BBH-kick-sim: A Simulator for Globular Cluster Black Hole Mergers

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Through many observations, both in light and through gravitational waves, we have found undeniable proof of the existence of black holes. Both stellar black holes of masses between $5M_{\odot} < m < 120M_{\odot}$, and supermassive black holes of masses in excess of $10^6 M_{\odot}$. However, the gap between these two is staggering. One might expect there to be black holes between these populations, often called Intermediate Mass Black Holes (IMBHs), their formation methods cannot be from stellar means alone. Two such formation models can be surmised: accretion and mergers.

For this paper, we explore the merger hypothesis in globular clusters. To do this, we developed a Monte Carlo simulator which progressively merges black holes together in a realistic way in order to evaluate whether IMBHs can be produced. This package is easy to use, expand, and features many parameters that can be customized by the user. Through the development of this tool, we hope to find which conditions most closely emulate observed quantities and which, if any, can produce IMBHs.

I. INTRO

In 2015, the Laser Interferometer Gravitational-wave Observatory (LIGO) directly detected gravitational waves for the first time in history, opening the door to gravitational studies of the Universe [1]. Since this first detection of a Binary Black Hole (BBH) merger, other stellar mass mergers have been detected (BHs with masses consistent with stellar progenitors). Finally in 2019, a merger was detected that produced an intermediate mass BH, one which could not be produced solely through the death of a high mass star [2].

Although electromagnetic observations have revealed a large population of supermassive ($10^6 - 10^9 M_{\odot}$) BHs at the centers of galaxies, it is not clear yet how these BHs grew to their observed size. Currently, the two ways theorized for BH growth is through accretion and hierarchical mergers. LIGO's detections of BH mergers have revealed detailed information about them including rates of merger, the populations of BHs that merge, as well as verifying the accuracy of numerical relativity simulations. Although mergers could be very efficient in increasing BH mass quickly, there are two fundamental roadblocks for this growth pathway.

The first is that when a BH binary forms, the decay of their orbit is entirely due to the radiation of gravitational waves and can be a very slow process. There are many binaries that will not merge in the age of the Universe due to this effect [3]. The second process that hinders BH growth through mergers is gravitational recoil, also known as kick, an effect of General Relativity that can impart high velocities on the remnant BH. These velocities could result in the BH leaving an environment conducive to merger growth [4].

One such environment that has been studied is globular clusters, as they contain a high amount of mass condensed into a relatively small volume in space. However, this depends on the BH remaining in the cluster for a long enough period to undergo several mergers to grow in size. If remnant BHs receive high kick velocities, they could escape the mass-dense cluster environment and have their growth stunted [5]. There are other potential environments in which hierarchical mergers could be an important pathway for BH growth, however, these are outside the scope of this work currently [6, 7].

These studies motivated the creation of a BH merger Monte Carlo simulation that utilizes a contemporary kick calculation software to determine possible BH populations and masses due to hierarchical growth alone. The calculations to determine a remnant BH's kick velocity are complicated numerical relativity simulations which prevent them from being used in such a massive study of BH populations. Recently, a software package called *surfinBH* was developed that utilizes some given numerical relativity calculations for BH kick and complements them with a neural network to interpolate for inputs outside of the initially calculated parameters. This has been proven to be accurate and quick, allowing for kick-based population simulation [8].

II. BBH-KICK-SIM

Our method for investigating black hole mergers in globular clusters is through a package we created called *BBH-kick-sim*. This Monte Carlo simulator allows for various tests of initial and merger conditions for black holes in globular clusters. Our methodology for this package's construction was for modularity and ease-of-use. Nearly every aspect of the simulator is built with parameters that you can tweak or methods that you can replace, changing either the initial conditions or the handling of

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the BH mergers.

A. What to merge

BBH-kick-sim is currently designed with a focus toward globular clusters, but we invite the community to build their own tools for it to explore other environments conducive to BH growth through hierarchical mergers. The characteristics of the globular cluster simulated is dependent on user input such as the cluster's mass, radius, and initial mass function. This creates an initial population of BHs with which to evolve in the cluster as well as the cluster's escape velocity. The characteristics of the first generation of BHs will be discussed in Section II B as they are related to our initial results of using *BBH-kick-sim*.

As the simulator begins to merge the BHs, the remnant's kick velocity will be calculated using *surfinBH* to determine whether it stays within the cluster ($V_{remnant} < V_{escape}$) to continue merging. If it escapes, it is removed from the cluster but is still used for population studies. Currently the simulator does not implement an accurate model of dynamical friction, instead applying a constant negative acceleration to all BHs and merging the first two to cross below a threshold velocity. This threshold velocity is arbitrarily set by the user currently. A future update to the package will implement a proper dynamical friction model. The simulator will continue to merge BHs, removing remnants with velocities greater than the cluster's escape velocity, until there are are no more mergers possible. This can occur when there is one BH left in the cluster or zero.

B. Initial conditions

Like many Monte Carlo simulations, *BBH-kick-sim* is heavily governed by the initial conditions of the simulator. Initial conditions like the mass of the globular cluster are important, but much of the variation in final parameters are controlled by the characteristics of the first generation of BHs. Despite each of these initial BHs only having two parameters: mass and spin, the choices for assigning these parameters are incredibly important to the end product, as will be shown in the toy model in Section III. While in the future we will allow users to use completely custom mass and spin distributions, in its current form, we programmed in several models for these parameters. Although the velocity of the BH is important for determining if it remains within the cluster after a merger, the first generation of BHs is assumed to have no initial velocity.

For initial BH masses, we use an initial mass function model to first calculate how many stars, within a range of star masses, can make BHs. From these masses of stars, we create a distribution of star masses to sample from. From the random samples of stars, we then either convert directly to BH mass using a simple mass fraction, or use a star mass to BH mass transfer function found in [9].

For initial BH spins, we currently support two simple models. Our simulator uses the dimensionless spin parameter $S = \frac{cJ}{GM^2}$ which has a minimum of 0 and maximum of 1. Our two models are either uniform initial spin, or zero initial spin. It is likely that neither of these models are correct, however, they are our two extremes, with the real initial spin model somewhere in the middle.

The uniform spin model hinges on the fact that due to the rotation of nearly every star we see, we would expect that the spin parameter will be non-zero for all initial BHs. The zero spin model suggests instead that these spins likely are not anywhere near the maximum spin when they form, instead, we might expect a large majority of spins to be close to 0. While both are likely to be incorrect, we would expect that the true initial spin distribution might look somewhat like a Gaussian, with few high spins and many low spins, leaving our models as the two extremes of this realistic model.

One final parameter that we have included in our package we call 'birth-ordering'. We define birth ordering as a method to sort the first black holes to merge based on the initial mass of the stars that made the black holes. The reason we include this parameter is because the largest stars in the cluster will form black holes first, allowing them the chance to merge before the smaller stars [10]. While this effect is real, the difference in black hole formations is fairly short compared with the time to first mergers [10]. It is because of this that we make this a parameter that you can turn on or off.

III. TOY MODEL

For a test of our package, we wanted to look at the effects of birth ordering. For this test, we used default parameters for everything besides birth ordering. These parameters are: the cluster mass to $10^6 M_\odot$, a cluster radius of 2 pc, an initial mass function alpha value of 2.25, a minimum star mass of $0.8 M_\odot$, a maximum star mass of $100 M_\odot$, using a uniform initial spin distribution, and telling the simulator to use the star mass to BH mass function from [9]. With all of these parameters in place, we ran 2,000 different simulations, 1,000 of which with birth ordering, and 1,000 without.

Thanks to the flexibility of the package, there are many parameters and statistics that you can create to analyze the output of each simulation. We chose to look at the final BH mass distribution, spin distribution, and velocity distribution for all BHs which have escaped the globular cluster.

Shown in Figures 1,2, and 3, the effect of birth ordering has a significant effect on the final masses of BH that have escaped the globular cluster. Birth ordering shows many more BHs at low masses escaping. This is likely due to the fact that mergers closer to an equal mass ratio ($q = 1$ where $q = m1/m2$) have higher kick velocities on average

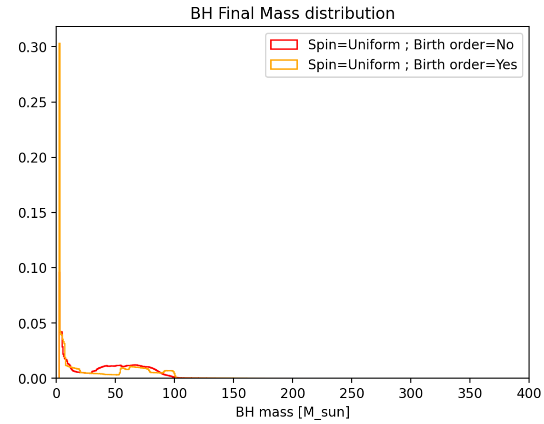

FIG. 1. The full mass distribution of all simulations

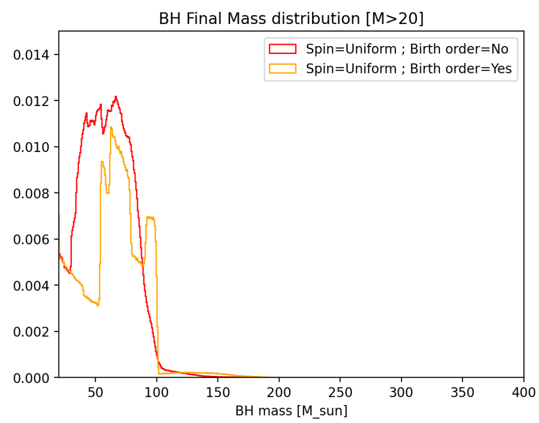

FIG. 2. The mass distribution of all simulations for masses $> 20 M_{\odot}$

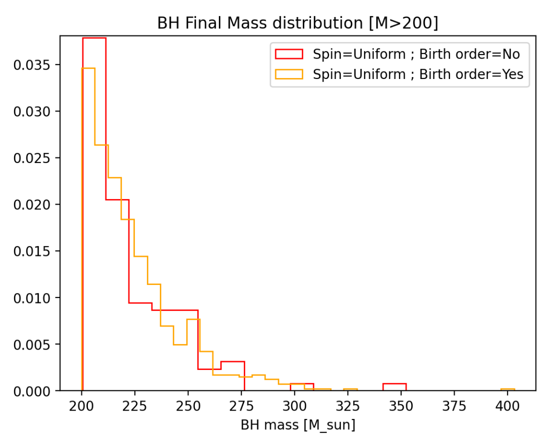

FIG. 3. The mass distribution of all simulations for masses $> 200 M_{\odot}$

[5]. Because of the large amount of BHs at these lower masses thanks to the initial mass function, a very large fraction escape on their first merger.

If we were to instead limit our view to $M > 20 M_{\odot}$

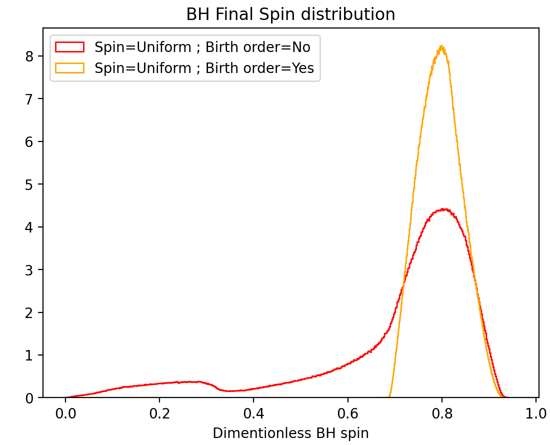

FIG. 4. The spin distribution of all simulations

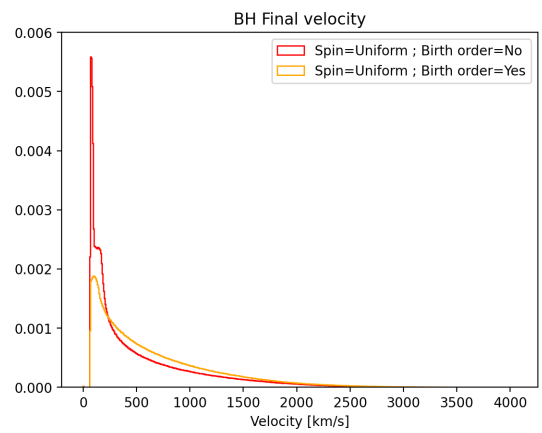

FIG. 5. The velocity distribution of all simulations

and $M > 200 M_{\odot}$, we find that the there are significantly more larger mass BHs with birth ordering than without. These can also likely be caused by low q mergers of large BHs. Without birth ordering the chance of low q merger with a large mass is fairly low, hence the scarcity at high masses.

We also looked at the percentage of BHs above $300 M_{\odot}$ and found that without birth ordering, only $2 * 10^{-5}$ % of BHs ever got that large, where enabling birth ordering resulted in a nearly order of magnitude increase in the number of BHs at $9 * 10^{-5}$ %.

Our final spin distributions in Fig. 4 showed a substantial difference. Birth ordering had a much higher spin magnitude, due to the larger number of low q mergers, which frequently result in spins > 0.7. Without birth ordering, we also see a much more even distribution below 0.7, due to high q mergers showing that the smaller BH has very little impact on the larger BHs spin.

Finally, our velocity distribution shows that without birth ordering, BHs have a lesser velocity, near the escape velocity of the globular cluster, whereas birth ordering results in more high velocity BHs on average. This is consistent with low q mergers, more frequent in the birth-ordering case, producing higher kick velocities

These show that, at least with out parameter selection, IMBHs of $M > 300M_{\odot}$ are very unlikely to be made in globular clusters.

IV. CONCLUSIONS

In order to evaluate the chances that IMBHs could form inside a globular cluster, we developed a brand new python package to simulate it. *BBH-kick-sim* is a Monte Carlo simulator for BHs in a globular cluster. By colliding these BHs together, we found that the influence of BH kicks are extraordinarily important to the merger trees within the cluster.

The flexibility of this package allows users to completely customize the globular cluster, the initial BH distribution, and the mechanics of the mergers. With more development, we also plan to introduce new ways to interface with the simulator and new ways to analyze the final results of the simulations.

Our toy model testing the effects of birth ordering show that even changing one variable drastically changed the final distribution of BH properties. We encourage others to explore how other parameters may effect these same values, and develop new ways to evaluate those results. We hope that through analyses of a combination of parameters, we might be able to narrow down the best fitting parameters to continued observations of BHs through both telescopes, and gravitational wave detectors.

While incomplete in its current state, we will continue its development with more realistic astrophysics and more modularity for a wider usage among the community.

V. FUTURE WORK

These initial tests have been extremely promising for the potential of this software package. There are several avenues of improvement that we plan to pursue for *BBH-kick-sim.* Of prime importance is implementing a realistic dynamical friction model. This would be more realistic than the current scheme for merging BHs and with it, we could compare rates of BH mergers with those observed by LIGO. The characteristics of the first generation of BHs also greatly affects the overall population, as seen in the Toy Model discussed in Section III, and soon the BH mass and spin distribution will have the capability to be user supplied. Additionally we hope to build more functionality for environments other than globular clusters, such as super or young star clusters, or galactic nuclei. To enable *BBH-kick-sim* to be more widely accessible, we plan on building a website with easy to use controls and visualization tools for the general public to experiment and learn more broadly about BH populations and the gaps in knowledge that still exist.

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[1] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), Observation of gravitational waves from a binary black hole merger, Phys. Rev. Lett. **116**, 061102 (2016).

[2] R. Abbott *et al.*, Properties and astrophysical implications of the 150 m binary black hole merger gw190521, The Astrophysical Journal Letters **900**, L13 (2020).

[3] N. C. Stone, B. D. Metzger, and Z. Haiman, Assisted inspirals of stellar mass black holes embedded in AGN discs: solving the 'final au problem', **464**, 946 (2017), arXiv:1602.04226 [astro-ph.GA].

[4] M. Campanelli, C. Lousto, Y. Zlochower, and D. Merritt, Large merger recoils and spin flips from generic black hole binaries, The Astrophysical Journal **659**, 5 (2007).

[5] K. Holley-Bockelmann, K. Gültekin, D. Shoemaker, and N. Yunes, Gravitational wave recoil and the retention of intermediate-mass black holes, The Astrophysical Journal **686**, 829 (2008).

[6] C. L. Rodriguez, S. Chatterjee, and F. A. Rasio, Binary black hole mergers from globular clusters: Masses, merger rates, and the impact of stellar evolution 10.1103/PhysRevD.93.084029 (2016).

[7] C. Kimball, C. Talbot, C. P. L. Berry, M. Carney, M. Zevin, E. Thrane, and V. Kalogera, Black hole genealogy: Identifying hierarchical mergers with gravitational waves, The Astrophysical Journal **900**, 177 (2020).

[8] V. Varma, D. Gerosa, F. HeÌbert, L. C. Stein, H. Zhang, V. Varma, D. Gerosa, F. HeÌbert, L. C. Stein, and H. Zhang, surfinbh: Surrogate final black hole properties for mergers of binary black holes, ascl , ascl:1809.007 (2018).

[9] M. Spera and M. Mapelli, Very massive stars, pair-instability supernovae and intermediate-mass black holes with the sevn code, Monthly Notices of the Royal Astronomical Society **470**, 4739 (2017).

[10] A. Dotter, Mesa isochrones and stellar tracks (mist) 0: Methods for the construction of stellar isochrones, The Astrophysical Journal Supplement Series **222**, 8 (2016).

[11] K. Jani and A. Loeb, Global stellar budget for ligo black holes, The Astrophysical Journal **889**, L35 (2020).