Math 120 Cheat Sheet

3D Coordinate Systems

Distance: $|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
Sphere: $(x-h)^2+(y-k)^2+(z-l)^2 = r^2$

Vectors

Properties: $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$
$c(\mathbf{a}+\mathbf{b}) = c\mathbf{a}+c\mathbf{b}$

Dot Product

Formula: $\mathbf{a}\cdot\mathbf{b} = a_1b_1+a_2b_2+a_3b_3$
Angle: $\cos(\theta) = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$
Orthogonality: $\mathbf{a}\cdot\mathbf{b}=0$
Projections: $\mathrm{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$
$\mathrm{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

Cross Product

Formula: $\mathbf{a}\times\mathbf{b} = \langle a_2b_3-a_3b_2, a_3b_1-a_1b_3, a_1b_2-a_2b_1\rangle$
Magnitude: $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$
Triple Product: $V = |\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$

Lines and Planes

Line: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$
Plane: $\mathbf{n}\cdot\mathbf{r} = \mathbf{n}\cdot\mathbf{r_0}$
$ax+by+cz+d=0$
Distance:

$$D = \frac{|a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

Curve Length: $s(t) = \int_a^t \sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2+\left(\frac{dz}{du}\right)^2}\,du$

Vector Value Functions

Form: $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle = f(t)\mathbf{i}+g(t)\mathbf{j}+h(t)\mathbf{k}$
Limit: $\lim_{t\to a}\mathbf{r}(t) = \langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)\rangle$
Derivative: $\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t)\rangle$
Orthogonality: If $|\mathbf{r}(t)| = c$, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$.
Definite Integral: $\int_a^b \mathbf{r}(t)\,dt = \mathbf{R}(t)\Big|_a^b = \mathbf{R}(b)-\mathbf{R}(a)$
Length: $L = \int_a^b |\mathbf{r}'(t)|\,dt$

Curvature

Form: $s(t) = \int_a^t |\mathbf{r}'(u)|du = \int_a^t \sqrt{\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2+\left(\frac{dz}{du}\right)^2}du$

Motion in space

Form: $\mathbf{v}(t) = \lim_{h\to 0}\frac{\mathbf{r}(t+h)-\mathbf{r}(t)}{h} = \mathbf{r}'(t)$

Partial Derivatives

Form: $f_x(a,b) = g'(a)$ where $g(x) = f(x,b)$ **Defintion:**

$$f_x(x,y) = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$$

Notation:

$$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}f(x,y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$

$$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}f(x,y) = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f$$

Rules
To find f_x, regard y as a constant and differentiate $f(x,y)$ with respect to x.
To find f_y, regard x as a constant and differentiate $f(x,y)$ with respect to y.
Higher Derivatives:

$$(f_x)_x = f_{xx} = f_{11} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 z}{\partial x^2}$$

$$(f_x)_y = f_{xy} = f_{12} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\partial x} = \frac{\partial^2 z}{\partial y\partial x}$$

$$(f_y)_x = f_{yx} = f_{21} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 z}{\partial x\partial y}$$

$$(f_y)_y = f_{yy} = f_{22} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 z}{\partial y^2}$$