

Math 244

PSET 3

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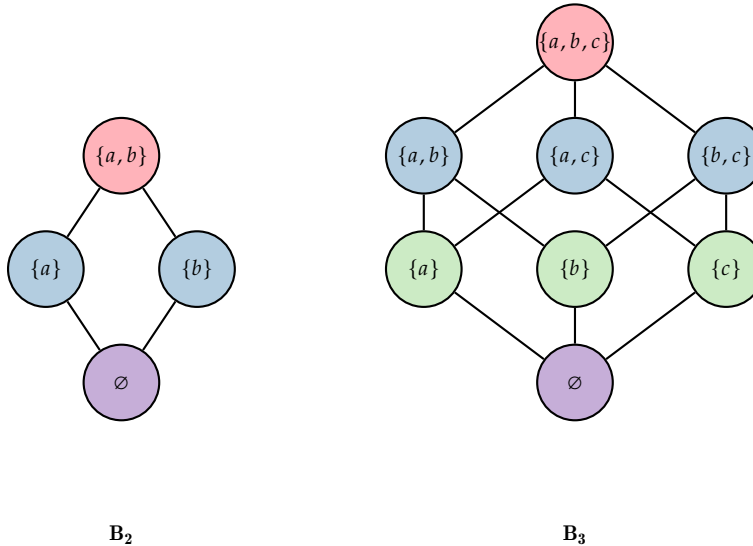
Contents

Prob 1	2
Bonus Prob	2
Prob 2	2
Prob 3	3
Prob 4	3
Prob 5	3

2.3 Prob 1

Question

How many linear extensions of \mathcal{B}_2 are there, and what about \mathcal{B}_3 ?



Remark For Prob 1

For \mathcal{B}_2 there are two linear extensions because the linear extension must respect and preserve the original partial order so the empty set must be first and the set $\{a, b\}$ must be at the top which means their variation in ordering the remaining elements is $2!$ as those are the elements below $\{a, b\}$ and above the empty set.

For \mathcal{B}_3 there are 36 possible linear extensions. Since the linear extension needs to preserve the original partial order than that means the unique top element and unique bottom element of the set $\{a, b, c\}$ and the empty set must be the last and first element of any linear extension respectively. For the remaining elements the sets with cardinalities 1 must all come before the sets of cardinality 2 but there are 3 sets of cardinality 1 that can be ordered in $3!$ ways as there are 3 options for the first one, 2 for the second, and 1 for the last one. This logic also applies to the 3 sets of cardinality of 2 that must all go after the sets of cardinality 1 and before the unique top element of cardinality 3 so there are $3!$ ways to order those. This makes the total possible linear extensions the result of multiplying the possibilities of each possible way of ordering the elements so $1 \cdot 3! \cdot 3! \cdot 1$ which is 36.



Bonus Problem

Question

Prove that not every finite poset admits an embedding into the poset (\mathbb{N}^2, \leq) , where $(x_1, y_1) \leq (x_2, y_2)$ if and only if $x_1 \leq x_2$ and $y_1 \leq y_2$.

2.4 Prob 3

Question

Find a sequence of real numbers of length 16 that contains no monotone subsequence of length 5.

Example (For Prob 2)

Array: [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16]



2.4 Prob 4

Question

Prove the following strengthening of Theorem 2.4.6: Let k, ℓ be natural numbers. Then every sequence of real numbers of length $k\ell + 1$ contains a nondecreasing subsequence of length $k + 1$ or a decreasing subsequence of length $\ell + 1$.

3.1 Prob 2

Question

Determine the number of ordered pairs (A, B) , where $A \subseteq B \subseteq \{1, 2, \dots, n\}$.

3.1 Prob 6

Question

Show that a natural number $n \geq 1$ has an odd number of divisors (including 1 and itself) if and only if \sqrt{n} is an integer. *The textbook has a hint to this problem in the back.*