# Math 120 QR

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# Contents

# Chapter 1

## 1.1 Day 1 notes

**Definition 1.1.1: Distance Formula**

Defintion:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

**Definition 1.1.2: Equation of a sphere**

Defintion: An equation of a sphere with center $C(h, k, l)$, and radius $r$ is

$$(x - h)^2 + (y - k)^2 + (z - l)^2$$

In particular, if the center is the origin $O$, than an equation of the sphere is

$$x^2 + y^2 + z^2$$

## 1.2 Day 2 Notes

**Definition 1.2.1: The lenght/magnitude of a vecotr**

In 2D, $\vec{v} =< a, b >$: $|\vec{v}| = \sqrt{a^2 + b^2}$
In 3D, $\vec{v}$, $\vec{v} =< a, b, c >$: $|\vec{v}| = \sqrt{a^2 + b^2 + c^2}$
A unit vector is a vector of length 1

**Question 1**

If $\vec{v}$ is a a vector and $a$ is a scalar, then what is $|a\vec{v}|$

***Solution:***

$$|a\vec{v}| = |a||\vec{v}|$$

**Definition 1.2.2: Vectors in $R^3$**

The standard basis vectors in $\mathbf{R}^3$ are

$$i =< 1, 0, 0 >$$

$$j =< 0, 1, 0 >$$

$$k =< 0, 0, 1 >$$

**Question 2**

What is special about i,j,k?

***Solution:***

- Cannot make any of them as a linear combibnation of the other three.
- Any vector $\vec{v}\epsilon\mathbf{R}^3$ can be written uniquily as a linear combibnation of i,j,k

**Example 1.2.1:**

$\vec{v} + \vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
$\vec{v} + \vec{u} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \end{pmatrix}$

**Definition 1.2.3: Dot product**

A dot product between $v$ and $w$ is:
In 2D: $v \cdot w = v_1 w_1 + v_2 w_2$
In 3D: $v \cdot 2 = v_1 w_1 + v_2 w_2 + v_3 w_3$

Geometric defintion:
$|u \cdot w| = |u||w| \cos(\theta)$ where $\theta$ is the angle between $v$ and $w$

**Example 1.2.2: Why the 2 defintions are the same**

$$v_1 w_1 + v_2 w_2 = |v||w| \cos(\theta) = p|v|$$

$$p = |w| \cos(\theta)$$

$$v_1 = |v| \cos(\theta)$$

$$v_2 = |v| \sin(\theta)$$

$$w_1 = |w| \cos(\theta)$$

$$w_2 = |w| \sin(\theta)$$

$$\text{LHS} = |v||w| \left(\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)\right)$$

$$\text{LHS} = |v||w| \sin(\alpha + \beta)$$

$$\text{LHS} = |v||w| \cos(\theta) = \text{RHS}$$

**Example 1.2.3: what does the def mean**

How much effect of $\vec{w}$ act along $\vec{v}$
Work: $w = \vec{F} \cdot \vec{S}$

**Question 3**

Find a relation between $|v|$ and $v \cdot v$

***Solution:*** $|v|^2 = v \cdot v$

**Question 4**

$v$, $w$ of fixed lenghts when is $v \cdot w$ largest?

***Solution:*** $v$ paralell: $\theta = 0$
$\cos(\theta) = 1$

**Example 1.2.4: Projections**

Given $\vec{v} \neq \vec{o}$
The project of $w$ on $v$ is proj $w = \left(\frac{\vec{w} \cdot \vec{v}}{|v|}\right) \frac{\vec{v}}{|v|}$
Directon of $v$ is $\frac{\vec{v}}{|v|}$
Dor project of $w$ with direction is:

$$\vec{w} \cdot \frac{\vec{v}}{|v|}$$

**Question 5**

TRUE or False:
$u, v, w$: vectors $(u \cdot v)w = u(v \cdot w)$

***Solution:*** false

**Question 6**

TRUE or FALSE:
$v - w| = |v| - |w|$ if $v \parallel w$

***Solution:*** false

**Question 7**

When is this ideal square sum happening?

$$|v + w|^2 = |v|^2 + |w|^2$$

***Solution:*** when $v \perp w$

**Definition 1.2.4: Cross Product**

The cross product of two vectors $v$ and $w$, $v * w$ is a vector u defined by $u \perp v$ and $u \perp w$.
Direction of $u$ is given by the right hand rule
Magnitude: $|u|$ = Area of the parallelogram spanned by $v$ and $w$.

## 1.3 Day 2 Reading notes

**Definition 1.3.1: Vector Addition**

If **u** and **v** are vectors positioned so the initial point of **v** is at the terminal point of **u**, then the **sum u + v** is the vector from the initial point of **u** to the terminal point of **v**.

**Definition 1.3.2: Scalar Multiplication**

If $c$ is a scalar and **v** is a vector, then the **scalar multiple** $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of **v** and whose direction is the same as **v** if $c > 0$ and is opposite to **v** if $c = 0$ or $\mathbf{v} = 0$, then $c\mathbf{v} = 0$

**Example 1.3.1:**

Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector **a** with represenation $\overrightarrow{AB}$ is:

$$a = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

**Example 1.3.2:**

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then:

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$$

$$\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$
$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

Similarily, for three demensional vectors,

$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1 + b_1, a_2 + a_3 + b_3 \rangle$$

$$\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle = \langle a_1 - b_1, a_2 - a_3 - b_3 \rangle$$
$$c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$$

**Note:-**

Properties of vectors: If **a**, **b**, and **c** are vectors in $V_n$ and $c$ and $d$ are scalars than

- $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
- $a + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
- $\mathbf{a} + 0 = \mathbf{a}$
- $\mathbf{a} + \mathbf{a} + \mathbf{-a} = 0$

- $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
- $(c + d)a = c\mathbf{a} + d\mathbf{a}$
- $(cd)\mathbf{a} = c(d\mathbf{a})$
- $l\mathbf{a} = \mathbf{a}$