Math 120 QR

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Contents

Chapter 1

1.1 Day 1 notes

Definition 1.1.1: Distance Formula

Defintion:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Definition 1.1.2: Equation of a sphere

Defintion: An equation of a sphere with center $C(h, k, l)$, and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2$$

In particular, if the center is the origin O, than an equation of the sphere is

$$x^2 + y^2 + z^2$$

1.2 Day 2 Notes

Definition 1.2.1: The lenght/magnitude of a vecotr

In 2D, $\vec{v} =< a, b >$: $|\vec{v}| = \sqrt{a^2 + b^2}$
In 3D, $\vec{v}$, $\vec{v} =< a, b, c >$: $|\vec{v}| = \sqrt{a^2 + b^2 + c^2}$
A unit vector is a vector of length 1

Question 1

If $\vec{v}$ is a a vector and a is a scalar, then what is $|a\vec{v}|$

Solution:

$$|a\vec{v}| = |a||\vec{v}|$$

Definition 1.2.2: Vectors in R^3

The standard basis vectors in $\mathbf{R}^3$ are

$$i =< 1, 0, 0 >$$
$$j =< 0, 1, 0 >$$
$$k =< 0, 0, 1 >$$

Question 2

What is special about i,j,k?

Solution:

- Cannot make any of them as a linear combibnation of the other three.
- Any vector $\vec{v}\epsilon\mathbf{R}^3$ can be written uniquily as a linear combibnation of i,j,k

Example 1.2.1:

$\vec{v} + \vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

$\vec{v} + \vec{u} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{pmatrix}$

Definition 1.2.3: Dot product

A dot product between v and w is:
In 2D: $v \cdot w = v_1w_1 + v_2w_2$
In 3D: $v \cdot 2 = v_1w_1 + v_2w_2 + v_3w_3$

Geometric defintion:
$|u \cdot w| = |u||w| \cos(\theta)$ where θ is the angle between v and w

Example 1.2.2: Why the 2 defintions are the same

$$v_1w_1 + v_2w_2 = |v||w| \cos(\theta) = p|v|$$
$$p = |w| \cos(\theta)$$
$$v_1 = |v| \cos(\theta)$$
$$v_2 = |v| \sin(\theta)$$
$$w_1 = |w| \cos(\theta)$$
$$w_2 = |w| \sin(\theta)$$
$$\text{LHS} = |v||w| \left(\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)\right)$$
$$\text{LHS} = |v||w| \sin(\alpha + \beta)$$
$$\text{LHS} = |v||w| \cos(\theta) = \text{RHS}$$

Example 1.2.3: what does the def mean

How much effect of $\vec{w}$ act along $\vec{v}$
Work: $w = \vec{F} \cdot \vec{S}$

Question 3

Find a relation between $|v|$ and $v \cdot v$

Solution: $|v|^2 = v \cdot v$

Question 4

v, w of fixed lenghts when is $v \cdot w$ largest?

Solution: v paralell: $\theta = 0$
$\cos(\theta) = 1$

Example 1.2.4: Projections

Given $\vec{v} \neq \vec{o}$
The project of w on v is proj $w = \left(\frac{\vec{w} \cdot \vec{v}}{|v|}\right) \frac{\vec{v}}{|v|}$
Directon of v is $\frac{\vec{v}}{|v|}$
Dor project of w with direction is:

$$\vec{w} \cdot \frac{\vec{v}}{|v|}$$

Question 5

TRUE or False:
u, v, w: vectors $(u \cdot v)w = u(v \cdot w)$

Solution: false

Question 6

TRUE or FALSE:
$v - w| = |v| - |w|$ if $v \parallel w$

Solution: false

Question 7

When is this ideal square sum happening?

$$|v + w|^2 = |v|^2 + |w|^2$$

Solution: when $v \perp w$

Definition 1.2.4: Cross Product

The cross product of two vectors v and w, $v * w$ is a vector u defined by $u \perp v$ and $u \perp w$.
Direction of u is given by the right hand rule
Magnitude: $|u|$ = Area of the parallelogram spanned by v and w.

1.3 Day 2 Reading notes

Definition 1.3.1: Vector Addition

If **u** and **v** are vectors positioned so the initial point of **v** is at the terminal point of **u**, then the **sum u + v** is the vector from the initial point of **u** to the terminal point of **v**.

Definition 1.3.2: Scalar Multiplication

If c is a scalar and **v** is a vector, then the **scalar multiple** $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of **v** and whose direction is the same as **v** if $c > 0$ and is opposite to **v** if $c = 0$ or $\mathbf{v} = 0$, then $c\mathbf{v} = 0$

Example 1.3.1:

Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector **a** with represenation $\vec{AB}$ is:

$$a = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

Example 1.3.2:

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then:

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$$

$$\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$

$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

Similarily for three demensional vectors