Math 120

PSet 4

Sep 22 2024

Contents

Chapter 1

1.1 PSet 4

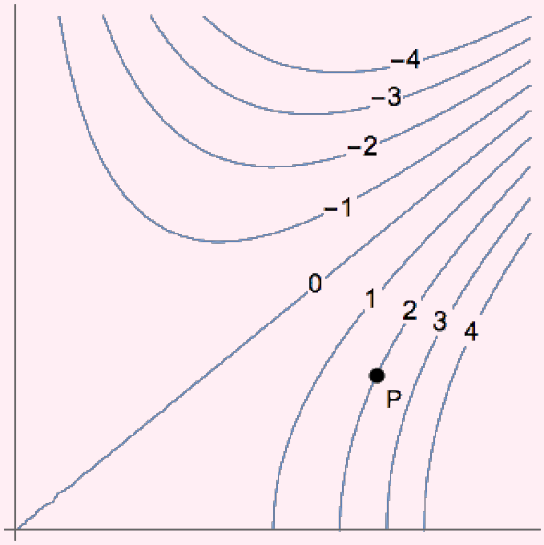
Question 1

By finding both $f_{xy} = (f_x)_y$ and $f_{yx} = (f_y)_x$, verify that Clairaut's Theorem holds for the function $f(x, y) = y \arctan(xy)$.

Solution:

Question 2

Level curves are shown below for a function f



(a) Determine the signs of f_x, f_y, f_{xx}, f_{yy} and f_{xy} at the point P. Explain your reasoning. *You should assume that the undrawn level curves are nicely and evenly distributed between the ones drawn.*

(b) Mark a point on the contour plot where $f_x = 0$. (You can either mark the point on a screenshot and insert the picture in your homework file, or just make a rough copy of the contour plot by hand.)

Question 3

Use the Chain Rule to find the indicated partial derivatives.

(a) Compute $\frac{dz}{dt}$ if $z = \tan(y/x)$, $x = e^t$, and $y = 1 - e^{-t}$.

(b) Compute $\frac{\partial M}{\partial u}$ and $\frac{\partial M}{\partial v}$ at $u = 3$ and $v = -1$ if $M = xe^{y-z^2}$, $x = 2uv$, $y = u - v$, and $z = u + v$.

Solution:

Question 4

Use implicit differentiation to compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the surface given by $z^3 + 3xyz + x^2 + y^2 = 0$ at the point $(1, -2, 1)$.

Solution:

Question 5

[(5.) (Stewart problem 14.5.36)] Wheat production W in a given year depends on the average temperature T and annual rainfall R. Scientists estimate that the average temperature is rising at a rate of 0.15°C/year and rainfall is decreasing at a rate of 0.1 cm/year. They also estimate that at current production levels, $\partial W/\partial T = -2$ and $\partial W/\partial R = 8$.

(a) What is the significance of the signs of these partial derivatives?

(b) Estimate the current rate of change of wheat production $\frac{dW}{dt}$.