Math 244

PSET 3

Feb 10 2025

Contents

2.3 Prob 1

Question

How many linear extensions of $\mathcal{B}_2$ are there, and what about $\mathcal{B}_3$?

Bonus Problem

Question

Prove that not every finite poset admits an embedding into the poset $(\mathbb{N}^2, \preceq)$, where $(x_1, y_1) \preceq (x_2, y_2)$ if and only if $x_1 \leq x_2$ and $y_1 \leq y_2$.

2.4 Prob 3

Question

Find a sequence of real numbers of length 16 that contains no monotone subsequence of length 5.

2.4 Prob 4

Question

Prove the following strengthening of Theorem 2.4.6: Let k, ℓ be natural numbers. Then every sequence of real numbers of length $k\ell + 1$ contains a nondecreasing subsequence of length $k + 1$ or a decreasing subsequence of length $\ell + 1$.

3.1 Prob 2

Question

Determine the number of ordered pairs (A, B), where $A \subseteq B$ $subseteq\{1, 2, \ldots, n\}$.

3.1 Prob 6

Question

Show that a natural number $n \geq 1$ has an odd number of divisors (including 1 and itself) if and only if $\sqrt{n}$ is an integer. *The textbook has a hint to this problem in the back.*