Math 120

PSet 2

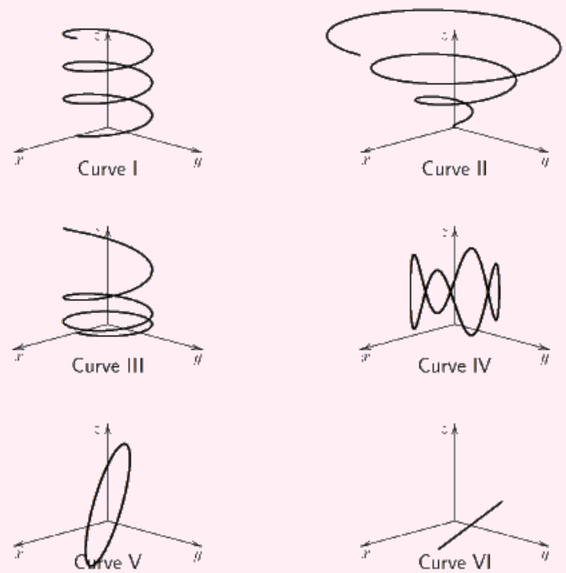
Sep 12 2024

Contents

Chapter 1

1.1 PSet 2

Question 1



(a) $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$

(b) $\vec{r}(t) = t\langle \cos t, \sin t, t \rangle$

(c) $\vec{r}(t) = \langle \cos t, \sin t, t^3 \rangle$

(d) $\vec{r}(t) = \langle \cos(t^3), \sin(t^3), t^3 \rangle$

(e) $\vec{r}(u) = \langle \cos u, \sin u, 1 + \sin(4u) \rangle$

(f) $\vec{r}(u) = \langle \cos u, \sin u, 1 + 4\sin(u) \rangle$

(g) $\vec{r}(t) = \langle 2\cos t, 1 + 4\cos t, 3\cos t \rangle$

Solution:

Question 2

2. Find a vector function that represents the curve of intersection of the plane $z = -2$ and the sphere $x^2 + (y-1)^2 + (z+1)^2 = 9$.

Solution:

Question 3

Consider the vector-valued function $\vec{r}_1(t) = \langle 2\sin t, -3\cos t, 0\rangle$, $0 \leqslant t \leqslant 2\pi$.

(a) Sketch the plane curve given by $\vec{r}_1(t)$.

(b) Compute and draw on your sketch from part (a) the position vector $\vec{r}_1\left(\frac{2\pi}{3}\right)$ and the tangent vector $\vec{r}_1'\left(\frac{2\pi}{3}\right)$.

(c) The vector-valued function $\vec{r}_2(t) = \langle 2\cos(3t), -3\sin(3t)\rangle$ parameterizes the same curve. Find the smallest $t^* > 0$ such that $\vec{r}_2(t^*) = \vec{r}_1\left(\frac{2\pi}{3}\right)$, and compute $\vec{r}_2'(t^*)$. Explain how and why $\vec{r}_2'(t^*)$ differs from the tangent vector $\vec{r}_1'\left(\frac{2\pi}{3}\right)$ you computed in part (b).

Solution:

Question 4

Find parametric equations for the tangent line to the curve parameterized by

$$x = 2t + 1, \quad y = e^{t^2-4}, \quad z = \ln(1+t^2)$$

at the point $(5, 1, \ln 5)$.

Solution:

Question 5

(a) Evaluate the integral $\int \left(\tan t\,\hat{i} + \sin^2 t\,\hat{j} + \sec^2 t\,\tan t\,\hat{k}\right) dt$.

(b) Suppose a particle is at the point $(-2, 1, 4)$ at time $t = 0$, and moves according to the velocity function $\vec{v}(t) = \tan t\,\hat{i} + \sin^2 t\,\hat{j} + \sec^2 t\,\tan t\,\hat{k}$. Find the particle's position at time $t = \frac{\pi}{4}$.

Solution:

Question 6

Consider the curve parameterized by $\vec{r}(t) = \langle e^{2t}, e^{-2t}, \sqrt{8}t \rangle$, $0 \leqslant t \leqslant 1$.

(a) Sketch the projections of $\vec{r}(t)$ in the xy-, zx-, and yz-planes.

(b) Find the length of the curve. *Hint:* To integrate, you will need to write $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2$ as a perfect square.

Solution:

Question 7

Let C be the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the plane $2x + y + z = 4$.

(a) Find a parameterization of C.

(b) Write down an integral for the length of C.

(c) Find the length accurate to five decimal places by using Desmos: `https://www.desmos.com/calculator`. (Click on the keyboard icon, then "functions", then "Misc", to find the integral symbol.)

Solution:

Question 8

Find the velocity and position vectors of a particle that has acceleration given by

$$\vec{a}(t) = 2\hat{i} + 6t\hat{j} + 12t^2\hat{k},$$

and initial velocity and position given by

$$\vec{v}(0) = \hat{i} \quad \text{and} \quad \vec{r}(0) = \hat{j} - \hat{k}.$$

Solution:

Question 9

Consider the function $f(x, y) = \frac{\sqrt{y} - 3x}{\ln(4 - x^2 - y^2)}$.

(a) Find and sketch the domain of f.

(b) On your sketch from part (a), mark where $f(x, y) = 0$, and indicate the region(s) where $f(x, y)$ is positive and negative.

Solution:

Question 10

Here are several surfaces.
Match each function with its graph. Justify your answers.

(a) $f(x,y) = x^2$

(b) $f(x,y) = \sqrt{x^2+y^2}$

(c) $f(x,y) = e^{x^2+y^2} - 1$

(d) $f(x,y) = y \sin x$

(e) $f(x,y) = \sin(x+y)$

(f) $f(x,y) = \sin\left(\sqrt{x^2+y^2}\right)$

Solution:

Question 11

Draw a contour map of the function $f(x,y) = x^2 e^{-y}$ showing several level curves.

Question 12

Match the function with its graph (labeled A-F below) and with its contour map (labeled I-VI). Give reasons for your choices.

(a) $z = e^x \cos y$

(b) $z = \sin x - \sin y$

(c) $z = \frac{x-y}{1+x^2+y^2}$