

Math 120

PSet 12

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# Contents

| Chapter | Homework | Page |
|---------|----------|------|
| 0.1     | PSet 12  | 2    |

# Homework

## 0.1 PSet 12

### Question 1

Evaluate the iterated integral

$$\int_0^{\pi/2} \int_0^x \int_0^{x^2} x^2 \sin y \, dy \, dz \, dx.$$

### Question 2

Evaluate the triple integral

$$\iiint_E xy \, dV,$$

where  $E$  is bounded by the parabolic cylinders  $y = x^2$ ,  $x = y^2$ , and the planes  $z = 0$  and  $z = x + y$ .

### Question 3

Write five other iterated integrals that are equal to the iterated integral

$$\int_0^2 \int_0^{x^2} \int_0^y f(x, y, z) dz dy dx.$$

#### Question 4

The average value of a function  $f(x, y, z)$  over a solid region  $E$  is defined to be

$$f_{\text{ave}} = \frac{1}{V(E)} \iiint_E f(x, y, z) dV,$$

where  $V(E)$  is the volume of  $E$ . Find the average value of the function  $f(x, y, z) = x^2z + y^2z$  over the region enclosed by the paraboloid  $z = 1 - x^2 - y^2$  and the plane  $z = 0$ .

### Question 5

Describe and sketch the solid whose volume is given by the integral. You do not need to evaluate the integral.

$$\int_0^{\pi/2} \int_0^2 \int_0^{\sqrt{9-r^2}} r \, dz \, dr \, d\theta$$
$$\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^2 \rho^2 \sin \phi \, d\phi \, d\theta \, d\rho.$$

### Question 6

Evaluate  $\iiint_E x \, dV$ , where  $E$  is enclosed by the planes  $z = 0$  and  $z = x + y + 5$  and by the cylinders  $x^2 + y^2 = 4$  and  $x^2 + y^2 = 9$ .



Question 7

Find the average height above the  $xy$ -plane of the points in the solid hemisphere  $x^2 + y^2 + z^2 \leq a^2$ ,  $z \geq 0$ .

### Question 8

1. Evaluate the integral by changing to cylindrical coordinates:

$$\int_{-1}^1 \int_{|x|}^{\sqrt{2-x^2}} \int_0^{\sqrt{x^2+y^2}} \sqrt{x^2+y^2} \, dz \, dy \, dx.$$

2. Evaluate the integral by changing to spherical coordinates:

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} \int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} (x^2 + y^2 z^2) \, dz \, dx \, dy.$$

### Question 9

Verify that the Divergence Theorem is true for the vector field  $\vec{F} = (x^2, -y^2, -z)$  and the solid  $E$  described by  $x^2 + z^2 \leq 1$ ,  $-2 \leq y \leq 2$ .