

Math 120

PSet 2

Sep 12 2024

Contents

Chapter 1

Page 2

1.1	12.1 Notes (Three Dimensional Coordinate Systems)	2
1.2	12.2 Notes (Vectors)	2
1.3	12.3 Dot Product	3
1.4	12.4 Cross Product	3
1.5	12.5 Equations of Lines and Planes	4

Chapter 1

1.1 12.1 Notes (Three Dimensional Coordinate Systems)

Definition 1.1.1: Distance Formula

Defintion:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



Definition 1.1.2: Equation of a sphere

Defintion: An equation of a sphere with center $C(h, k, l)$, and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2$$



1.2 12.2 Notes (Vectors)

Note:-

Properties of vectors: If \mathbf{a} , \mathbf{b} , and \mathbf{c} are vectors in V_n and c and d are scalars than

- $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
- $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
- $\mathbf{a} + \mathbf{0} = \mathbf{a}$
- $\mathbf{a} + \mathbf{a} + -\mathbf{a} = \mathbf{0}$
- $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
- $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
- $(cd)\mathbf{a} = c(d\mathbf{a})$
- $l\mathbf{a} = \mathbf{a}$



1.3 12.3 Dot Product

Definition 1.3.1: Dot Product

Definition:

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

$$\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Two vectors \mathbf{a} and \mathbf{b} are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$



Definition 1.3.2: Projections

Definition:

Scalar projection of \mathbf{b} onto \mathbf{a} : $\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

Vector projection of \mathbf{b} onto \mathbf{a} : $\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \right) \mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$



1.4 12.4 Cross Product

Definition 1.4.1: Cross Product

Definition

$$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$$

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = |\mathbf{a}||\mathbf{b}| \sin(\theta)$$

$$\begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix} i - \begin{bmatrix} a_1 & a_3 \\ b_1 & b_3 \end{bmatrix} j + \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} k \\ = (a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)k$$

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$$



Definition 1.4.2: Triple Product

Triple Product:

$$V = Ah = |\mathbf{b} \times \mathbf{c}||\mathbf{a}| \cos \theta = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$V = Ah = |\mathbf{b} \times \mathbf{c}||\mathbf{a}| \cos \theta = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$$



1.5 12.5 Equations of Lines and Planes

Definition 1.5.1: Line

Line:

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$$



Definition 1.5.2: Equation of a plane

Plane:

$$\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$$

$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$ax + by + cz + d = 0$$



Definition 1.5.3: Distance of a plane

Distance of a Plane:

$$\begin{aligned} D &= |\text{comp}_{\mathbf{n}} \mathbf{b}| = \frac{|\mathbf{n} \cdot \mathbf{b}|}{|\mathbf{n}|} \\ &= \frac{|a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)|}{\sqrt{a^2 + b^2 + c^2}} \\ &= \frac{|(ax_1 + by_1 + cz_1) - (ax_0 + by_0 + cz_0)|}{\sqrt{a^2 + b^2 + c^2}} \end{aligned}$$



Definition 1.5.4: Length of curve

Length:

$$s(t) = \int_a^t |\mathbf{r}'(u)| du = \int_a^t \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2 + \left(\frac{dz}{du}\right)^2} du$$

