Math 244

PSET 1

JAN 24 2025

Contents

Section 1.2 Problem 5

Question: Section 1.2 Problem 5

Is a "cancellation" possible for the Cartesion Product? That is, if $X \times Y = X \times Z$ holds for some sets, X, Y, and Z, does it follow that $Y = Z$?

Remark What is the Cartesian Product?

The Cartesian product of X and Y is the set of all ordered pairs of the form (x, y), where $x \in X$ and $y \in Y$.

Remark Asnwer

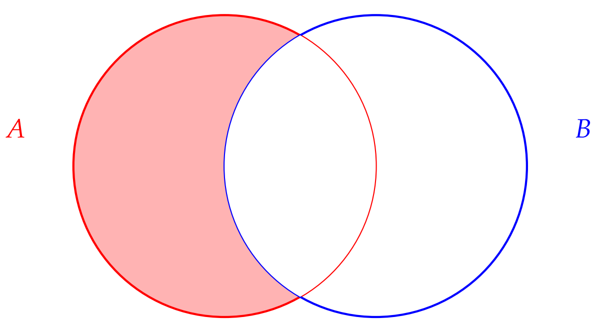
The "cancellation" is not possible for the Cartesian Product unless it is stated that X is not an empty set. For if X is an empty set, then the Cartesian Product of X and another set would always be the empty set. In this scenario, Y and Z could be different and their Cartesian Products with X would still be the empty set.

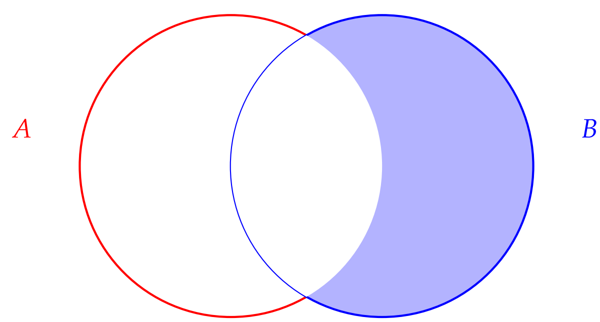
Section 1.2 Problem 6

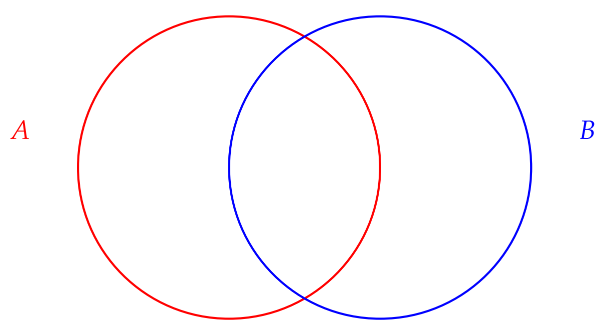
Question: Section 1.2 Problem 6

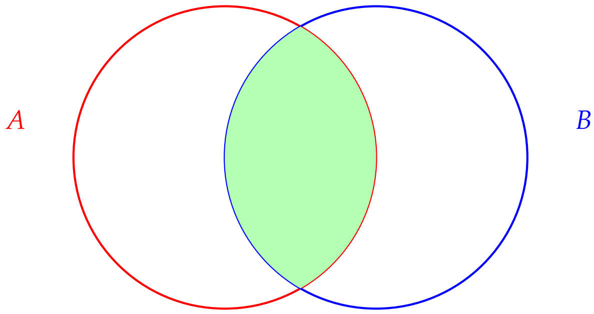
Prove that for any two sets A, B we have

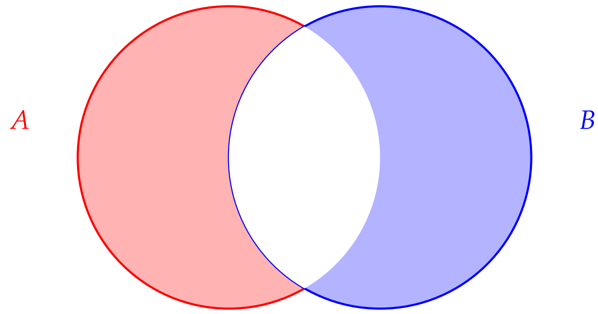
$$(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$$











Section 1.3 Problem 2

Question: Section 1.3 Problem 2

The numbers F_0, F_1, F_2, F_3, ... are defined as follows:

$$F_0 = 0, F_1 = 1, F_{n+2} = F_{n+1} + F_n \text{for } n = 0, 1, 2, \ldots$$

Prove that for any $n \geq 0$ we have $F_n \leq \left(\frac{1+\sqrt{5}}{2}\right)^{n-1}$

1.4 Problem 2

Question: Section 1.4 Problem 2

Find an example of:

(a) A one-to-one function $f : \mathbb{N} \to \mathbb{N}$ that is not onto.

(b) A function $f : \mathbb{N} \to \mathbb{N}$ that is onto but not one-to-one.

1.4 Problem 6

Question

Prove that the following two statements about a function $g_1 : Z \to X$ and $g_2 : Z \to X$ the composed functions $f \circ g_1$ and $f \circ g_2$ are also distinct.