Math 120 QR

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Contents

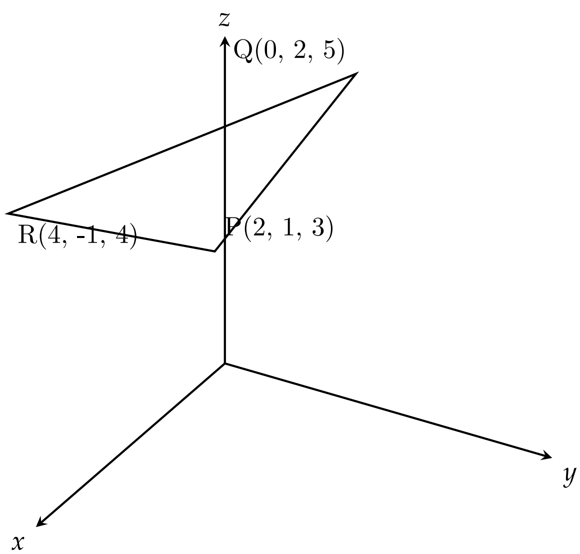
Chapter 1

1.1 PSet 1

Question 1

Find the lengths of the sides of the triangle with vertices P (2, 1, 3), Q(0, 2, 5) and R(4, -1, 4). Is the triangle an acute triangle (all sides less than 90°), a right triangle, or an obtuse triangle (one angle greater than 90°)

Solution: Image:



$$\overrightarrow{PQ} = \langle 0-2, 2-1, 5-3\rangle = \langle -2, 1, 2\rangle$$

$$\overrightarrow{QR} = \langle 4-0, -1-2, 4-5\rangle = \langle 4, -3, -1\rangle$$

$$\overrightarrow{RP} = \langle 2-4, 1--1, 3-4\rangle = \langle -2, -2, 1\rangle$$

$$|\overrightarrow{PQ}| = \sqrt{(-2)^2+1^2+(-2)^2} = \sqrt{9} = 3$$

$$|\overrightarrow{QR}| = \sqrt{(4)^2+(-3)^2+(1)^2} = \sqrt{26}$$

$$|\overrightarrow{RP}| = \sqrt{(-2)^2+(-2)^2+(1)^2} = \sqrt{9} = 3$$

angles:

$$|\overrightarrow{PQ}|^2 = 26 = 9+9-2(3)(3)\cos\theta$$

$$8 = -18\cos\theta$$
$$\frac{8}{18} = -\cos(\theta)$$
$$\theta = \arccos\left(-\frac{8}{18}\right) \approx 116$$

Trianlge is obtuse

Question 2

Find the equation of the sphere for which the line segment between the points $A(1,1,1)$ and $B(3,-7,3)$ is a diameter. (This means that that A and B are antipodal points on the sphere.)

Solution:
Center of sphere:

$$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} = \frac{1+3}{2}, \frac{1+-7}{2}, \frac{1+3}{2} = (2,-3,2)$$

Dimaeter:

$$\sqrt{(3-1)^2+(-7-1)^2+(3-1)^2} = 6\sqrt{2}$$

Radius:

$$r = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

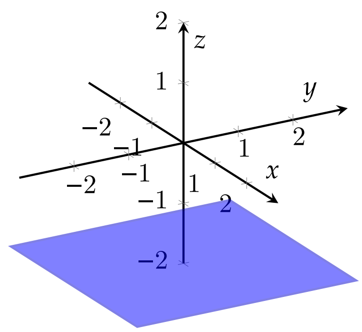
Equation of sphere:

$$(x-2)^2+(y+3)^2+(z-2)^2 = (3\sqrt{2})^2$$
$$(x-2)^2+(y+3)^2+(z-2)^2 = 18$$

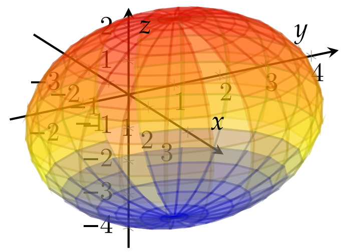
Question 3

a) Describe in words and with a sketch the regions in $\mathbb{R}^3$ represented by
i. The equation $z = -2$
ii. The inequality $x^2+(y-1)^2+(z+1)^2 \leqslant 9$
b) In your sketch, shade in the intersection of the two regions you drew in part (a), i.e., the set of all points (x,y,z) in $\mathbb{R}^3$ satisfying both $z = -2$ and $x^2+(y-1)^2+(z+1)^2 \leqslant 9$

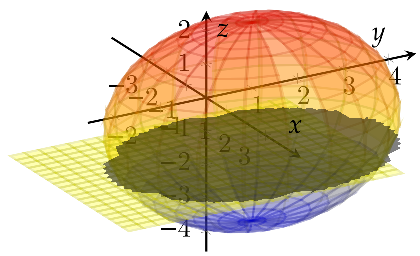
Solution:
a)
i. Represents a plane in $\mathbb{R}^3$ that is parallel to the xy-plane and liest at a height of -2 units below the xy-plane. This plane contains all points where the z-cooridnate is -2, regardless of the x and y values.



ii. A solid sphere in $\mathbb{R}^3$ with the center at point (0,1,-1) and a radius of 3. This indluces all the points inside and on the surface of the sphere.



b)



Question 4

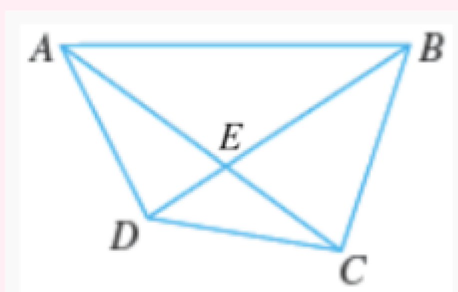
Describe in words the region(s) $\mathbb{R}^3$ represented by the inequality $x^2 \leqslant 9$

Solution: It is the set off all points that lite within the planes $x = -3$ and $x = 3$. It is a 3-dimensional slab that extends infinitely in teh y and $z-$ directions, but is bounded by $x = -3$ and $x = 3$ along the x-axis.

Question 5

Let A, B, C, D, and E be the points in the diagram below. Write each combintation of vectors as a single vector.

1. $\overrightarrow{AE} + \overrightarrow{ED}$
2. $\overrightarrow{AC} - \overrightarrow{AB}$
3. $\overrightarrow{AC} + \overrightarrow{EA}$
4. $\overrightarrow{AC} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA}$



Solution:

1. $\overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}$
2. $\overrightarrow{AC} - \overrightarrow{AB} = (\overrightarrow{AC} + \overrightarrow{BC}) - \overrightarrow{AB} = \overrightarrow{BC}$
3. $\overrightarrow{AC} + \overrightarrow{EA} = \overrightarrow{AC} - \overrightarrow{AE} = \overrightarrow{EC}$
4. $\overrightarrow{AC} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA}$:

$$\overrightarrow{ED} + \overrightarrow{DC} = \overrightarrow{EC}$$

$$\overrightarrow{CB} + \overrightarrow{BA} = \overrightarrow{CA}$$
$$\overrightarrow{AC} + \overrightarrow{EC} + \overrightarrow{CA} = \overrightarrow{EA}$$

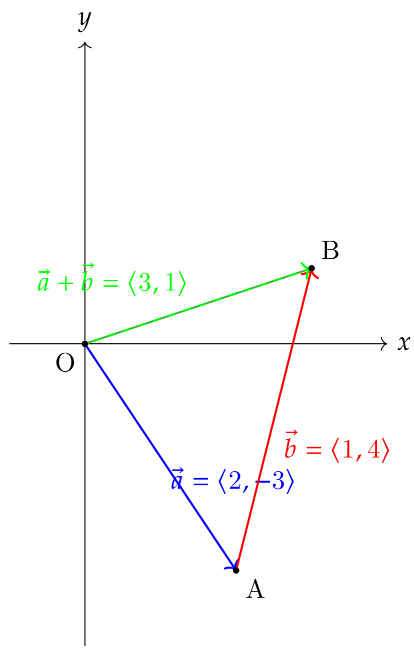
Question 6

Find the sum of the vectors $\vec{a} = \langle 2, -3\rangle$ and $\vec{b} = \langle 1, 4\rangle$ and illustrate geometrically

Solution:

$$\vec{a} + \vec{b} = \langle 2, -3\rangle + \langle 1, 4\rangle$$

$$\vec{a} + \vec{b} = \langle 2+1, -3+4\rangle = \langle 3, 1\rangle$$



Question 7

Find the vector that has the same direction as $\hat{\imath} + 3\hat{\jmath} - \hat{k}$ but has length 6

Solution:

$$\hat{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{11}}\hat{\imath} + \frac{3}{\sqrt{11}}\hat{\jmath} - \frac{1}{\sqrt{11}}\hat{k}$$

$$\mathbf{u} \times \hat{u} = 6 \times \left(\frac{1}{\sqrt{11}}\hat{\imath} + \frac{3}{\sqrt{11}}\hat{\jmath} - \frac{1}{\sqrt{11}}\hat{k}\right)$$

$$\mathbf{u} = \frac{6}{\sqrt{11}}\hat{\imath} + \frac{18}{\sqrt{11}}\hat{\jmath} - \frac{6}{\sqrt{11}}\hat{k}$$