Math 244

PSET 2

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Section 1.5 Problem 5

Ouestion

Prove the associativity of composing relations: If R, S, and T are relations such that $(R \circ S) \circ T$ is well defined, then $R \circ (S \circ T)$ is well defined and equal to $(R \circ S) \circ T$.

Remark For prob 1

We know (w,z) lies in $(R \circ S) \circ T$ iff there is some y such that

1 $(w, y) \in R \circ S$ and

 $2 (y,z) \in T$

 $(w,y) \in R \circ S$ means:

$$(w,x) \in R$$
 and $(x,y) \in S$

So we have $(x,y) \in S$ and $(y,z) \in T$, which implies $(y,z) \in S \circ T$ So, $(w,x) \in R$ and $(y,z) \in S \circ T$ means that $(w,z) \in R \circ (S \circ T)$ $(R \circ S) \circ T \subseteq R \circ (S \circ T)$

Section 1.6 Problem 3

Question

Prove that a relation R is transitive if and only if $R \circ R \subseteq R$.

Key Idea

We need to show

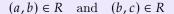
- \Rightarrow If R is transitive, then, $R \circ R \subseteq R$
- \Leftarrow If $R \circ R \subseteq R$, then R is transitive.



Remark \Rightarrow

We assume that R is transitive.

By def or relational composition we know that for a pair $(a,c) \in R \circ R$ there exists some $b \ s.t.$





Section 1.6 Problem 6

Question

Describe all relations on a set X that are equivalences and orderings at the same time.

Section 2.1 Problem 4

Question

Let (X, \leq) , (Y, \leq) be ordered sets. We say that they are *isomorphic* if there exists a bijection $f: X \to Y$ such that for every $x, y \in X$, we have $x \leq y$ if and only if $f(x) \leq f(y)$.

- a) Draw Hasse diagrams for all non-isomorphic ordered sets with 3 elements posets.
- b) Prove that any two n-elements linearly ordered sets are isomorphic.

Section 2.2 Problem 3

Question

- a) Consider the set $\{1,2,\ldots n\}$ ordered by the divisibility relation |. What is the maximum possible number of elements of a set $X\subseteq\{1,2,\ldots n\}$ that is ordered linearly by the relation |
- b) Solve the same question for the set $2^{\{1,2,\dots n\}}$ ordered by the inclusion relation \subseteq .