

# Math 120 QR

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Fall 2024

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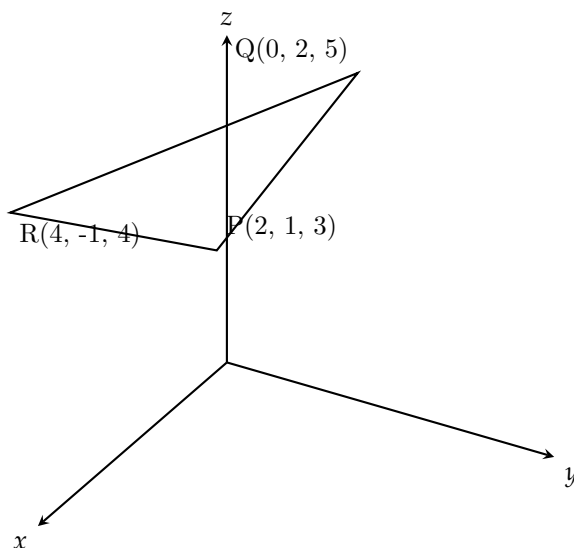
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# Chapter 1

## 1.1 PSet 1

### Question 1

Find the lengths of the sides of the triangle with vertices P (2, 1, 3), Q(0, 2, 5) and R(4, -1, 4). Is the triangle an acute triangle (all sides less than  $90^\circ$ ), a right triangle, or an obtuse triangle (one angle greater than  $90^\circ$ )



**Solution:** Image:

$$\overrightarrow{PQ} = \langle 0 - 2, 2 - 1, 5 - 3 \rangle = \langle -2, 1, 2 \rangle$$

$$\overrightarrow{QR} = \langle 4 - 0, -1 - 2, 4 - 5 \rangle = \langle 4, -3, -1 \rangle$$

$$\overrightarrow{RP} = \langle 2 - 4, 1 - (-1), 3 - 4 \rangle = \langle -2, -2, 1 \rangle$$

$$|\overrightarrow{PQ}| = \sqrt{(-2)^2 + 1^2 + (-2)^2} = \sqrt{9} = 3$$

$$|\overrightarrow{QR}| = \sqrt{(4)^2 + (-3)^2 + (-1)^2} = \sqrt{26}$$

$$|\overrightarrow{RP}| = \sqrt{(-2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$$

angles:

$$|\overrightarrow{PQ}|^2 = 26 = 9 + 9 - 2(3)(3) \cos \theta$$

$$8 = -18 \cos \theta$$

$$\frac{8}{18} = -\cos(\theta)$$

$$\theta = \arccos\left(-\frac{8}{18}\right) \approx 116$$

Triangle is obtuse

### Question 2

Find the equation of the sphere for which the line segment between the points  $A(1, 1, 1)$  and  $B(3, -7, -3)$  is a diameter. (This means that  $A$  and  $B$  are antipodal points on the sphere.)

**Solution:**

Center of sphere:

$$\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} = \frac{1 + 3}{2}, \frac{1 + (-7)}{2}, \frac{1 + (-3)}{2} = (2, -3, -1)$$

Diameter:

$$\sqrt{(3 - 1)^2 + (-7 - 1)^2 + (-3 - 1)^2} = 2\sqrt{21}$$

Radius:

$$r = \frac{2\sqrt{21}}{2} = \sqrt{21}$$

Equation of sphere:

$$(x - 2)^2 + (y + 3)^2 + (z + 1)^2 = (\sqrt{21})^2$$

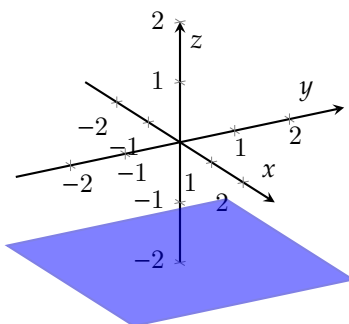
$$(x - 2)^2 + (y + 3)^2 + (z + 1)^2 = 21$$

### Question 3

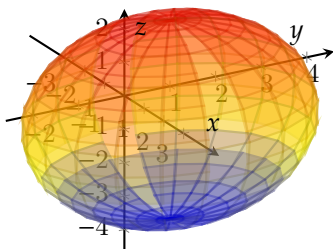
- a) Describe in words and with a sketch the regions in  $\mathbb{R}^3$  represented by
- The equation  $z = -2$
  - The inequality  $x^2 + (y - 1)^2 + (z + 1)^2 \leq 9$
- b) In your sketch, shade in the intersection of the two regions you drew in part (a), i.e., the set of all points  $(x, y, z)$  in  $\mathbb{R}^3$  satisfying both  $z = -2$  and  $x^2 + (y - 1)^2 + (z + 1)^2 \leq 9$

**Solution:**

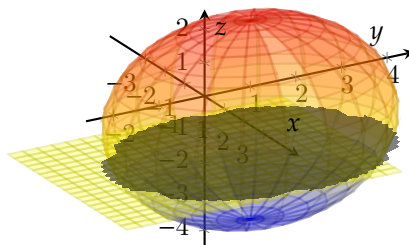
- a)
- Represents a plane in  $\mathbb{R}^3$  that is parallel to the  $xy$ -plane and lies at a height of -2 units below the  $xy$ -plane. This plane contains all points where the  $z$ -coordinate is -2, regardless of the  $x$  and  $y$  values.



- A solid sphere in  $\mathbb{R}^3$  with the center at point  $(0, 1, -1)$  and a radius of 3. This includes all the points inside and on the surface of the sphere.



b)



### Question 4

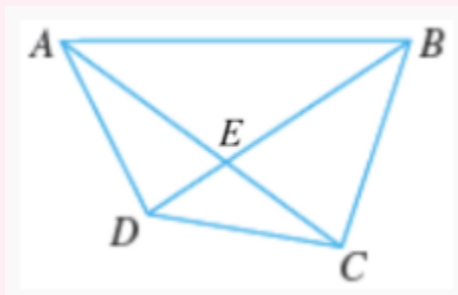
Describe in words the region(s)  $\mathbb{R}^3$  represented by the inequality  $x^2 \leq 9$

**Solution:** It is a 3-dimensional slab that extends infinitely in the  $y$  and  $z$ - directions, but is bounded by  $x = -3$  and  $x = 3$  along the  $x$ -axis.

### Question 5

Let  $A, B, C, D$ , and  $E$  be the points in the diagram below. Write each combination of vectors as a single vector.

1.  $\overrightarrow{AE} + \overrightarrow{ED}$
2.  $\overrightarrow{AC} - \overrightarrow{AB}$
3.  $\overrightarrow{AC} + \overrightarrow{EA}$
4.  $\overrightarrow{AC} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA}$



**Solution:**

1.  $\overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD}$
2.  $\overrightarrow{AC} - \overrightarrow{AB} = (\overrightarrow{AC} + \overrightarrow{BC}) - \overrightarrow{AB} = \overrightarrow{BC}$
3.  $\overrightarrow{AC} + \overrightarrow{EA} = \overrightarrow{AC} - \overrightarrow{AE} = \overrightarrow{EC}$
4.  $\overrightarrow{AC} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CB} + \overrightarrow{BA} = \vec{0}$

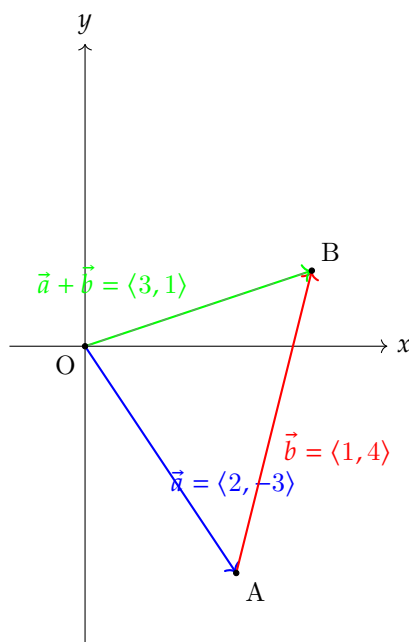
### Question 6

Find the sum of the vectors  $\vec{a} = \langle 2, -3 \rangle$  and  $\vec{b} = \langle 1, 4 \rangle$  and illustrate geometrically

**Solution:**

$$\vec{a} + \vec{b} = \langle 2, -3 \rangle + \langle 1, 4 \rangle$$

$$\vec{a} + \vec{b} = \langle 2 + 1, -3 + 4 \rangle = \langle 3, 1 \rangle$$



### Question 7

Find the vector that has the same direction as  $\hat{i} + 3\hat{j} - \hat{k}$  but has length 6

**Solution:**

$$\begin{aligned}\hat{u} &= \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{11}}\hat{i} + \frac{3}{\sqrt{11}}\hat{j} - \frac{1}{\sqrt{11}}\hat{k} \\ \mathbf{u} \times \hat{u} &= 6 \times \left( \frac{1}{\sqrt{11}}\hat{i} + \frac{3}{\sqrt{11}}\hat{j} - \frac{1}{\sqrt{11}}\hat{k} \right) \\ \mathbf{u} &= \frac{6}{\sqrt{11}}\hat{i} + \frac{18}{\sqrt{11}}\hat{j} - \frac{6}{\sqrt{11}}\hat{k}\end{aligned}$$

### Question 8

complete the Ximera assignment on vector projections linked from the Problem Set 1 assignment in the Canvas umbrella site. Write “I have completed the Ximera assignment” for Problem 8 on the file you submit for this problem set.

**Solution:** “I have completed the Ximera assignment”