SOLUTIONS TO CONCEPTS
CHAPTER 14

1. $F = mg$

   Stress $= \dfrac{F}{A}$

   Strain $= \dfrac{\Delta L}{L}$

   $Y = \dfrac{FL}{A\Delta L} \Rightarrow \dfrac{\Delta L}{L} = \dfrac{F}{YA}$

2. ρ = stress = mg/A

   e = strain = ρ/Y

   Compression $\Delta L = eL$

3. $y = \dfrac{F}{A}\dfrac{L}{\Delta L} \Rightarrow \Delta L = \dfrac{FL}{AY}$

4. $L_{steel} = L_{cu}$ and $A_{steel} = A_{cu}$

   a) $\dfrac{\text{Stress of cu}}{\text{Stress of st}} = \dfrac{F_{cu}}{A_{cu}}\dfrac{A_g}{F_g} = \dfrac{F_{cu}}{F_{st}} = 1$

   b) Strain $= \dfrac{\Delta Lst}{\Delta lcu} = \dfrac{F_{st}L_{st}}{A_{st}Y_{st}} \cdot \dfrac{A_{cu}Y_{cu}}{F_{cu}l_{cu}}$ ($\because L_{cu} = l_{st}$; $A_{cu} = A_{st}$)

5. $\left(\dfrac{\Delta L}{L}\right)_{st} = \dfrac{F}{AY_{st}}$

   $\left(\dfrac{\Delta L}{L}\right)_{cu} = \dfrac{F}{AY_{cu}}$

   $\dfrac{\text{strain steel wire}}{\text{Strain om copper wire}} = \dfrac{F}{AY_{st}} \times \dfrac{AY_{cu}}{F} (\because A_{cu} = A_{st}) = \dfrac{Y_{cu}}{Y_{st}}$

6. Stress in lower rod $= \dfrac{T_1}{A_1} \Rightarrow \dfrac{m_1 g + \omega g}{A_1} \Rightarrow$ w = 14 kg

   Stress in upper rod $= \dfrac{T_2}{A_u} \Rightarrow \dfrac{m_2 g + m_1 g + wg}{A_u} \Rightarrow$ w = .18 kg

   For same stress, the max load that can be put is 14 kg. If the load is increased the lower wire will break first.

   $\dfrac{T_1}{A_1} = \dfrac{m_1 g + \omega g}{A_1} = 8 \times 10^8 \Rightarrow$ w = 14 kg

   $\dfrac{T_2}{A_u} \Rightarrow \dfrac{m_2 g + m_1 g + \omega g}{A_u} = 8 \times 10^8 \Rightarrow \omega_0 = 2$ kg

   The maximum load that can be put is 2 kg. Upper wire will break first if load is increased.

7. $Y = \dfrac{F}{A}\dfrac{L}{\Delta L}$

8. $Y = \dfrac{F}{A}\dfrac{L}{\Delta L} \Rightarrow F = \dfrac{YA\,\Delta L}{L}$

9. $m_2 g - T = m_2 a$ …(1)

   and $T - F = m_1 a$ …(2)

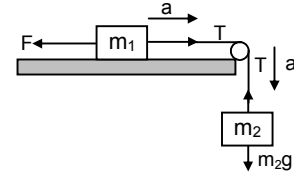 $\Rightarrow a = \dfrac{m_2 g - F}{m_1 + m_2}$

From equation (1) and (2), we get $\dfrac{m_2 g}{2(m_1 + m_2)}$

Again, T = F + m_1a

$\Rightarrow T = \dfrac{m_2 g}{2} + m_1 \dfrac{m_2 g}{2(m_1 + m_2)} \Rightarrow \dfrac{m_2^2 g + 2m_1 m_2 g}{2(m_1 + m_2)}$

Now Y = $\dfrac{FL}{A\,\Delta L} \Rightarrow \dfrac{\Delta L}{L} = \dfrac{F}{AY}$

$\Rightarrow \dfrac{\Delta L}{L} = \dfrac{(m_2^2 + 2m_1 m_2)g}{2(m_1 + m_2)AY} = \dfrac{m_2 g(m_2 + 2m_1)}{2AY(m_1 + m_2)}$

10. At equilibrium ⇒ T = mg
    When it moves to an angle θ, and released, the tension the T′ at lowest point is

    $\Rightarrow$ T′ = mg + $\dfrac{mv^2}{r}$

    The change in tension is due to centrifugal force $\Delta T = \dfrac{mv^2}{r}$ …(1)

    $\Rightarrow$ Again, by work energy principle,

    $\Rightarrow \dfrac{1}{2}mv^2 - 0 = mgr(1 - \cos\theta)$

    $\Rightarrow v^2 = 2gr\,(1 - \cos\theta)$ …(2)

    So, $\Delta T = \dfrac{m[2gr(1 - \cos\theta)]}{r} = 2mg(1 - \cos\theta)$

    $\Rightarrow$ F = ΔT

    $\Rightarrow$ F = $\dfrac{YA\,\Delta L}{L}$ = 2mg – 2mg cos θ ⇒ 2mg cos θ = 2mg – $\dfrac{YA\,\Delta L}{L}$

    = cos θ = 1 – $\dfrac{YA\,\Delta L}{L(2mg)}$

11. From figure cos θ = $\dfrac{x}{\sqrt{x^2 + l^2}}$ = $\dfrac{x}{l}\left[1 + \dfrac{x^2}{l^2}\right]^{-1/2}$

    = x / l   … (1)
    Increase in length ΔL = (AC + CB) – AB
    Here, AC = $(l^2 + x^2)^{1/2}$
    So, ΔL = $2(l^2 + x^2)^{1/2}$ – 100   …(2)

    Y = $\dfrac{F}{A}\dfrac{l}{\Delta l}$ …(3)
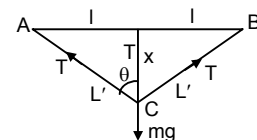
    From equation (1), (2) and (3) and the freebody diagram,
    2l cosθ = mg.

12. Y = $\dfrac{FL}{A\Delta L} \Rightarrow \dfrac{\Delta L}{L} = \dfrac{F}{Ay}$

    σ = $\dfrac{\Delta D / D}{\Delta L / L} \Rightarrow \dfrac{\Delta D}{D} = \dfrac{\Delta L}{L}$

    Again, $\dfrac{\Delta A}{A} = \dfrac{2\Delta r}{r}$

    $\Rightarrow \Delta A = \dfrac{2\Delta r}{r}$

13. $B = \dfrac{Pv}{\Delta v} \Rightarrow P = B\left(\dfrac{\Delta v}{v}\right)$

14. $\rho_0 = \dfrac{m}{V_0} = \dfrac{m}{V_d}$

so, $\dfrac{\rho_d}{\rho_0} = \dfrac{V_0}{V_d}$ \qquad\qquad ...(1)

vol.strain $= \dfrac{V_0 - V_d}{V_0}$

$B = \dfrac{\rho_0 gh}{(V_0 - V_d)/V_0} \Rightarrow 1 - \dfrac{V_d}{V_0} = \dfrac{\rho_0 gh}{B}$

$\Rightarrow \dfrac{vD}{v_0} = \left(1 - \dfrac{\rho_0 gh}{B}\right)$ \qquad ...(2)

Putting value of (2) in equation (1), we get

$\dfrac{\rho_d}{\rho_0} = \dfrac{1}{1 - \rho_0 gh/B} \Rightarrow \rho_d = \dfrac{1}{(1 - \rho_0 gh/B)} \times \rho_0$

15. $\eta = \dfrac{F}{A\theta}$

Lateral displacement $= l\theta$.

16. $F = T l$

17. a) $P = \dfrac{2T_{Hg}}{r}$ \qquad b) $P = \dfrac{4T_g}{r}$ \qquad c) $P = \dfrac{2T_g}{r}$

18. a) $F = P_0 A$

b) Pressure $= P_0 + (2T/r)$

$F = P'A = (P_0 + (2T/r)A$

c) $P = 2T/r$

$F = PA = \dfrac{2T}{r}A$

19. a) $h_A = \dfrac{2T\cos\theta}{r_A - \rho g}$ \qquad b) $h_B = \dfrac{2T\cos\theta}{r_B \rho g}$ \qquad c) $h_C = \dfrac{2T\cos\theta}{r_C \rho g}$

20. $h_{Hg} = \dfrac{2T_{Hg}\cos\theta_{Hg}}{r\rho_{Hg}g}$

$h_\omega = \dfrac{2T_\omega \cos\theta_\omega}{r\rho_\omega g}$ where, the symbols have their usual meanings.

$\dfrac{h_\omega}{h_{Hg}} = \dfrac{T_\omega}{T_{Hg}} \times \dfrac{\rho_{Hg}}{\rho_\omega} \times \dfrac{\cos\theta_\omega}{\cos\theta_{Hg}}$

21. $h = \dfrac{2T\cos\theta}{r\rho g}$

22. $P = \dfrac{2T}{r}$

$P = F/r$

23. $A = \pi r^2$

24. $\dfrac{4}{3}\pi R^3 = \dfrac{4}{3}\pi r^3 \times 8$

$\Rightarrow r = R/2 = 2$

Increase in surface energy $= TA' - TA$

25. $h = \dfrac{2T\cos\theta}{r\rho g}$, $h' = \dfrac{2T\cos\theta}{r\rho g}$

$\Rightarrow \cos\theta = \dfrac{h'r\rho g}{2T}$

So, $\theta = \cos^{-1}(1/2) = 60°$.

26. a) $h = \dfrac{2T\cos\theta}{r\rho g}$

b) $T \times 2\pi r \cos\theta = \pi r^2 h \times \rho \times g$

$\therefore \cos\theta = \dfrac{hr\rho g}{2T}$

27. $T(2l) = [1 \times (10^{-3}) \times h]\rho g$

28. Surface area $= 4\pi r^2$

29. The length of small element $= r\, d\theta$

$dF = T \times r\, d\theta$

considering symmetric elements,

$dF_y = 2T\, rd\theta \cdot \sin\theta$ $[dF_x = 0]$

so, $F = 2Tr \displaystyle\int_0^{\pi/2} \sin\theta d\theta = 2Tr[\cos\theta]_0^{\pi/2} = T \times 2\,r$

Tension $\Rightarrow 2T_1 = T \times 2r \Rightarrow T_1 = Tr$

30. a) Viscous force $= 6\pi\eta rv$

b) Hydrostatic force $= B = \left(\dfrac{4}{3}\right)\pi r^3 \sigma g$

c) $6\pi\eta\, rv + \left(\dfrac{4}{3}\right)\pi r^3 \sigma g = mg$

$v = \dfrac{2}{9}\dfrac{r^2(\rho-\sigma)g}{\eta} \Rightarrow \dfrac{2}{9}r^2 \dfrac{\left(\dfrac{m}{(4/3)\pi r^3} - \sigma\right)g}{n}$

31. To find the terminal velocity of rain drops, the forces acting on the drop are,

i) The weight $(4/3)\pi r^3 \rho g$ downward.

ii) Force of buoyancy $(4/3)\pi r^3 \sigma g$ upward.

iii) Force of viscosity $6\pi\eta r v$ upward.

Because, σ of air is very small, the force of buoyancy may be neglected.

Thus,

$6\pi\eta r v = \left(\dfrac{4}{3}\right)\pi r^2 \rho g$ or $v = \dfrac{2r^2 \rho g}{9\eta}$

32. $v = \dfrac{R\eta}{\rho D} \Rightarrow R = \dfrac{v\rho D}{\eta}$

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