Math 244

PSET 2

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Contents

Section 1.5 Problem 5

Question

Prove the associativity of composing relations: If R, S, and T are relations such that $(R \circ S) \circ T$ is well defined, then $R \circ (S \circ T)$ is well defined and equal to $(R \circ S) \circ T$.

Remark For prob 1

We know (w, z) lies in $(R \circ S) \circ T$ *iff* there is some y such that

1 $(w, y) \in R \circ S$ and

2 $(y, z) \in T$

$(w, y) \in R \circ S$ means:

$$(w, x) \in R \quad \text{and} \quad (x, y) \in S$$

So we have $(x, y) \in S$ and $(y, z) \in T$, which implies $(y, z) \in S \circ T$ So, $(w, x) \in R$ and $(y, z) \in S \circ T$ means that $(w, z) \in R \circ (S \circ T)$
$(R \circ S) \circ T \subseteq R \circ (S \circ T)$

Section 1.6 Problem 3

Question

Prove that a relation R is transitive if and only if $R \circ R \subseteq R$.

Key Idea

We need to show

$\Rightarrow$ If R is transitive, then, $R \circ R \subseteq R$

$\Leftarrow$ If $R \circ R \subseteq R$, then R is transitive.

Remark $\Rightarrow$

We assume that R is transitive.
By def or relational composition we know that for a pair $(a, c) \in R \circ R$ there exists some b $s.t.$

$$(a, b) \in R \quad \text{and} \quad (b, c) \in R$$

Section 1.6 Problem 6

Question

Describe all relations on a set X that are equivalences and orderings at the same time.

Section 2.1 Problem 4

Question

Let $(X, \preceq)$, $(Y, \preceq)$ be ordered sets. We say that they are *isomorphic* if there exists a bijection $f : X \longrightarrow Y$ such that for every $x, y \in X$, we have $x \preceq y$ if and only if $f(x) \preceq f(y)$.

a) Draw Hasse diagrams for all non-isomorphic ordered sets with 3 elements posets.

b) Prove that any two n-elements linearly ordered sets are isomorphic.

Section 2.2 Problem 3

Question

a) Consider the set $\{1, 2, \dots n\}$ ordered by the divisibility relation $|$. What is the maximum possible number of elements of a set $X \subseteq \{1, 2, \dots n\}$ that is ordered linearly by the relation $|$

b) Solve the same question for the set $2^{\{1,2,\dots n\}}$ ordered by the inclusion relation $\subseteq$.