

Math 120 QR

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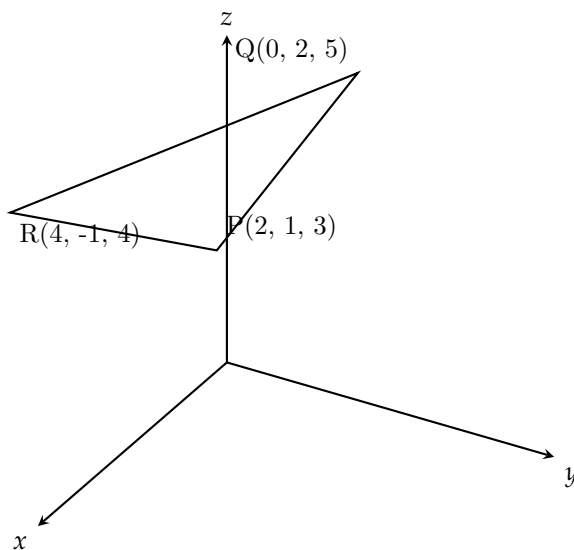
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Chapter 1

1.1 PSet 1

Question 1

Find the lengths of the sides of the triangle with vertices P (2, 1, 3), Q(0, 2, 5) and R(4, -1, 4). Is the triangle an acute triangle (all sides less than 90°), a right triangle, or an obtuse triangle (one angle greater than 90°)



Solution: Image:

$$\overrightarrow{PQ} = \langle 0 - 2, 2 - 1, 5 - 3 \rangle = \langle -2, 1, 2 \rangle$$

$$\overrightarrow{QR} = \langle 4 - 0, -1 - 2, 4 - 5 \rangle = \langle 4, -3, -1 \rangle$$

$$\overrightarrow{RP} = \langle 2 - 4, 1 - (-1), 3 - 4 \rangle = \langle -2, -2, 1 \rangle$$

$$|\overrightarrow{PQ}| = \sqrt{(-2)^2 + 1^2 + (-2)^2} = \sqrt{9} = 3$$

$$|\overrightarrow{QR}| = \sqrt{(4)^2 + (-3)^2 + (-1)^2} = \sqrt{26}$$

$$|\overrightarrow{RP}| = \sqrt{(-2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$$

angles:

$$|\overrightarrow{PQ}|^2 = 26 = 9 + 9 - 2(3)(3) \cos \theta$$

$$8 = -18 \cos \theta$$

$$\frac{8}{18} = -\cos(\theta)$$

$$\theta = \arccos\left(-\frac{8}{18}\right) \approx 116$$

Triangle is obtuse

Question 2

Find the equation of the sphere for which the line segment between the points $A(1, 1, 1)$ and $B(3, -7, 3)$ is a diameter. (This means that A and B are antipodal points on the sphere.)

Solution:

Center of sphere:

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} = \frac{1 + 3}{2}, \frac{1 + (-7)}{2}, \frac{1 + 3}{2} = (2, -3, 2)$$

Diameter:

$$\sqrt{(3 - 1)^2 + (-7 - 1)^2 + (3 - 1)^2} = 6\sqrt{2}$$

Radius:

$$r = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

Equation of sphere:

$$(x - 2)^2 + (y + 3)^2 + (z - 2)^2 = (3\sqrt{2})^2$$

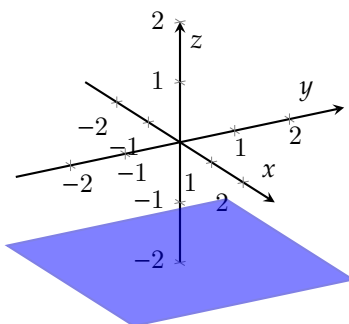
$$(x - 2)^2 + (y + 3)^2 + (z - 2)^2 = 18$$

Question 3

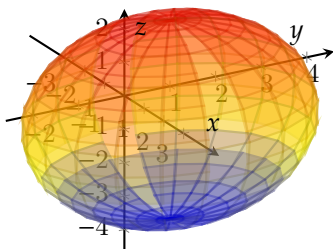
- a) Describe in words and with a sketch the regions in \mathbb{R}^3 represented by
- The equation $z = -2$
 - The inequality $x^2 + (y - 1)^2 + (z + 1)^2 \leq 9$
- b) In your sketch, shade in the intersection of the two regions you drew in part (a), i.e., the set of all points (x, y, z) in \mathbb{R}^3 satisfying both $z = -2$ and $x^2 + (y - 1)^2 + (z + 1)^2 \leq 9$

Solution:

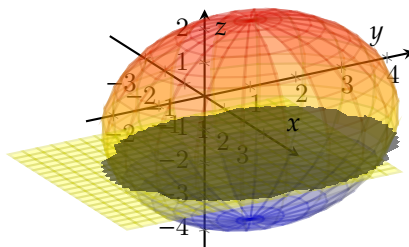
- a)
- Represents a plane in \mathbb{R}^3 that is parallel to the xy -plane and lies at a height of -2 units below the xy -plane. This plane contains all points where the z -coordinate is -2, regardless of the x and y values.



- A solid sphere in \mathbb{R}^3 with the center at point $(0, 1, -1)$ and a radius of 3. This includes all the points inside and on the surface of the sphere.



b)



Question 4

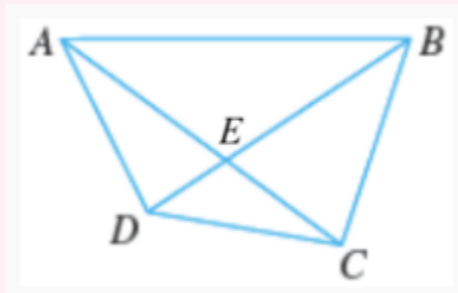
Describe in words the region(s) \mathbb{R}^3 represented by the inequality $x^2 \leq 9$

Solution: It is the set of all points that lie within the planes $x = -3$ and $x = 3$. It is a 3-dimensional slab that extends infinitely in the y and z -directions, but is bounded by $x = -3$ and $x = 3$ along the x -axis.

Question 5

Let A, B, C, D , and E be the points in the diagram below. Write each combination of vectors as a single vector.

1. $\overrightarrow{AE} + \overrightarrow{ED}$
2. $\overrightarrow{AC} - \overrightarrow{AB}$
3. $\overrightarrow{AC} + \overrightarrow{EA}$
4. $\overrightarrow{AC} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA}$



Solution:

1. $\overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}$
2. $\overrightarrow{AC} - \overrightarrow{AB} = (\overrightarrow{AC} + \overrightarrow{BC}) - \overrightarrow{AB} = \overrightarrow{BC}$
3. $\overrightarrow{AC} + \overrightarrow{EA} = \overrightarrow{AC} - \overrightarrow{AE} = \overrightarrow{EC}$
4. $\overrightarrow{AC} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA} :$

$$\overrightarrow{ED} + \overrightarrow{DC} = \overrightarrow{EC}$$

$$\overrightarrow{CB} + \overrightarrow{BA} = \overrightarrow{CA}$$

$$\overrightarrow{AC} + \overrightarrow{EC} + \overrightarrow{CA} = \overrightarrow{EA}$$

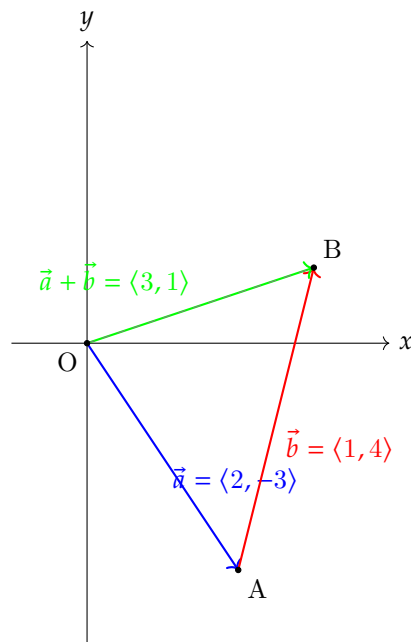
Question 6

Find the sum of the vectors $\vec{a} = \langle 2, -3 \rangle$ and $\vec{b} = \langle 1, 4 \rangle$ and illustrate geometrically

Solution:

$$\vec{a} + \vec{b} = \langle 2, -3 \rangle + \langle 1, 4 \rangle$$

$$\vec{a} + \vec{b} = \langle 2 + 1, -3 + 4 \rangle = \langle 3, 1 \rangle$$



Question 7

Find the vector that has the same direction as $\hat{i} + 3\hat{j} - \hat{k}$ but has length 6

Solution:

$$\hat{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{11}}\hat{i} + \frac{3}{\sqrt{11}}\hat{j} - \frac{1}{\sqrt{11}}\hat{k}$$

$$\mathbf{u} \times \hat{u} = 6 \times \left(\frac{1}{\sqrt{11}}\hat{i} + \frac{3}{\sqrt{11}}\hat{j} - \frac{1}{\sqrt{11}}\hat{k} \right)$$

$$\mathbf{u} = \frac{6}{\sqrt{11}}\hat{i} + \frac{18}{\sqrt{11}}\hat{j} - \frac{6}{\sqrt{11}}\hat{k}$$

Question 8

complete the Ximera assignment on vector projections linked from the Problem Set 1 assignment in the Canvas umbrella site. Write “I have completed the Ximera assignment” for Problem 8 on the file you submit for this problem set.

Solution: “I have completed the Ximera assignment”