$$\|x\|_2 \le \|x\|_1 \le \sqrt{n}\,\|x\|_2$$

$$|\langle x, y\rangle| \le \|x\|_2 \|y\|_2$$

$$\|x\|_1 = \sum_{i=1}^{n} |x_i|$$

$$e = (1, 1, \dots, 1)$$

$$|\langle x, e\rangle| = \left|\sum x_i\right| \le \sum |x_i|$$

$$\|x\|_2 \|e\|_2 = \sqrt{n}\,\|x_2\|$$

$$|\langle x, e\rangle| = \left|\sum x_i\right| = \sum |x_i| \leq \sqrt{n}\,\|x\|_2$$

$$" |x_i| " \quad x_i > 0 \ \forall i$$

$$x_i < 0 \ \forall i$$

$$\left(\sqrt{n}\,\|x\|_2\right)^2 \overset{?}{\geq} \left(\|x\|_1\right)^2$$

$$n \cdot \sum_{i=1}^{n} x_i^2 \qquad \left(\sum |x_i|\right)^2$$

$$\hat{x} = (|x_1|, \dots, |x_n|) \neq x$$

$$\cancel{e = (1, \dots, 1)}, \quad \hat{e} = (\operatorname{sign}(x_1), \dots, \operatorname{sign}(x_n))$$

$$\langle x, \hat{e} \rangle = \sum_{i=1}^{n} x_i \cdot \operatorname{sign}(x_i) = \sum_{i=1}^{n} |x_i| = \|x\|_1$$

$$\leq \sqrt{n}\, \|x\|_2$$

Cauchy schwarz.

$$(1,-1) \Rightarrow \max(1,-1) = (1,0)$$

$$x = (x_1, \dots, x_n), \quad y = (y_1, \dots, y_n)$$

$$d(x+y) = d(x_1+y_1, x_2+y_2, \dots, x_n+y_n) =$$

$$= \|(\max(x_1+y_1, 0), \dots, \max(x_n+y_n, 0))\|_2$$

$$- \|\max(x,0)\|_2 + \|\max(y,0)\|_2$$

$$\left(\|\max(x,0)\|_2 + \|\max(y,0)\|_2\right)^2 = \|\max(x,0)\|^2 + \|\max(y,0)\|$$

$$+ \; 2 \; \|\max(x,0)\|_2 \|\max(y,0)\|_2 \, ,$$

$$\|\max(x+y,0)\|_2^2 = \sum_{i=1}^{n} \max(x_i+y_i,0)^2$$

$$x_1^2 + y_1^2 + 2|x_1||y_1|$$

$x_1 > 0 \qquad y_1 > 0 \quad \Rightarrow (x_1+y_1)^2$

$x_1 \not> 0 \qquad y_1 < 0 \quad \times$

$$\text{Si } x_i + y_i \geq 0 \Rightarrow \max(x_i + y_i, 0) = x_i + y_i$$

$$\text{Si } x_i + y_i < 0 \Rightarrow \max(x_i + y_i, 0) = 0$$

$$d(x+y)^2 = \sum_{i=1}^{n} \max(x_i + y_i, 0)^2 = \sum_{i \in I} (x_i + y_i)^2 + \sum_{j \in J} 0$$

$$I = i \in \{1, \dots, n\} : x_i + y_i > 0$$

$$J = j \in \{1, \dots, n\} : x_j + y_j \leq 0$$

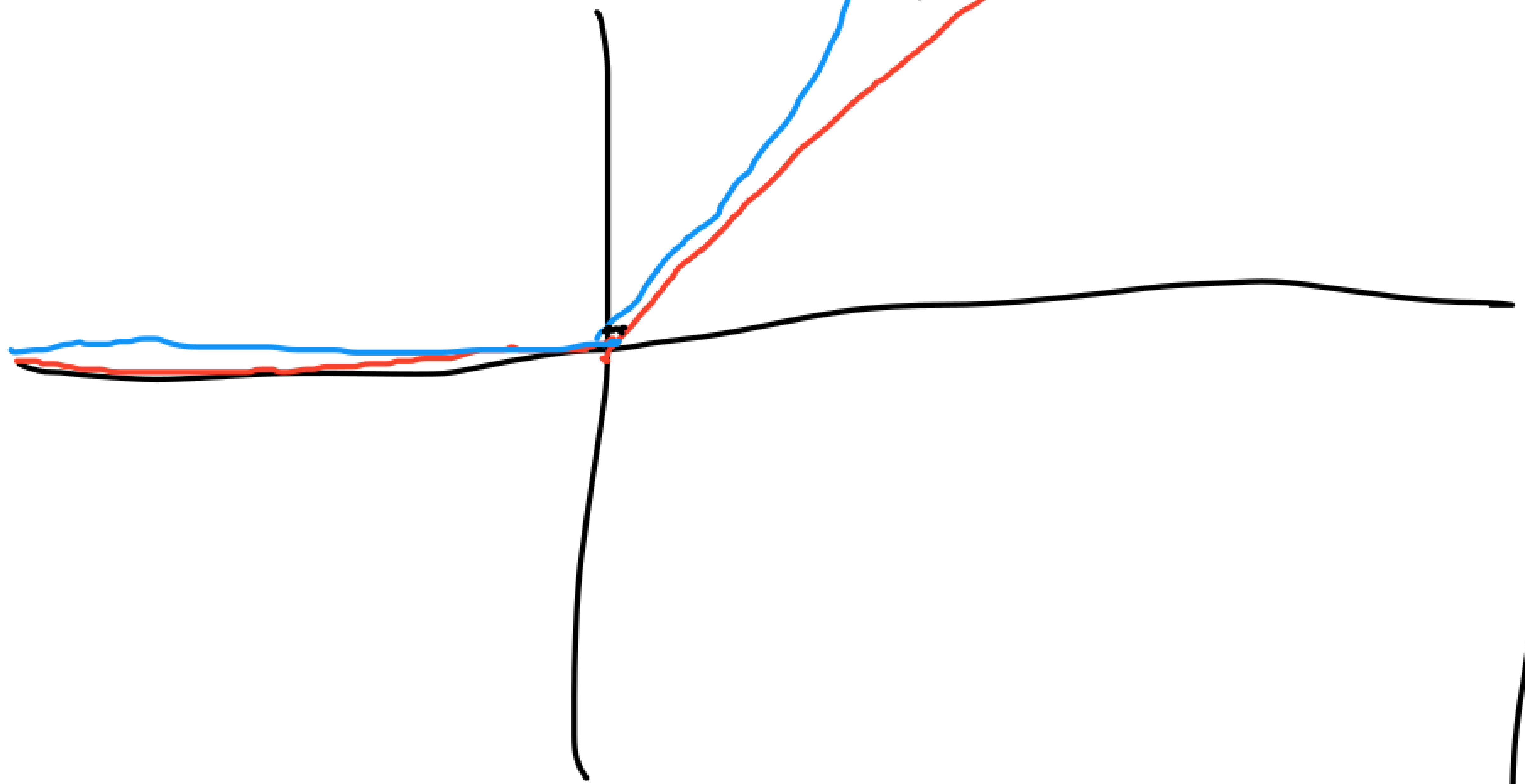
$$\left(d(x)+d(y)\right)^2 = d(x)^2 + d(y)^2 + 2\,d(x)\,d(y)$$

$$= \sum_{i=1}^{n} \max(x_i, 0)^2 + \sum_{i=1}^{n} \max(y_i, 0)^2 + 2\sqrt{\sum \max(x_i, 0)^2}\,\sqrt{\sum_{i=1}^{n} \max(y_i, 0)^2}$$

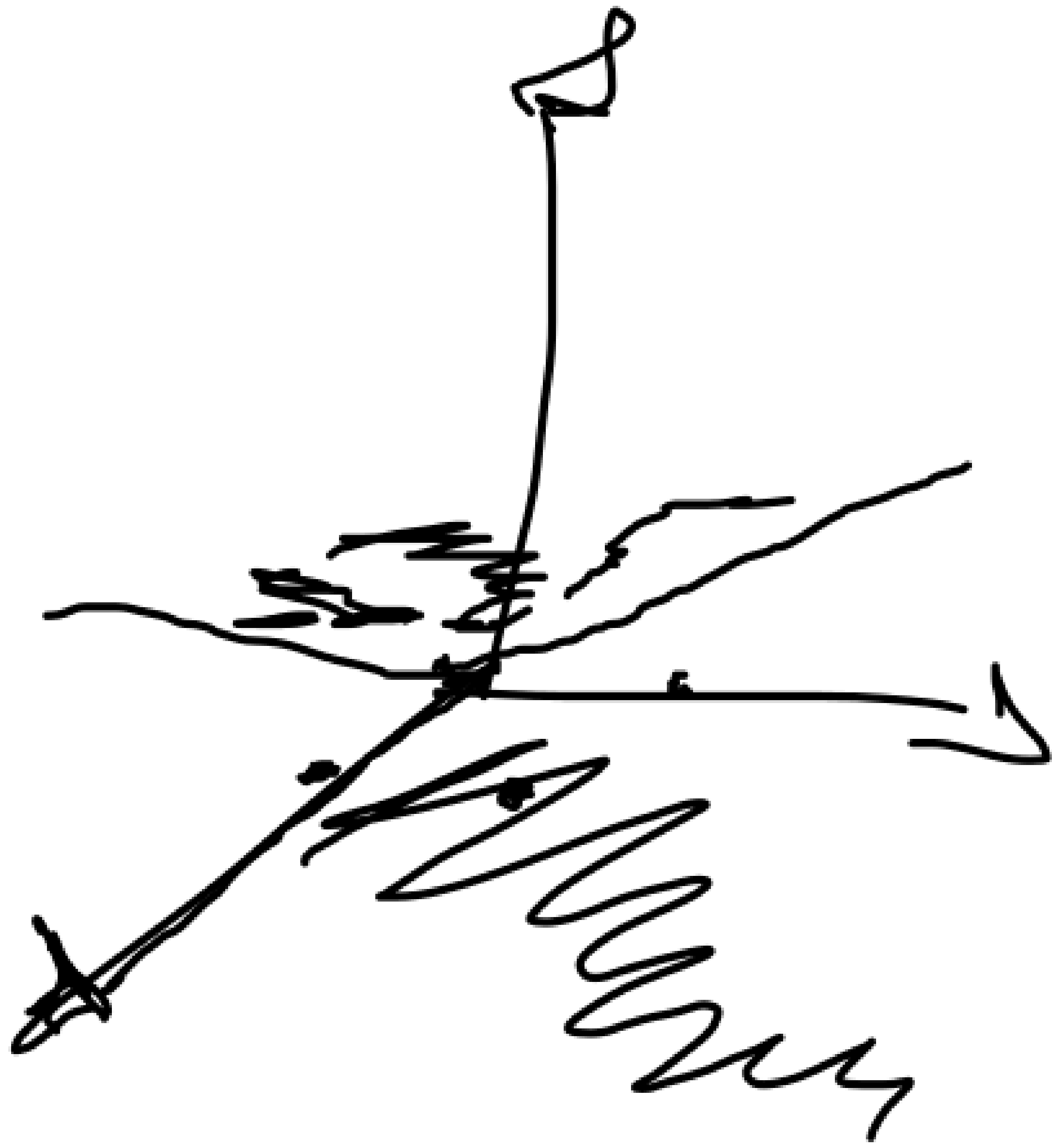
si $x_i, y_i > 0 \Rightarrow$

$$x_i^2 + y_i^2 + 2|x_i||y_i|$$

$$f(x)^2 = \max(x,0)^2$$



$$f((x,y)) = \|\max((x,y),(0,0))\|_2$$



$$x = (5,5), \quad y = (-2,-3)$$

$$d(x,0) = \boxed{\sqrt{50}}, \quad d(y,0) = 0$$

$$5\sqrt{2}$$

$$d(x+y,0) = \sqrt{13}$$

$$d(x,0) = \|\max(x,0)\|_2 = \sqrt{\sum_i}$$

6) $\|\cdot\|$ norma vectorial y $A \in \mathbb{R}^{m \times n}$

$\|\cdot\|$ norma inducida

$$\|A\| = \sup_{x \neq 0} \frac{\|Ax\|}{\|x\|} = \boxed{\max_{\|x\|=1} \|Ax\|}$$

$$\exists\, x : \|x\| = 1 \text{ y } \|A\| \underbrace{\|x\|}_{=1} = \|Ax\|$$

$$z_i = \max(x_i + y_i, 0)$$

$$x_i^+ = \max(x_i, 0), \quad y_i^+ = \max(y_i, 0)$$

$$z_i \leq x_i^+ + y_i^+$$

$$\|z\|_2 \leq \|x^+ + y^+\|_2 \leq \|x^+\|_2 + \|y^+\|_2$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \sqrt{2}$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \sqrt{2}$$