

Math 120

PSet 6

Oct 9 2024

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Question 1

Find all the (local) maximum and minimum values and saddle points of the function.

(a) $f(x, y) = xy + \frac{1}{x} + \frac{1}{y}$

(b) $f(x, y) = e^y(x^2 - y^2)$

Solution:

Question 2

Find the absolute maximum and minimum values of the function

$$f(x, y) = x + y - xy$$

on the closed triangular region with vertices $(0, 0)$, $(0, 2)$, and $(4, 0)$.

Solution:

Question 3

Find the absolute maximum and minimum values of the function

$$f(x, y) = xy^2$$

on the region $x^2 + y^2 \leq 3$, $x \geq 0$, $y \geq 0$.

Solution:

$$f_x = \frac{\partial}{\partial x} xy^2 = y^2$$

$$f_y = \frac{\partial}{\partial y} xy^2 = 2xy$$

$$f_x = y^2 = 0 \Rightarrow y = 0$$

$$f_y = 2xy = 0 \Rightarrow x = 0, y = 0$$

$$x^2 + y^2 \leq 3 \quad x \geq 0 \quad y \geq 0$$

$$x = \sqrt{3} \cos \theta$$

$$y = \sqrt{3} \sin \theta$$

$$f(\theta) = (\sqrt{3} \cos \theta) (\sqrt{3} \sin \theta)^2 = 3\sqrt{3} \cos \theta \sin^2 \theta$$

$$\frac{d}{d\theta} = 3\sqrt{3} \sin \theta \cos \theta (2 \cos \theta \sin \theta)$$

$$3\sqrt{3} \sin \theta \cos \theta (2 \cos \theta \sin \theta) = 0$$

$$\sin \theta = 0 \Rightarrow \theta = 0$$

$$\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

$$2 \cos \theta = \sin \theta \Rightarrow \tan \theta = 2 \Rightarrow \theta = \arctan(2)$$

$$f(0) = 3\sqrt{3} \cos(0) \sin^2(0) = 0$$

$$f\left(\frac{\pi}{2}\right) = 3\sqrt{3} \cos\left(\frac{\pi}{2}\right) \sin^2\left(\frac{\pi}{2}\right) = 0$$

$$f(\arctan(2)) = 3\sqrt{3} \left(\frac{1}{\sqrt{5}}\right) \left(\frac{2}{\sqrt{5}}\right)^2 = \frac{24\sqrt{3}}{25}$$

max of $f = \frac{24\sqrt{3}}{25}$ and min of 0.

Question 4

Find the maximum and minimum values of the function $f(x, y) = x + 4y$ subject to the constraint

$$\sqrt{x} + \sqrt{y} = 3.$$

Question 5

Consider the function $f(x, y) = e^{xy}$ and the constraint $x^3 + y^3 = 16$.

- Use Lagrange multipliers to find the coordinates (x, y) of any points on the constraint where the function f could attain a maximum or minimum.
- For each point you found in part (a), is the point a maximum, a minimum, both or neither? Explain your answer carefully. What are the minimum and maximum values of f on the constraint? Please explain your answers carefully.
- The Extreme Value Theorem, which we covered last week, guarantees that under the right circumstances, we are guaranteed to find absolute minima and maxima for a function f on a certain constraint. Please explain why parts (a) and (b) don't violate the Extreme Value Theorem.

Solution:

Question 6

Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x^2 y^2 z^2$ subject to the constraint $x^2 + y^2 + z^2 = 1$.

Solution:

Question 7

Use Lagrange multipliers to find the maximum and minimum values of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x^4 + y^4 + z^4 = 1$.

Solution:

Question 8

Find the absolute minimum and maximum values of the function $f(x, y) = x^2 - (y - 2)^2$ on the region

$$D = \{x^2 + y^2 \leq 9 \text{ and } y \geq 0\},$$

and the points at which those extrema occur.

Solution: