

Math 115 QR

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Contents

Chapter 1

Page 2

1.1	Problem 1: Stewart 11.2.20	2
1.2	Problem 2	2
1.3	Problem 3	2
1.4	Problem 4	3
1.5	Problem 5	3

Chapter 1

1.1 Problem 1: Stewart 11.2.20

Question 1

Determine whether the series is convergent or divergent by expressing S_n as a telescoping sum. If it is convergent find its sum

$$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$$

Solution:

$$S_n = \ln\left(\frac{1}{1+1}\right) + \ln\left(\frac{2}{2+1}\right)$$

1.2 Problem 2

Question 2

What does the Test for Divergence tell you about convergence or divergence of each of the following series?

- (a). $\sum_{n=30}^{\infty} \frac{1}{n^n}$
- (b). $\sum_{n=500}^{\infty} (\cos(1))^n$
- (c). $\sum_{n=500}^{\infty} \cos(n)$
- (d). $\sum_{k=2}^{\infty} \frac{2^k+1}{3^k+1}$

1.3 Problem 3

Question 3

Consider the Alternating Harmonic Series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$, which can be written in summation notation as $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$

- (a). Plot the partial sums $s_n = \sum_{k=1}^n \frac{(-1)^{k+1}}{k}$ versus n for $n = 1, 2, \dots, 8$
- (b). Explain why $S_1 > S_3 > S_5 > \dots$, ie., why the odd partial sums are decreasing.
- (c). Explain why $S_2 < S_4 < S_6 > \dots$, i.e., why the even partial sums are increasing.
- (d). Explain why $\frac{1}{2} \leq s_n \leq 1$ for all n .
- (e). The Monotonic Convergence Theorem states that any increasing sequence that is bounded above must converge, as must any decreasing sequence that is bounded below. Since the odd partial sums of the Alternating Harmonic Series are decreasing and bounded below by $\frac{1}{2}$ the odd partial sums converge.

Since the even partial sums are increasing and bounded above by 1, the even partial sums also converge. Explain why the odd partial sums and the even partial sums must converge to the same number. Hint: What is $\lim_{n \rightarrow \infty} (s_{2n+1} - s_{2n})$

1.4 Problem 4

Question 4

Use the Comparison Test to decide whether the series

$$\sum_{k=3}^{\infty} \frac{2 + (-1)^k}{k}$$

converges or diverges. Explain your reasoning. (In particular, explain why it is valid to use comparison and what you are comparing to.)

1.5 Problem 5

Question 5

A ball is dropped out of a window from a height of 100 feet. Assume that each time the ball falls from a height h that it rebounds to 30% of height h . Assuming the ball bounces indefinitely, determine the total distance it travels.