10) Si sto $y, A \in \mathbb{R}^n$, entonces

$$\left\| A\left(I - \frac{ss^t}{s^t s}\right)\right\|_F^2 = \|A\|_F^2 - \frac{\|As\|^2}{\|s\|^2}$$

Sabemos $\|A\|_F = \sqrt{tr(A^t A)}$

Entonces $\left\| A\left(I - \frac{ss^t}{s^t s}\right)\right\|_F^2 = tr\left(\left[A\left(I - \frac{ss^t}{s^t s}\right)\right]^t \left[A\left(I - \frac{ss^t}{s^t s}\right)\right]\right)$

$$= tr\left(\left(I - s s^t\right)^t A^t A\left(I - ss^t\right)\right) = tr\left(\left(I - \frac{ss^t}{s^t s}\right)A^t A\left(I - \frac{ss^t}{s^t s}\right)\right)$$

$\underbrace{}$
simétrica!

La traza es conmutativa ⇓

$$\mathrm{tr}\left(\left(I-\frac{ss^t}{s^ts}\right)A^tA\left(I-\frac{ss^t}{s^ts}\right)\right) = \mathrm{tr}\left(\left(I-\frac{ss^t}{s^ts}\right)^2 A^tA\right) =$$

$$\left(I-\frac{ss^t}{s^ts}\right)^2 = I - 2\frac{ss^t}{s^ts} + \frac{ss^t s s^t}{s^ts\, s^ts} = I - \frac{ss^t}{s^ts}$$

$$= \mathrm{tr}\left(\left(I-\frac{ss^t}{s^ts}\right)A^tA\right) = \mathrm{tr}\left(A^tA - \frac{ss^t}{s^ts}A^tA\right) = \mathrm{tr}(A^tA) - \mathrm{tr}\left(\frac{ss^t}{s^ts}A^tA\right)$$

La suma nos permite
separar en la traza.

$$\text{tr}\left(A^t A\right) - \text{tr}\left(\frac{ss^t}{s^t s} A^t A\right) = \|A\|_F^2 - \frac{1}{s^t s} \text{tr}\left(s s^t A^t A\right) =$$

$$\underbrace{\phantom{\text{tr}\left(A^t A\right)}}_{\|A\|_F^2} \qquad \underbrace{s^t s}_{\text{número sale!}} \qquad \underbrace{s^t s}_{\|s\|_2^2}$$

$$= \|A\|_F^2 - \frac{1}{\|s\|_2^2} \text{tr}\left(s^t A^t A s\right) = \|A\|_F^2 - \frac{\|A s\|_2^2}{\|s\|_2^2} \quad \text{!!}$$

$$\underset{\text{números!}}{\uparrow}$$