

Math 120

PSet 7

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Question 1

Calculate the given iterated integrals.

1. $\int_0^1 \int_0^1 x\sqrt{1+4y} \, dy \, dx$

2. $\int_0^1 \int_1^2 \frac{xe^x}{y} \, dy \, dx$

Solution:

1)

$$\int_0^1 \int_0^1 x\sqrt{1+4y} \, dy \, dx$$

$$\int_0^1 x\sqrt{1+4y} \, dy$$

$$1+4y = t \quad r = dt$$

$$x \int_0^1 \frac{1}{4} \sqrt{t} \, dt$$

$$\frac{1}{4}x \int_0^1 \sqrt{t} \, dt$$

$$\frac{1}{4}x \cdot \frac{2t\sqrt{t}}{3} \Big|_0^1$$

$$\frac{1}{4}x \cdot \frac{2(1+4y)\sqrt{1+4y}}{3} \Big|_0^1$$

$$\frac{x\sqrt{1+4y}(1+4y)}{6} \Big|_0^1$$

$$\frac{x\sqrt{1+4}(1+4)}{6} - \frac{x\sqrt{1}1}{6}$$

$$\frac{5x\sqrt{5}}{6} - \frac{x}{6}$$

$$\int_0^1 \frac{5x\sqrt{5}}{6} - \frac{x}{6} \, dx$$

$$\frac{1}{6} \int_0^1 5\sqrt{5}x - x \, dx$$

$$\begin{aligned} & \frac{1}{6} \left(\int_0^1 5\sqrt{5}x dx - \int_0^1 x dx \right) \\ & \int_0^1 5\sqrt{5}x dx \Rightarrow \frac{5\sqrt{5}x^2}{2} \Big|_0^1 \\ & \frac{5\sqrt{5}(1)^2}{2} - 0 = \frac{5\sqrt{5}}{2} \\ & \int_0^1 x dx \Rightarrow \frac{x^2}{2} \Big|_0^1 \\ & \frac{1}{2} - 0 = \frac{1}{2} \\ & \frac{1}{6} \left(\frac{5\sqrt{5}}{2} - \frac{1}{2} \right) = \frac{5\sqrt{5} - 1}{12} \end{aligned}$$

2)

$$\begin{aligned} & \int_0^1 \int_1^2 \frac{xe^x}{y} dy dx \\ & xe^x \int_1^2 \frac{1}{y} dy \\ & xe^x \ln(y) \Big|_1^2 \Rightarrow xe^x \ln(2) - xe^x \ln(1) = xe^x \ln(2) \\ & \ln(2) \int_0^1 xe^x dx \\ & \ln(2) (xe^x - e^x) \Big|_0^1 \\ & (\ln(2)e - \ln(2)e) - (\ln(2)(0) - \ln(2)e^0) = 0 - (-\ln(2)(1)) = \ln(2) \end{aligned}$$

Question 2

(a) Sketch the solid whose volume is given by the iterated integral

$$\int_0^1 \int_0^2 e^{-x^2-y^2} dy dx.$$

(b) Explain why

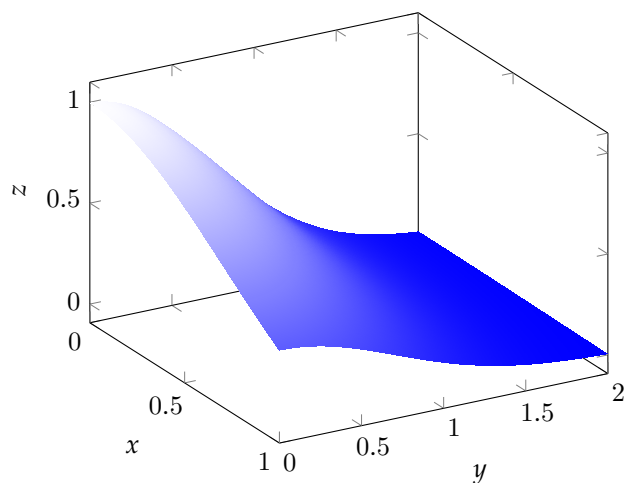
$$\int_0^1 \int_0^2 e^{-x^2-y^2} dy dx = \int_0^1 e^{-x^2} dx \cdot \int_0^2 e^{-y^2} dy.$$

(c) Use Desmos to compute

$$\int_0^1 \int_0^2 e^{-x^2-y^2} dy dx.$$

(Desmos will give a numerical approximation, but this is fine. In fact, there is no way to compute the antiderivatives necessary to get an exact answer.)

Solution:



Question 3

- Find the average value of the function $f(x, y) = \sin x \cos y$ on the rectangle $R = [0, \pi] \times [-\pi/2, \pi/2]$.
- Use symmetry to find the average value of $f(x, y) = \frac{4 \sin y}{e^{x^2}} - \frac{\cos x}{\ln y} + 3$ on the region $R = [2\pi, 4\pi] \times [2\pi, 6\pi]$. Please explain your answer carefully.

Question 4

In each part, draw the region D , and evaluate the integral.

- $\iint_D \frac{y}{x^5+1} dA$, where D is the region $D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x^2\}$.
- $\iint_D x^3 dA$, where $D = \{(x, y) \mid 1 \leq x \leq e, 0 \leq y \leq \ln x\}$.

Question 5

Draw the region D . Set up the iterated integrals for both orders of integration. Then evaluate the double integral using the easier order and explain why it's easier.

$$\iint_D x^2 e^{-xy} dA \quad \text{where } D \text{ is bounded by } y = x, x = 4, \text{ and } y = 0.$$

Question 6

- Find the volume of the solid in the first octant enclosed by the parabolic cylinder $y = 1 - x^2$ and the planes $z = 2 - y$ and $z = y$.
- Sketch the solid whose volume is given by the iterated integral

$$\int_0^1 \int_0^{1-x} (2 - y^2) dy dx.$$

Question 7

Sketch the region of integration and change the order of integration.

- $\int_0^1 \int_{4x}^4 f(x, y) dy dx$
- $\int_0^3 \int_{\sqrt{9-y}}^3 f(x, y) dx dy$

3. $\int_0^4 \int_0^{\ln 2x} f(x, y) dy dx$

Question 8

Evaluate the integral

$$\int_0^1 \int_x^1 \frac{e^x}{y} dy dx$$

by reversing the order of integration.

Question 9

Evaluate the given integral by converting to polar coordinates. Be sure to draw the region of integration in each part.

1. $\iint_R (x + y) dA$, where R is the region that lies to the left of the y -axis between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
2. $\iint_R ye^x dA$, where R is the region in the first quadrant enclosed by the circle $x^2 + y^2 = 25$.

Question 10

Use polar coordinates to find the volume of the given solid.

- (a) Inside the sphere $x^2 + y^2 + z^2 = 4$ and outside the cylinder $x^2 + y^2 = 1$.
- (b) Bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

Question 11

Evaluate the iterated integral

$$\int_0^b \int_{-\sqrt{b^2-y^2}}^0 x^2 y dx dy$$

by converting to polar coordinates.

Question 12

Let D be the disk with center at the origin and radius a .

- (a) Use your intuition: what do you expect is the average distance from points on the disk to the origin?
 - less than $a/2$
 - $a/2$
 - between $a/2$ and a
 - more than a

Give an intuitive explanation of your answer.

- (b) What is the average distance from points in the disk to the origin?