

Math 120

PSet 6

Oct 9 2024

Contents

Chapter 1

Page 2

1.1 PSet 6

2

Chapter 1

1.1 PSet 6

Question 1

Find all the (local) maximum and minimum values and saddle points of the function.

(a) $f(x, y) = xy + \frac{1}{x} + \frac{1}{y}$

(b) $f(x, y) = e^y(x^2 - y^2)$

Solution:

Question 2

Find the absolute maximum and minimum values of the function

$$f(x, y) = x + y - xy$$

on the closed triangular region with vertices $(0, 0)$, $(0, 2)$, and $(4, 0)$.

Solution:

Question 3

Find the absolute maximum and minimum values of the function

$$f(x, y) = xy^2$$

on the region $x^2 + y^2 \leq 3$, $x \geq 0$, $y \geq 0$.

Solution:

Question 4

Find the maximum and minimum values of the function $f(x, y) = x + 4y$ subject to the constraint

$$\sqrt{x} + \sqrt{y} = 3.$$

Question 5

Consider the function $f(x, y) = e^{xy}$ and the constraint $x^3 + y^3 = 16$.

- (a) Use Lagrange multipliers to find the coordinates (x, y) of any points on the constraint where the function f could attain a maximum or minimum.
- (b) For each point you found in part (a), is the point a maximum, a minimum, both or neither? Explain

your answer carefully. What are the minimum and maximum values of f on the constraint? Please explain your answers carefully.

- (c) The Extreme Value Theorem, which we covered last week, guarantees that under the right circumstances, we are guaranteed to find absolute minima and maxima for a function f on a certain constraint. Please explain why parts (a) and (b) don't violate the Extreme Value Theorem.

Solution:

Question 6

Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x^2 y^2 z^2$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

Solution:

Question 7

Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x^4 + y^4 + z^4 = 1$.

Solution:

Question 8

Find the absolute minimum and maximum values of the function $f(x, y) = x^2 - (y - 2)^2$ on the region

$$D = \{x^2 + y^2 \leq 9 \text{ and } y \geq 0\},$$

and the points at which those extrema occur.

Solution: