

# Math 120

## PSet 7

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# Chapter 1

## 1.1 PSet 7

### Question 1

Calculate the given iterated integrals.

1.  $\int_0^1 \int_0^1 x\sqrt{1+4y} \, dy \, dx$

2.  $\int_0^1 \int_1^2 \frac{xe^x}{y} \, dy \, dx$

**Solution:**

1)

$$\int_0^1 \int_0^1 x\sqrt{1+4y} \, dy \, dx$$

$$\int_0^1 x\sqrt{1+4y} \, dy$$

$$1+4y = t \quad r = dt$$

$$x \int_0^1 \frac{1}{4} \sqrt{t} \, dt$$

$$\frac{1}{4}x \int_0^1 \sqrt{t} \, dt$$

$$\frac{1}{4}x \cdot \frac{2t\sqrt{t}}{3} \Big|_0^1$$

$$\frac{1}{4}x \cdot \frac{2(1+4y)\sqrt{1+4y}}{3} \Big|_0^1$$

$$\frac{x\sqrt{1+4y}(1+4y)}{6} \Big|_0^1$$

$$\frac{x\sqrt{1+4}(1+4)}{6} - \frac{x\sqrt{1}1}{6}$$

$$\frac{5x\sqrt{5}}{6} - \frac{x}{6}$$

$$\int_0^1 \frac{5x\sqrt{5}}{6} - \frac{x}{6} \, dx$$

$$\frac{1}{6} \int_0^1 5\sqrt{5}x - x \, dx$$

$$\begin{aligned} & \frac{1}{6} \left( \int_0^1 5\sqrt{5}x dx - \int_0^1 x dx \right) \\ & \int_0^1 5\sqrt{5}x dx \Rightarrow \frac{5\sqrt{5}x^2}{2} \Big|_0^1 \\ & \frac{5\sqrt{5}(1)^2}{2} - 0 = \frac{5\sqrt{5}}{2} \\ & \int_0^1 x dx \Rightarrow \frac{x^2}{2} \Big|_0^1 \\ & \frac{1}{2} - 0 = \frac{1}{2} \\ & \frac{1}{6} \left( \frac{5\sqrt{5}}{2} - \frac{1}{2} \right) = \frac{5\sqrt{5} - 1}{12} \end{aligned}$$

2)

$$\begin{aligned} & \int_0^1 \int_1^2 \frac{xe^x}{y} dy dx \\ & xe^x \int_1^2 \frac{1}{y} dy \\ & xe^x \ln(y) \Big|_1^2 \Rightarrow xe^x \ln(2) - xe^x \ln(1) = xe^x \ln(2) \\ & \ln(2) \int_0^1 xe^x dx \\ & \ln(2) (xe^x - e^x) \Big|_0^1 \\ & (\ln(2)e - \ln(2)e) - (\ln(2)(0) - \ln(2)e^0) = 0 - (-\ln(2)(1)) = \ln(2) \end{aligned}$$

### Question 2

(a) Sketch the solid whose volume is given by the iterated integral

$$\int_0^1 \int_0^2 e^{-x^2-y^2} dy dx.$$

(b) Explain why

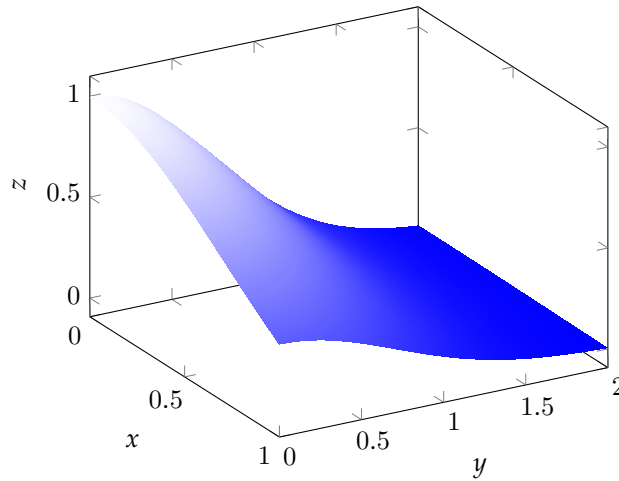
$$\int_0^1 \int_0^2 e^{-x^2-y^2} dy dx = \int_0^1 e^{-x^2} dx \cdot \int_0^2 e^{-y^2} dy.$$

(c) Use Desmos to compute

$$\int_0^1 \int_0^2 e^{-x^2-y^2} dy dx.$$

(Desmos will give a numerical approximation, but this is fine. In fact, there is no way to compute the antiderivatives necessary to get an exact answer.)

**Solution:**



### Question 3

- (a) Find the average value of the function  $f(x, y) = \sin x \cos y$  on the rectangle  $R = [0, \pi] \times [-\pi/2, \pi/2]$ .
- (b) Use symmetry to find the average value of  $f(x, y) = \frac{4 \sin y}{e^{x^2}} - \frac{\cos x}{\ln y} + 3$  on the region  $R = [2\pi, 4\pi] \times [2\pi, 6\pi]$ . Please explain your answer carefully.

**Solution:** a)

$$f(x, y) = \sin x \cos y$$

$$R = [0, \pi] \times \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$f_{avg} = \frac{1}{A(R)} \iint_R f(x, y) dA$$

$$A(R) = (\pi - 0) \times \left(\frac{\pi}{2} - -\frac{\pi}{2}\right) = \pi^2$$

$$\frac{1}{\pi^2} \int_0^\pi \int_{-\pi/2}^{\pi/2} \sin x \cos y dy dx$$

$$\sin x \int_{-\pi/2}^{\pi/2} \cos y dy$$

$$(\sin x) \sin y \Big|_{-\pi/2}^{\pi/2}$$

$$(\sin x) \sin \left(\frac{\pi}{2}\right) - (\sin x) \sin \left(-\frac{\pi}{2}\right) = 2 \sin x$$

$$\int_0^\pi 2 \sin x dx$$

$$-2 \cos x \Big|_0^\pi$$

$$- \cos \pi - (-2) \cos(0) = 4$$

$$\frac{1}{\pi^2} \cdot 4 = \frac{4}{\pi^2}$$

### Question 4

In each part, draw the region  $D$ , and evaluate the integral.

1.  $\iint_D \frac{y}{x^5+1} dA$ , where  $D$  is the region  $D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x^2\}$ .

2.  $\iint_D x^3 dA$ , where  $D = \{(x, y) \mid 1 \leq x \leq e, 0 \leq y \leq \ln x\}$ .

#### Question 5

Draw the region  $D$ . Set up the iterated integrals for both orders of integration. Then evaluate the double integral using the easier order and explain why it's easier.

$$\iint_D x^2 e^{-xy} dA \quad \text{where } D \text{ is bounded by } y = x, x = 4, \text{ and } y = 0.$$

#### Question 6

- (a) Find the volume of the solid in the first octant enclosed by the parabolic cylinder  $y = 1 - x^2$  and the planes  $z = 2 - y$  and  $z = y$ .
- (b) Sketch the solid whose volume is given by the iterated integral

$$\int_0^1 \int_0^{1-x} (2 - y^2) dy dx.$$

#### Question 7

Sketch the region of integration and change the order of integration.

1.  $\int_0^1 \int_{4x}^4 f(x, y) dy dx$
2.  $\int_0^3 \int_{\sqrt{9-y}}^3 f(x, y) dx dy$
3.  $\int_0^4 \int_0^{\ln 2x} f(x, y) dy dx$

#### Question 8

Evaluate the integral

$$\int_0^1 \int_x^1 \frac{e^x}{y} dy dx$$

by reversing the order of integration.

#### Question 9

Evaluate the given integral by converting to polar coordinates. Be sure to draw the region of integration in each part.

1.  $\iint_R (x + y) dA$ , where  $R$  is the region that lies to the left of the  $y$ -axis between the circles  $x^2 + y^2 = 1$  and  $x^2 + y^2 = 4$ .
2.  $\iint_R ye^x dA$ , where  $R$  is the region in the first quadrant enclosed by the circle  $x^2 + y^2 = 25$ .

#### Question 10

Use polar coordinates to find the volume of the given solid.

- (a) Inside the sphere  $x^2 + y^2 + z^2 = 4$  and outside the cylinder  $x^2 + y^2 = 1$ .

(b) Bounded by the paraboloids  $z = 3x^2 + 3y^2$  and  $z = 4 - x^2 - y^2$ .

#### Question 11

Evaluate the iterated integral

$$\int_0^b \int_{-\sqrt{b^2-y^2}}^0 x^2 y \, dx \, dy$$

by converting to polar coordinates.

#### Question 12

Let  $D$  be the disk with center at the origin and radius  $a$ .

(a) Use your intuition: what do you expect is the average distance from points on the disk to the origin?

- less than  $a/2$
- $a/2$
- between  $a/2$  and  $a$
- more than  $a$

Give an intuitive explanation of your answer.

(b) What is the average distance from points in the disk to the origin?