Math 120 Cheat Sheet

3D Coordinate Systems

Distance: $|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
Sphere: $(x-h)^2+(y-k)^2+(z-l)^2=r^2$

Vectors

Properties: $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$
$c(\mathbf{a}+\mathbf{b})=c\mathbf{a}+c\mathbf{b}$

Dot Product

Formula: $\mathbf{a}\cdot\mathbf{b}=a_1b_1+a_2b_2+a_3b_3$
Angle: $\cos(\theta)=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$
Orthogonality: $\mathbf{a}\cdot\mathbf{b}=0$
Projections:

$$\text{comp}_{\mathbf{a}}\mathbf{b}=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$$

$$\text{proj}_{\mathbf{a}}\mathbf{b}=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$$

Cross Product

Formula:

$$\mathbf{a}\times\mathbf{b}=\langle a_2b_3-a_3b_2, a_3b_1-a_1b_3, a_1b_2-a_2b_1\rangle$$

Magnitude: $|\mathbf{a}\times\mathbf{b}|=|\mathbf{a}||\mathbf{b}|\sin\theta$
Triple Product: $V=|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$

Lines and Planes

Line: $\mathbf{r}=\mathbf{r}_0+t\mathbf{v}$
Plane: $\mathbf{n}\cdot\mathbf{r}=\mathbf{n}\cdot\mathbf{r_0}$
$ax+by+cz+d=0$
Distance: '

$$D=\frac{|a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

Curve Length: $s(t)=\int_a^t\sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2+\left(\frac{dz}{du}\right)^2}\,du$

Equation for Planes containing three points

$$\vec{PQ}\times\vec{PR}$$

Normal Vector of a Plane

$$ax+by+cz+d=0 \quad \text{with normal vector} \quad \vec{n}=\langle a,b,c\rangle$$

Vector Value Functions

Form:

$$\mathbf{r}(t)=\langle f(t),g(t),h(t)\rangle=f(t)\mathbf{i}+g(t)\mathbf{j}+h(t)\mathbf{k}$$

Limit:

$$\lim_{t\to a}\mathbf{r}(t)=\left\langle\lim_{t\to a}f(t),\lim_{t\to a}g(t),\lim_{t\to a}h(t)\right\rangle$$

Derivative: $\mathbf{r}'(t)=\langle f'(t),g'(t),h'(t)\rangle$
Unit Tangent Vector $\mathbf{T}(t)=\frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$
Orthogonality: If $|\mathbf{r}(t)|=c$, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$.
Definite Integral: $\int_a^b\mathbf{r}(t)\,dt=\mathbf{R}(t)\Big|_a^b=\mathbf{R}(b)-\mathbf{R}(a)$
Length: $L=\int_a^b|\mathbf{r}'(t)|\,dt$

Curvature

Form: $s(t)=\int_a^t|\mathbf{r}'(u)|du=\int_a^t\sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2+\left(\frac{dz}{du}\right)^2}du$

Motion in space

Form: $\mathbf{v}(t)=\lim_{h\to0}\frac{\mathbf{r}(t+h)-\mathbf{r}(t)}{h}=\mathbf{r}'(t)$

Partial Derivatives

Form: $f_x(a,b)=g'(a) \quad \text{where} \quad g(x)=f(x,b)$
Defintion:

$$f_x(x,y)=\lim_{h\to0}\frac{f(x+h,y)-f(x,y)}{h}$$

$$f_y(x,y)=\lim_{h\to0}\frac{f(x,y+h)-f(x,y)}{h}$$

Notation:

$$f_x(x,y)=f_x=\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}f(x,y)=\frac{\partial z}{\partial x}=f_1=D_1f=D_xf$$

$$f_y(x,y)=f_y=\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}f(x,y)=\frac{\partial z}{\partial y}=f_2=D_2f=D_yf$$

Rules
To find f_x, regard y as a constant and differentiate $f(x,y)$ with respect to x.
To find f_y, regard x as a constant and differentiate $f(x,y)$ with respect to y.
Higher Derivatives:

$$(f_x)_x=f_{xx}=f_{11}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial^2 f}{\partial x^2}=\frac{\partial^2 z}{\partial x^2}$$

$$(f_x)_y=f_{xy}=f_{12}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial^2 f}{\partial y\partial x}=\frac{\partial^2 z}{\partial y\partial x}$$

$$(f_y)_x=f_{yx}=f_{21}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial^2 z}{\partial x\partial y}$$

$$(f_y)_y=f_{yy}=f_{22}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial^2 f}{\partial y^2}=\frac{\partial^2 z}{\partial y^2}$$

Clairaut's Theorem $f_{xy}(a,b)=f_{yx}(a,b)$

Chain Rule

Case 1:

$$\frac{dz}{dt}=\frac{\partial f}{\partial x}\frac{dx}{dt}+\frac{\partial f}{\partial y}\frac{dy}{dt}$$

Case 2:

$$\frac{\partial z}{\partial s}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

General Form:

$$\frac{\partial u}{\partial t_i}=\frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i}+\frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial t_i}+\cdots+\frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i}$$

Implicit Differentiation

Form:

$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{F_x}{F_y}$$

Three variables:

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = -\frac{F_x}{F_z} \quad \frac{\partial z}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = -\frac{F_y}{F_z}$$

Directional Derivative and Gradient Vector

Directional Derivative:

$$D_{\mathbf{u}} f(x, y) = f_x(x, y)a + f_y(x, y)b$$

Gradient For 2 variables

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y)\rangle = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j}$$

Gradient for 3 Variables

$$\nabla f = \langle f_x, f_y, f_z\rangle = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

Properties of Gradient Vecotr
$\nabla f(\mathbf{x})$ is perpendicular to the level curve or level surface of f through $\mathbf{x}$.

Tange Planes and Surfaces

Form:

$$F_x(x_0,y_0,z_0)(x-x_0)+F_y(x_0,y_0,z_0)(y-y_0)+F_z(x_0,y_0,z_0)(z-z_0)=0$$