Math 115 QR
PSet 2

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Chapter 1

1.1 Problem 1: Gottlieb 27.1.4

Question 1

(a) A farmer has planted corn on a rectangular plot of land 800 meters by 1000 meters. A straight stream runs alongside one of the long borders of the plot, and the farmer's irrigation system is such that his yield decreases with the distance from the stream. Suppose his yield is given by $f(x) = 50 - 0.3\sqrt{x}$ ears of corn per square meter, where x is the distance from the stream in meters. What is the farmer's yield from the plot?

(b) A second farmer plants his corn in a circular plot with radius 80 meters and he has a centralized irrigation system located in the middle of his field. His yield drops with the distance from the center of the field. Suppose his yield is also given by $f(x) = 50 - 0.3\sqrt{x}$ ears of corn per square meter, this time x being the distance from the center of the field. What is the farmers yield for this plot?

Solution:

(a)

$f(x) = 50 - 0.3\sqrt{x}$

area of the i^{th} slice = $1000\Delta x$

ears of corn in the i^{th} slice = $1000\Delta x \cdot \left(50 - 0.3\sqrt{x}\right)$

total corn = $\sum_{i=1}^{n} \left(50 - 0.3\sqrt{x}\right) \cdot 1000\Delta x$

total corn = $\int_0^{800} \left(50 - 0.3\sqrt{x}\right) \cdot 1000dx$

$$\int_0^{800} \left(50 - 0.3\sqrt{x}\right) \cdot 1000dx = 1000\int_0^{800} \left(50 - 0.3\sqrt{x}\right) dx$$

$$1000\int_0^{800} \left(50 - 0.3\sqrt{x}\right) dx = \int_0^{800} 50000 - 300\sqrt{x}dx$$

$$\int_0^{800} 50000 - 300\sqrt{x}dx = 50000x - 200x\sqrt{x}\Big|_0^{800}$$

$$50000x - 200x\sqrt{x}\Big|_0^{800} = 50000(800) - 200(800)\sqrt{800}$$

$$50000(800) - 200(800)\sqrt{800} = 35,474,516.6004$$

(b)

$f(x) = 50 - 0.3\sqrt{x}$

area of the i^{th} slice = $2\pi x\Delta x$

ears of corn in the i^{th} slice = $2\pi x\Delta x \left(50 - 0.3\sqrt{x}\right)$

total corn = $\sum_{i=1}^{n} 2\pi \left(50x - 0.3x\sqrt{x}\right) \Delta x$

total corn = $\int_0^{80} 2\pi \left(50x - 0.3x\sqrt{x}\right) dx$

$$\int_0^{80} 2\pi\left(50x - 0.3x\sqrt{x}\right)dx = 2\pi\int_0^{80} 50x - 0.3x\sqrt{x}dx$$
$$2\pi\int_0^{80} 50x - 0.3x\sqrt{x}dx = \int_0^{80} 100\pi x - 0.3\cdot 2\pi x^{\frac{3}{2}}dx$$
$$\int_0^{80} 100\pi x - 0.3\cdot 2\pi x^{\frac{3}{2}}dx = 50\pi x^2 - \frac{3\pi x^2\sqrt{x}}{25}\Big|_0^{80}$$
$$50\pi x^2 - \frac{3\pi x^2\sqrt{x}}{25}\Big|_0^{80} = 983,729.4183$$

1.2 Problem 2: Gottlieb 27.1.5

Question 2

Consider a box of cereal with raisins. The box is 5 centimeters deep, 25 centimeters tall, and 16 centimeters wide. The raisins tend to fall toward the bottom; assume their density is given by $\rho(h) = \frac{4}{h+10}$ raisins per cubic centimeter, where h is the above the bottom of the box. How many raisins are in the box?

Solution:
$w = 5$
$h = 25$
$l = 16$
area of the i^{th} slice is $16\cdot 5\Delta h = 80\Delta x$
cereal in the i^{th} slice is $80\Delta x\frac{4}{h+10}$
$\int_0^{25} 80\frac{4}{h+10}dx$

$$\int_0^{25} 80\frac{4}{h+10}dh = \int_0^{25}\frac{320}{h+10}dh$$
$$\int_0^{25}\frac{320}{h+10}dh = 320\ln(x+10)\Big|_0^{25}$$
$$320\ln(x+10)\Big|_0^{25} = 320ln(25+10) - 320ln(0+10) \approx 400.884$$

1.3 Problem 3: Gottlieb 27.1.14

Question 3

Liquid is being stored in a large spherical tank of radius 2 meters. The tank is completely full and has been left standing for a long time. A mineral suspended in the liquid is setting. Its density at a depth of h meters from the top is given by $5h$ milligrams per cubic meter. Determine the number of milligrams of the mineral contained in the tank.

Solution: volume of i^{th} slice $= \pi r^2\Delta h$
$r_i = \sqrt{4 - h_i^2}$
raisins in i^{th} slice $= \pi\left(\sqrt{4-h_i^2}\right)^2\cdot\Delta h\cdot 5h$

$$\int_0^2 \pi\left(4-h^2\right)5hdh + \int_0^2 \pi(4-h^2)5(h+2)dh$$
$$5\pi\int_0^2 4x - x^3dx = 10\pi x^2 - \frac{5\pi x^4}{4}\Big|_0^2 = 20\pi$$

1.4 Problem 4: Gottlieb 27.1.21

Question 4

A circus tent has cylindrical symmetry about its center pole. The height a distance of x feet from the center pole is given by $h(x) = \frac{8}{1+\frac{x^2}{16}}$ feet. What is the volume of the enclosed by the tent of radius 4.

Solution: Volume of i^{th} slice is: $\pi x h_i \Delta x$

$$\int_0^4 2\pi x \frac{8}{1+\frac{x^2}{16}} dx = 278.731$$

1.5 Problem 5: Stewart 6.4.8

Question 5

A spring has a natural length of 40 cm.
(a) If a 60-N force is required to keep the spring compressed 10 cm, how much work is done during this compression?
(b) How much work is required to compress the spring to a length of 25 cm?

Solution:
(a)

$$f(x) = kx$$

$$60N = k.1$$

$$k = \frac{60}{0.1} = 600$$

$$\int_0^{0.1} 600x dx = \left(300x^2\right)\Big|_0^{10}$$

$$\left(300x^2\right)\Big|_0^{10} = 300(0.1)^2 = 3 \text{ J}$$

(b)

$$\int_0^{0.15} 600x dx = \left(300x^2\right)\Big|_0^{10}$$

$$\left(300x^2\right)\Big|_0^{10} = 300(0.15)^2 = 6.75 \text{ J}$$

1.6 Problem 6: Stewart 6.4.12

Question 6

If 6 J of work is needed to stretch a spring from 10 cm to 12 cm and another 10 J is needed to stretch it from 12 cm to 14 cm, what is the natural length of the spring?

Solution:

$$\left(yx^2\right)\Big|_a^{a+2} = y(a+2)^2 - y(a)^2 = 6 \text{ J}$$

$$y(a+2)^2 - y(a)^2 = 6$$

$$y(a+4)^2 - y(a+2)^2 = 10$$

$$y(l-0.14)^2 - y(l-.012)^2 = 10$$

$$y\left(l-\frac{7}{50}\right)^2 - y\left(l-\frac{3}{25}\right)^2 = 10$$

$$y\left(l-\frac{3}{25}\right)^2 - y\left(l-\frac{1}{10}\right)^2 = 6$$

$$y\left(\left(l-\frac{7}{50}\right)^2 - \left(l-\frac{3}{25}\right)^2\right) = 10$$

$$y\left(\left(l-\frac{3}{25}\right)^2 - y\left(l-\frac{1}{10}\right)^2\right) = 6$$

$$y\left(\left(l-\frac{7}{50}-l+\frac{3}{25}\right)\left(l-\frac{7}{50}+l+\frac{3}{25}\right)\right) = 10$$

$$y\left(\left(l-\frac{3}{25}-l+\frac{1}{10}\right)\left(l-\frac{3}{25}+l+\frac{1}{10}\right)\right) = 6$$

$$y\left(\left(-\frac{7}{50}+\frac{3}{25}\right)\left(l-\frac{7}{50}+l-\frac{3}{25}\right)\right) = 10$$

$$y\left(\left(-\frac{3}{25}+\frac{1}{10}\right)\left(l-\frac{3}{25}+l-\frac{1}{10}\right)\right) = 6$$

$$y\left(-\frac{1}{50}\left(2l-\frac{13}{50}\right)\right) = 10$$

$$y\left(-\frac{1}{50}\left(2l-\frac{11}{50}\right)\right) = 6$$

$$\frac{2l-\frac{13}{50}}{2l-\frac{11}{50}} = \frac{5}{3}$$

$$\frac{100l-13}{100l-11} = \frac{5}{3}$$

$$300l - 39 = 500l - 55$$

$$-200l = -16$$

$$l = \frac{2}{25}$$

1.7 Problem 7: Stewart 6.4.14

Question 7

A thick cable, 60 ft long and weighing 180 lb, hangs from a winch on a crane. Compute in two different ways the work done if the winch winds up 25 ft of the cable.
(a) Follow the method of Example 4
(b) Write a function for the weight of the remaining cable after x feet has been wound up by the winch. Estimate the amount of work done when the winch pulls up Δx feet of cable

Solution: (a)
Weight per foot: $\frac{180}{60} = 3$ lb/ft

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} 3x_i \Delta x$$

$$\int_0^{25} 3x dx = \frac{3}{2}x^2\Big|_0^{25}$$

$$\frac{3}{2}x^2\Big|_0^{25} = \frac{3}{2}(25)^2 = 937.5$$

(b)

$$w = 180 - 3x$$

What does that second part mean?

1.8 Problem 8: Stewart 6.4.24

Question 8

A tank is full of water. Find the work required to pump the water out of the spout. $r = 3$m and spout = 1m

Solution: Volume of i^{th} slice is $\pi r^2 dh$

$$\left(\frac{r_i}{2}\right)^2 = 3^2 - h_i^2$$

$$\frac{r_i^2}{4} = 3^2 - h_i^2$$

$$r_i^2 = 4(3)^2 - 4h_i^2$$

$$r_i = \sqrt{36 - 4h_i^2}$$

$\int_0^7 \left(\sqrt{36 - 4h_i^2}\right) dh$
$\pi \int_0^7 36 - 4h_i^2 dh$

1.9

Question 9

A tank is full of water. Find the work required to pump the water out of the spout. In Exercises 25 and 26 use the fact that water weighs 62.5 lb/ft^3. Traingular prism with $h = 6$, $w = 10$, and $l = 12$

Solution: The ratio of the legs is $\frac{12}{6}$ which is 2 so the length will always be 2 times the height. This is important as we are going to use the property of similar traignles.
Height = h_i
Width = 10
Length = $2h_i$
Volume of i$^[th]$ slice is $(10)(2h_i)\Delta h$ Amount of water in ith slice is $(10)(2h_i)(62.5)\Delta h$

$$\text{Total work required to pump the water: } \int_0^6 (62.5)(20h)(6-h)dh = 45000$$