Math 120

PSet 2

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Contents

Chapter 1

1.1 PSet 2

Question 1

Consider the line L_1 given by $x + 2y = 7$ and the line L_2 given by $5x - y = 2$.

1. There are two unit vectors that are parallel to L_1. What are they?
2. There are two unit vectors that are perpendicular to L_1. What are they?
3. Find the acute angle between the lines L_1 and L_2. First find an exact expression and then approximate to the nearest degree.

Question 2

Find all values of x such that the angle between the vectors $\langle 1, -1, 0\rangle$ and $\langle 2, x, 1\rangle$ is $\frac{\pi}{3}$.

Question 3

Find the scalar and vector projections of $\vec{b} = \hat{\imath} + \hat{\jmath}$ onto $\vec{a} = -\hat{\imath} + 3\hat{\jmath}$, and illustrate your answers with a sketch.

Question 4

Find two vectors of length 2 that are orthogonal to both $\vec{v} = \langle 2, 4, 4\rangle$ and $\vec{w} = \langle 1, -1, -3\rangle$.

Question 5

Let $\vec{a} = \langle 3, 1, 0\rangle$. Find all vectors $\vec{b} = \langle b_1, b_2, b_3\rangle$ such that $\vec{a} \times \vec{b}$ is parallel to the z-axis and pointing in the positive z direction. Illustrate with a sketch, in which all vectors are drawn as position vectors, i.e., with the tail at the origin.

Question 6

Consider the four points in $\mathbb{R}^3$, $K(1, 2, 3)$, $L(1, 3, 6)$, $M(3, 8, 6)$, and $N(3, 7, 3)$.

1. Show that the vectors $\overrightarrow{KL}$, $\overrightarrow{KM}$, and $\overrightarrow{KN}$ are coplanar. Explain why this means that K, L, M, and N all lie in the same plane.
2. From part (a), we know that K, L, M, and N are the vertices of a quadrilateral. Explain how you can tell that this quadrilateral is actually a parallelogram.
3. Find the area of the parallelogram with vertices K, L, M, and N.

Question 7

Find the vector equation and parametric equations for the line through the point $(1, 2, -2)$ parallel to the line $x = t - 2$, $y = -2t + 1$, $z = 3$.

Question 8

Consider the lines $L_1 : x = t + 3$, $y = 2t - 1$, $z = -t$, and $L_2 : x = t - 1$, $y = t - 4$, $z = -t + 4$. Determine whether the L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

Question 9

Consider the planes $x + y + 2z = 4$ and $2x - y - 2z = 1$.

1. Find a vector equation for the line of intersection of the planes.
2. Find the angle between the planes. First find an exact expression and then approximate to the nearest degree.

Question 10

Let P be the plane $x + y + 2z = 1$ and let A be the point $(1, 1, 1)$.

(a) Find an equation of the plane through point A parallel to plane P.

(b) Find a vector equation for the line through the point A which is perpendicular to the plane P. Call this line L.

(c) Find the point of intersection of the line L (from part (b)) and the plane P.

(d) Find the point on the plane P closest to the point A, and then find the shortest distance from the point A to the plane P.