal{N}(s_j^r; q^r, \eta)\\
&\int \mathcal{N}(q^{r-1}; \widehat{\mu}^{r-1}, \widehat{\sigma}^{r-1})\mathcal{N}(q^r; aq^{r-1}, \gamma)\mathrm{d}q^{r-1}.
\end{aligned} \tag{28}$$

Applying Lemma 5 to the integral item in Eq. (28) we have

$$\begin{aligned}
&p(\mathcal{S}^r|\mathbf{S^{r-1}})\mathcal{N}(q^r; \widehat{\mu}^r, \widehat{\sigma}^r)\\
=&\prod_j \mathcal{N}(s_j^r; q^r, \eta)\mathcal{N}(q^r; a\widehat{\mu}^{r-1}, a^2\widehat{\sigma}^{r-1}+\gamma).
\end{aligned} \tag{29}$$

Taking logarithm of both sides of Eq. (29):

$$-\frac{(q^r-\widehat{\mu}^r)^2}{2\widehat{\sigma}^r} = -\sum_j \frac{(s_j^r - q^r)^2}{2\eta} - \frac{(q^r - a\widehat{\mu}^{r-1})^2}{2(a^2\widehat{\sigma}^{r-1}+\gamma)} + C, \tag{30}$$

where C contains terms independent of q^r. Compare the first-order and second-order coefficients of q^r on both sides of Eq. (30) we have

$$\eta(a^2\widehat{\sigma}^{r-1}+\gamma) = \widehat{\sigma}^r|\mathcal{S}^r|(a^2\widehat{\sigma}^{r-1}+\gamma) + \widehat{\sigma}^r\eta, \tag{31}$$

$$\eta(a^2\widehat{\sigma}^{r-1}+\gamma)\widehat{\mu}^r = \widehat{\sigma}^r(a^2\widehat{\sigma}^{r-1}+\gamma)\sum_j s_j^r + \widehat{\sigma}^r\eta a\widehat{\mu}^{r-1}. \tag{32}$$

Solving the above two equations we derive formula (17) and (18).

Applying Lemma 5 to Eq. (3) we can easily prove formula (19). □

Appendix C
Description of Opt-UB

Opt-UB is given as the largest i such that $\sum_{j\leq i} Q_j \leq Q_a$ (all Q_j are sorted in ascending order), where Q_a is the total available quality of workers with respect to budget constraint bounded by following three ways:

- If $\sum_{i\in\mathcal{W}} n_i c_i \leq B$, then all bids are possibly acceptable and $Q_a = \sum_{i\in\mathcal{W}} n_i\mu_i$ is the corresponding estimated quality of all workers contributed to all bidding tasks.
- If $\sum_{i\in\mathcal{W}} n_i c_i > B$, then we can modify the above Q_a by subtracting a correction item $(\sum_{i\in\mathcal{W}} n_i c_i - B)\cdot \min_{i\in\mathcal{W}}\frac{\mu_i}{c_i}$, which is the least loss of total available quality due to the budget constraint.
- $Q_a = B\cdot\max_{i\in\mathcal{W}}\frac{\mu_i}{c_i}$ is an estimated total available quality considering the budget constraint directly.

Overall, Opt-UB is the largest i such that $\sum_{j\leq i} Q_j \leq \min\Big\{\sum_{i\in\mathcal{W}} n_i\mu_i - \big[\sum_{i\in\mathcal{W}} n_i c_i - B\big]^+ \cdot \min_{i\in\mathcal{W}}\frac{\mu_i}{c_i}, B\cdot \max_{i\in\mathcal{W}}\frac{\mu_i}{c_i}\Big\}$, where $[x]^+$ returns the larger value between x and 0.

Appendix D
Proof of Truthfulness of Random

Note that although Random is a randomized mechanism, worker i cannot increase the probability of being selected into the set of $k+1$ workers by bidding untruthfully. Therefore, we prove that Random is strongly truthful for workers under any circumstance.

Proof:

1) Cost-truthfulness: We prove that $u_{i,j}(\overline{c}_i) \geq u_{i,j}(c_i)$ holds for the following two cases.

Case 1: Worker i is not selected into the set of $k+1$ workers for task j. In this case, $u_{i,j}(\overline{c}_i) = u_{i,j}(c_i) = 0$.

Case 2: Worker i is selected into the set of $k+1$ workers. The proof in this case is the same as in Theorem 4.

In summary, we have $u_{i,j}(\overline{c}_i) \geq u_{i,j}(c_i)$ for all task t_j. Therefore, $u_i(\overline{c}_i) \geq u_i(c_i)$.

2) Frequency-truthfulness: The proof is the same as in Theorem 4. □