Lecture 8

02/10/2022

Lots of definitions today!

$$p(x,y) = p(x|y)p(y) = p(y|x)p(x)$$

$$\hookrightarrow p(y|x) = \frac{p(x|y)p(y)}{p(x)}$$

$$p(y|x) = \frac{p(x|y)p(y)}{\int p(x|y)p(y)\,dy}$$

BAYES' RULE

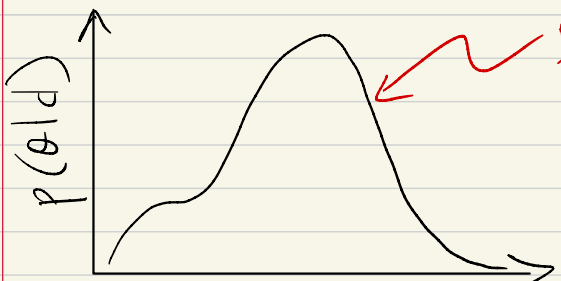
reverses what is being inferred

$$\text{Prob}(\text{Parameters}|\text{Data}) = \frac{\text{Prob}(\text{Data}|\text{Parameters}) \times \text{Prob}(\text{Parameters})}{\text{Prob}(\text{Data})}$$

$$p(\theta|d) = \frac{p(d|\theta) \times p(\theta)}{p(d)}$$

$$\text{Posterior Probability} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Evidence}}$$

- Likelihood = fitting function (as before)
- Prior = constraints on parameters
- Evidence = does not depend on model parameters, so not important for parameter estimation.



POSTERIOR is not a distribution over simulations or experimental trials ----

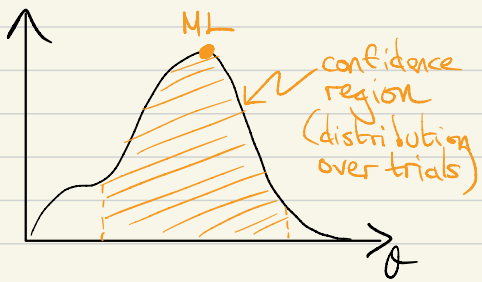
it is a "belief spread" in parameters given some data + priors.

- Priors
  - we want the posterior to be DATA DOMINATED
  - priors should be weakly informative.

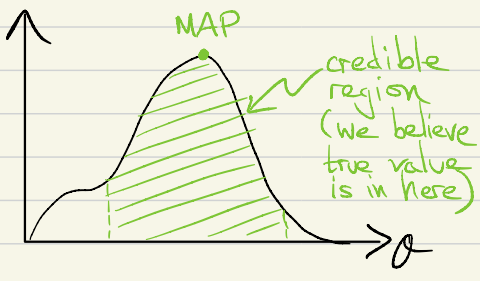
→ uniform over a range
→ gaussian
→ scale-invariant ⇒ $p(\sigma) \propto 1/\sigma$
→ conjugate ⇒ posterior belongs to the same family of distributions as the prior.

⇒ priors can be physical constraints, or the outcome of previous experiments.

- # Credible Regions

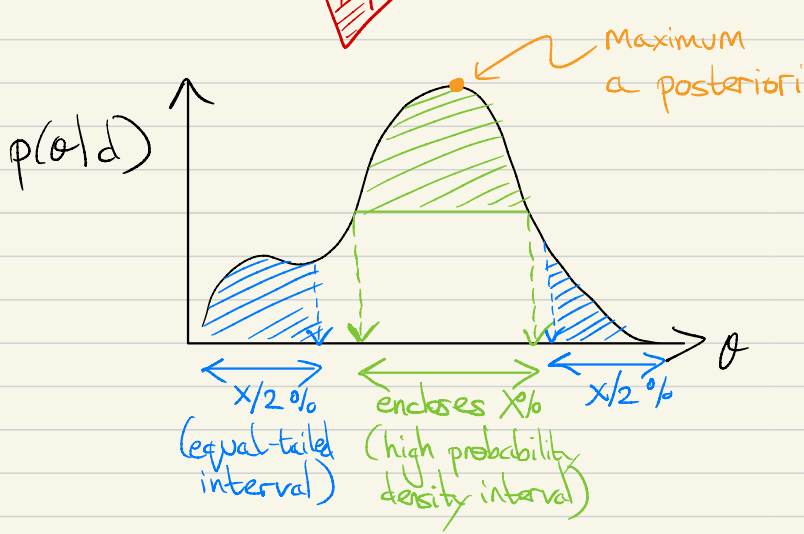


CLASSICAL STATISTICS



BAYESIAN STATISTICS

Bayesian credible regions are not unique



⇒ we usually use equal-tailed intervals.