Math 120

PSet 12

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Contents

Homework

0.1 PSet 12

Question 1

Evaluate the iterated integral
$$\int_0^{\sqrt{\pi}} \int_0^{x} \int_0^{xz} x^2 \sin y \, dy \, dz \, dx.$$

Question 2

Evaluate the triple integral

$$\iiint_E xy\, dV,$$

where E is bounded by the parabolic cylinders $y = x^2$, $x = y^2$, and the planes $z = 0$ and $z = x + y$.

Question 3

Write five other iterated integrals that are equal to the iterated integral

$$\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)\,dz\,dy\,dx.$$

Question 4

The average value of a function $f(x, y, z)$ over a solid region E is defined to be

$$f_{\text{ave}} = \frac{1}{V(E)} \iiint_E f(x, y, z)\, dV,$$

where $V(E)$ is the volume of E. Find the average value of the function $f(x, y, z) = x^2 z + y^2 z$ over the region enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$.

Question 5

Describe and sketch the solid whose volume is given by the integral. You do not need to evaluate the integral.

$$\int_0^{\pi/2} \int_0^2 \int_0^{9-r^2} r\, dz\, dr\, d\theta$$

$$\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^2 \rho^2 \sin\phi\, d\phi\, d\theta\, d\rho.$$

Question 6

Evaluate $\iiint_E x \, dV$, where E is enclosed by the planes $z = 0$ and $z = x + y + 5$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

Question 7

Find the average height above the xy-plane of the points in the solid hemisphere $x^2 + y^2 + z^2 \leqslant a^2$, $z \geqslant 0$.

Question 8

1. Evaluate the integral by changing to cylindrical coordinates:

$$\int_{-1}^{1}\int_{|x|}^{\sqrt{2-x^2}}\int_{0}^{x^2+y^2}\sqrt{x^2+y^2}\,dz\,dy\,dx.$$

2. Evaluate the integral by changing to spherical coordinates:

$$\int_{-a}^{0}\int_{0}^{\sqrt{a^2-y^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}}(xz^2+yz^2)\,dz\,dx\,dy.$$

Question 9

Verify that the Divergence Theorem is true for the vector field $\vec{F} = (x^2, -y^2, -z)$ and the solid E described by $x^2 + z^2 \leqslant 1$, $-2 \leqslant y \leqslant 2$.